High Performance Computing : Home Work 2

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Question 2
Optimizing matrix-matrix multiplication

(1) MACHINE SPECS:

First, let us look at the processor/machine details on which **all the** codes were run on. A brief summary of the processor information would look something like the following:

1. **CPU** :
   (a) Machine: Lenovo ThinkStation P330 (2nd Gen)
   (b) Processor Company: Genuine Intel
   (c) CPU Family: 6
   (d) Model Name: Intel(R) Core(TM) i9-9900 CPU @ 3.10GHz
   (e) Stepping: 1
   (f) Siblings / CPU cores : 8 / 16 processors
   (g) Number of Threads: 16
   (h) CPU MHz = 1600.006 MHz
   (i) CPU Max Frequency: 3.1 GHz
   (j) Max Turbo Speed : 5 GHz
   (k) Max FLOPS : $\approx$ 300 GFlops/s
2. **MEMORY** :
   (a) Mem Total (Slow Access) : 1 TB
   (b) RAM: 32 GB
   (c) L1 Cache : 512 KB
   (d) Cache Size: 16384 KB
   (e) Rated Memory Speed : 2.66 MHz
   (f) g++/gcc compiler version : gcc version 7.5.0 (Ubuntu 7.5.0-3ubuntu1 18.04)

(2) LOOP ORDER TIMINGS:

The loop order matters very crucially and can vary the timings and performance. All the loop orders were tried (the first 3 outer I,J,P loops and the inner three i,j,p loops of the blocked version of matrix multiplication). Now for every three i,j,p loops of a matix multiplication it was found that each one has a different performance per iteration:

1. pij and ipj : 0.5 cache misses (2 Load + 1 Store)
2. ijp and jip : 1.25 cache misses (2 Load + 0 Store)
3. jpi and pji : 2.0 cache misses (2 Load + 1 Store)

Conclusion: So, it is found that the ipj/IPJ version is better.

Inner 3 Loops: The timings are as follows -

Dimension	Time	Gflop/s	GB/s	Error
12	0.230762	8.666955	11.555939	0.000000e+00
60	0.233796	8.555136	6.844109	0.000000e+00
108	0.231822	8.629137	6.391954	0.000000e+00
156	0.229603	8.730308	6.267913	0.000000e+00
204	0.232540	8.615967	6.081859	0.000000e+00
252	0.239964	8.402852	5.868659	0.000000e+00
300	0.241534	8.495705	5.890355	0.000000e+00
348	0.238941	8.466211	5.838766	0.000000e+00
396	0.251214	8.404686	5.772916	0.000000e+00
444	0.253169	8.297536	5.681196	0.000000e+00
492	0.254138	8.435245	5.760655	0.000000e+00
540	0.260334	8.467954	5.770754	0.000000e+00
588	0.247000	8.230674	5.599098	0.000000e+00
636	0.253072	8.132366	5.523871	0.000000e+00
684	0.319580	8.010860	5.434268	0.000000e+00
732	0.299873	7.847797	5.317633	0.000000e+00
780	0.358760	7.936543	5.372429	0.000000e+00
828	0.300582	7.554187	5.109112	0.000000e+00
876	0.351031	7.659973	5.176603	0.000000e+00
924	0.440575	7.162364	4.836921	0.000000e+00
972	0.527920	6.958097	4.696000	0.000000e+00
1020	0.299613	7.083851	4.778127	0.000000e+00
1068	0.357787	6.809562	4.590716	0.000000e+00

Figure 1: **i,p,j order**

Dimension	Time	Gflop/s	GB/s	Error
12	0.350059	5.713329	7.617772	0.000000e+00
60	0.356442	5.611464	4.489171	0.000000e+00
108	0.356222	5.615663	4.159750	0.000000e+00
156	0.367037	5.461319	3.920947	0.000000e+00
204	0.369952	5.415727	3.822866	0.000000e+00
252	0.370045	5.449007	3.805656	0.000000e+00
300	0.382322	5.367203	3.721260	0.000000e+00
348	0.382743	5.285328	3.645054	0.000000e+00
396	0.405213	5.210519	3.578943	0.000000e+00
444	0.445986	4.710192	3.224996	0.000000e+00
492	0.438026	4.894044	3.342274	0.000000e+00
540	0.469167	4.698749	3.202111	0.000000e+00
588	0.403482	5.038570	3.427599	0.000000e+00
636	0.411701	4.998961	3.395521	0.000000e+00
684	0.513047	4.990002	3.385031	0.000000e+00
732	0.470067	5.006388	3.392306	0.000000e+00
780	0.601026	4.737415	3.206866	0.000000e+00
828	0.455918	4.980403	3.368389	0.000000e+00
876	0.540367	4.976039	3.362803	0.000000e+00
924	0.643071	4.907012	3.313826	0.000000e+00
972	0.756914	4.853020	3.275289	0.000000e+00
1020	0.444535	4.774468	3.220426	0.000000e+00
1068	0.501754	4.855715	3.273516	0.000000e+00

Figure 2: **i,j,p order**

Dimension	Time	Gflop/s	GB/s	Error
12	0.409244	4.887059	6.516079	0.000000e+00
60	0.415375	4.815310	3.852248	0.000000e+00
108	0.415254	4.817352	3.568409	0.000000e+00
156	0.416953	4.807511	3.451547	0.000000e+00
204	0.449226	4.460032	3.148258	0.000000e+00
252	0.426461	4.728167	3.302212	0.000000e+00
300	0.440680	4.656444	3.228468	0.000000e+00
348	0.440067	4.596852	3.170243	0.000000e+00
396	0.481078	4.388829	3.014549	0.000000e+00
444	0.486628	4.316809	2.955653	0.000000e+00
492	0.487916	4.393625	3.000524	0.000000e+00
540	0.499224	4.415847	3.009318	0.000000e+00
588	0.456377	4.454594	3.030336	0.000000e+00
636	0.460353	4.470642	3.036662	0.000000e+00
684	0.571283	4.481331	3.039967	0.000000e+00
732	0.558213	4.215841	2.856636	0.000000e+00
780	0.677417	4.203190	2.845237	0.000000e+00
828	0.518909	4.375827	2.959497	0.000000e+00
876	0.614173	4.378057	2.958687	0.000000e+00
924	0.731156	4.315847	2.914598	0.000000e+00
972	0.850002	4.321543	2.916597	0.000000e+00
1020	0.500558	4.240098	2.859988	0.000000e+00
1068	0.564836	4.313415	2.907920	0.000000e+00

Figure 3: **j,p,i order**

Outer 3 Loops: The timings are as follows -

Dimension	Time	Gflop/s	GB/s	Error
12	0.242509	8.247126	10.996168	0.000000e+00
60	0.226832	8.817808	7.054246	0.000000e+00
108	0.228756	8.744804	6.477632	0.000000e+00
156	0.228040	8.790152	6.310878	0.000000e+00
204	0.230186	8.704105	6.144074	0.000000e+00
252	0.233277	8.643728	6.036890	0.000000e+00
300	0.236208	8.687255	6.023163	0.000000e+00
348	0.238342	8.487488	5.853440	0.000000e+00
396	0.250544	8.427152	5.788347	0.000000e+00
444	0.246580	8.519276	5.833018	0.000000e+00
492	0.247653	8.656133	5.911505	0.000000e+00
540	0.252557	8.728710	5.948454	0.000000e+00
588	0.240446	8.455029	5.751720	0.000000e+00
636	0.241353	8.527242	5.792089	0.000000e+00
684	0.296206	8.643012	5.863096	0.000000e+00
732	0.292672	8.040872	5.448460	0.000000e+00
780	0.345110	8.250454	5.584923	0.000000e+00
828	0.273536	8.301117	5.614282	0.000000e+00
876	0.341273	7.878987	5.324612	0.000000e+00
924	0.383683	8.224387	5.554131	0.000000e+00
972	0.434442	8.455251	5.706424	0.000000e+00
1020	0.285900	7.423643	5.007320	0.000000e+00
1068	0.293051	8.313824	5.604825	0.000000e+00

