<u>ME302: 2024-25 - II</u>
COURSE PROJECT REPORT

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1 Problem Description

The objective of this project is to determine the maximum allowable geometric scale for testing a turbomachine component in a closed-loop, high-pressure flow facility. The model must dynamically represent the actual engine conditions by matching both the Mach number and the Reynolds number of the full-scale prototype. The model scale can be defined as:

$$\text{scale} = \frac{l_\text{m}}{l_\text{p}},.$$

m=model

p=prototype(lp=0.2m)

The testing facility is constrained by a maximum allowable stagnation pressure of 250 kPa, and the compressor driving the flow is characterized by a known set of:

- **Stagnation pressure ratio (p02 / p01)**
- **Mass flow rate ($\dot{m}_{\mathbf{ref}}$)**
- **Stagnation temperature ratio (T_{02}/T_{01})**

To accommodate instrumentation, the **inlet diameter** of the test section is given by:

$$D_4 = 2 \times lm,$$

The goal is to compute the maximum scale ($0 \leq \text{scale} \leq 1$) that satisfies:

- **Mach and Reynolds number similarity**
- **Pressure constraint ($P_0 \leq 250\,\mathbf{kPa}$)**
- **Compressor operating requirements for a stable closed-loop flow**

2 Methodology

The Examination follows a systematic approach to determine the largest allowable model scale that maintains dynamic similarity. This process includes calculating the associated inlet stagnation pressure and choosing the most suitable compressor operating point based on the limitations of the testing facility.

2.1 Scaling and Geometric Relations

The geometric scale of the model is defined as the ratio of the model length to the prototype length:

$$\text{scale} = \frac{l_\text{m}}{l_\text{p}},$$

To ensure adequate space for flow development and instrumentation, the inlet diameter of the test section is specified as:

$$D_4 = 2\, l_\text{m},$$

Accordingly, the cross-sectional area at the test section inlet is given by:

$$A_4 = \frac{\pi \ \times D_4^2}{4} = \pi \ \times l_m^2$$

This relationship links the model scale to the test section geometry and is critical for further flow analysis and performance calculations.

2.2 Calculation of Stagnation Quantities at Each Station

To enable a comprehensive analysis, the stagnation (total) pressure and temperature are evaluated at all critical stations within the test facility.

- **Station 1 (Inlet):**

  $$T_{01} = 293\,\text{K}$$

  The stagnation temperature is fixed at T01 =293K, while the stagnation pressure p_{01} is to be determined based on the maximum facility pressure limit and the compressor's performance characteristics.

- **Station 2 (Compressor Exit):** Stagnation properties at the compressor outlet are given by:

  $$T_{02} = T_0\,\text{ratio} \times T_{01} \quad \textit{and} \quad p_{02} = p_0\,\text{ratio} \times p_{01}.$$

- **Station 3 (Post Minor Losses:)** A minor pressure drop of 1.5% occurs downstream of the compressor, resulting in:

  $$p_{03} = 0.985\, p_{02}, \quad T_{03} = T_{02} \quad \text{(since adiabatic conditions)}.$$

- **Station 4 (Diffuser Exit):** Assuming negligible viscous losses in the diffuser, the stagnation properties remain unchanged:

  $$p_{04} = p_{03}, \quad T_{04} = T_{03}.$$

- **Station 5 (Post Throttle Loss):** A pressure drop of 5% occurs after the diffuser due to throttle effects:

  $$p_{05} = 0.95\, p_{04}, \quad T_{05} = T_{04}.$$

2.3 Flow Characterization at the Diffuser Exit

At Station 4 (diffuser exit), the static flow properties are derived from the known stagnation conditions using standard isentropic flow relations. The flow is assumed to be steady, adiabatic, and isentropic up to this point. The isentropic flow factor is defined as:

