

## EXERCISE-I

1. Find the equation to the hyperbola whose directrix is  $2x + y = 1$ , focus  $(1, 1)$  & eccentricity  $\sqrt{3}$ . Find also the length of its latus rectum.
2. The hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  passes through the point of intersection of the lines,  $7x + 13y - 87 = 0$  and  $5x - 8y + 7 = 0$  & the latus rectum is  $32\sqrt{2}/5$ . Find 'a' & 'b'.
3. For the hyperbola  $\frac{x^2}{100} - \frac{y^2}{25} = 1$ , prove that
  - (i) eccentricity  $= \sqrt{5}/2$
  - (ii)  $SA \cdot S'A = 25$ , where S & S' are the foci & A is the vertex.
4. Find the centre, the foci, the directrices, the length of the latus rectum, the length & the equations of the axes of the hyperbola  $16x^2 - 9y^2 + 32x + 36y - 164 = 0$ .
5. Find the equation of the tangent to the hyperbola  $x^2 - 4y^2 = 36$  which is perpendicular to the line  $x - y + 4 = 0$ .
6. Tangents are drawn to the hyperbola  $3x^2 - 2y^2 = 25$  from the point  $(0, 5/2)$ . Find their equations.
7. If C is the centre of a hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , S, S' its foci and P a point on it. Prove that  $SP \cdot S'P = CP^2 - a^2 + b^2$ .
8. If  $\theta_1$  &  $\theta_2$  are the parameters of the extremities of a chord through  $(ae, 0)$  of a hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then show that  $\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} + \frac{e-1}{e+1} = 0$ .
9. Tangents are drawn from the point  $(\alpha, \beta)$  to the hyperbola  $3x^2 - 2y^2 = 6$  and are inclined at angles  $\theta$  and  $\phi$  to the x-axis. If  $\tan \theta \cdot \tan \phi = 2$ , prove that  $\beta^2 = 2\alpha^2 - 7$ .
10. If two points P & Q on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  whose centre is C be such that CP is perpendicular to CQ &  $a < b$ , then prove that  $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$ .
11. An ellipse has eccentricity  $1/2$  and one focus at the point P  $(1/2, 1)$ . Its one directrix is the common tangent, nearer to the point P, to the circle  $x^2 + y^2 = 1$  and the hyperbola  $x^2 - y^2 = 1$ . Find the equation of the ellipse in the standard form.
12. The tangents & normal at a point on  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  cut the y-axis at A & B. Prove that the circle on AB as diameter passes through the foci of the hyperbola.

13. The perpendicular from the centre upon the normal on any point of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  meets at R. Find the locus of R.
14. If the normal at a point P to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  meets the x - axis at G, show that  $SG = e \cdot SP$ , S being the focus of the hyperbola.
15. Show that the locus of the middle points of normal chords of the rectangular hyperbola  $x^2 - y^2 = a^2$  is  $(y^2 - x^2)^3 = 4 a^2 x^2 y^2$ .
16. If a chord joining the points P ( $a \sec \theta$ ,  $a \tan \theta$ ) & Q ( $a \sec \phi$ ,  $a \tan \phi$ ) on the hyperbola  $x^2 - y^2 = a^2$  is a normal to it at P, then show that  $\tan \phi = \tan \theta (4 \sec^2 \theta - 1)$ .
17. Chords of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  are tangents to the circle drawn on the line joining the foci as diameter. Find the locus of the point of intersection of tangents at the extremities of the chords.
18. Let 'p' be the perpendicular distance from the centre C of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  to the tangent drawn at a point R on the hyperbola. If S & S' are the two foci of the hyperbola, then show that  $(RS + RS')^2 = 4 a^2 \left( 1 + \frac{b^2}{p^2} \right)$ .
19. Prove that the part of the tangent at any point of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  intercepted between the point of contact and the transverse axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point.
20. An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by a distance  $2\sqrt{13}$ , the difference of their focal semi axes is equal to 4. If the ratio of their eccentricities is  $3/7$ . Find the equation of these curves.

## EXERCISE-II

1. Prove that the locus of the middle point of the chord of contact of tangents from any point of the circle

$$x^2 + y^2 = r^2 \text{ to the hyperbola } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ is given by the equation } \left( \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2 = \frac{(x^2 + y^2)}{r^2}.$$

2. The graphs of  $x^2 + y^2 + 6x - 24y + 72 = 0$  &  $x^2 - y^2 + 6x + 16y - 46 = 0$  intersect at four points. Compute the sum of the distances of these four points from the point  $(-3, 2)$ .