Figure 4: **I,P,J order**

Dimension	Time	Gflop/s	GB/s	Error
12	0.226530	8.828874	11.771832	0.000000e+00
60	0.227163	8.804954	7.043963	0.000000e+00
108	0.232784	8.593484	6.365544	0.000000e+00
156	0.239737	8.361286	6.002975	0.000000e+00
204	0.234257	8.552849	6.037305	0.000000e+00
252	0.234828	8.586621	5.997005	0.000000e+00
300	0.241306	8.503737	5.895925	0.000000e+00
348	0.235776	8.579827	5.917122	0.000000e+00
396	0.251890	8.382099	5.757402	0.000000e+00
444	0.252474	8.320372	5.696831	0.000000e+00
492	0.264953	8.090931	5.525514	0.000000e+00
540	0.264184	8.344549	5.686656	0.000000e+00
588	0.246010	8.263782	5.621620	0.000000e+00
636	0.251254	8.191220	5.563847	0.000000e+00
684	0.314402	8.142790	5.523764	0.000000e+00
732	0.294641	7.987136	5.412048	0.000000e+00
780	0.357518	7.964113	5.391092	0.000000e+00
828	0.298623	7.603744	5.142629	0.000000e+00
876	0.348736	7.710379	5.210667	0.000000e+00
924	0.428067	7.371642	4.978252	0.000000e+00
972	0.493774	7.439273	5.020744	0.000000e+00
1020	0.294993	7.194790	4.852956	0.000000e+00
1068	0.344888	7.064252	4.762417	0.000000e+00

Figure 5: **I,J,P order**

Dimension	Time	Gflop/s	GB/s	Error
12	0.230487	8.677298	11.569731	0.000000e+00
60	0.229214	8.726164	6.980931	0.000000e+00
108	0.236031	8.475259	6.277969	0.000000e+00
156	0.245029	8.180694	5.873319	0.000000e+00
204	0.252125	7.946709	5.609442	0.000000e+00
252	0.257353	7.835071	5.472113	0.000000e+00
300	0.259508	7.907258	5.482365	0.000000e+00
348	0.264530	7.647236	5.273956	0.000000e+00
396	0.290228	7.274861	4.996874	0.000000e+00
444	0.294421	7.134967	4.885202	0.000000e+00
492	0.302169	7.094427	4.844975	0.000000e+00
540	0.313506	7.031762	4.792015	0.000000e+00
588	0.305749	6.649154	4.523234	0.000000e+00
636	0.281416	7.313287	4.967515	0.000000e+00
684	0.349622	7.322501	4.967310	0.000000e+00
732	0.327205	7.192245	4.873434	0.000000e+00
780	0.396609	7.179147	4.859730	0.000000e+00
828	0.326402	6.956612	4.704955	0.000000e+00
876	0.388426	6.922518	4.678231	0.000000e+00
924	0.464702	6.790494	4.585788	0.000000e+00
972	0.540560	6.795399	4.586195	0.000000e+00
1020	0.317221	6.690647	4.512907	0.000000e+00
1068	0.367211	6.634806	4.472903	0.000000e+00

Figure 6: **J,P,I order**

(3) BLOCK SIZE VARIATION TIMINGS:

After implementing the Blocked version of matrix multiplication, the block sizes were varied to see how the performance change. The performance is shown as follows:

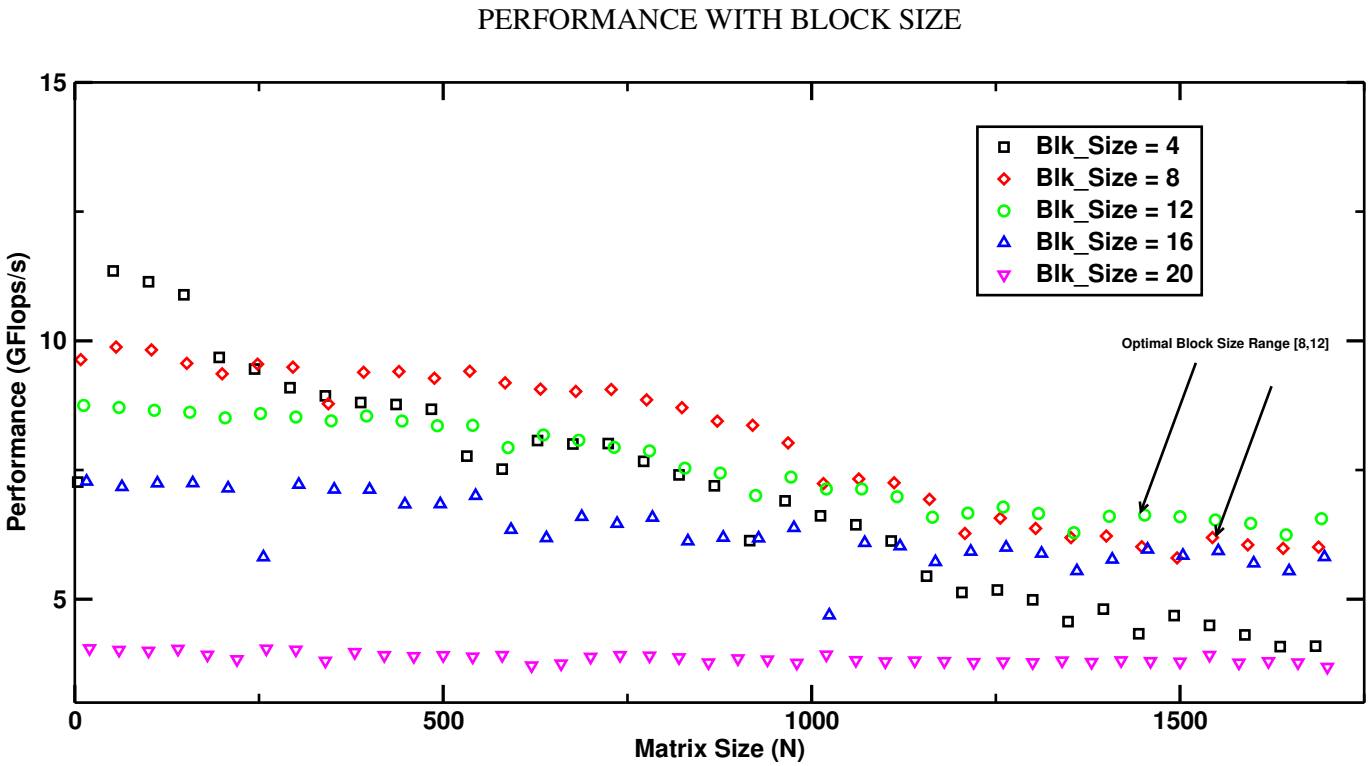


Figure 7: **Performance with different block sizes with varying matrix dimensions**

It is seen that as we vary block sizes the optimal block size lies somewhere between 8 and 12. This also seems correct since as deduced in - Irony/Tiskin/Toeldo (2004) + James Demmel + HBL(Hölder-Brascamp-Lieb) Bound - we find that the {optimal block memory size} $\approx$ (L1 cache)$^{0.5}$. Now for the machine being used, L1 cache is 512 KB. Which means the block should be $\approx 10 \times 10$ matrix of elements with a size of **double**. This fits very well with our observation in Figure 7 that the optimal size peaks performance somewhere between 8 and 12!!

(4) TIMINGS FOR BLOCKED AND BLOCKED + OpenMP VERSIONS:
Here the timings for both Bocked only and Blocked + OpenMP versions are shown for both compilation flags -O2 and -O3. In the OpenMP version both static scheduling and collapsed version were tried, but only one of them is presented here.
BEST PERFORMANCE $\approx$ 13.33% of PEAK THEORETICAL LIMIT (40 GFPs/300 GFPs)

Dimension	Time	Gflop/s	GB/s	Error
8	0.198287	10.086378	20.172756	0.000000e+00
56	0.193540	10.335164	11.811615	0.000000e+00
104	0.195125	10.249870	11.038322	0.000000e+00
152	0.201693	9.924636	10.446986	0.000000e+00
200	0.204403	9.862891	10.257407	0.000000e+00
248	0.201938	9.970344	10.291968	0.000000e+00
296	0.204020	9.915107	10.183083	0.000000e+00
344	0.205733	9.893295	10.123371	0.000000e+00
392	0.210164	9.744930	9.943807	0.000000e+00
440	0.208871	9.787953	9.965915	0.000000e+00
488	0.216098	9.680142	9.838833	0.000000e+00
536	0.218637	9.860480	10.007651	0.000000e+00
584	0.244251	9.785525	9.919574	0.000000e+00
632	0.209011	9.662121	9.784426	0.000000e+00
680	0.260531	9.655118	9.768708	0.000000e+00
728	0.244502	9.468103	9.572148	0.000000e+00
776	0.292857	9.573712	9.672410	0.000000e+00
824	0.236215	9.474031	9.566012	0.000000e+00
872	0.281160	9.433131	9.519674	0.000000e+00
920	0.337304	9.234259	9.314557	0.000000e+00
968	0.397051	9.137749	9.213268	0.000000e+00
1016	0.253318	8.280273	8.345472	0.000000e+00
1064	0.272767	8.832088	8.898495	0.000000e+00
1112	0.315086	8.728015	8.790806	0.000000e+00
1160	0.387726	8.051547	8.107075	0.000000e+00
1208	0.450199	7.831157	7.883019	0.000000e+00
1256	0.513507	7.717079	7.766233	0.000000e+00
1304	0.605334	7.326018	7.370963	0.000000e+00
1352	0.725391	6.813780	6.854098	0.000000e+00
1400	0.786077	6.981503	7.021398	0.000000e+00
1448	0.876354	6.928771	6.967052	0.000000e+00
1496	0.978041	6.846484	6.883096	0.000000e+00
1544	1.080335	6.814180	6.849486	0.000000e+00