$$A = 1 + \frac{\gamma - 1}{2} \times M_4^2,$$

$$\gamma = 1.4, (\text{ratio of specific heats for air})$$

$$M_4 = 0.55 \quad (\text{M}_4 \text{ is the Mach number at station 4})$$

Using this, the static temperature at station 4 is obtained as

$$T_4 = \frac{T_{04}}{A}.$$

Using this, the static temperature at station 4 is obtained as:

$$p_4 = \frac{p_{04}}{A^{\frac{\gamma}{\gamma-1}}}.$$

The static density is then calculated using the ideal gas law:

$$\rho_4 = \frac{p_4}{RT_4}, \quad R = 287\,\text{J}/(\text{kg}\,\text{K}).$$

The speed of sound at station 4 is given by:

$$a_4 = (\gamma \times R \times T_4)^{1/2}$$

Finally, the flow velocity is computed using the Mach number definition:

$$V_4 = M_4 \times a_4.$$

These relations collectively provide all necessary static flow properties at station 4, which are essential for evaluating the Reynolds number and assessing model similarity.

2.4 Constraint for Maintaining Closed-Loop Operation

To ensure continuous flow circulation within the closed-loop test facility, the stagnation pressure at the end of the loop (station 5) must be greater than or equal to the inlet stagnation pressure (station 1):

$$p_{05} \geq p_{01}.$$

Using the known pressure drops through the system:

- A 5% pressure loss occurs after the diffuser:

$$p_{05} = 0.95\, p_{04}$$

$$= 0.95 \times 0.985 \times p_0 ratio \times p_{01}$$

$$\approx 0.93575 \times p_0 ratio \times p_{01}.$$

- And a 1.5% loss after the compressor:

Also, the compressor exit pressure is:

$$p_{02} = p_0 ratio \times p_{01}$$

Substituting back, we get:

$$0.93575\, p_0 ratio \times p_{01} \geq p_{01}$$
$$\Rightarrow \quad p_0 ratio \geq 1.068.$$

For the loop to operate properly:

$$p_{05} \geq p_{01}$$
$$\Rightarrow 0.9357 \times p_0 ratio \times p_{01} \geq p_{01}$$

Dividing both sides by:

$$0.93575 \times p_0 ratio \geq 1 \Rightarrow p_0 ratio \geq \frac{1}{0.93575} \approx 1.068$$

Thus, the minimum required compressor pressure ratio for maintaining closed-loop operation is:

$$p_0 ratio \geq 1.068$$

According to the compressor performance data, this condition is satisfied, confirming that closed-loop operation is feasible under the evaluated conditions.

2.5 Constraints for Flow Similarity and Scaling

To ensure that the scaled model exhibits dynamic similarity with the full-scale prototype, two critical constraints must be satisfied: mass flow consistency and Reynolds number matching.

1. **Mass Flow Balance:** The compressor mass flow rate is defined relative to a standard atmospheric reference as:

   $$\dot{m} = \dot{m}_{\text{ref}} \times \left(\frac{p_{01}}{101.325}\right),$$

   where:

   $$\dot{m}_{\text{ref}} : \text{Reference mass flow rate from compressor data}$$

   $$p_{01} : \text{Inlet stagnation pressure (in kPa)}$$
   $$101.325 kPa : \text{Standard atmospheric pressure}$$

   At the test section (station 4), the mass flow rate is also expressed as:

   $$\dot{m} = \rho_4\, V_4\, A_4 = \rho_4\, V_4\, \pi\, l_{\text{model}}^2.$$

   Equating these expressions(cancels p_{01} from the equation) and solving for the scale yields the mass flow based scale:

   $$\text{scale}_{\max} = \sqrt{\frac{\dot{m}}{\rho_4\, V_4\, \pi\, l_{\text{prototype}}^2}} = max\ f(\dot{m}_{\text{ref}},\, T_0 ratio,\, p_0 ratio).$$

