Agenda

Factorization

Let's say we have a number 42

Questions

Can we decompose this number into product of it's another smaller numbers?

$$A_{n \times m} \implies B_{n \times d} \cdot C_{d \times m}$$

$$A_{n \times m} \implies B_{n \times d} \cdot C_{d \times k} \cdot D_{k \times m}$$

So, How do we find these U and I matrix such that $A = U \cdot I^{Transpose}$

$I_1, I_2 I_3 - - - I_m$

U_1
U_2
$- - -$
U_n

B,B3
Null

U

U_1
U_2
U_3
$- - -$
U_n

$0 \quad 0$

$n \times d$

I

$m_1, m_2 - - - m$

$0 0 0$

$d \times m$

U matrix can be interpreted as Containing User information

I matrix can be interpreted as Containing Item information

Hypothesis:

$$A_{ij} \approx U_i \cdot M_j$$

For ex:

The Rating User 2 will give to item 3 can be Calculated as

$$P_{23} = U_2 \cdot I_3$$

SS

A_{23}

==MSE==

$$\sum_{i,j}^{0,M} (A_{ij} - U_i \cdot I_j)^2$$

Output from interaction

S.t. $A_{ij} \neq$ Null

* SGD to find value of U_i and I_j

Step1: Initialize matrix U and Matrix I randomly

Step2: Take derivative of loop and update U and I

* We calculate loss only for values where user has given some ratings
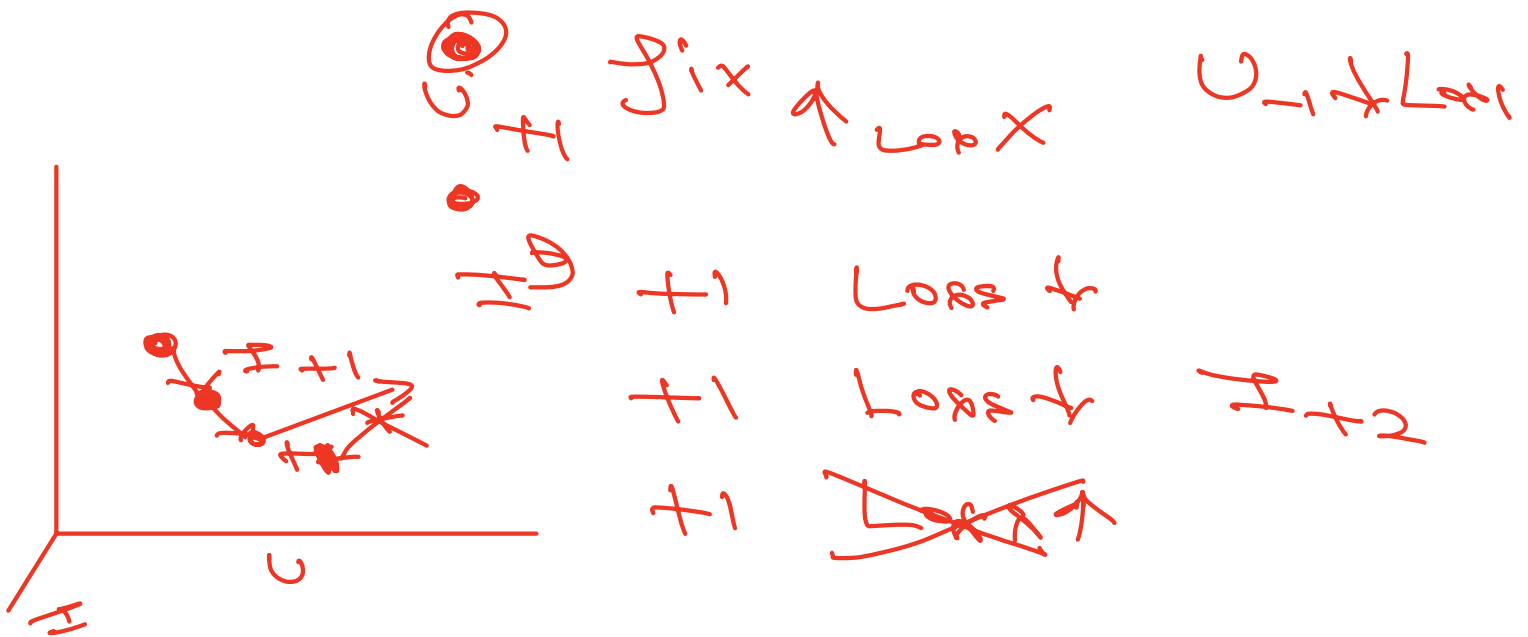
Alternative: Co-ordinate Ascent/Descent Algorithm

