

Show that the normals at the points (4a, 4a) & at the upper end of the latus ractum of the parabola $y^2 = 4ax$ intersect on the same parabola.



Prove that the locus of the middle point of portion of a normal to y2 = 4ax intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.



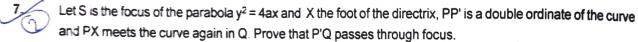
Find the equations of the tangents to the parabola $y^2 = 16x$, which are parallel & perpendicular respectively to the line 2x - y + 5 = 0. Find also the coordinates of their points of contact.

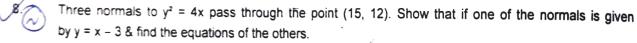
A circle is described whose centre is the vertex and whose diameter is three-quarters of the latus rectum of a parabola $y^2 = 4ax$. Prove that the common chord of the circle and parabola bisects the distance between the vertex and the focus.



Find the equations of the tangents of the parabola $y^2 = 12x$, which passes through the point (2,5).

Through the vertex O of a parabola y2 = 4x, chords OP & OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.





- Find the equations of the chords of the parabola $y^2 = 4ax$ which pass through the point (-6a, 0) and which 9. subtends an angle of 45° at the vertex.
- $^{\circ}$ O' is the vertex of the parabola $y^2 = 4ax \& L$ is the upper end of the latus rectum. If LH is drawn perpendicular to OL meeting OX in H prove that the length of the double ordinate through H is $4a\sqrt{5}$.
- The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2 = constant$.

If the normal at P(18, 12) to the parabola y^2 = 8x cuts it again at Q, show that 9PQ = $80\sqrt{10}$

Prove that, the normal to $y^2 = 12x$ at (3-6) meets the parabola again in (27, -18) & circle on this normal chord as diameter is $x^2 + y^2 - 30x + 12y - 27 = 0$.

- Find the equation of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the 14. point (6, 9).
- P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P prove that TP is bisected by the directrix.
- From the point (-1, 2) tangent lines are drawn to the parabola $y^2 = 4x$. Find the equation of the chord of contact. Also find the area of the triangle formed by the chord of contact & the tangents.
- From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ & parabola $y^2 = 4ax$. Find the area 17. of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola.

Show that the locus of a point, such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are perpendicular is $y^2 = a(x - 3a)$.

- 19. Prove that the two parabolas $y^2 = 4ax \& y^2 = 4c (x b)$ cannot have a common normal, other than the axis, unless $\frac{b}{(a-c)} > 2$.
- Find the condition on 'a' & 'b' so that the two tangents drawn to the parabola $y^2 = 4ax$ from a point are normals to the parabola $x^2 = 4by$.

EXERCISE-II



In the parabola $y^2 = 4ax$, the tangent at the point P, whose abscissa is equal to the latus ractum meets the axis in T & the normal at P cuts the parabola again in Q. Prove that PT: PQ = 4:5.



Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P & Q. If PQ = 4 units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8 (x + 2)$.