2. **Reynolds Number Matching:** To preserve similarity in viscous effects, the Reynolds number at station 4 must match the target engine condition:

$$Re = \frac{\rho_4 V_4 l_\mathrm{m}}{\mu} = 3 \times 10^6,$$

where:

$$\mu = 1.83 \times 10^{-5}\,\mathrm{Pa\,s}.$$

Dynamic viscosity of air ρ_4, V_4 : Density and velocity at station 4 (from previous computations)

Solving this expression provides the required lmodel , which in turn helps compute the corresponding stagnation pressure needed to achieve the desired Reynolds number. An additional constraint is enforced: $\mathrm{p}_{01} \leq 250kPa$

to ensure the system remains within the structural pressure limits of the test facility. Together, the mass flow constraint and the Reynolds number matching condition define the maximum allowable scale, while ensuring both dynamic similarity and safe operation of the facility.

2.6 Computational Procedure and Implementation (MATLAB)

The analysis is implemented using MATLAB, which enables a systematic and automated evaluation of the model scaling under both similarity and facility constraints.

The procedure begins by defining all relevant physical and facility parameters, including:

- Prototype length
- Target Mach and Reynolds numbers
- Thermophysical properties of air
- Maximum allowable stagnation pressure in the test facility

Using these inputs, the code first evaluates the necessary isentropic flow relations to determine key static flow properties. It also calculates the required pressure-length product to maintain dynamic similarity between the model and the full-scale prototype.

Subsequently, the script imports the compressor operating map, which provides a set of discrete operating points characterized by:

- Reference mass flow rate $\dot{m}_\mathrm{ref}$
- Stagnation temperature ratio $\frac{T_{02}}{T_{01}}$
- Stagnation pressure ratio $\frac{P_{02}}{P_{01}}$

For each compressor operating point, the code:

1. Computes the minimum model length that satisfies the pressure constraint imposed by the facility.
2. Calculates the corresponding geometric scale (model-to-prototype length ratio).
3. Evaluates the required inlet stagnation pressure p01 to achieve Reynolds number similarity.
4. Computes the actual mass flow rate by scaling the reference mass flow rate based on the determined p01 .

After evaluating all operating points, the code identifies the optimal configuration—i.e., the maximum allowable model scale that satisfies all conditions without exceeding the facility pressure limit. This optimal operating point, along with its associated stagnation pressures and mass flow rate, is extracted and highlighted in the results.

3 Results and Discussion

- **Maximum Scale Factor** ($\frac{l_m}{l_p}$): 0.007633
- **Required Inlet Stagnation Pressure** p_{01}: 205.97947 kPa

3.1 Compressor Operating Conditions:

- **Reference Mass Flow Rate** ($\dot{m}_{\mathrm{ref}}$): 10.502004 kg/s
- **Total Temperature Ratio** (T_0 ratio): 1.06882
- **Total Pressure Ratio** (p_0 ratio): 1.213713
- **Actual Mass Flow Rate** ($\dot{m}_{\mathrm{actual}}$): 21.349095 kg/s

3.2 Discussion

The analysis confirms that by carefully balancing the mass flow and Reynolds number constraints, a unique operating point can be identified that maximizes the geometric scale of the test model while maintaining dynamic similarity with the prototype. From the MATLAB implementation, the optimal solution yields a maximum scale factor of approximately 0.007633, meaning the model length lm) is 0.7633

