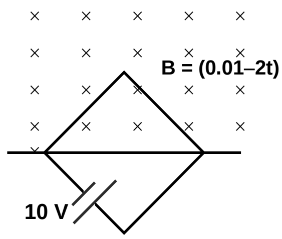
ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

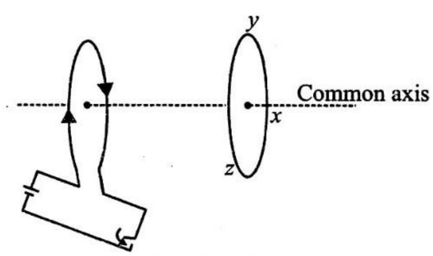
MULTIPLE CHOICE QUESTIONS

1. A square loop of side 1 m is placed in a perpendicular magnetic field. Half of the area of the loop lies inside the magnetic field. A battery of emf 10 V and negligible internal resistance is connected in the loop. The magnetic field changes with time according to the relation B = (0.01– 2t) tesla. The resultant emf of the circuit is



(A) 1 V (B) 11 V
(C) 9 V (D) 10 V

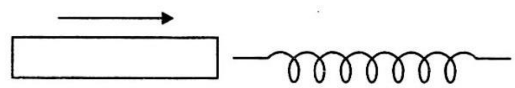
2. The direction of induced current in the right loop in the situation shown by the given figure is

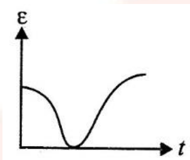


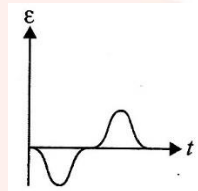
(Tapping key just closed)

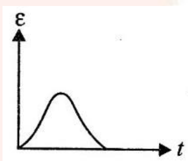
(A) along the common axis (B) along *xzy*
(C) along *xyz* (D) none of these

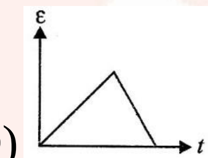
3. The variation of induced emf (ε) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as



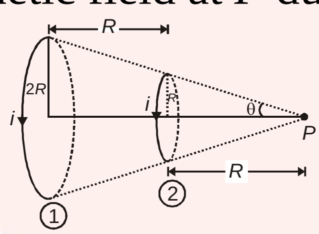
(A) 

(B) 

(C) 

(D) 

4. Figure shows two coils carrying equal currents. The ratio of magnetic field at P due to coil-1 to coil-2 is



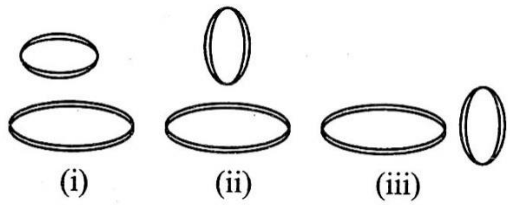
a) $\frac{1}{\sqrt{2}}$

b) 1 : 2

c) $\frac{1}{4}$

d) $\left(\frac{1}{2}\right)^{\frac{1}{4}}$

5. Two circular coils can be arranged in any of three situations as shown in the figure. Their mutual inductance will be



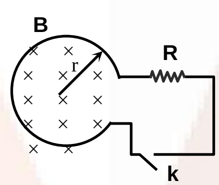
(A) maximum in situation (i)

(B) maximum in situation (ii)

(C) maximum in situation (iii)

(D) same in all situations

6. Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil. If the key (K) is closed, the electrical power developed right after closing the switch is equal to



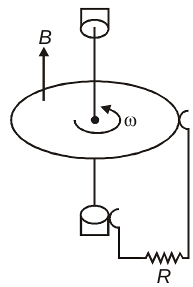
(A) $\frac{B_0^2 \pi r^2}{R}$

(B) $\frac{B_0 \pi r^3}{R}$

(C) $\frac{B_0^2 \pi^2 r^4 R}{5}$

(D) $\frac{B_0^2 \pi^2 r^4}{R}$

7. The arrangement shown here is a Faraday's disc of radius *a* which is rotating in a uniform magnetic field *B* perpendicular to the plane of disc. The current passing through the resistance is