3. Find the equations of the tangents to the hyperbola  $x^2 - 9y^2 = 9$  that are drawn from  
(3 2) Find the area of the triangle that these tangents form with their chord of contact.

4. A line through the origin meets the circle  $x^2 + y^2 = a^2$  at P & the hyperbola  $x^2 - y^2 = a^2$  at Q. Prove that the locus of the point of intersection of the tangent at P to the circle and the tangent at Q to the hyperbola is curve  $a^4(x^2 - a^2) + 4x^2y^4 = 0$ .



5. A tangent to the parabola  $x^2 = 4ay$  meets the hyperbola  $xy = k^2$  in two points P & Q. Prove that the middle point of PQ lies on another parabola.
6. The normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  drawn at an extremity of its latus rectum is parallel to an asymptote.  
Show that the eccentricity is equal to the square root of  $(1 + \sqrt{5})/2$ .
7. Ascertain the co-ordinates of the two points Q & R, where the tangent to the hyperbola  $\frac{x^2}{45} - \frac{y^2}{20} = 1$  at the point P(9, 4) intersects the two asymptotes. Finally prove that P is the middle point of QR. Also compute the area of the triangle CQR where C is the centre of the hyperbola.
8. A point P divides the focal length of the hyperbola  $9x^2 - 16y^2 = 144$  in the ratio S'P : PS = 2 : 3 where S & S' are the foci of the hyperbola. Through P a straight line is drawn at an angle of  $135^\circ$  to the axis OX. Find the points of intersection of this line with the asymptotes of the hyperbola.
9. Find the length of the diameter of the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  perpendicular to the asymptote of the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  passing through the first & third quadrants.
10. The tangent at P on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  meets one of the asymptote in Q. Show that the locus of the mid point of PQ is a similar hyperbola.
11. A transversal cuts the same branch of a hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  in P, P' and the asymptotes in Q, Q'. Prove that (i)  $PQ = P'Q'$  & (ii)  $PQ' = P'Q$
12. A series of hyperbolas is drawn having a common transverse axis of length 2a. Prove that the locus of a point P on each hyperbola, such that its distance from the transverse axis is equal to its distance from an asymptote, is the curve  $(x^2 - y^2)^2 = 4x^2(x^2 - a^2)$ .
13. From any point of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , tangents are drawn to another hyperbola which has the same asymptotes. Show that the chord of contact cuts off a constant area from the asymptotes.
14. Through any point P of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  a line QPR is drawn with a fixed gradient m, meeting the asymptotes in Q & R. Show that the product,  $(QP) \cdot (PR) = \frac{a^2 b^2 (1 + m^2)}{b^2 - a^2 m^2}$ .
15. If a rectangular hyperbola have the equation,  $xy = c^2$ , prove that the locus of the middle points of the chords of constant length 2d is  $(x^2 + y^2)(xy - c^2) = d^2 xy$ .
16. A triangle is inscribed in the rectangular hyperbola  $xy = c^2$ . Prove that the perpendiculars to the sides at the points where they meet the asymptotes are concurrent. If the point of concurrence is  $(x_1, y_1)$  for one asymptote and  $(x_2, y_2)$  for the other, then prove that  $x_2 y_1 = c^2$ .
17. Prove that infinite number of triangles can be inscribed in the rectangular hyperbola,  $xy = c^2$  whose sides touch the parabola,  $y^2 = 4ax$ .

18. The normals at three points P, Q, R on a rectangular hyperbola  $xy = c^2$  intersect at a point on the curve. Prove that the centre of the hyperbola is the centroid of the triangle PQR.

19. Tangents are drawn from any point on the rectangular hyperbola  $x^2 - y^2 = a^2 - b^2$  to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

Prove that these tangents are equally inclined to the asymptotes of the hyperbola.

20. P & Q are two variable points on a rectangular hyperbola  $xy = c^2$  such that the tangent at Q passes through the foot of the ordinate of P. Show that the locus of the point of intersection of tangent at P & Q is a hyperbola with the same asymptotes as the given hyperbola.