Figure 8: **Block Size 8 ; Only Blocked; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
8	3.419173	0.584937	1.169874	0.000000e+00
56	0.618221	3.235518	3.697735	0.000000e+00
104	0.377051	5.304339	5.712365	0.000000e+00
152	0.276544	7.238370	7.619337	0.000000e+00
200	0.215723	9.345298	9.719110	0.000000e+00
248	0.171704	11.725949	12.104205	0.000000e+00
296	0.146991	13.761924	14.133868	0.000000e+00
344	0.128736	15.810526	16.178213	0.000000e+00
392	0.118915	17.222662	17.574144	0.000000e+00
440	0.104810	19.505985	19.860640	0.000000e+00
488	0.105427	19.841825	20.167101	0.000000e+00
536	0.092449	23.319518	23.667571	0.000000e+00
584	0.092650	25.797251	26.150638	0.000000e+00
632	0.083779	24.105039	24.410166	0.000000e+00
680	0.085229	29.514246	29.861472	0.000000e+00
728	0.074001	31.283100	31.626870	0.000000e+00
776	0.094651	29.621710	29.927088	0.000000e+00
824	0.064302	34.802820	35.140711	0.000000e+00
872	0.072280	36.693542	37.030180	0.000000e+00
920	0.082458	37.773814	38.102282	0.000000e+00
968	0.089561	40.510491	40.845288	0.000000e+00
1016	0.049707	42.198038	42.530306	0.000000e+00
1064	0.075729	31.812078	32.051266	0.000000e+00
1112	0.093082	29.544508	29.757058	0.000000e+00
1160	0.108583	28.750291	28.948569	0.000000e+00
1208	0.147430	23.913635	24.072003	0.000000e+00
1256	0.124232	31.898207	32.101380	0.000000e+00
1304	0.149107	29.741587	29.924051	0.000000e+00
1352	0.169268	29.200104	29.372886	0.000000e+00
1400	0.180008	30.487527	30.661742	0.000000e+00
1448	0.188219	32.260504	32.438739	0.000000e+00
1496	0.220114	30.421230	30.583911	0.000000e+00
1544	0.240214	30.646017	30.804804	0.000000e+00

Figure 9: **Block Size 8 ; Blocked+ OMP; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
12	0.220743	9.060301	12.080401	0.000000e+00
60	0.217705	9.187481	7.349985	0.000000e+00
108	0.220083	9.089389	6.732881	0.000000e+00
156	0.221703	9.041415	6.491272	0.000000e+00
204	0.223191	8.976886	6.336625	0.000000e+00
252	0.225377	8.946686	6.248479	0.000000e+00
300	0.228759	8.970140	6.219297	0.000000e+00
348	0.227315	8.899188	6.137371	0.000000e+00
396	0.238389	8.856818	6.083471	0.000000e+00
444	0.235767	8.909974	6.100523	0.000000e+00
492	0.242041	8.856838	6.048572	0.000000e+00
540	0.246996	8.925229	6.082378	0.000000e+00
588	0.228924	8.880550	6.041191	0.000000e+00
636	0.232707	8.844057	6.007284	0.000000e+00
684	0.299350	8.552234	5.801515	0.000000e+00
732	0.271125	8.679906	5.881467	0.000000e+00
780	0.329821	8.632906	5.843814	0.000000e+00
828	0.269116	8.437459	5.706494	0.000000e+00
876	0.315570	8.520719	5.758294	0.000000e+00
924	0.379110	8.323600	5.621133	0.000000e+00
972	0.438817	8.370952	5.649531	0.000000e+00
1020	0.266226	7.972247	5.377359	0.000000e+00
1068	0.301725	8.074804	5.443688	0.000000e+00
1116	0.367590	7.562395	5.095807	0.000000e+00
1164	0.425247	7.417335	4.995868	0.000000e+00
1212	0.483324	7.367152	4.960063	0.000000e+00
1260	0.528100	7.575747	5.098598	0.000000e+00
1308	0.610413	7.332120	4.932925	0.000000e+00
1356	0.706445	7.058798	4.747510	0.000000e+00
1404	0.772588	7.164460	4.817130	0.000000e+00
1452	0.860024	7.119007	4.785228	0.000000e+00
1500	0.952277	7.088277	4.763322	0.000000e+00

Figure 10: **Block Size 12 ; Only Blocked; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
12	1.500857	1.332573	1.776763	0.000000e+00
60	0.708336	2.823744	2.258995	0.000000e+00
108	0.691396	2.893309	2.143192	0.000000e+00
156	0.408254	4.909948	3.525091	0.000000e+00
204	0.319095	6.278886	4.432155	0.000000e+00
252	0.309096	6.523479	4.556080	0.000000e+00
300	0.327950	6.257052	4.338223	0.000000e+00
348	0.181791	11.127735	7.674300	0.000000e+00
396	0.223584	9.443280	6.486293	0.000000e+00
444	0.160203	13.112621	8.978011	0.000000e+00
492	0.134790	15.904124	10.861353	0.000000e+00
540	0.110389	19.970214	13.609331	0.000000e+00
588	0.097572	20.835693	14.173941	0.000000e+00
636	0.084034	24.490883	16.635317	0.000000e+00
684	0.102136	25.065711	17.003640	0.000000e+00
732	0.119946	19.619991	13.294420	0.000000e+00
780	0.143643	19.822081	13.418024	0.000000e+00
828	0.106687	21.283399	14.394569	0.000000e+00
876	0.122645	21.924072	14.816268	0.000000e+00
924	0.136823	23.063093	15.575076	0.000000e+00
972	0.153353	23.953416	16.166091	0.000000e+00
1020	0.086395	24.566385	16.570268	0.000000e+00
1068	0.091653	26.582523	17.920802	0.000000e+00
1116	0.100394	27.689477	18.658142	0.000000e+00
1164	0.111431	28.306229	19.065364	0.000000e+00
1212	0.118356	30.084764	20.255088	0.000000e+00
1260	0.128853	31.048904	20.896405	0.000000e+00
1308	0.138878	32.226986	21.681764	0.000000e+00
1356	0.151365	32.944659	22.157470	0.000000e+00
1404	0.158935	34.826749	23.416276	0.000000e+00
1452	0.181832	33.671319	22.633063	0.000000e+00
1500	0.189450	35.629503	23.943026	0.000000e+00

Figure 11: **Block Size 12 ; Blocked+ OMP; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
16	0.265345	7.537368	7.537368	0.000000e+00
64	0.264945	7.549345	4.718341	0.000000e+00
112	0.263618	7.589073	4.336613	0.000000e+00
160	0.270108	7.430506	4.086778	0.000000e+00
208	0.272673	7.392577	3.980619	0.000000e+00
256	0.331944	6.065074	3.222071	0.000000e+00
304	0.269072	7.517696	3.956682	0.000000e+00
352	0.272213	7.370148	3.852577	0.000000e+00
400	0.275869	7.423800	3.860376	0.000000e+00
448	0.299664	7.201298	3.729243	0.000000e+00
496	0.305654	7.186016	3.708911	0.000000e+00
544	0.309379	7.285063	3.749665	0.000000e+00
592	0.290330	7.146165	3.669652	0.000000e+00
640	0.320305	6.547369	3.355527	0.000000e+00
688	0.375295	6.941962	3.551701	0.000000e+00
736	0.341235	7.010220	3.581308	0.000000e+00
784	0.412884	7.002791	3.572853	0.000000e+00
832	0.334937	6.878079	3.505175	0.000000e+00
880	0.404739	6.734920	3.428687	0.000000e+00
928	0.472259	6.768981	3.442844	0.000000e+00
976	0.548508	6.779955	3.445551	0.000000e+00
1024	0.443649	4.840500	2.458067	0.000000e+00
1072	0.367674	6.701190	3.400604	0.000000e+00
1120	0.426631	6.586158	3.340123	0.000000e+00
1168	0.517665	6.156163	3.120247	0.000000e+00
1216	0.564988	6.364895	3.224322	0.000000e+00
1264	0.637151	6.339116	3.209679	0.000000e+00
1312	0.721732	6.258288	3.167304	0.000000e+00
1360	0.844514	5.957169	3.013627	0.000000e+00
1408	0.913661	6.110165	3.089799	0.000000e+00
1456	0.970399	6.361563	3.215735	0.000000e+00
1504	1.094751	6.215241	3.140680	0.000000e+00