- A variable chord $t_1 t_2$ of the parabola $y^2 = 4ax$ subtends a right angle at a fixed point t_0 of the curve. Show that it passes through a fixed point. Also find the co-ordinates of the fixed point.
- Two perpendicular straight lines through the focus of the parabola y² = 4ax meet its directrix in T & T' respectively. Show that the tangents to the parabola parallel to the perpendicular lines intersect in the mid point of T T'.
- Two straight lines one being a tangent to $y^2 = 4ax$ and the other to $x^2 = 4by$ are right angles. Find the locus of their point of intersection.
- A variable chord PQ of the parabola $y^2 = 4x$ is drawn parallel to the line y = x. If the parameters of the points P & Q on the parabola are p & q respectively, show that p + q = 2. Also show that the locus of the point of intersection of the normals at P & Q is 2x y = 12.
- 7. Show that an infinite number of triangles can be inscribed in either of the parabolas $y^2 = 4ax & x^2 = 4by$ whose sides touch the other.
- 8. If (x_1, y_1) , (x_2, y_2) and (x_3, y_3) be three points on the parabola $y^2 = 4ax$ and the normals at these points meet in a point then prove that $\frac{x_1 x_2}{y_3} + \frac{x_2 x_3}{y_1} + \frac{x_3 x_1}{y_2} = 0$.
- 9. Show that the normals at two suitable distinct real points on the parabola $y^2 = 4ax$ (a > 0) intersect at a point on the parabola whose abscissa > 8a.
- 10. If $Q(x_1, y_1)$ is an arbitrary point in the plane of a parabola $y^2 = 4ax$, show that there are three points on the parabola at which QQ subtends a right angle, where Q is the origin. Show further that the normal at these three points are concurrent at a point Q, determine the coordinates of Q in terms of those of Q.
- 11. A quadrilateral is inscribed in a parabola $y^2 = 4ax$ and three of its sides pass through fixed points on the axis. Show that the fourth side also passes through fixed point on the axis of the parabola.
- 12. Prove that the parabola $y^2 = 16x$ & the circle $x^2 + y^2 40x 16y 48 = 0$ meet at the point P(36, 24) & one other point Q. Prove that PQ is a diameter of the circle. Find Q.
- 13. A variable tangent to the parabola $y^2 = 4ax$ meets the circle $x^2 + y^2 = r^2$ at P & Q. Prove that the locus of the mid point of PQ is $x(x^2 + y^2) + ay^2 = 0$.
- 14. Find the locus of the foot of the perpendicular from the origin to chord of the parabola $y^2 = 4ax$ subtending an angle of 45^0 at the vertex.



- 16. A fixed parabola y² = 4 ax touches a variable parabola. Find the equation to the locus of the vertex of the variable parabola. Assume that the two parabolas are equal and the axis of the variable parabola remains parallel to the x-axis.
- Show that the circle through three points the normals at which to the parabola $y^2 = 4ax$ are concurrent at the point (h, k) is $2(x^2 + y^2) 2(h + 2a) x ky = 0$.
- 18. Prove that the locus of the centre of the circle, which passes through the vertex of the parabola $y^2 = 4ax & through its intersection with a normal chord is <math>2y^2 = ax a^2$.

EXERCISE-III

1.	Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabolic field $x^2 + y^2 - 6y + 4 = 0$	ola $y^2 = x$.
		[REE '99, 6]

2/	(a) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of 'k' is				
	(A) 1/8	(B) 8	(C)4	(D) 1/4	
	(b) If x + y	= k is normal to y ² = 12	x, then 'k' is:	[JEE'2000 (Scr), 1+	1]
	(A) 3	(B) 9	(C) - 9	(D) – 3	

- Find the locus of the points of intersection of tangents drawn at the ends of all normal chords of the parabola $y^2 = 8(x 1)$. [REE '2001, 3]
- (a) The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x axis is

 (A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x + 3)$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x + 1)$
 - (b) The equation of the directrix of the parabola, $y^2 + 4y + 4x + 2 = 0$ is

 (A) x = -1(B) x = 1(C) x = -3/2(D) x = 3/2[JEE'2001(Scr), 1+1]
 - The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix / [JEE'2002 (Scr.), 3]