1) Random initialization

② Calculate Loss

③ We fix value U for some iteration and update I such that loss Reduce

④ After few iteration freeze
U and update I

⑤ Repeat until Convergence

==Alternate Least SQuare==
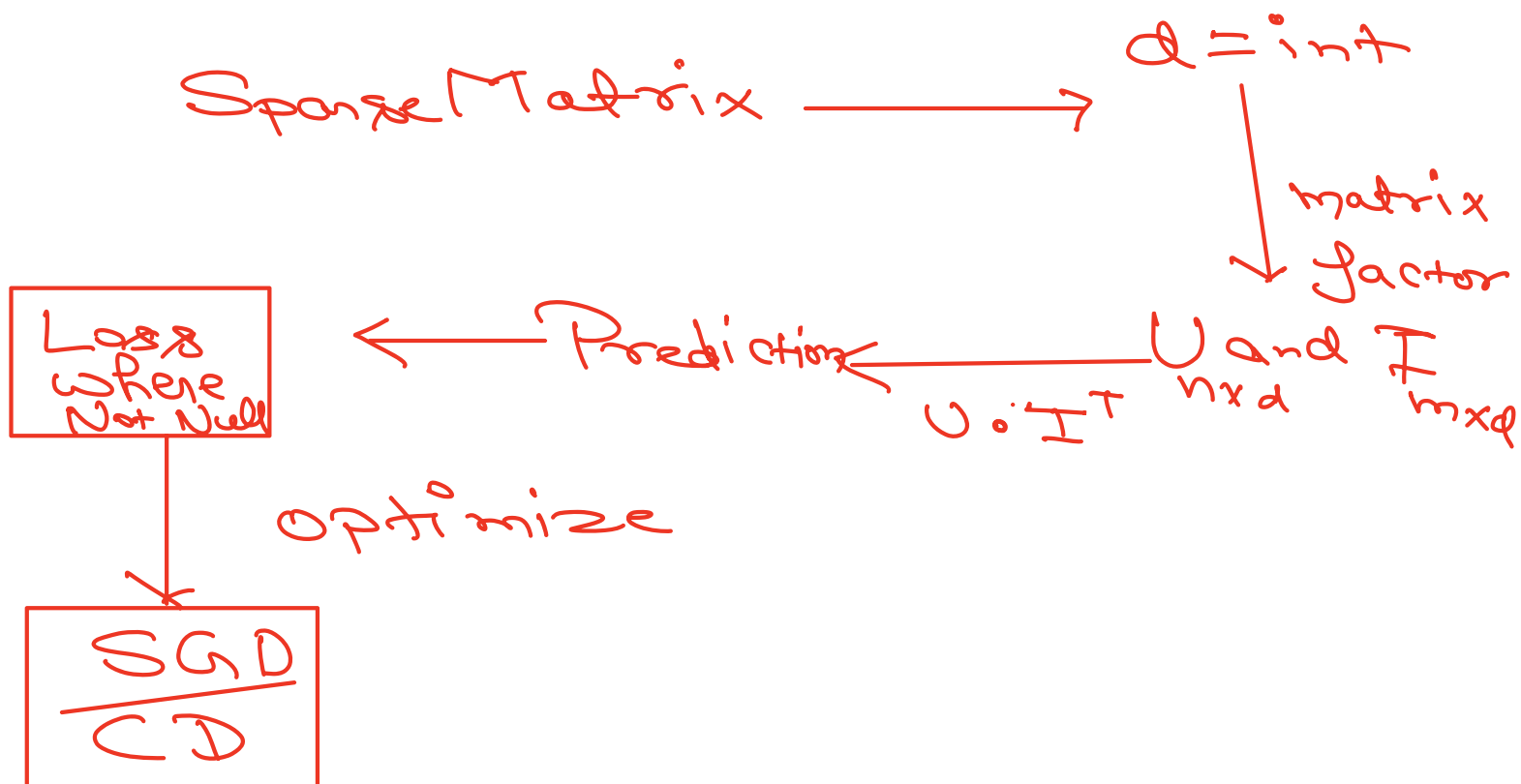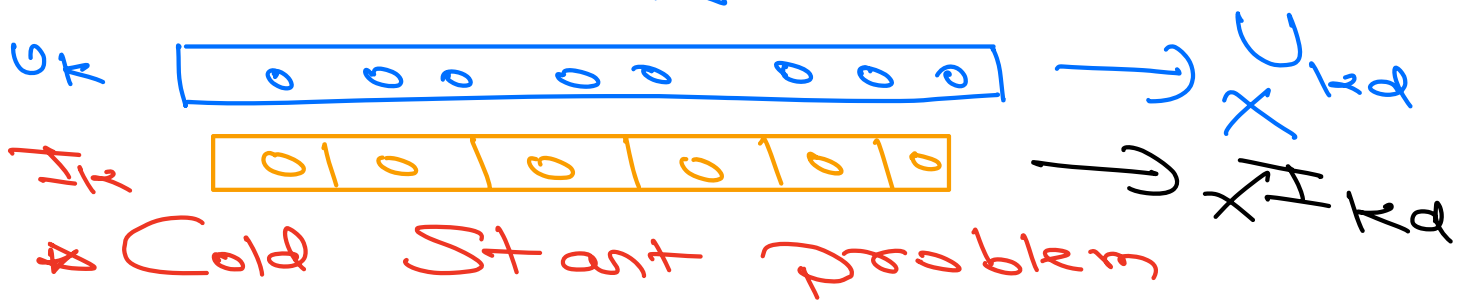
$$U_{-i+L_{21}}$$