Figure 12: **Block Size 16 ; Only Blocked; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
16	1.065947	1.876269	1.876269	0.000000e+00
64	0.858279	2.330428	1.456518	0.000000e+00
112	0.861023	2.323536	1.327735	0.000000e+00
160	0.776119	2.585994	1.422297	0.000000e+00
208	0.543795	3.706835	1.995988	0.000000e+00
256	0.595023	3.383507	1.797488	0.000000e+00
304	0.323650	6.249958	3.289452	0.000000e+00
352	0.294896	6.803247	3.556243	0.000000e+00
400	0.244607	8.372611	4.353758	0.000000e+00
448	0.251192	8.590914	4.448866	0.000000e+00
496	0.189526	11.589072	5.981456	0.000000e+00
544	0.192968	11.679892	6.011709	0.000000e+00
592	0.146228	14.188390	7.285930	0.000000e+00
640	0.191361	10.959165	5.616572	0.000000e+00
688	0.154929	16.815988	8.603529	0.000000e+00
736	0.166407	14.375171	7.343837	0.000000e+00
784	0.188894	15.306654	7.809517	0.000000e+00
832	0.163563	14.084595	7.177726	0.000000e+00
880	0.162512	16.773460	8.539216	0.000000e+00
928	0.222268	14.382235	7.315102	0.000000e+00
976	0.219240	16.962499	8.620286	0.000000e+00
1024	0.266424	8.060394	4.093169	0.000000e+00
1072	0.113462	21.715195	11.019651	0.000000e+00
1120	0.155808	18.034057	9.145843	0.000000e+00
1168	0.161739	19.703489	9.986700	0.000000e+00
1216	0.195333	18.410075	9.326156	0.000000e+00
1264	0.191584	21.082044	10.674453	0.000000e+00
1312	0.214772	21.030698	10.643585	0.000000e+00
1360	0.220998	22.764554	11.516186	0.000000e+00
1408	0.297615	18.757839	9.485498	0.000000e+00
1456	0.243235	25.379812	12.829356	0.000000e+00
1504	0.274701	24.769282	12.516393	0.000000e+00

Figure 13: **Block Size 16 ; Blocked+ OMP; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
20	0.480365	4.163533	3.330826	0.000000e+00
60	0.475555	4.205950	2.243174	0.000000e+00
100	0.475765	4.207964	2.019823	0.000000e+00
140	0.475779	4.210189	1.924658	0.000000e+00
180	0.483544	4.148966	1.843985	0.000000e+00
220	0.478503	4.183511	1.825532	0.000000e+00
260	0.476948	4.201012	1.809667	0.000000e+00
300	0.492869	4.163378	1.776375	0.000000e+00
340	0.510673	4.002188	1.695044	0.000000e+00
380	0.502121	4.152660	1.748488	0.000000e+00
420	0.510264	4.065472	1.703626	0.000000e+00
460	0.515367	4.155078	1.734293	0.000000e+00
500	0.547573	4.109043	1.709362	0.000000e+00
540	0.540632	4.077625	1.691459	0.000000e+00
580	0.571709	4.095341	1.694624	0.000000e+00
620	0.579461	4.112926	1.698240	0.000000e+00
660	0.557878	4.122711	1.699057	0.000000e+00
700	0.502376	4.096530	1.685430	0.000000e+00
740	0.592860	4.101041	1.684752	0.000000e+00
780	0.689682	4.128440	1.693719	0.000000e+00
820	0.538491	4.095652	1.678219	0.000000e+00
860	0.630606	4.034572	1.651360	0.000000e+00
900	0.719220	4.054395	1.657797	0.000000e+00
940	0.809388	4.104749	1.676834	0.000000e+00
980	0.921354	4.086126	1.667807	0.000000e+00
1020	0.523273	4.056039	1.654228	0.000000e+00
1060	0.583901	4.079512	1.662594	0.000000e+00
1100	0.654319	4.068349	1.656928	0.000000e+00

Figure 14: **Block Size 20 ; Only Blocked; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
20	1.067050	1.874341	1.499473	0.000000e+00
60	0.932330	2.145334	1.144178	0.000000e+00
100	0.983606	2.035367	0.976976	0.000000e+00
140	0.760628	2.633507	1.203889	0.000000e+00
180	0.672323	2.983994	1.326219	0.000000e+00
220	0.569816	3.513108	1.532992	0.000000e+00
260	0.480885	4.166619	1.794851	0.000000e+00
300	0.446429	4.596480	1.961165	0.000000e+00
340	0.352122	5.804260	2.458275	0.000000e+00
380	0.293761	7.098072	2.988662	0.000000e+00
420	0.274482	7.557752	3.167058	0.000000e+00
460	0.234416	9.135007	3.812872	0.000000e+00
500	0.226687	9.925577	4.129040	0.000000e+00
540	0.204321	10.789352	4.475583	0.000000e+00
580	0.205665	11.384252	4.710725	0.000000e+00
620	0.188232	12.661409	5.227937	0.000000e+00
660	0.169947	13.533440	5.577418	0.000000e+00
700	0.139993	14.700773	6.048318	0.000000e+00
740	0.201081	12.091368	4.967265	0.000000e+00
780	0.172290	16.526288	6.780016	0.000000e+00
820	0.138011	15.980361	6.548051	0.000000e+00
860	0.172461	14.752502	6.038233	0.000000e+00
900	0.190270	15.325550	6.266447	0.000000e+00
940	0.214745	15.471045	6.320087	0.000000e+00
980	0.229024	16.438309	6.709514	0.000000e+00
1020	0.122105	17.381948	7.089108	0.000000e+00
1060	0.126976	18.759764	7.645489	0.000000e+00
1100	0.140642	18.927527	7.708666	0.000000e+00

Figure 15: **Block Size 20 ; Blocked+ OMP; -O3 Flag**

Dimension	Time	Gflop/s	GB/s	Error
8	3.651720	0.547687	1.095375	0.000000e+00
56	0.863368	2.316819	2.647793	0.000000e+00
104	0.640383	3.123145	3.363387	0.000000e+00
152	0.363444	5.507674	5.797551	0.000000e+00
200	0.259674	7.763568	8.074111	0.000000e+00
248	0.257224	7.827410	8.079907	0.000000e+00
296	0.247364	8.177743	8.398763	0.000000e+00
344	0.167492	12.152092	12.434699	0.000000e+00
392	0.155873	13.139144	13.407289	0.000000e+00
440	0.150491	13.585017	13.832017	0.000000e+00
488	0.125622	16.651995	16.924978	0.000000e+00
536	0.150318	14.342085	14.556146	0.000000e+00
584	0.133308	17.929357	18.174965	0.000000e+00
632	0.103468	19.518052	19.765116	0.000000e+00
680	0.121725	20.665063	20.908181	0.000000e+00
728	0.105715	21.898229	22.138869	0.000000e+00
776	0.121507	23.074581	23.312463	0.000000e+00
824	0.091122	24.559487	24.797929	0.000000e+00
872	0.102308	25.923824	26.161657	0.000000e+00
920	0.116031	26.844230	27.077658	0.000000e+00
968	0.126661	28.644722	28.881456	0.000000e+00
1016	0.070207	29.876615	30.111863	0.000000e+00
1064	0.105469	22.841831	23.013574	0.000000e+00
1112	0.132176	20.806105	20.955789	0.000000e+00
1160	0.159297	19.597291	19.732445	0.000000e+00
1208	0.159432	22.113348	22.259794	0.000000e+00
1256	0.186447	21.254098	21.389474	0.000000e+00
1304	0.184727	24.006639	24.153919	0.000000e+00
1352	0.209616	23.579533	23.719057	0.000000e+00
1400	0.234125	23.440448	23.574394	0.000000e+00
1448	0.257093	23.618100	23.748587	0.000000e+00
1496	0.266226	25.152096	25.286599	0.000000e+00
1544	0.295523	24.910422	25.039492	0.000000e+00