(A)
$$x = -a$$
 (B) $x = -a/2$ (C) $x = 0$ (D) $x = a/2$

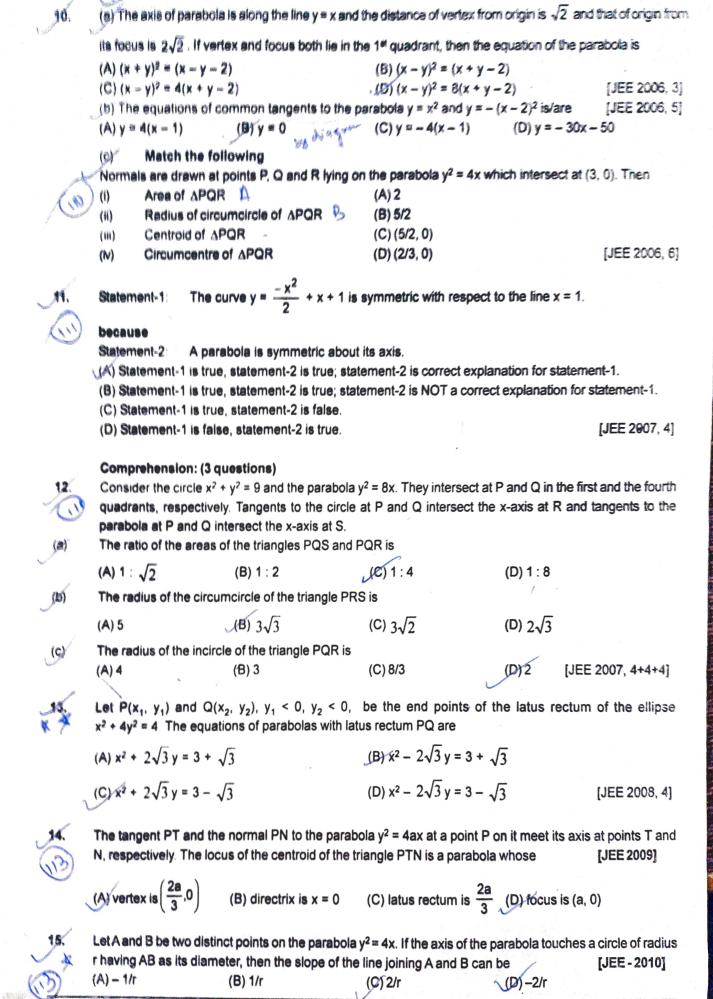
The equation of the common tangent to the curves $y^2 = 8x$ and xy = -1 is [JEE'2002 (Scr), 3]

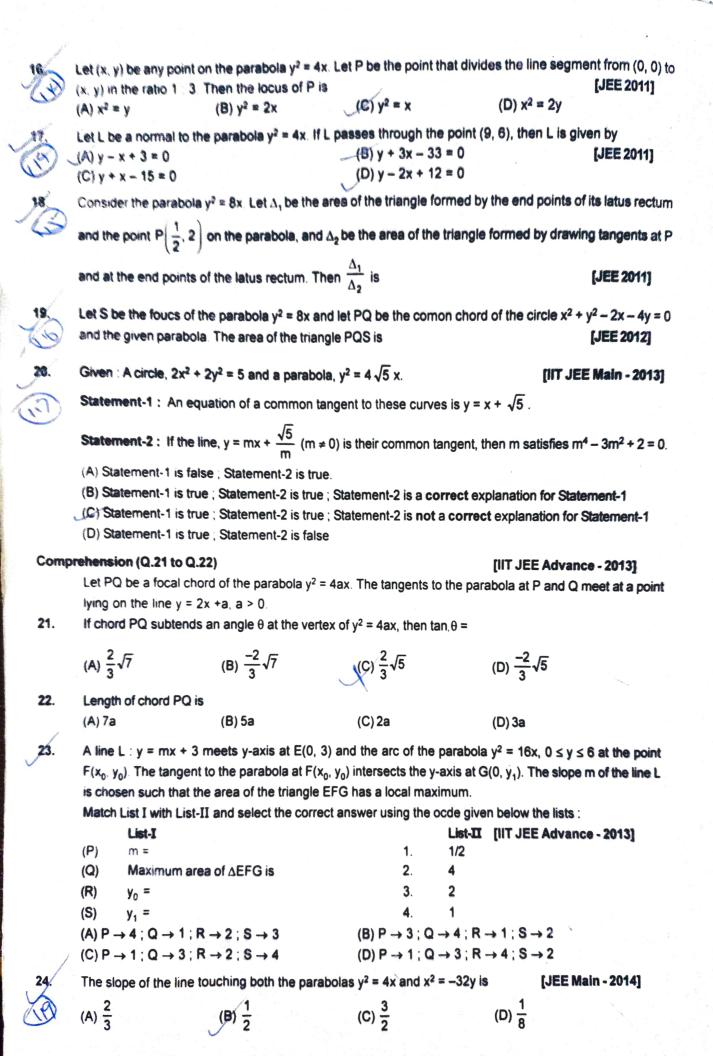
- (A) 3y = 9x + 2 (B) y = 2x + 1 (C) 2y = x + 8 (D) y = x + 2 (a) The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle
- (a) The slope of the local chords of the parabola $y^2 = 10x$ which are tangents to the chords $(x-6)^2 + y^2 = 2$ are [JEE'2003, (Scr.)] (A) ± 2 (B) -1/2, 2 (C) ± 1 (D) -2, 1/2
 - (b) Normals are drawn from the point 'P' with slopes m_1 , m_2 , m_3 to the parabola $y^2 = 4x$. If locus of P with m_1 $m_2 = \alpha$ is a part of the parabola itself then find α . [JEE 2003, 4 out of 60]
- The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

 (A) $\pi/2$ (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$ [JEE 2004, (Scr.)]
- Let P be a point on the parabola $y^2 2y 4x + 5 = 0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2}$:1.