a) $\frac{B\omega^2 a}{R}$

b) $\frac{B\omega a^2}{2R}$

c) $\frac{B\omega^2 a}{2R}$

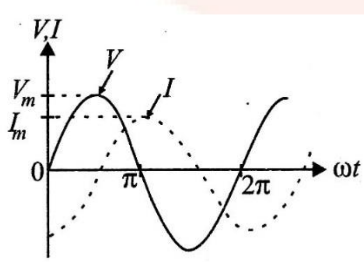
d) $\frac{B\omega a^2}{R}$

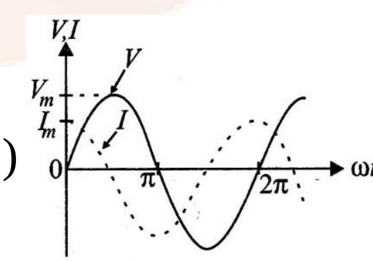
8. A square of side x m lies in the x-y plane in a region, where the magnetic field is given by $\vec{B} = B_0\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)T,$ where B0 is constant. The magnitude of flux passing through the square is

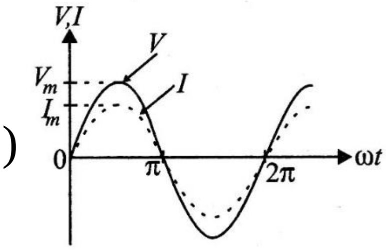
(A) $5B_0x^2Wb$ (B) $3B_0x^2Wb$

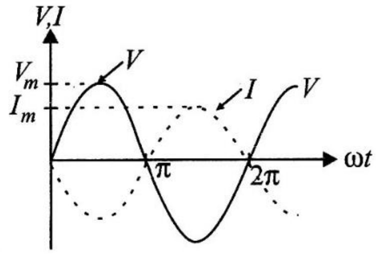
(C) $2B_0x^2Wb$ (D) B_0x^2Wb

9. The phase relationship between current and voltage in a pure resistive circuit is best represented by

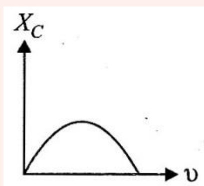
(A) 

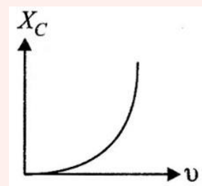
(B) 

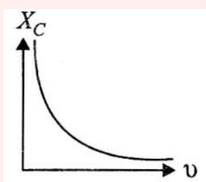
(C) 

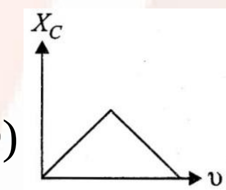
(D) 

10. Which of the following graphs represents the correct variation of capacitive reactance X_C with frequency υ?

(A) 

(B) 

(C) 

(D) 

11. The natural frequency (ω_0) of oscillations in LC circuit is given by

(A) $\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

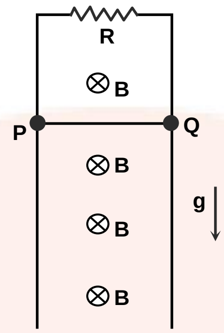
(B) $\frac{1}{\pi}\frac{1}{\sqrt{2LC}}$

(C) $\frac{1}{\sqrt{LC}}$

(D) $\sqrt{LC}$

12. A rod PQ mass 'm' and length ℓ can slide without friction on two vertical conducting semi-infinite rails. It is given a velocity v_0 downwards, so that it continues to move downward

with the same speed v_0 on its own at any later instant of time. Assuming g to be constant every where, the value of v_0 is :-



(A) $\frac{mgR}{2B^2L^2}$ (B) $\frac{mgR}{B^2L^2}$

(C) zero (D) Any value

13. Alternating voltage (V) is represented by the equation

(A) $V(t) = V_m e^{\omega t}$ (B) $V(t) = V_m \sin \omega t$

(C) $V(t) = V_m \cot \omega t$ (D) $V(t) = V_m \tan \omega t$

where V_m is the peak voltage

14. In the case of an inductor

(A) voltage lags the current by $\frac{\pi}{2}$

(B) voltage leads the current by $\frac{\pi}{2}$

(C) voltage leads the current by $\frac{\pi}{3}$

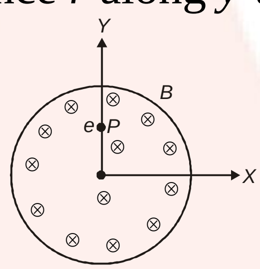
(D) voltage leads the current by $\frac{\pi}{4}$