fix

Loop X

+1	Loss ↑
+1	Loss ↓
+1	

$I+2$

$$P_{ij} = U_{i} \cdot I_{j}$$

Containing Interaction
info about
User i

Contains
info about
ratings
of
item j

* Issue with this

A new User or User that never rates

GR

HT

$$0 \; 0 \; 0 \quad 0 \; 0 \quad 0 \; 0 \; 0$$

$$U \; X \; U_{kd}$$

$$0 \mid 0 \mid 0 \mid 0 \mid 0 \mid 0$$

$$V \; X \; H_{kd}$$

* Cold Start Problem

SparseMatrix $\longrightarrow$ $d = int$

$\downarrow$ matrix factor

Prediction $\longleftarrow$ U and H
$U \cdot H^T$ $n \times d$ $m \times d$

Loss Where Not Null $\longleftarrow$

Optimize

$\dfrac{SGD}{CD}$

MF in PCA

$X_{n \times d}$ $\longrightarrow$ $cov(X) \Rightarrow S_{d \times d}$

row $\nwarrow$ n×d
↘ features

$$\left(\frac{X^T_{d \times n} \cdot X_{n \times d}}{n-1} \right)$$
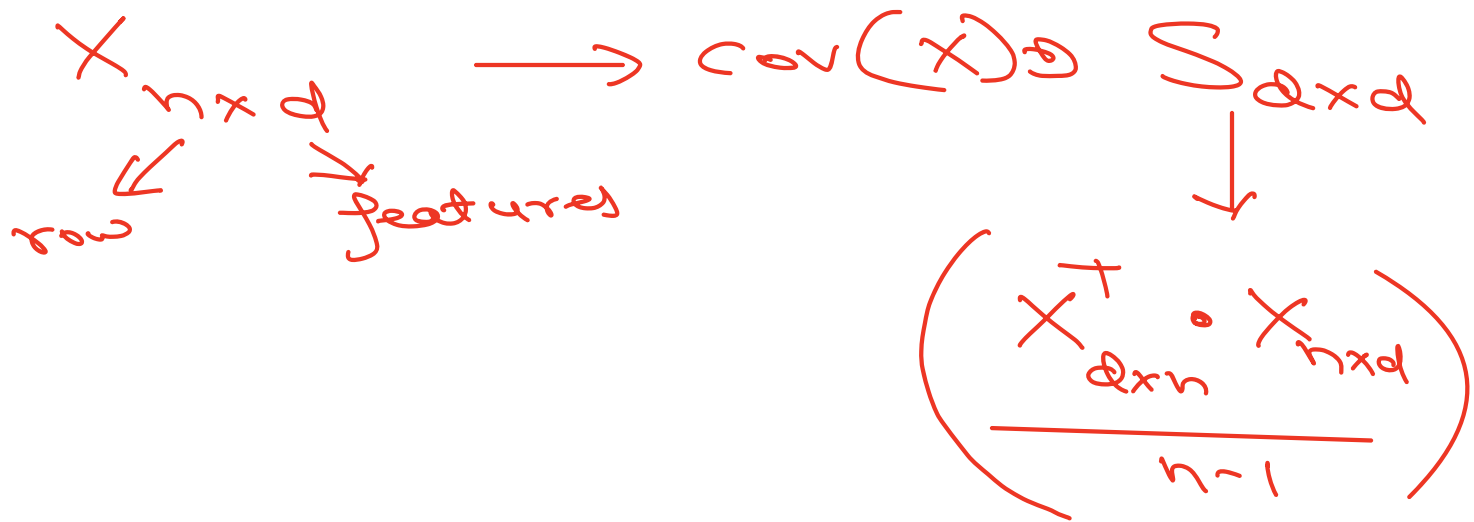
E·V·D : **Matrix Needs to Square**

$$S_{d \times d} \longrightarrow W_{d \times d} \cdot \Lambda_{d \times d} \cdot W^T_{d \times d}$$