Figure 16: **Block Size 8 ; Blocked+ OMP ; -O2 Flag**

Dimension	Time	Gflop/s	GB/s	Error
12	1.654250	1.209007	1.612010	0.000000e+00
60	0.989177	2.022045	1.617636	0.000000e+00
108	0.734019	2.725303	2.018743	0.000000e+00
156	0.563634	3.556400	2.553313	0.000000e+00
204	0.374752	5.346358	3.773900	0.000000e+00
252	0.302428	6.667299	4.656526	0.000000e+00
300	0.253460	8.095942	5.613187	0.000000e+00
348	0.214725	9.420996	6.497239	0.000000e+00
396	0.195428	10.803815	7.420802	0.000000e+00
444	0.164372	12.780057	8.750309	0.000000e+00
492	0.166132	12.903715	8.812293	0.000000e+00
540	0.144332	15.273822	10.408827	0.000000e+00
588	0.143091	14.207595	9.665031	0.000000e+00
636	0.097634	21.079471	14.318131	0.000000e+00
684	0.112452	22.766319	15.443819	0.000000e+00
732	0.092375	25.475845	17.262321	0.000000e+00
780	0.113695	25.043517	16.952535	0.000000e+00
828	0.111842	20.302410	13.731098	0.000000e+00
876	0.135547	19.837356	13.406067	0.000000e+00
924	0.152480	20.694853	13.975745	0.000000e+00
972	0.186373	19.709550	13.301919	0.000000e+00
1020	0.101854	20.837800	14.055300	0.000000e+00
1068	0.111006	21.948155	14.796509	0.000000e+00
1116	0.120202	23.126548	15.583481	0.000000e+00
1164	0.129251	24.403584	16.436778	0.000000e+00
1212	0.141377	25.185986	16.956902	0.000000e+00
1260	0.153564	26.052713	17.533890	0.000000e+00
1308	0.164000	27.290304	18.360449	0.000000e+00
1356	0.176475	28.256975	19.004691	0.000000e+00
1404	0.188350	29.387665	19.759228	0.000000e+00
1452	0.213874	28.626790	19.242250	0.000000e+00
1500	0.215433	31.332228	21.055257	0.000000e+00

Figure 17: **Block Size 12 ; Blocked+ OMP; -O2 Flag**

Dimension	Time	Gflop/s	GB/s	Error
16	1.145526	1.745925	1.745925	0.000000e+00
64	0.807184	2.477947	1.548717	0.000000e+00
112	0.824988	2.425027	1.385730	0.000000e+00
160	0.741213	2.707779	1.489279	0.000000e+00
208	0.504783	3.993310	2.150244	0.000000e+00
256	0.593063	3.394692	1.803430	0.000000e+00
304	0.357165	5.663488	2.980783	0.000000e+00
352	0.309356	6.485260	3.390022	0.000000e+00
400	0.250836	8.164682	4.245634	0.000000e+00
448	0.281968	7.653229	3.963279	0.000000e+00
496	0.208249	10.547138	5.443684	0.000000e+00
544	0.203074	11.098661	5.712546	0.000000e+00
592	0.153543	13.512450	6.938826	0.000000e+00
640	0.179729	11.668443	5.980077	0.000000e+00
688	0.190454	13.679360	6.998742	0.000000e+00
736	0.210704	11.353032	5.799918	0.000000e+00
784	0.175264	16.497025	8.416849	0.000000e+00
832	0.169481	13.592829	6.927115	0.000000e+00
880	0.167449	16.278934	8.287457	0.000000e+00
928	0.196279	16.286564	8.283684	0.000000e+00
976	0.215824	17.230984	8.756730	0.000000e+00
1024	0.217339	9.880812	5.017600	0.000000e+00
1072	0.110674	22.262170	11.297220	0.000000e+00
1120	0.148553	18.914822	9.592517	0.000000e+00
1168	0.165586	19.245791	9.754716	0.000000e+00
1216	0.201941	17.807634	9.020973	0.000000e+00
1264	0.195041	20.708320	10.485225	0.000000e+00
1312	0.218205	20.699809	10.476123	0.000000e+00
1360	0.226068	22.253937	11.257874	0.000000e+00
1408	0.311969	17.894759	9.049054	0.000000e+00
1456	0.271142	22.767633	11.508913	0.000000e+00
1504	0.287130	23.697080	11.974588	0.000000e+00

Figure 18: **Block Size 16 ; Blocked+ OMP; -O2 Flag**

Dimension	Time	Gflop/s	GB/s	Error
20	1.196139	1.672060	1.337648	0.000000e+00
60	0.835539	2.393857	1.276724	0.000000e+00
100	0.759241	2.636842	1.265684	0.000000e+00
140	0.794808	2.520256	1.152117	0.000000e+00
180	0.676612	2.965081	1.317814	0.000000e+00
220	0.566066	3.536379	1.543147	0.000000e+00
260	0.456443	4.389739	1.890964	0.000000e+00
300	0.456973	4.490418	1.915912	0.000000e+00
340	0.358772	5.696677	2.412710	0.000000e+00
380	0.295695	7.051647	2.969114	0.000000e+00
420	0.259901	7.981752	3.344734	0.000000e+00
460	0.235285	9.101282	3.798796	0.000000e+00
500	0.234475	9.595902	3.991895	0.000000e+00
540	0.204291	10.790962	4.476251	0.000000e+00
580	0.198497	11.795364	4.880840	0.000000e+00
620	0.189886	12.551129	5.182401	0.000000e+00
660	0.170355	13.501046	5.564068	0.000000e+00
700	0.140927	14.603256	6.008197	0.000000e+00
740	0.157274	15.459320	6.350856	0.000000e+00
780	0.173745	16.387925	6.723251	0.000000e+00
820	0.139677	15.789795	6.469965	0.000000e+00
860	0.173082	14.699553	6.016561	0.000000e+00
900	0.197106	14.794107	6.049146	0.000000e+00
940	0.210637	15.772779	6.443348	0.000000e+00
980	0.247693	15.199361	6.203821	0.000000e+00
1020	0.126883	16.727363	6.822140	0.000000e+00
1060	0.135523	17.576551	7.163274	0.000000e+00
1100	0.133270	19.974527	8.135080	0.000000e+00

Figure 19: **Block Size 20 ; Blocked+ OMP ; -O2 Flag**

Dimension	Time	Gflop/s	GB/s	Error
12	0.424348	4.713116	58.128430	0.000000e+00
60	0.559387	3.575631	43.145941	0.000000e+00
108	0.684716	2.921537	35.166647	0.000000e+00
156	0.770652	2.601053	31.279335	0.000000e+00
204	0.813436	2.463083	29.605295	0.000000e+00
252	0.832989	2.420655	29.086278	0.000000e+00
300	0.858037	2.391504	28.729935	0.000000e+00
348	0.853251	2.370841	28.477340	0.000000e+00
396	0.898747	2.349238	28.214591	0.000000e+00
444	0.899716	2.334827	28.038963	0.000000e+00
492	0.929863	2.305412	27.683693	0.000000e+00
540	0.957807	2.301607	27.636336	0.000000e+00
588	0.893093	2.276330	27.331442	0.000000e+00
636	0.903848	2.277014	27.338495	0.000000e+00
684	1.127747	2.270109	27.254587	0.000000e+00
732	1.051642	2.237777	26.865546	0.000000e+00
780	1.250181	2.277519	27.341913	0.000000e+00
828	1.005278	2.258733	27.115710	0.000000e+00
876	1.187966	2.263436	27.171568	0.000000e+00
924	1.398689	2.256081	27.082734	0.000000e+00
972	1.665560	2.205457	26.474557	0.000000e+00
1020	1.005511	2.110783	25.337674	0.000000e+00
1068	1.105551	2.203763	26.453409	0.000000e+00
1116	1.274019	2.181960	26.191343	0.000000e+00
1164	1.509866	2.089058	25.075876	0.000000e+00
1212	1.654144	2.152606	25.838374	0.000000e+00
1260	1.877641	2.130733	25.575556	0.000000e+00
1308	2.529002	1.769718	21.242029	0.000000e+00
1356	2.630846	1.895456	22.751063	0.000000e+00
1404	3.190116	1.735101	20.826161	0.000000e+00
1452	4.377298	1.398697	16.788221	0.000000e+00
1500	4.427739	1.524480	18.297828	0.000000e+00

Figure 20: **Simple Multiplication; -O3 Flag**

Speedup: From the above simple multiplication routine, the observed speed up with respect to its corresponding Blocked + OpenMp version is roughly **23.365 times faster!**

Question 4
OpenMP version of 2D Jacobi/Gauss-Seidel smoothing

(1) MACHINE SPECS: Is the same as mentioned above in Question 2.