## EXERCISE-III

1. (a) The curve described parametrically by,  $x = t^2 + t + 1$ ,  $y = t^2 - t + 1$  represents:  
 (A) a parabola (B) an ellipse (C) a hyperbola (D) a pair of straight lines  
 (b) Let P ( $a \sec \theta$ ,  $b \tan \theta$ ) and Q ( $a \sec \phi$ ,  $b \tan \phi$ ), where  $\theta + \phi = \frac{\pi}{2}$ , be two points on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If (h, k) is the point of intersection of the normals at P & Q, then k is equal to:  
 (A)  $\frac{a^2 + b^2}{a}$  (B)  $-\left(\frac{a^2 + b^2}{a}\right)$  (C)  $\frac{a^2 + b^2}{b}$  (D)  $-\left(\frac{a^2 + b^2}{b}\right)$   
 (c) If  $x = 9$  is the chord of contact of the hyperbola  $x^2 - y^2 = 9$ , then the equation of the corresponding pair of tangents, is :  
 (A)  $9x^2 - 8y^2 + 18x - 9 = 0$  (B)  $9x^2 - 8y^2 - 18x + 9 = 0$   
 (C)  $9x^2 - 8y^2 - 18x - 9 = 0$  (D)  $9x^2 - 8y^2 + 18x + 9 = 0$   
[JEE '99, 2 + 2 + 2 (out of 200)]
2. The equation of the common tangent to the curve  $y^2 = 8x$  and  $xy = -1$  is  
 (A)  $3y = 9x + 2$  (B)  $y = 2x + 1$  (C)  $2y = x + 8$  (D)  $y = x + 2$  [JEE 2002 Screening]
3. Given the family of hyperbolas  $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$  for  $\alpha \in (0, \pi/2)$  which of the following does not change with varying  $\alpha$ ?  
 (A) abscissa of foci (B) eccentricity  
 (C) equations of directrices (D) abscissa of vertices [JEE 2003 (Scr.)]
4. The line  $2x + \sqrt{6}y = 2$  is a tangent to the curve  $x^2 - 2y^2 = 4$ . The point of contact is  
 (A)  $(4, -\sqrt{6})$  (B)  $(7, -2\sqrt{6})$  (C)  $(2, 3)$  (D)  $(\sqrt{6}, 1)$  [JEE 2004 (Scr.)]
5. Tangents are drawn from any point on the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  to the circle  $x^2 + y^2 = 9$ . Find the locus of midpoint of the chord of contact.  
[JEE 2005 (Mains), 4]



6. If a hyperbola passes through the focus of the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  and its transverse and conjugate axis coincides with the major and minor axis of the ellipse, and product of their eccentricities is 1, then

(A) equation of hyperbola  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(B) equation of hyperbola  $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(C) focus of hyperbola (5, 0)

(D) focus of hyperbola is  $(5\sqrt{3}, 0)$  [JEE 2006, 5]

**Comprehension: (3 questions)**

7. Let ABCD be a square of side length 2 units.  $C_2$  is the circle through vertices A, B, C, D and  $C_1$  is the circle touching all the sides of the square ABCD. L is a line through A

- (a) If P is a point on  $C_1$  and Q in another point on  $C_2$ , then  $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$  is equal to

(A) 0.75

(B) 1.25

(C) 1

(D) 0.5

- (b) A circle touches the line L and the circle  $C_1$  externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

(A) ellipse

(B) hyperbola

(C) parabola

(D) parts of straight line

- (c) A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at  $T_2$  and  $T_3$  and AC at  $T_1$ , then area of  $\Delta T_1 T_2 T_3$  is

(A)  $1/2$  sq. units

(B)  $2/3$  sq. units

(C) 1 sq. unit

(D) 2 sq. units

[JEE 2006, 5 marks each]

8. (a) A hyperbola, having the transverse axis of length  $2 \sin \theta$ , is confocal with the ellipse  $3x^2 + 4y^2 = 12$ . Then its equation is

(A)  $x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$

(B)  $x^2 \sec^2 \theta - y^2 \operatorname{cosec}^2 \theta = 1$

(C)  $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$

(D)  $x^2 \cos^2 \theta - y^2 \sin^2 \theta = 1$

[JEE 2007, 3]

- (b) Match the statements in Column I with the properties in Column II.

**Column I**

(A) Two intersecting circles

(B) Two mutually external circles

(C) Two circles, one strictly inside the other

(D) Two branches of a hyperbola

**Column II**

(P) have a common tangent

(Q) have a common normal

(R) do not have a common tangent

(S) do not have a common normal

[JEE 2007, 3 + 6]

9. (a) Let a and b be non-zero real numbers. Then, the equation  $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$  represents

(A) four straight lines, when  $c = 0$  and a, b are of the same sign.

(B) two straight lines and a circle, when  $a = b$ , and c is of sign opposite to that of a.

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a.