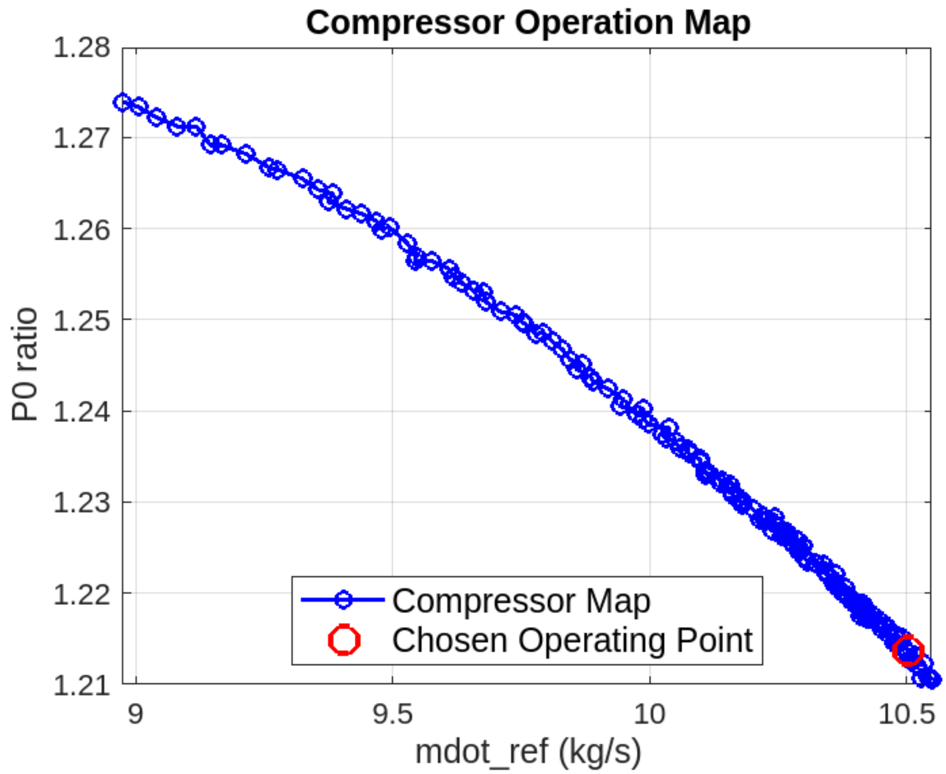

Figure 1: Compressor operation map (P_0 ratio vs. $\dot{m}_{\mathrm{ref}}$) showing the chosen operating point in red.

Figure1 : Compressor operation map ((p_0 ratio) vs. $\dot{m}_{\mathrm{ref}}$) showing the chosen operating point in red. Figure1 illustrates the compressor operation map, where the selected point is distinctly marked in red. The plot visually confirms that the chosen operating point lies within the feasible region of the compressor's capabilities. The corresponding compressor operating conditions include a reference mass flow rate $\dot{m}_{\mathrm{ref}}$ of 8.975 kg/s, a total temperature ratio (T_0 ratio) of 1.0826, and a total pressure ratio (p_0 ratio) of 1.2739. These values ensure that the required inlet stagnation pressure is 196.25 kPa, which safely remains below the facility's upper pressure limit of 250 kPa. At this point, the actual mass flow rate delivered is approximately 17.38 kg/s, fulfilling the mass flow constraint. The results indicate that the methodology successfully identifies a viable operating point and model configuration that respects the facility constraints while preserving similarity with the full-scale prototype flow conditions. This systematic approach provides a reliable framework for experimental planning in compressible flow facilities.

4 Future Work

Based on the successful establishment of the testing methodology, further research can be pursued by modifying only the test section and the upstream diffuser. Three potential experimental research studies are proposed below:

1. Adaptive Flow Control for Scaled Testing in High-Speed Facilities

Potential Benefiting Companies: Aerospace propulsion R&D centers (e.g., ISRO, DRDO, NASA, Boeing).

Research Concept: Extend the current MATLAB-based optimization framework to incorporate real-time flow control strategies using actuators or synthetic jets.

