Scalable Optimization of Multivariate Performance Measures in Multi-instance Multi-label Learning

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Goal: Scalable algorithm for optimizing multivariate performance measures for multi-instance multi-label (MIML) learning problems



Performance Measures

Univariate or Decomposable Measures:

• Ill suited in the presence of label imbalance or a heavy tailed label distribution. Tend to neglect performance on rare labels

Multivariate Performance Measures:

- Typically non-decomposable as their evaluation does not decompose over individual points
- Some performance measures such as F-macro force predictor to do well on rare labels as well.

Non Decomposable Loss Functions

Dataset:
$$\{(x_i, y_i)\}_{i=1}^N$$

$$\begin{cases} x_i = \{x_i^{(1)}, \dots, x_i^{(n_i)}\} \in X \\ y_i = [y_{i,1}, y_{i,2}, \dots, y_{i,L}] \in Y = \{0, 1\}^L \end{cases}$$

Macro F-measure:

Class-wise F-measure

$$F_{\beta}^{j}(\mathbf{f};X,Y) := \left(\frac{\beta}{\operatorname{Prec}^{j}(\mathbf{f};X,Y)} + \frac{1-\beta}{\operatorname{Rec}^{j}(\mathbf{f};X,Y)}\right)^{-1}$$

• Averaged over all classes
$$F_{\beta}^{macro}(\mathbf{f};X,Y):=\frac{1}{L}\sum_{j=1}^{L}F_{\beta}^{j}(\mathbf{f};X,Y)$$
 on rare labels

Micro F-measure:

$$F_{\beta}^{micro}(\mathbf{f}; X, Y) := \left(\frac{\beta}{\operatorname{Prec}(\mathbf{f}; X, Y)} + \frac{1 - \beta}{\operatorname{Rec}(\mathbf{f}; X, Y)}\right)^{-1}$$

MIML_perf: A Novel Plugin Classifier Learning Framework

Drawbacks of Existing Approaches

- Training Objective ≠ Evaluation Measure
- Not scalable to large, web scale datasets
- Ill suited for label imbalance problems

Plug-in Classifiers

- Learn a CPE model to predict $g(x) \approx P(y=1)$
- ullet Tune a threshold η to obtain a classifier $f(x) = \text{sign}(g(x) - \eta)$ to maximize perf. measure Δ , e.g. classfn accuracy, F-measure
- Challenge: lack of instance level training in MIML. Learning CPE model itself a challenge
 - **Solution**: EM-style alternating approach
 - Model instance level labels using latent variables
 - $z_k^{(i,j)}$ models if instance k in bag i expresses label j or not
 - Alternately, improve latent variable assign and model
 - Fix hidden variables, update CPE models
 - CPE-train ($D^j \leftarrow \{\{(x_i^{(k)}, z_k^{(i,j)})\}_{k=1}^{n_i}\}_{i=1}^n$)
 - ullet Tune threshold(s) s.t. Δ is maximized (EUM)
 - Fix plug-in models, update hidden variables
 - Re-estimate hidden labels probabilistically according to CPE-train scores

CPE-Train

- Treat hidden variables as instance level labels $z_{k}^{(i,j)} \in \{0,1\}$
- Train a CPE classifier g^{\jmath} for each label j
- Use $g^j(x_i^{(k)})$ to model probability that the k^{th} instance in bag iexpresses label j

Tuning the Thresholds [EUM Approach]

- Optimizing F-macro \rightarrow threshold η_j for label j
- Optimizing F-micro \rightarrow Single threshold η



Threshold

 $\hat{y}_j \leftarrow \bigvee_{k=1}^{n_t} I\{j : g^j(x^{(k)}) \ge \eta_j\}$

(Bag Level, label j) (Instance Level)

(Maximize Performance Measure)

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Research

Speed up while tuning the thresholds

- No. of CPE Scores = No. of instances →
- Tune the threshold over the largest **CPE Scores** in the bags only



Millions

(no. of instances)
$$\sum_{i=1}^N n_i o N$$
 (no. of bags)

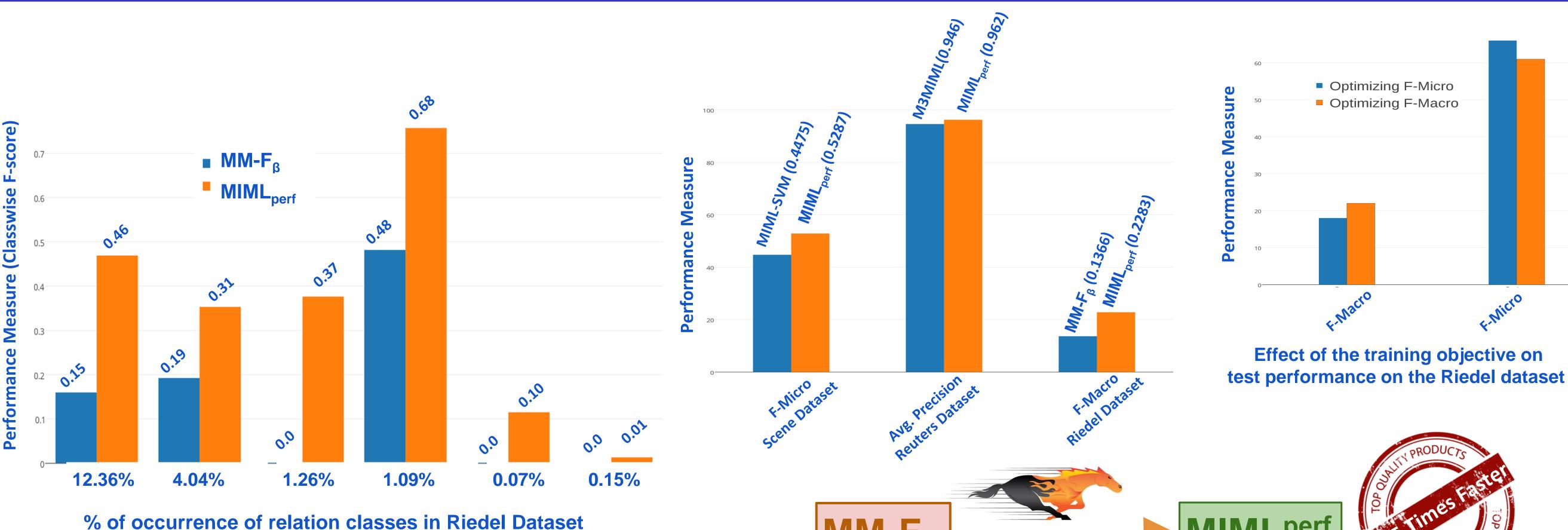
- F-scores can be written as f(TP,TN)
- Sort the CPEs and do a linear scan to find the best threshold

$$O(N^2 \cdot L) \to O(N \log N \cdot L)$$

Estimating the Hidden Variables

- If bag i does not have label $j \to z^{(i,j)} = 0$ (extreme sparsity)
- If bag i has label j, choose $c^{(i,j)}$ instances probabilistically according to CPE scores and make them 1
- Choice of $c^{(i,j)}$

Initialization – $c^{(i,j)} \leftarrow k.n_i$ Subsequently – $c^{(i,j)} \leftarrow \sum_{k=1}^{n_i} \mathbb{I}\{g^j(x_i^{(k)}) \geq \eta_j\}$



Experiments