15. The magnetic flux linked with a coil of N turns of area of cross section A held with its plane parallel to the field B is

(A) $\frac{NAB}{2}$ (B) NAB

(C) $\frac{NAB}{4}$ (D) zero

16. In the cylindrical region shown, magnetic field is diminishing at the rate of α(T/s). The force on the electron at a distance *r* along *y*-axis is



a) $\frac{r}{2}\alpha e(-\hat{i})$

b) $\frac{r}{2}\alpha e(\hat{i})$

c) $\frac{r^2}{2}\alpha e(-\hat{i})$

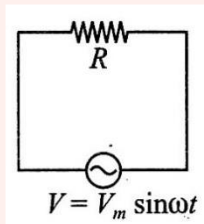
d) $\frac{r^2}{2}\alpha e(\hat{i})$

17. An ideal inductor is in turn put across 220 V, 50 Hz and 220 V, 100 Hz supplies. The current flowing through it in the two cases will be

(A) equal (B) different

(C) zero (D) infinite

18. An ac source of voltage V = V_m sin ωt is connected across the resistance R as shown in figure. The phase relation between current and voltage for this circuit is



(A) both arc in phase

(B) both are out of phase by 90°

(C) both are out of phase by 120°

(D) both are out of phase by 180°

19. A coil of wire having finite inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time t = 0, so that a time-dependent current I_1(t) starts flowing through the coil. If I_2(t) is the current induced in the ring, and B(t) is the magnetic field at the axis of coil due to I_1(t), then as a function of time (t > 0), the product I_2(t) B(t)

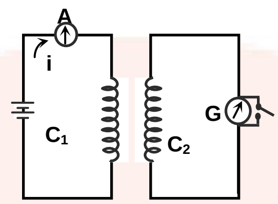
(A) increases with time

(B) decreases with time

(C) does not vary with time

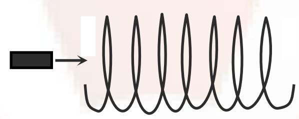
(D) passes through a maximum.

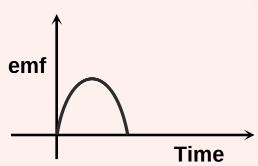
20. In figure when key is pressed the ammeter A reads i ampere. The charge passing in the galvanometer circuit of total resistance R is Q. The mutual inductance of the two coils is :

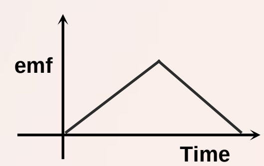


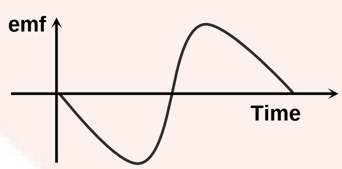
(A) Q/R (B) QR
(C) QR/i (D) i/QR

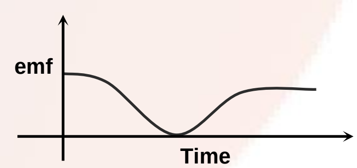
21. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced time?



(A)


(B)


(C)


(D)


22. Two circular coils of radii R_1 and R_2 having N_1 and N_2 turns are placed concentrically in the same plane. If $R_2 << R_1$, then the mutual inductance between them is

(A) $\frac{\mu_0 \pi R_2^2}{2R_1}$ (B) $\frac{\mu_0 \pi R_2^2 N_1 N_2}{2R_1}$

(C) $\frac{\mu_0 \pi R_2 N_1 N_2}{2R_1}$ (D) $\frac{\mu_0 \pi R_1 N_1 N_2}{2R_2}$

23. The frequency for which a $5.0\mu F$ capacitor has a reactance of 1000Ω is given by

(A) $\frac{1000}{\pi}$ cycles/sec
(B) $\frac{100}{\pi}$ cycles/sec
(C) 200 cycle /s
(D) 5000 cycles /sec

24. A copper rod of length ℓ is rotated about one end perpendicular to the uniform magnetic field B with constant angular velocity ω. The induced emf between the end is

(A) $\frac{1}{2}B\omega\ell^2$
(B) $\frac{3}{2}B\omega\ell^2$
(C) $B\omega\ell^2$
(D) $2B\omega\ell^2$

25. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now the radius of the loop starts shrinking at the rate (dr/dt). Then, the induced emf at the instant when the radius is r, is

(A) $\pi rB\left(\frac{dr}{dt}\right)$
(B) $2\pi rB\left(\frac{dr}{dt}\right)$
(C) $\pi r^2\frac{dB}{dt}$
(D) $\frac{\pi r^2}{2}B\frac{dr}{dt}$

INGTEGERS TYPE QUESTION

26. Find the mutual inductance if number of turns in primary and secondary coils is increased to two times each.

27. In an ac circuit, V and I are given by

$$V = 150\sin(150t)\,V \text{ and } I = 150\sin\left(150t + \frac{\pi}{3}\right)A.$$

Find the power dissipated in the circuit.

28. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply. What is the rms value of current in the circuit?

29. Find the capacitive reactance if a 5 μF capacitor is connected to a 200 V, 100 Hz ac source.

30. A conductor is moving with the velocity *v* in the magnetic field and induced current is I. If the velocity of conductor becomes double, the induced current will be __________

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. (C)

$e_{induced} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt}$

$= \frac{\ell^2}{2}\frac{d(0.01-2t)}{dt}$ = 1 Volt

$\therefore$ Resultant emf = 10 – 1 = 9V

2. (C)

The induced current in the right loop will be along *xyz*.

3. (B)

The polarity of emf will be opposite in the two cases while the magnet enters the coil and while the magnetic leaves the coil. Only in option (B) polarity is changing.

4. (b)

Explanation:

$B_1 = \frac{\mu_0}{2} i \frac{(2R)^2}{[(2R)^2 + (2R)^2]^{3/2}}; \; B_2 = \frac{\mu_0}{2}\frac{iR^2}{[R^2 + R^2]^{3/2}}$

$\frac{B_1}{B_2} = \frac{4/8^{3/2}}{1/2\sqrt{2}} = \frac{4\times 2\sqrt{2}}{8\times 2\sqrt{2}} = 1:2$

5. (A)

A mutual inductance occurs when the magnetic field generated by a coil induces a voltage in a secondary coil.

$M = \varepsilon_m / \left(\frac{dI}{dt}\right)$

Since, in option (A), there is maximum emf is induced in secondary coil. So, mutual inductance is maximum.

6. (D)

The induced emf = E = $\frac{d\phi}{dt}$

$= \frac{d}{dt}(B.A) = A\ \frac{dB}{dt}$

$= (\pi r^2)B_0 \frac{d}{dt}(e^{-t}) = -\pi r^2 B_0 e^{-t}$

$\Rightarrow E_0 = B_0\pi\ r^2 e^{-t}|_{t=0} = B_0\pi\ r^2$

$\therefore$ The electrical power developed in the resister just at the instant of closing the key = P = $\frac{E_0^2}{R} = \frac{B_0^2\pi^2\ r^4}{R}$

7. (b)

Explanation:

Emf induced in the disc will be same as that induced in a rod rotating in a magnetic field.

$\therefore\ e = \frac{1}{2}B\omega a^2$

$\therefore\ i = \frac{B\omega a^2}{2R}$

8. (A)

Here, $\vec{A} = x^2\hat{k}\ m^2$ and $\vec{B} = B_0\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$ T

As $\phi = \vec{B}\cdot\vec{A} = B_0\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)\cdot x^2\hat{k}$

$\therefore\quad \phi = 5B_0x^2 Wb$

9. (C)

In the pure resistive circuit current and voltage both are in phase. Hence graph (C) is correct.

10. (C)

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} \Rightarrow X_C = \frac{1}{\upsilon}$

With increase in frequency, X_C decreases.

Hence, option (C) represents the correct graph.

11. (C)

12. (B)

$$\frac{B^2V_0L^2}{R} = mg \Rightarrow V_0 = \frac{mgr}{B^2L^2}$$

13. (B)

The equation of the alternating voltage is $V(t) = V_m \sin \omega t$

14. (B)

In an inductor voltage leads the current by $\frac{\pi}{2}$ or current lags the voltage by $\frac{\pi}{2}$.

15. (D)

Magnetic flux linked with a coil

$$\phi = NBA\cos\theta$$

Since the magnetic field B is parallel to the area A, i.e., $\theta = 90°$.