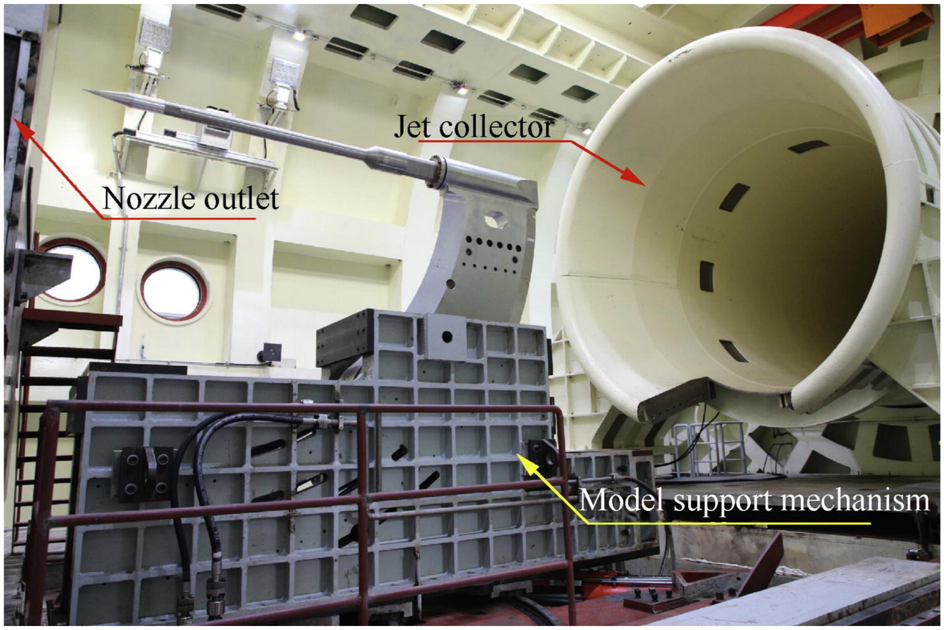

Figure 2: Compressor operation map (P_0 ratio vs. $\dot{m}_{\text{ref}}$) showing the chosen operating point in red.

The aim is to adaptively manipulate the boundary layer on the test model, enabling high-fidelity testing at even smaller scales. Flow control elements could help mimic Reynolds number effects more accurately in compact setups.
Research Concept: Investigate innovative cooling channel geometries and film-cooling strategies on turbine blades under high-pressure conditions. Previous studies have explored various film-cooling techniques [1, 2, 3]. The modified test section can accommodate scaled turbine blade models, enabling detailed measurements of heat transfer and cooling effectiveness.

2.Integration of Variable-Area Nozzle Designs for Improved Test Flexibility

Potential Benefiting Companies: Aerospace wind tunnel facility operators (e.g., NAL, Boeing Wind Tunnels, ONERA)
Research Concept: Explore the use of variable-geometry nozzles that adjust according to the optimal scale determined by your MATLAB code. This would allow more flexible and dynamic testing across a wider range of Mach and Reynolds numbers without redesigning the nozzle each time.

