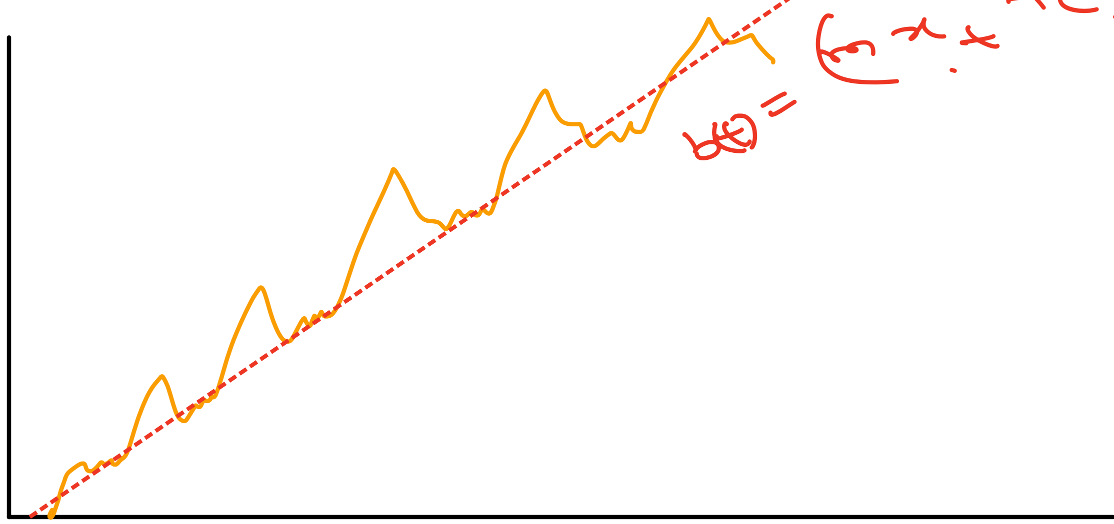
Agenda

- Making Time Series Stationary
- ACF and PACF plot
- ARIMA Family models:
  - AR
  - MA
  - ARMA
  - ARIMA
  - SARIMA



$b(t) = (m \cdot x + C)$

Q. How do we make this time series Stationary?

① Decomposition ⇒ $y(t) - s(t) - b(t)$

② Differencing

$$\hat{y}_t \Rightarrow b(t) + s(t) + e(t)$$

$$\hat{y}_t \Rightarrow m \cdot x_t + c + s(t) + e(t)$$

ab initio

$$\frac{\partial \hat{y}}{\partial t} \Rightarrow \frac{m}{t} + c + \frac{\partial s(t)}{\partial (t)} + \frac{\partial (e_t)}{\partial t}$$

Constant Value

$$\left(\frac{m x_t}{\partial t} \right.$$

$$\boxed{\frac{\partial \hat{y}}{\partial t} = \lim_{\Delta t \to 0} \frac{y(t + \Delta t) - y(t)}{\Delta t}}$$

$\Delta t \Rightarrow 1$ unit

$$\Rightarrow \frac{y(t+1) - y(t)}{1}$$

t_1	S_1
t_2	S_2
t_3	S_3

$$\longrightarrow \frac{S_2 - S_1}{1}$$

differencing

1000 $\Rightarrow$ 999 differenced

d=1

Pre-Processing

Ex:

Time	Sales
T_1	20
T_2	25
T_3	23
T_4	29
"	

$\xrightarrow{}$ diff(1) $\xrightarrow{}$

Time	Sales
T_1	NaN
T_2	5
T_3	-2
T_4	6
"	"

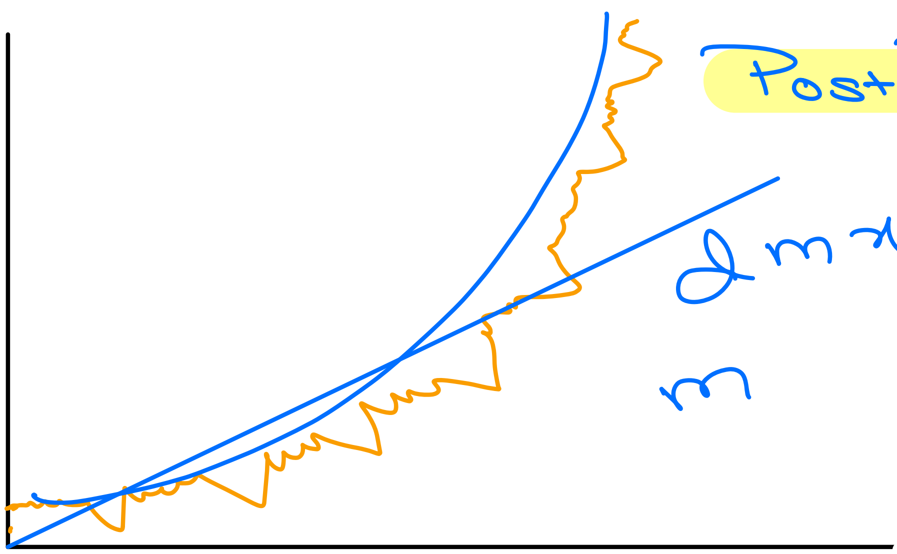
cumsum()

integration(1)