Diagonal Matrix

$$\begin{bmatrix} | & | & & | \\ V_1 & V_2 & --- & V_d \\ | & | & & | \end{bmatrix}$$

Eigen Vector

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_d \end{bmatrix}$$

Eigen Values
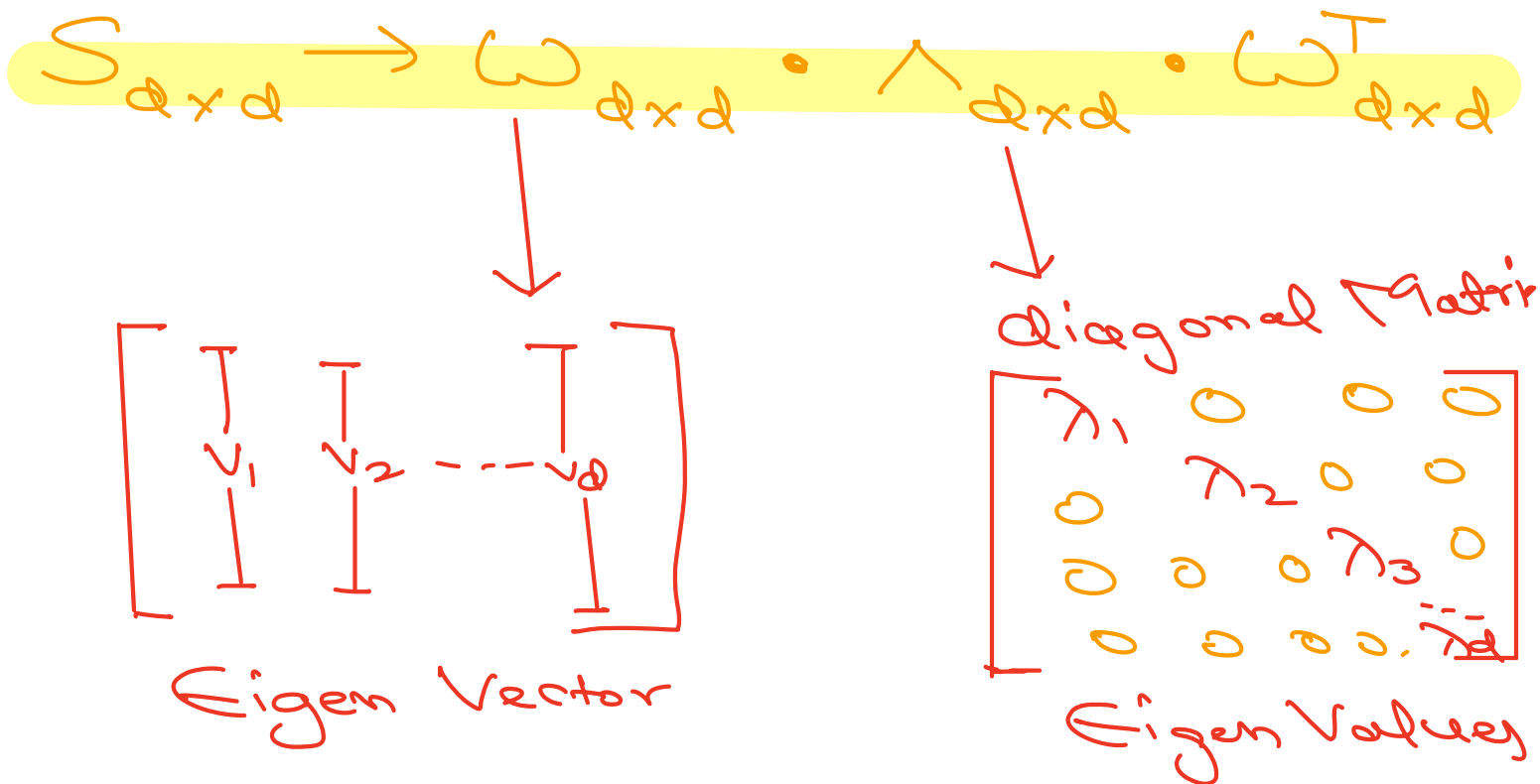
* Special case of MF
  * Constraint 1 : V's are Orthogonal's
  * Constraint 2 : Λ is diagonal

$$X_{n \times d} = U_{n \times n} \; \Sigma_{n \times d} \; V^T_{d \times d}$$

$$\begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ U_1 & U_2 & \cdots & U_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

n × n

$$\begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ V_1 & V_2 & \cdots & V_d \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

Singular Value

$$\begin{bmatrix} S_{11} & & & \\ & S_{22} & & \\ & & \ddots & \\ & & & S_{nd} \end{bmatrix}$$

n × d

$$S_i \Rightarrow \sqrt{\lambda_i}$$