$\therefore \quad \phi = 0$

16. (a)

Explanation:

$E \times 2\pi r = \pi r^2.\alpha$

$\therefore \; E = \frac{r}{2}\alpha$,

At P, E will be along $\hat{i}$

$\therefore \quad F = \frac{r}{2}\alpha e(-\hat{i})$

17. An ideal inductor is in turn put across 220 V, 50 Hz and 220 V, 100 Hz supplies. The current (B)

The current in the inductor coil is given by $I = \frac{V}{X_L} = \frac{V}{2\pi\upsilon L}$

Since frequency υ in the two cases is different, hence the current in two cases will be different.

18. (A)

The given circuit is a pure resistive circuit. In this circuit the voltage and current both are in phase.

19. (D)

$i_2(t)\ B(t) \propto i_2\ (t)i_1(t) \propto (1-e^{-t/\tau})e^{-t/\tau}$

At $t = 0$, $i_1(t).i_2 = 0$

Also $i_1(t).i_2\ (t) \rightarrow 0$ as $t \rightarrow \infty$

20. (C)

21. (C)

22. (B)

Let I be the current through the coil of radius R_1. The magnetic induction at the centre of the coil is $B = \frac{\mu_0}{4\pi}\frac{2\pi N_1 I}{R_1}$

Magnetic flux linked with the coil of radius R_1 is

$\phi = B_1\pi R_2^2 N_2 = \frac{\mu_0}{4\pi}\left(\frac{2\pi N_1 I}{R_1}\right)\pi R_2^2 N_2$

$\phi = MI = \therefore\ M = \frac{\phi}{I} = \frac{\mu_0}{4\pi}\left(\frac{2\pi N_1}{R_1}\right)\pi R_2^2 N_2 = \frac{\mu_0 \pi R_2^2 N_1 N_2}{2R_1}$

23. (B)

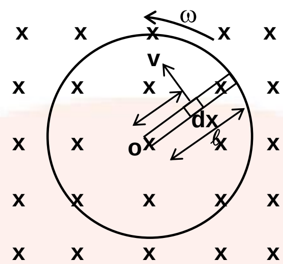
$X_C = \frac{1}{\omega C}$

$\Rightarrow \quad \omega = \frac{1}{X_C C} = \frac{1}{1000 \times 5 \times 10^{-6}}$

$f = \frac{1}{2\pi \times 5 \times 10^{-3}} = \frac{100}{\pi}$ cycle/sec

24. (A)

Consider a small element of the rod of length dx at a distance x from the centre O. Let v be the linear velocity of the element at the right angle to the magnetic field B. The emf developed across the element is



$de = Bvdx = B\omega xdx$

The emf across the entire length ℓ

is given by

$$e = \int de = B\omega \int_0^{\ell} xdx = \frac{B\omega \ell^2}{2}$$

25. (B)

If the radius is r at a time t, then the ins flux ϕ is given by $\phi = \pi r^2 B$

Now induced emf e is given by

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\pi r^2 B\right) = -\pi B\left(2r\frac{dr}{dt}\right)$$

$$= -2\pi Br\frac{dr}{dt}$$

Induced emf = $2\pi Br\frac{dr}{dt}$ (Numerically)

INTEGERS TYPE QUESTION

26. (4 times)

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

$\therefore$ M becomes 4 times.

27. (5625 W)

Compare V = $150\sin(150t)$ with $V = V_0 \sin \omega t$, we get V_0 = 150 V

Compare I $= 150\sin\left(150t + \frac{\pi}{3}\right)$ with $I = I_0 \sin(\omega t + \phi)$, we get $I = 150\,A,\ \phi = \frac{\pi}{3} = 60°$

The power dissipated in ac circuit is

$$P = \frac{1}{2}V_0 I_0 \cos\phi = \frac{1}{2}\times 150\times 150\times \cos 60° = \frac{1}{2}\times 150\frac{1}{2}\times 150\times\frac{1}{2} = 5625\ W$$

28. (2.8 A)

Here, R = 100 Ω, V_{rms} = 220 V, υ = 50 Hz

$$\therefore \quad I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2\,A$$

29. (318 Ω)

30. (2I)

When the velocity of conductor becomes double, area intercepted becomes twice. Therefore induced current becomes twice.