Time	Sales
T_1	20
T_2	25
T_3	23
T_4	29
"	

Q What if trend is Non-Linear

df.sales.diff().diff()



Post-Processing

dmx, +C

x_0

m

Q: How do we get to know Seasonality period?

m-differencing

↓

seasonal_window_size

df.sales.diff(12)

Jan 2023 — Jan 2022

Jan 2022 — Jan 2021

ACF and PACF

ACF: Auto Correlation Factor

Time	Sales
T_1	S_1
T_2	S_2
T_3	S_3
T_4	S_4
T_5	S_5
T_6	S_6
"	"

m=1

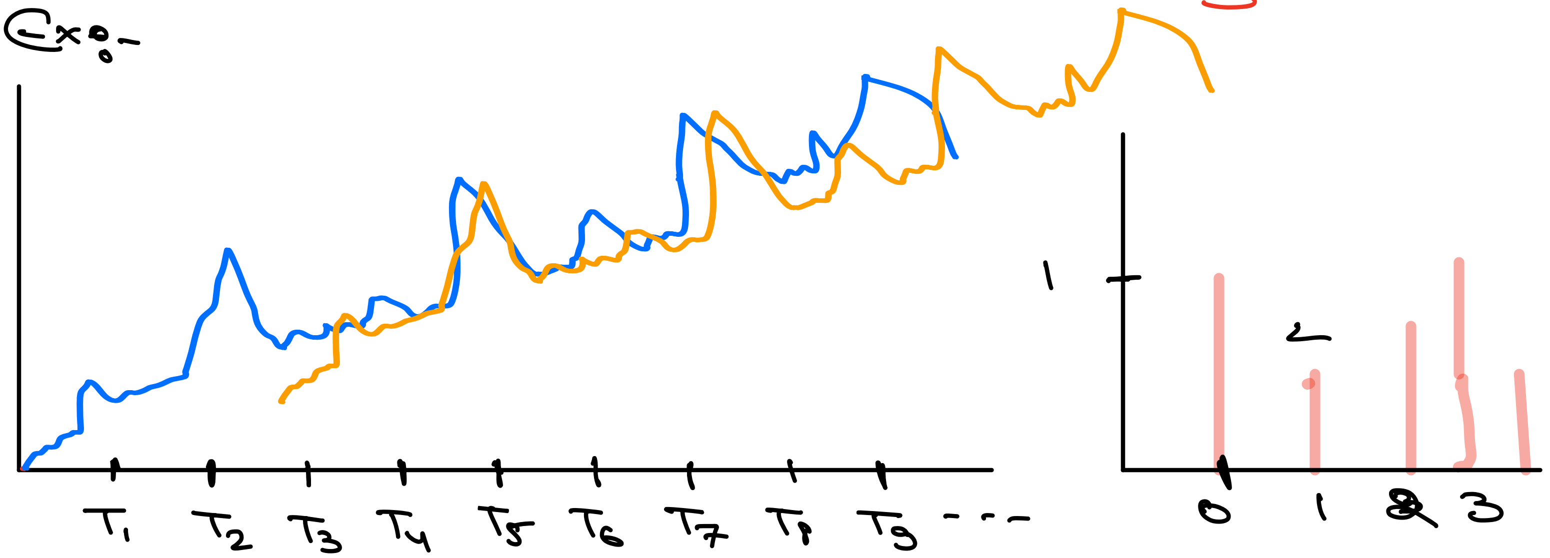
$\longrightarrow \boxed{\text{Shift(m)}} \longrightarrow$

