# CS663
# Assignment 5 - Q2

Kalpesh Krishna
Pranav Sankhe
Mohit Madan

## 1 Solution - 1D

For a 1D image, we can assume the gradient kernel to be $h = [-1 \;\; 0 \;\; 1]$ (zero-crossing kernel). Since we have $g = h \star f$, we have $G = HF$ after taking their Discrete Fourier Transform. Hence this would imply $f = F^{-1}(G/H)$. Let's try to compute the DFTs of these signals to understand the issue with this operation.

Let's assume the length of our image is $K$. Hence we would need to zero-pad our filter $K-1$ times before finding its DFT. Hence we would have to take an $N$-point DFT with $N = K - 1 + 3 = K + 2$

$$\begin{aligned} H(k) &= \sum_n h[n] e^{-j\frac{2\pi}{N}kn} \\ &= -1 + e^{-j\frac{4\pi}{N}k}, k \in \{0, 1, 2 \dots N-1\} \end{aligned}$$

Now for $k = 0$, we would have $H(k) = 0$. (In other words, a gradient operation removes all DC components from a signal). Also for large values of $K$, $H(k)$ will be close to 1 (in the complex domain).
Hence the student will not be able to recover the DC components in the signal and might have a hard time uncovering the larger frequencies if the signal is very long and has non-trivial high frequency components.

## 2 Solution - 2D

Similar to the 1D case, we attempt to calculate the DFT of the 2-D kernel. We use the kernel $h_x = [-1 \; 0 \; 1; -2 \; 0 \; 2; -1 \; 0 \; 1]$ for derivatives along the X-axis and $h_y = [1 \; 2 \; 1; 0 \; 0 \; 0; -1 - 2 - 1]$ for derivatives along the Y-axis. Assuming we need to take a $N_1, N_2$-DFT after zero padding we obtain,

$$\begin{aligned} H_x(k_1, k_2) &= \sum_x \sum_y h_x[x, y] e^{-j\frac{2\pi}{N_1}k_1 x} e^{-j\frac{2\pi}{N_2}k_2 y} \\ &= (-1 + e^{-j\frac{4\pi}{N_1}k_1})(1 + 2e^{-j\frac{2\pi}{N_2}k_2} + e^{-j\frac{4\pi}{N_2}k_2}), k_1 \in \{0, 1, 2 \dots N_1 - 1\}, k_2 \in \{0, 1, 2 \dots N_2 - 1\} \\ H_y(k_1, k_2) &= \sum_x \sum_y h_y[x, y] e^{-j\frac{2\pi}{N_1}k_1 x} e^{-j\frac{2\pi}{N_2}k_2 y} \\ &= (1 - e^{-j\frac{4\pi}{N_2}k_2})(1 + 2e^{-j\frac{2\pi}{N_1}k_1} + e^{-j\frac{4\pi}{N_1}k_1}), k_1 \in \{0, 1, 2 \dots N_1 - 1\}, k_2 \in \{0, 1, 2 \dots N_2 - 1\} \end{aligned}$$

Quite clearly, for the DC case ($k_1 = k_2 = 0$) we obtain $H_x = H_y = 0$ and we won't be able to recover the DC components correctly. Again, for large widths (for X-derivatives) and large heights (for Y-derivatives), for $k_1 \to N_1$ and $k_2 \to N_2$.