 Find the locus of R.

 [JEE 2004, 4 out of 60]





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The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P_i Q_i and the parabola at the points R, S. Then the area of the quadrileteral PORS is

[JIT JEE Advance - 2014]

(A) 3

(8)6

(C) 9

(5) 15

Comprehension (Q.26 to Q.27)

Let a, r, s, t be nonzero real numbers. Let P(at2, 2at), Q, R(ar2, 2ar) and S(as2, 2as) be distinct points on the parabola y2 = 4ax. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point [JIT JEE Advance - 2014]



The value of r is :



(B)
$$\frac{t^2+1}{t}$$

 $(C)^{\frac{1}{4}}$

$$(D)^{\frac{3^2-1}{1}}$$

if st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A)
$$\frac{(t^2+1)^2}{2t^3}$$

(B)
$$\frac{a(t^2+1)^2}{2t^3}$$

(C)
$$\frac{a(t^2+1)^2}{t^3}$$

(B)
$$\frac{a(t^2+1)^2}{2t^3}$$
 (C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

28.

Let O be the vertex and Q be any point on the parabola, x².≈ 8y. If the point P divides the line segment OQ [JEE Main - 2015] internally in the ratio 1 : 3, then the locus of P is:

(B)
$$x^2 = y$$

(C)
$$y^2 = x$$

$$(D) y^2 = 2x$$

Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle $\triangle OPQ$ is $3\sqrt{2}$, then [IIT JEE Advance - 2015] which of the following is(are) the coordinates of P?

$$(C)\left(\frac{1}{4},\frac{1}{\sqrt{2}}\right) \qquad (D)\left(1,\sqrt{2}\right)$$

Let the curve C be the mirror image of the parabola $y^2 = 4x$ with rescpect to the line x + y + 4 = 0. If A and B are the points of intersection of C with the line y = -5, then the distance between A and B is

[IIT JEE Advance - 2015]

 ω if the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle [IIT JEE Advance - 2015] $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is:

Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(A)
$$x^2 + y^2 - 4x + 8y + 12 = 0$$

(B)
$$x^2 + y^2 - x + 4y - 12 = 0$$
 [JEE Main - 2016]

(C)
$$x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$$

(D)
$$x^2 + y^2 - 4x + 9y + 18 = 0$$

33. The circle $C_1: x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C₁ at P touches other two circles C₂ and C₃ at R₂ and R₃, respectively. Suppose

 C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3 , respectively. If Q_2 and Q_3 lie on the y-axis, then

(A)
$$Q_2Q_3 = 12$$

(B)
$$R_2 R_3 = 4\sqrt{6}$$

(C) area of the triangle
$$OR_2R_3$$
 is $6\sqrt{2}$

(D) area of the triangle
$$PQ_2Q_3$$
 is $4\sqrt{2}$

Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

(A) SP =
$$2\sqrt{5}$$

[IIT JEE Advance - 2016]

(B) SQ: QP = $(\sqrt{5} + 1)$: 2

(C) The x-intercept of the normal to the parabola at P is 6

(D) The slope of the tangent to the circle at Q is 1/2

If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation 2x + y = p, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k?

[JEE Advanced - 2017]

(A) p = -2, h = 2, k = -4(B) p = 5, h = 4, k = -3(C) p = -1, h = 1, k = -3(D) p = 2, h = 3, k = -4

36. Tangent and normal are drawn at P(16, 16) on the parabola y² = 16x, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and ∠CPB = θ, then a value of tanθ is:

A)
$$\frac{4}{3}$$
 (B) $\frac{1}{2}$ (C) 2 (D) 3