CS663
Assignment 5 - Q1

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1 Solution

From the model given in the question we have,

$$\begin{aligned}
&& g_1 = f_1 + h_2 \star f_2 \;\; \text{and} \\
&& g_2 = h_1 \star f_1 + f_2 \\
\Longrightarrow && G_1 = F_1 + H_2 F_2 \;\; \text{and} \\
\Longrightarrow && G_2 = H_1 F_1 + F_2
\end{aligned}$$

Solving the linear equations in F_1 and F_2,

$$\begin{aligned}
F_1 &= \frac{G_1 - H_2 G_2}{1 - H_1 H_2} \\
F_2 &= \frac{G_2 - H_1 G_1}{1 - H_1 H_2}
\end{aligned}$$

With some noise n_1 and n_2 we would get,

$$\begin{aligned}
F_1 &= \frac{G_1 - H_2 G_2}{1 - H_1 H_2} - \frac{N_1}{1 - H_1 H_2} + \frac{H_2 N_2}{1 - H_1 H_2} \\
F_2 &= \frac{G_2 - H_1 G_1}{1 - H_1 H_2} - \frac{N_2}{1 - H_1 H_2} + \frac{H_1 N_1}{1 - H_1 H_2}
\end{aligned}$$

Hence we can compute f_1 and f_2 as,

$$\begin{aligned}
f_1 &= F^{-1}(F_1) \\
f_2 &= F^{-1}(F_2)
\end{aligned}$$

2 Issue with Solution

The issue with this solution is the denominator $1 - H_1 H_2$. H_1 and H_2 are typically blur kernels and do not amplify the images. As a result, for lower frequencies both $H_1 \to 1$ and $H_2 \to 1$. This makes the denominator $(1 - H_1 H_2) \to 0$, making the system ill-conditioned at lower frequencies. The DC component of frequency (average grayscale level) isn't affected by blur (due to energy conservation) and hence $H_1(0,0) = H_2(0,0) = 1$, which leads to infinite solutions for f_1 and f_2.
Small values of $1 - H_1 H_2$ are also more sensitive to noise, as seen in the solution above.