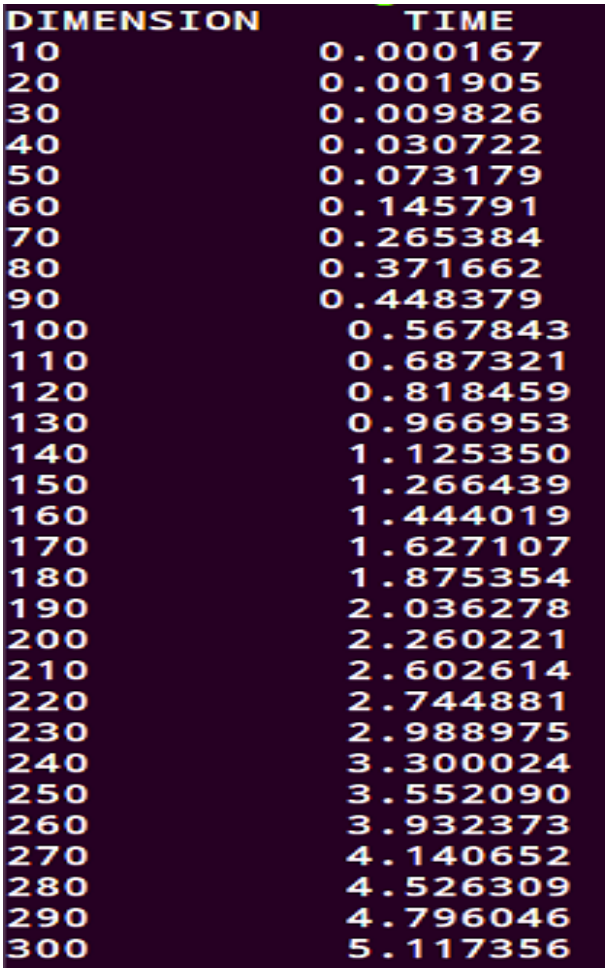
(2) TIMINGS FOR DIFFERENT DISCRETISATIONS AND THREADS:

Both the timings are given below, for Jacobi and Gauss-Seidel, in the following pages, The maximum iterations of 15000 for all the runs. Both the serial code and the OpenMP code timings for different matrix sizes and different number of threads are reported below.

(A) **2D JACOBI SMOOTHING**: Figures 21, 23-26

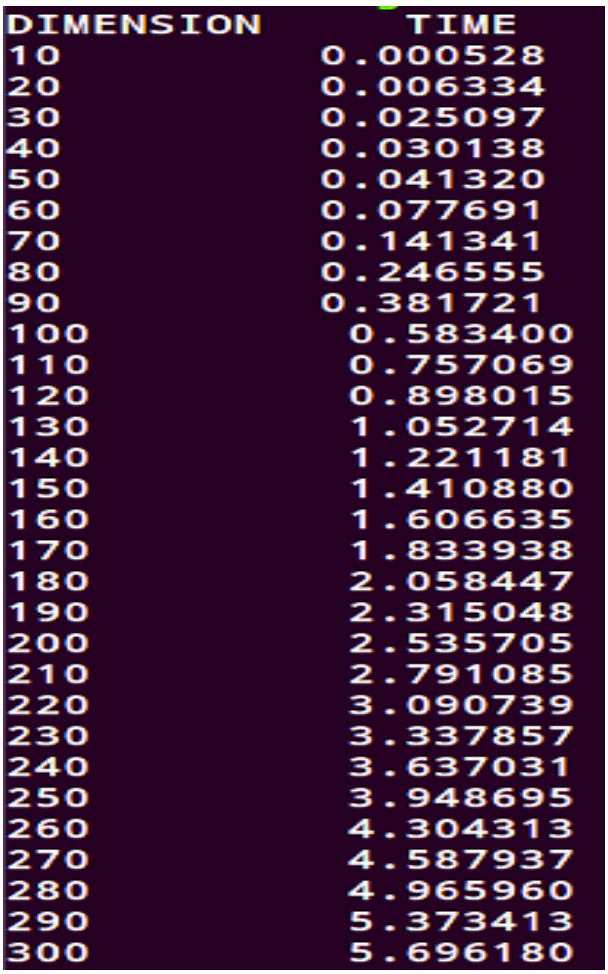
(B) **2D GAUSS-SEIDEL SMOOTHING**: Figures 22, 27-30

Both these timings are given below, in the follwoing pages. The maximum iterations was set to 15000 for all runs. Both the serial code and the OpenMP code timings for different matrix sizes and different number of threads are reported below.



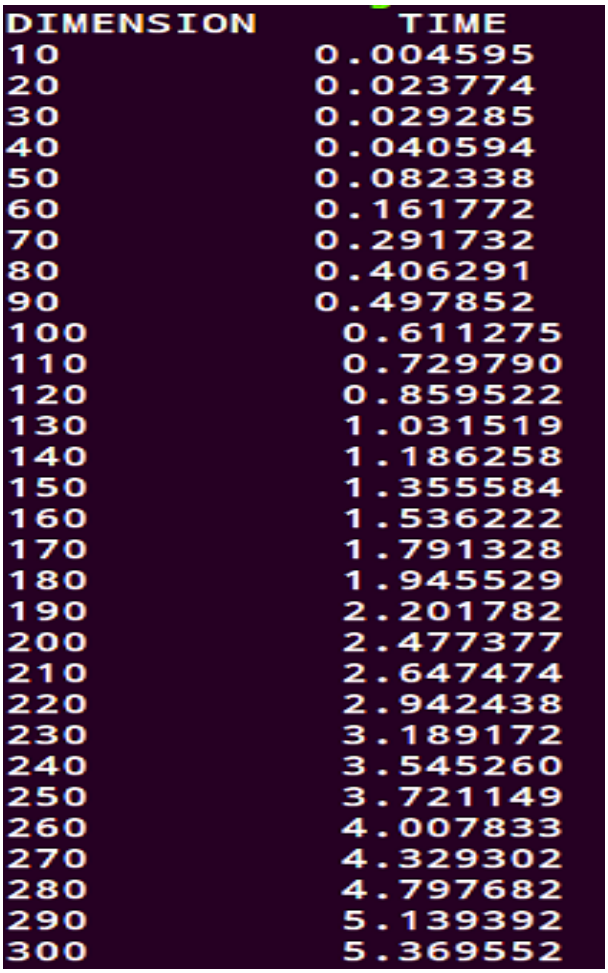
DIMENSION	TIME
10	0.000167
20	0.001905
30	0.009826
40	0.030722
50	0.073179
60	0.145791
70	0.265384
80	0.371662
90	0.448379
100	0.567843
110	0.687321
120	0.818459
130	0.966953
140	1.125350
150	1.266439
160	1.444019
170	1.627107
180	1.875354
190	2.036278
200	2.260221
210	2.602614
220	2.744881
230	2.988975
240	3.300024
250	3.552090
260	3.932373
270	4.140652
280	4.526309
290	4.796046
300	5.117356

Figure 21: **SERIAL JACOBI**



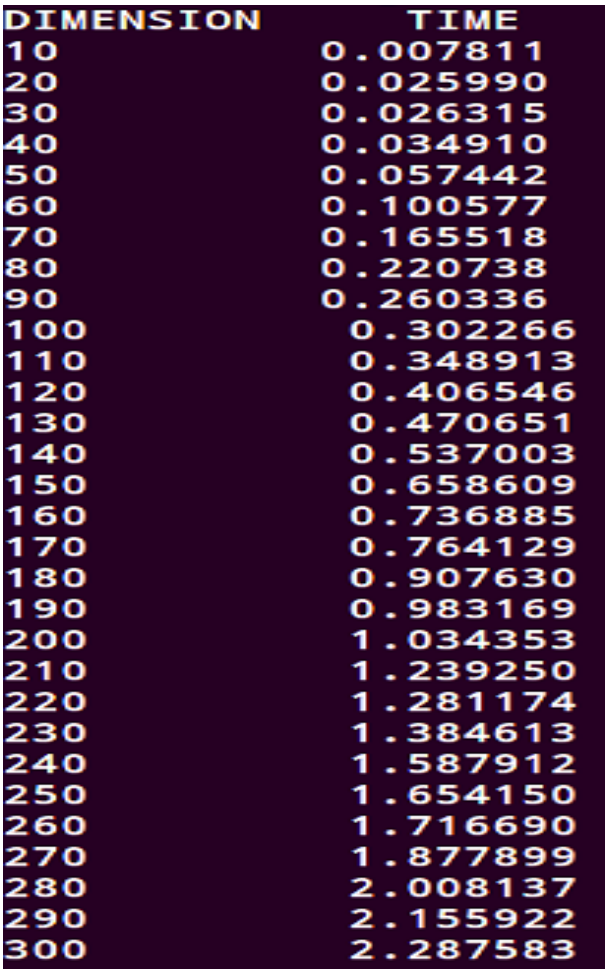
DIMENSION	TIME
10	0.000528
20	0.006334
30	0.025097
40	0.030138
50	0.041320
60	0.077691
70	0.141341
80	0.246555
90	0.381721
100	0.583400
110	0.757069
120	0.898015
130	1.052714
140	1.221181
150	1.410880
160	1.606635
170	1.833938
180	2.058447
190	2.315048
200	2.535705
210	2.791085
220	3.090739
230	3.337857
240	3.637031
250	3.948695
260	4.304313
270	4.587937
280	4.965960
290	5.373413
300	5.696180