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

- (b) Consider a branch of the hyperbola,  $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$  with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

[JEE 2008, 3+3]

(A)  $1 - \sqrt{\frac{2}{3}}$

(B)  $\sqrt{\frac{3}{2}} - 1$

(C)  $1 + \sqrt{\frac{2}{3}}$

(D)  $\sqrt{\frac{3}{2}} + 1$

10. Match the conics in Column I with the statements/expressions in Column II.

[JEE 2009]

Column I

Column II

(A) Circle

(P) The locus of the point  $(h, k)$  for which the line  $hx + ky = 1$  touches the circle  $x^2 + y^2 = 4$

(B) Parabola

(Q) Points  $z$  in the complex plane satisfying  $|z + 2| - |z - 2| = \pm 3$

(C) Ellipse

(R) Points of the conic have parametric representation

$$x = \sqrt{3} \left( \frac{1-t^2}{1+t^2} \right), y = \frac{2t}{1+t^2}$$

(D) Hyperbola

(S) The eccentricity of the conic lies in the interval  $1 \leq e < \infty$

(T) Points  $z$  in the complex plane satisfying  $\operatorname{Re}(z + 1)^2 = |z|^2 + 1$

11. An ellipse intersects the hyperbola  $2x^2 - 2y^2 = 1$  orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then [JEE 2009]

(A) Equation of ellipse is  $x^2 + 2y^2 = 2$

(B) The foci of ellipse are  $(\pm 1, 0)$

(C) Equation of ellipse is  $x^2 + 2y^2 = 4$

(D) The foci of ellipse are  $(\pm \sqrt{2}, 0)$

Paragraph for Questions 12 to 13

[JEE 2010]

The circle  $x^2 + y^2 - 8x = 0$  and hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  intersect at the points A and B.

12. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A)  $2x - \sqrt{5}y - 20 = 0$

(B)  $2x - \sqrt{5}y + 4 = 0$

(C)  $3x - 4y + 8 = 0$

(D)  $4x - 3y + 4 = 0$

13. Equation of the circle with AB as its diameter is

(A)  $x^2 + y^2 - 12x + 24 = 0$

(B)  $x^2 + y^2 + 12x + 24 = 0$

(C)  $x^2 + y^2 + 24x - 12 = 0$

(D)  $x^2 + y^2 - 24x - 12 = 0$

14. The line  $2x + y = 1$  is tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is [JEE 2010]

15. Let  $P(6, 3)$  be a point on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If the normal at the point P intersects the x-axis at  $(9, 0)$ , then the eccentricity of the hyperbola is [JEE 2011]

(A)  $\sqrt{\frac{5}{2}}$

(B)  $\sqrt{\frac{3}{2}}$

(C)  $\sqrt{2}$

(D)  $\sqrt{3}$

16. Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be reciprocal to that of the ellipse  $x^2 + 4y^2 = 4$ . If the hyperbola passes through a focus of the ellipse, then [JEE 2011]

(A) the equation of the hyperbola is  $\frac{x^2}{3} - \frac{y^2}{2} = 1$  (B) a focus of the hyperbola is  $(2, 0)$

(C) the eccentricity of the hyperbola is  $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is  $x^2 - 3y^2 = 3$



17. Tangents are drawn to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$ , parallel to the straight line  $2x - y = 1$ . The points of contact of the tangents on the hyperbola are [JEE 2012]

(A)  $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$  (B)  $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$  (C)  $(3\sqrt{3}, -2\sqrt{2})$  (D)  $(-3\sqrt{3}, 2\sqrt{2})$

18. Consider the hyperbola  $H: x^2 - y^2 = 1$  and a circle  $S$  with center  $N(x_2, 0)$ . Suppose that  $H$  and  $S$  touch each other at a point  $P(x_1, y_1)$  with  $x_1 > 1$  and  $y_1 > 0$ . The common tangent to  $H$  and  $S$  at  $P$  intersects the  $x$ -axis at point  $M$ . If  $(\ell, m)$  is the centroid of the triangle  $\triangle PMN$ , then the correct expression(s) is(are)

[IIT JEE Advance - 2015]

(A)  $\frac{d\ell}{dx_1} = 1 - \frac{1}{3x_1^2}$  for  $x_1 > 1$  (B)  $\frac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$  for  $x_1 > 1$   
 (C)  $\frac{d\ell}{dx_1} = 1 + \frac{1}{3x_1^2}$  for  $x_1 > 1$  (D)  $\frac{dm}{dy_1} = \frac{1}{3}$  for  $y_1 > 0$