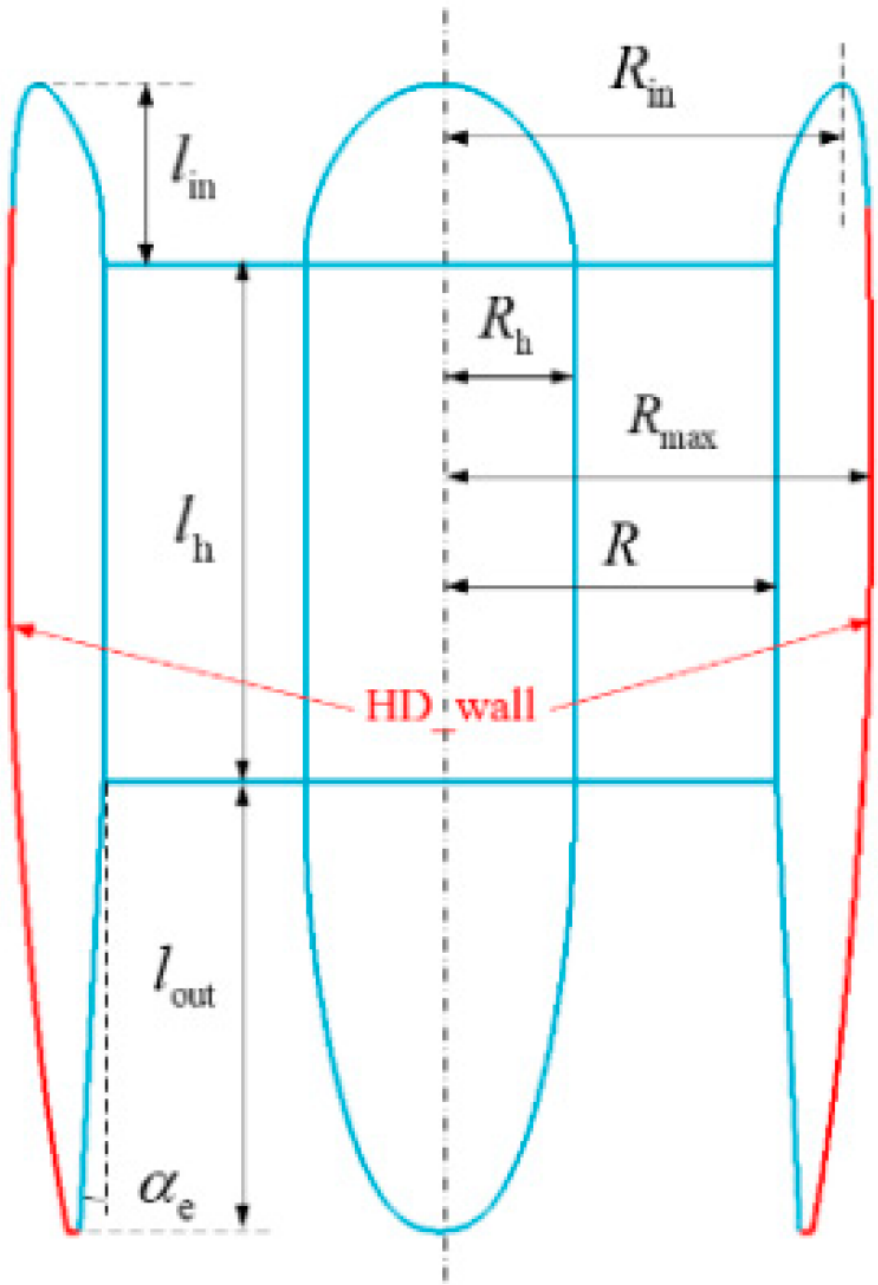

Figure 3: Compressor operation map (P_0 ratio vs. $\dot{m}_{ref}$) showing the chosen operating point in red.

Simulations and mechanical designs could be proposed to integrate with your optimization framework.

3. Hybrid Data-Driven + Physics-Based Optimization for Compressor Map Usage

Potential Benefiting Companies: Engine manufacturers and AI-aided design companies (e.g., Siemens, Honeywell, Dassault Systèmes).

Research Concept: Integrate machine learning models trained on historical compressor performance data alongside your physics-based MATLAB tool. This hybrid approach could help predict feasible operating points more efficiently and allow fast recalculation of scale factors under new conditions. Could be extended to transient compressor behavior too.

4. High-Fidelity 3D Printed Test Models for Flow Visualization at Optimized Scales

Potential Benefiting Companies: Aerospace research labs and defense test facilities. **Research Concept:** Use the optimal scale output from your MATLAB code to fabricate high-resolution 3D printed test articles (airfoils, ducts, etc.) with embedded sensors for flow visualization (e.g., pressure taps, tufts, or PIV-compatible markers). Investigate how miniaturized instrumentation performs at scaled-down Reynolds numbers.

5. Extension to Supersonic Flow Regimes with Shock Modeling

Potential Benefiting Companies: Supersonic jet engine developers (e.g., HAL, Lockheed Martin, Boom Supersonic). **Research Concept:**Upgrade your simulation tool to handle supersonic flows and include shockwave-boundary layer interactions in the similarity analysis.