Figure 22: **SERIAL GAUSS SEIDEL**



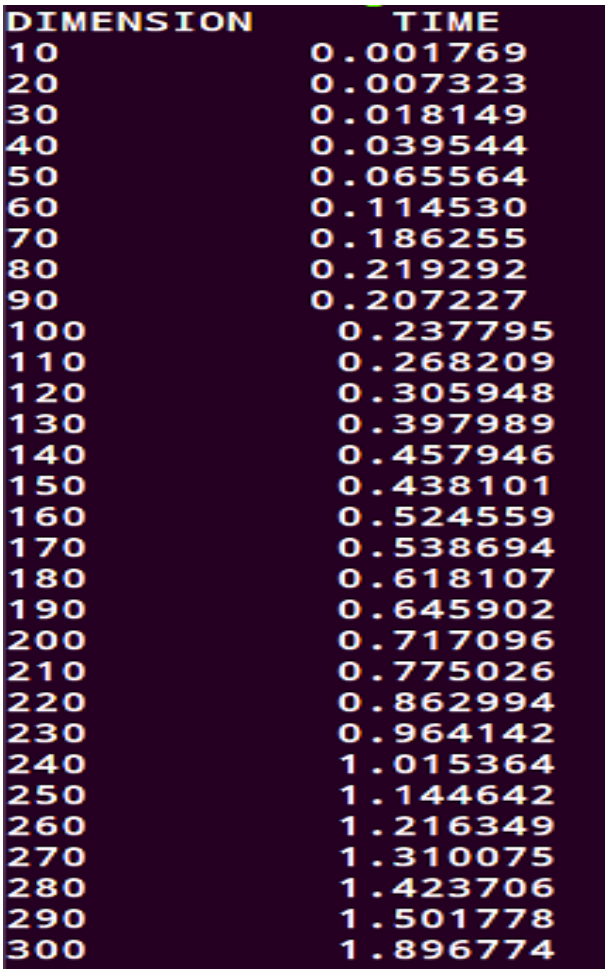
DIMENSION	TIME
10	0.004595
20	0.023774
30	0.029285
40	0.040594
50	0.082338
60	0.161772
70	0.291732
80	0.406291
90	0.497852
100	0.611275
110	0.729790
120	0.859522
130	1.031519
140	1.186258
150	1.355584
160	1.536222
170	1.791328
180	1.945529
190	2.201782
200	2.477377
210	2.647474
220	2.942438
230	3.189172
240	3.545260
250	3.721149
260	4.007833
270	4.329302
280	4.797682
290	5.139392
300	5.369552

Figure 23: **2 THREAD JACOBI**



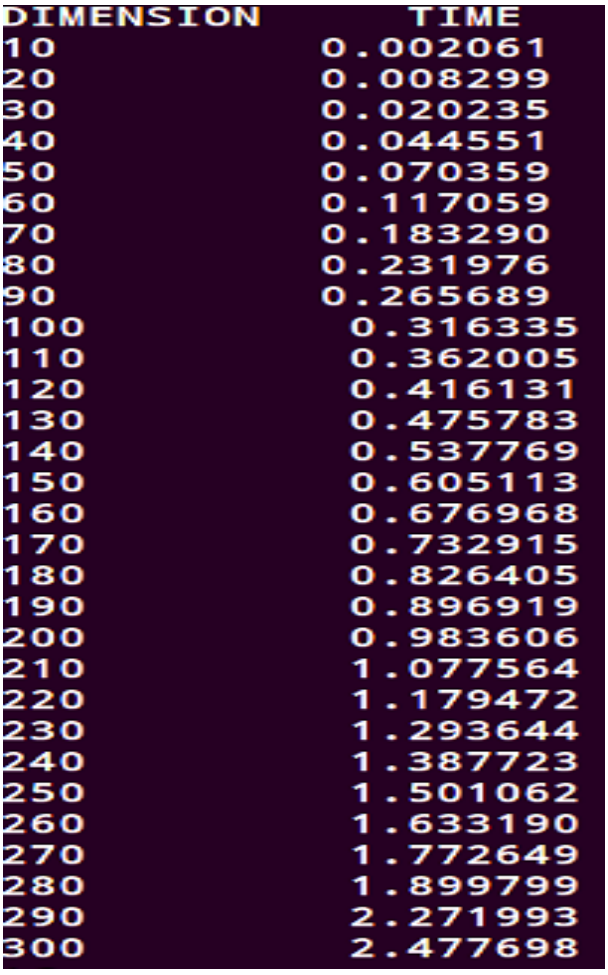
DIMENSION	TIME
10	0.007811
20	0.025990
30	0.026315
40	0.034910
50	0.057442
60	0.100577
70	0.165518
80	0.220738
90	0.260336
100	0.302266
110	0.348913
120	0.406546
130	0.470651
140	0.537003
150	0.658609
160	0.736885
170	0.764129
180	0.907630
190	0.983169
200	1.034353
210	1.239250
220	1.281174
230	1.384613
240	1.587912
250	1.654150
260	1.716690
270	1.877899
280	2.008137
290	2.155922
300	2.287583

Figure 24: **5 THREAD JACOBI**



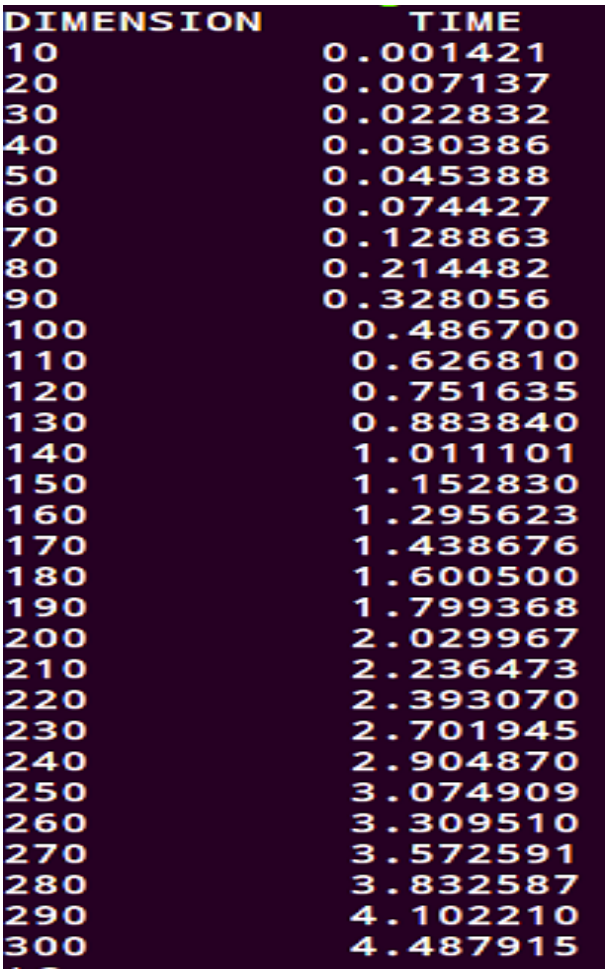
DIMENSION	TIME
10	0.001769
20	0.007323
30	0.018149
40	0.039544
50	0.065564
60	0.114530
70	0.186255
80	0.219292
90	0.207227
100	0.237795
110	0.268209
120	0.305948
130	0.397989
140	0.457946
150	0.438101
160	0.524559
170	0.538694
180	0.618107
190	0.645902
200	0.717096
210	0.775026
220	0.862994
230	0.964142
240	1.015364
250	1.144642
260	1.216349
270	1.310075
280	1.423706
290	1.501778
300	1.896774