19. The centres of those circles which touch the circle,  $x^2 - y^2 - 8x - 8y - 4 = 0$ , externally and also touch the  $x$ -axis, lie on :

- (A) a circle (B) an ellipse which is not a circle  
 (C) a hyperbola (D) a parabola

[JEE Main - 2016]

20. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

[JEE Main - 2016]

(A)  $\frac{4}{3}$  (B)  $\frac{4}{\sqrt{3}}$  (C)  $\frac{2}{\sqrt{3}}$  (D)  $\sqrt{3}$

21. A hyperbola passes through the point  $P(\sqrt{2}, \sqrt{3})$  and has foci at  $(\pm 2, 0)$ . Then the tangent to this hyperbola at  $P$  also passes through the point :

[JEE Main - 2017]

(A)  $(\sqrt{3}, \sqrt{2})$  (B)  $(-\sqrt{2}, -\sqrt{3})$   
 (C)  $(3\sqrt{2}, 2\sqrt{3})$  (D)  $(2\sqrt{2}, 3\sqrt{3})$

22. If  $2x - y + 1 = 0$  is a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ , then which of the following CANNOT be sides of a right angled triangle?

[JEE Advanced 2017]

(A)  $a, 4, 2$  (B)  $2a, 8, 1$  (C)  $2a, 4, 1$  (D)  $a, 4, 1$

**Answer Q. 23, Q. 24 and Q. 25 by appropriately matching the information given in the three columns of the following table.**

[JEE Advanced 2017]

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column-1

Column-2

Column-3

(I)  $x^2 + y^2 = a^2$

(i)  $my = m^2x + a$

(P)  $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

(II)  $x^2 + a^2y^2 = a^2$

(ii)  $y = mx + a\sqrt{m^2 + 1}$

(Q)  $\left(\frac{-ma}{\sqrt{m^2 + 1}}, \frac{a}{\sqrt{m^2 + 1}}\right)$

(III)  $y^2 = 4ax$

(iii)  $y = mx + \sqrt{a^2m^2 - 1}$

(R)  $\left(\frac{-a^2m}{\sqrt{a^2m^2 + 1}}, \frac{1}{\sqrt{a^2m^2 + 1}}\right)$

(IV)  $x^2 - a^2y^2 = a^2$

(iv)  $y = mx + \sqrt{a^2m^2 + 1}$

(S)  $\left(\frac{-a^2m}{\sqrt{a^2m^2 - 1}}, \frac{-1}{\sqrt{a^2m^2 - 1}}\right)$

23. For  $a = \sqrt{2}$ , if a tangent is drawn to a suitable conic (Column 1) at the point of contact  $(-1, 1)$ , then which of the following options is the only CORRECT combination for obtaining its equation?

(A) (II) (ii) (Q)

(B) (III) (i) (P)

(C) (I) (ii) (Q)

(D) (I) (i) (P)

24. If a tangent to a suitable conic (Column 1) is found to be  $y = x + 8$  and its point of contact is  $(8, 16)$ , then which of the following options is the only CORRECT combination?

(A) (III) (i) (P)

(B) (II) (iv) (R)

(C) (III) (ii) (Q)

(D) (I) (ii) (Q)

25. The tangent to a suitable conic (Column 1) at  $\left(\sqrt{3}, \frac{1}{2}\right)$  is found to be  $\sqrt{3}x + 2y = 4$ , then which of the following options is the only CORRECT combination?

(A) (IV) (iii) (S)

(B) (II) (iv) (R)

(C) (IV) (iv) (S)

(D) (II) (iii) (R)

26. Tangents are drawn to the hyperbola  $4x^2 - y^2 = 36$  at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of  $\Delta PTQ$  is : [JEE Main 2018]

(A)  $36\sqrt{5}$

(B)  $45\sqrt{5}$

(C)  $54\sqrt{3}$

(D)  $60\sqrt{3}$

27. Let  $H \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , where  $a > b > 0$ , be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of  $60^\circ$  at one of its vertices N. Let the area of the triangle LMN be  $4\sqrt{3}$ . [JEE Advance 2018]

List - I

List - II

(P) The length of the conjugate axis of H is

(1) 8

(Q) The eccentricity of H is

(2)  $\frac{4}{\sqrt{3}}$

(R) The distance between the foci of H is

(3)  $\frac{2}{\sqrt{3}}$

(S) The length of the latus rectum of H is

(4) 4

The correct option is :

(A)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$

(B)  $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 2$

(C)  $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 2$

(D)  $P \rightarrow 3; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 1$