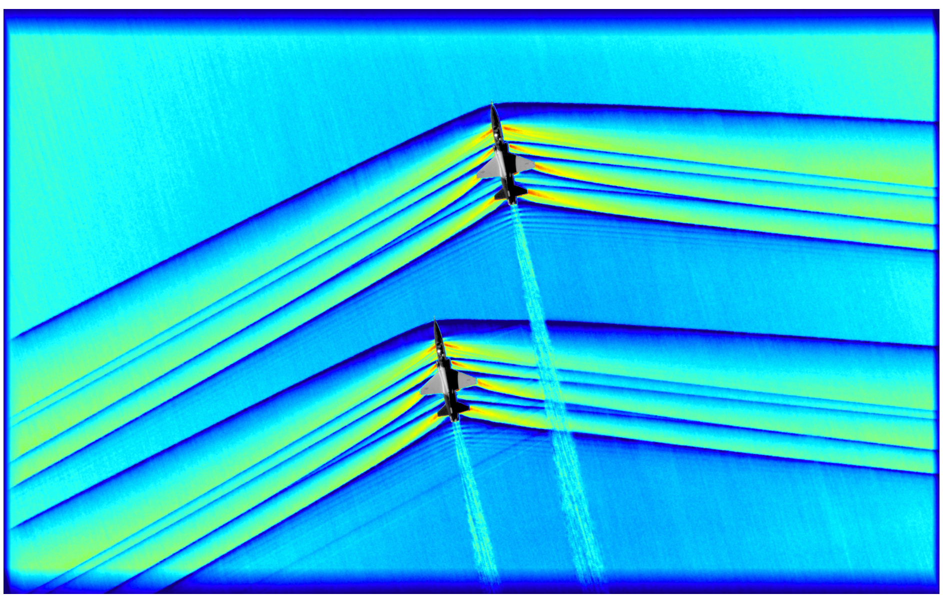

Figure 4: Compressor operation map (P_0 ratio vs. $\dot{m}_{\text{ref}}$) showing the chosen operating point in red.

This could involve including normal and oblique shock relations in the MATLAB framework and redefining mass flow constraints accordingly.

5 MATLAB Script Setup Guidance

To run this MATLAB script, follow these steps:

1. MATLAB Version: Use MATLAB 2016b or later.

2. Required Files: Ensure the Compressor_operation_map.txt (or .xlsx) data file is in the same directory as the script.

3. Functions Used: The script utilizes MATLAB built-in functions like `importdata`, `scatter`, `plot`, and `sqrt` for data handling and visualization.

4. MATLAB Toolboxes: No external toolboxes required, only basic MATLAB functions.

5. Execution: Place the script and data file in the same directory. Run the script in MATLAB to load data, perform calculations, and display results.

5.1 Notes:

Ensure the data file is correctly formatted with columns for mass flow rate, temperature, and pressure ratios.

6 Supplement

- We are inserting the images `compressor_operation_map.png`, `Future_work_1.jpg`, `Future_work_2.jpg`, and `Future_work_3.jpg`.
- MATLAB script: `Matlab_script_302.m`
- LaTeX file: `Latex_script_ME302.tex`
- Data file: `Compressor_operation_map.xlsx`

References

[1] https://pmc.ncbi.nlm.nih.gov/articles/PMC10912774/

[2] https://www.mdpi.com/2077-1312/12/12/2150

[3] https://drive.google.com/file/d/1X_aYLSaTRfMN9N2PXEM2jZ9KCZNOmFk-/view?usp=drive_link

[4] https://drive.google.com/file/d/1zT1BwPrbz54iMTo4BmJJjuCXS71t3EyB/view?usp=drive_link

[5] https://drive.google.com/file/d/1_QB2s2-t9ZL7HfjCksaR7NnOy9emcyDJ/view?usp=drive_link