Figure 25: **8 THREAD JACOBI**



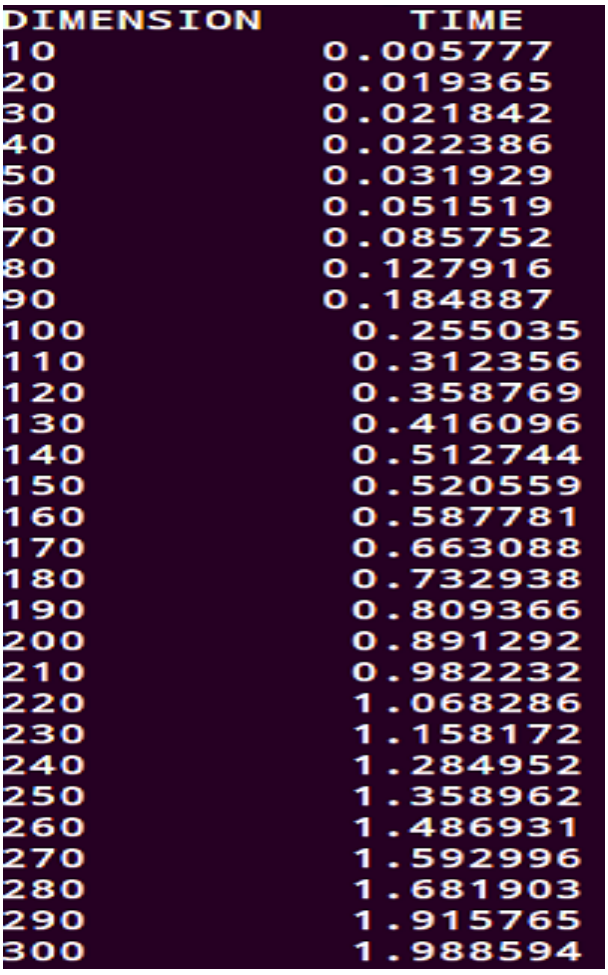
DIMENSION	TIME
10	0.002061
20	0.008299
30	0.020235
40	0.044551
50	0.070359
60	0.117059
70	0.183290
80	0.231976
90	0.265689
100	0.316335
110	0.362005
120	0.416131
130	0.475783
140	0.537769
150	0.605113
160	0.676968
170	0.732915
180	0.826405
190	0.896919
200	0.983606
210	1.077564
220	1.179472
230	1.293644
240	1.387723
250	1.501062
260	1.633190
270	1.772649
280	1.899799
290	2.271993
300	2.477698

Figure 26: **10 THREAD JACOBI**



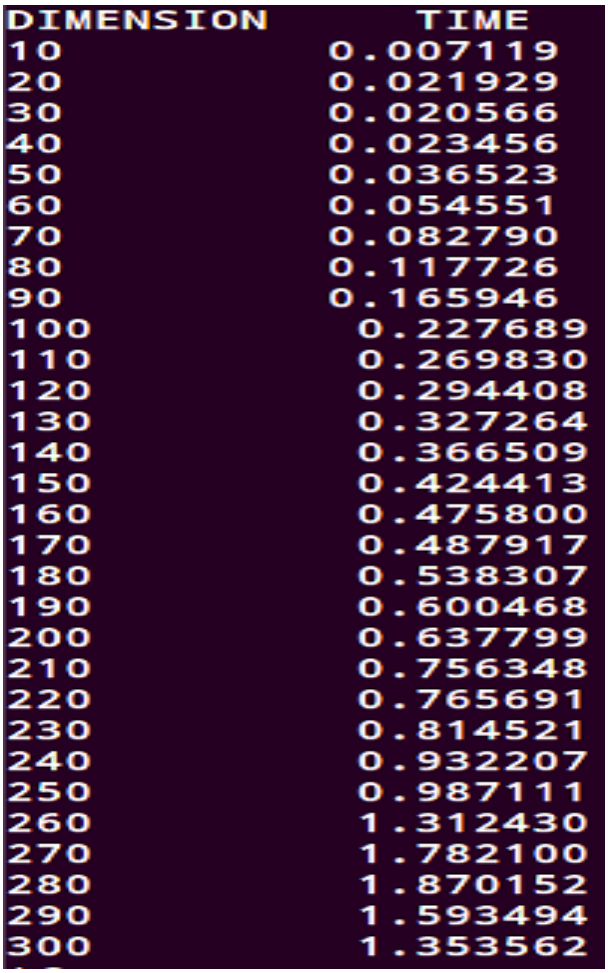
DIMENSION	TIME
10	0.001421
20	0.007137
30	0.022832
40	0.030386
50	0.045388
60	0.074427
70	0.128863
80	0.214482
90	0.328056
100	0.486700
110	0.626810
120	0.751635
130	0.883840
140	1.011101
150	1.152830
160	1.295623
170	1.438676
180	1.600500
190	1.799368
200	2.029967
210	2.236473
220	2.393070
230	2.701945
240	2.904870
250	3.074909
260	3.309510
270	3.572591
280	3.832587
290	4.102210
300	4.487915

Figure 27: **2 THREAD GAUSS SEIDEL**



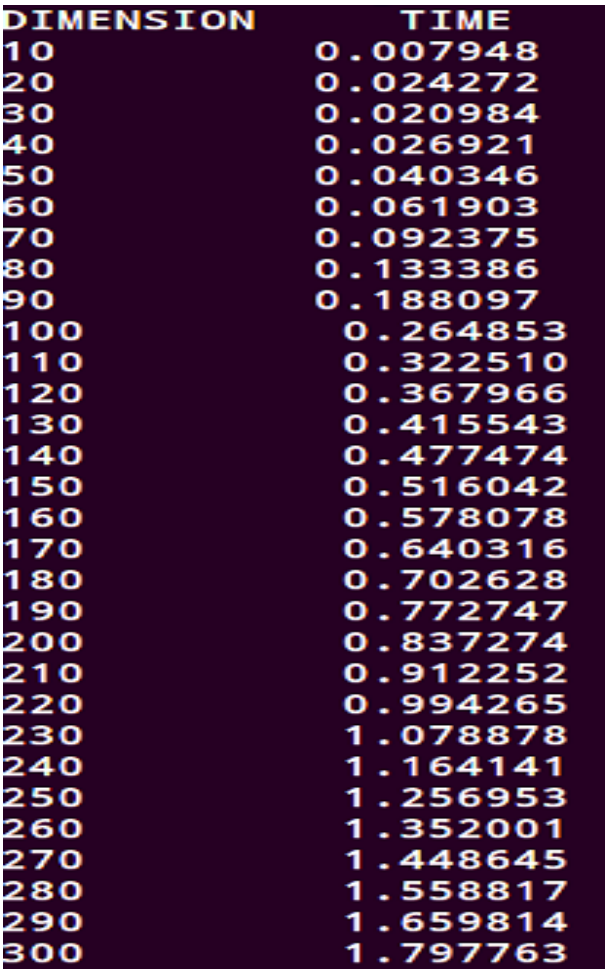
DIMENSION	TIME
10	0.005777
20	0.019365
30	0.021842
40	0.022386
50	0.031929
60	0.051519
70	0.085752
80	0.127916
90	0.184887
100	0.255035
110	0.312356
120	0.358769
130	0.416096
140	0.512744
150	0.520559
160	0.587781
170	0.663088
180	0.732938
190	0.809366
200	0.891292
210	0.982232
220	1.068286
230	1.158172
240	1.284952
250	1.358962
260	1.486931
270	1.592996
280	1.681903
290	1.915765
300	1.988594

Figure 28: **5 THREAD GAUSS SEIDEL**



DIMENSION	TIME
10	0.007119
20	0.021929
30	0.020566
40	0.023456
50	0.036523
60	0.054551
70	0.082790
80	0.117726
90	0.165946
100	0.227689
110	0.269830
120	0.294408
130	0.327264
140	0.366509
150	0.424413
160	0.475800
170	0.487917
180	0.538307
190	0.600468
200	0.637799
210	0.756348
220	0.765691
230	0.814521
240	0.932207
250	0.987111
260	1.312430
270	1.782100
280	1.870152
290	1.593494
300	1.353562

Figure 29: **8 THREAD GAUSS SEIDEL**



DIMENSION	TIME
10	0.007948
20	0.024272
30	0.020984
40	0.026921
50	0.040346
60	0.061903
70	0.092375
80	0.133386
90	0.188097
100	0.264853
110	0.322510
120	0.367966
130	0.415543
140	0.477474
150	0.516042
160	0.578078
170	0.640316
180	0.702628
190	0.772747
200	0.837274
210	0.912252
220	0.994265
230	1.078878
240	1.164141
250	1.256953
260	1.352001
270	1.448645
280	1.558817
290	1.659814
300	1.797763

Figure 30: **10 THREAD GAUSS SEIDEL**