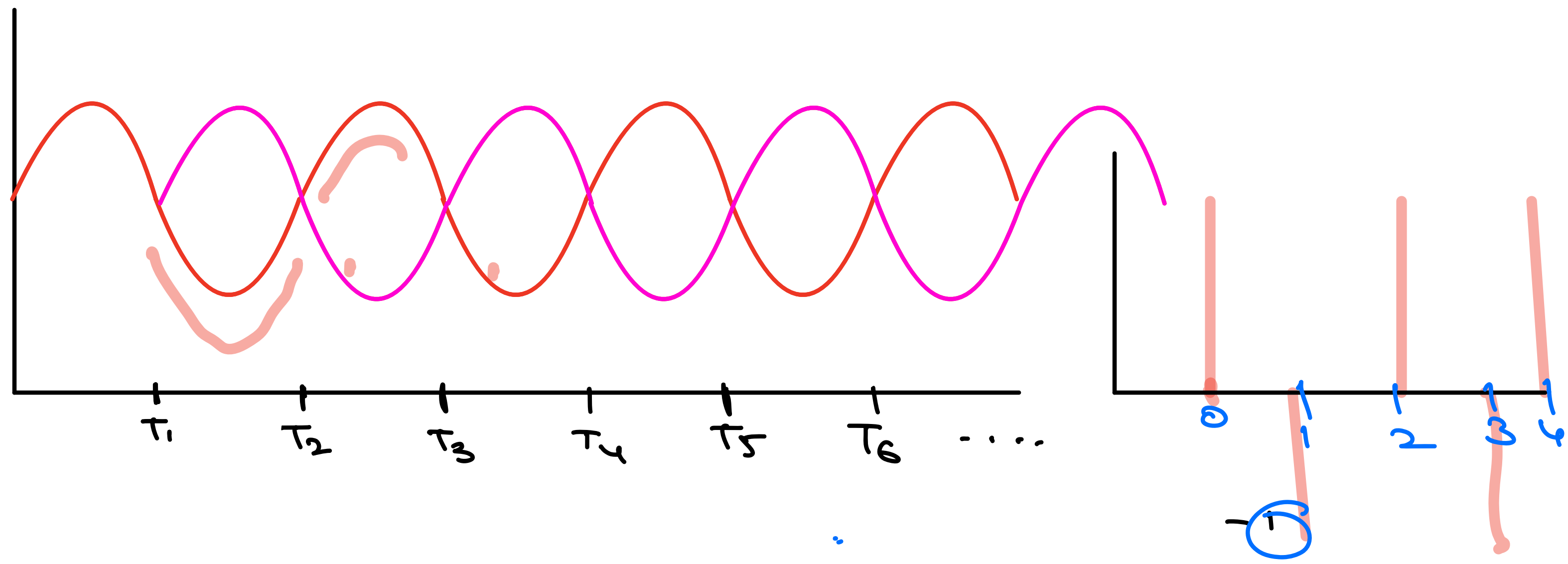
Time	Sales
T_1	
T_2	
T_3	S_1
T_4	S_2
T_5	S_3
T_6	S_4
"	S_5
	"

corr(1) = 0.5

corr(2) = 0.7

Ex:-





* PACF : Same as ACF with effect of previous correlations removed

ACF(0,2) ⇒ Corr(0,1) + Corr(1,2) + Corr(0,2)

shift

remove these 2

PACF(0,2) will ensure resultant is independent of Past Corr

ACF vs PACF

	ACF	PACF
Considers Correlation	direct and indirect	Only Direct
Reveals	Overall pattern in Data	Specific Lags with Direct Influence

Note: PACF removes the impact of intermediate Correlations using AR Model

AR(p)

⇒ p : hyperparameter (order of AR)

⇒ Residuals are dependent on previous Value

⇒ We can build a Linear Regression Model to predict Residuals

Ex:

Time	Sales
t_0	r_0
t_1	r_1
t_2	r_2
t_3	r_3
⋮	⋮
t_n	r_n

$\Rightarrow \boxed{p} \Rightarrow$

↓

3

⟵ features ⟶			Sales
r_0,	r_1,	r_2 ... r_{p-1}	r_3
r_1,	r_2,	r_3 ...	r_4
r_2	r_3	r_4	r_5
⋮	⋮		⋮

P = 3 , weights ⇒ r_1

$\hat{y}_t \Rightarrow \alpha_0 + \alpha_1 r_1 + \alpha_2 r_2 + \alpha_3 r_3$

bias term

LoR + Gradient

Q Diff b/w EMA and AR

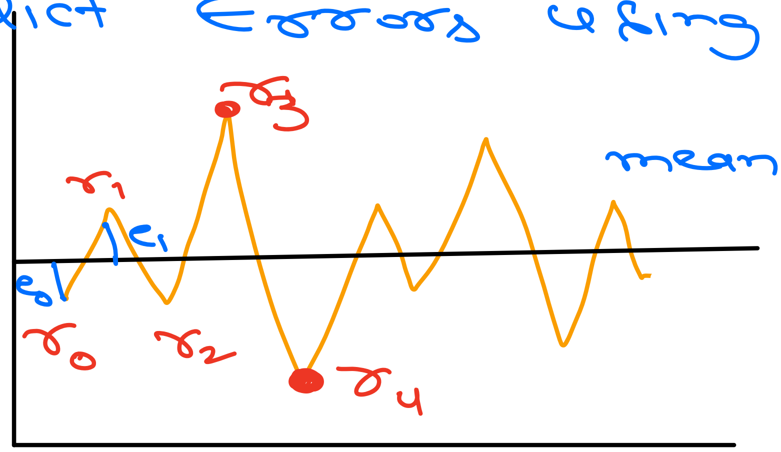
fixed (α)

learnt (α)

$\boxed{MA(q)}$ → moving Average

⇒ q ⇒ order of ma

⇒ What if predict Errors using past Errors



$\hat{y}_t = \beta_0 + \beta_1 e_{t-1}$

ma (1)

$e_0 \Rightarrow r_0 - \text{mean}$

$e_1 \Rightarrow r_1 - \text{mean}$

$\hat{y} \Rightarrow \beta_0 + \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3$ (MA(3))

$\boxed{ARMA\ (p,q)}$

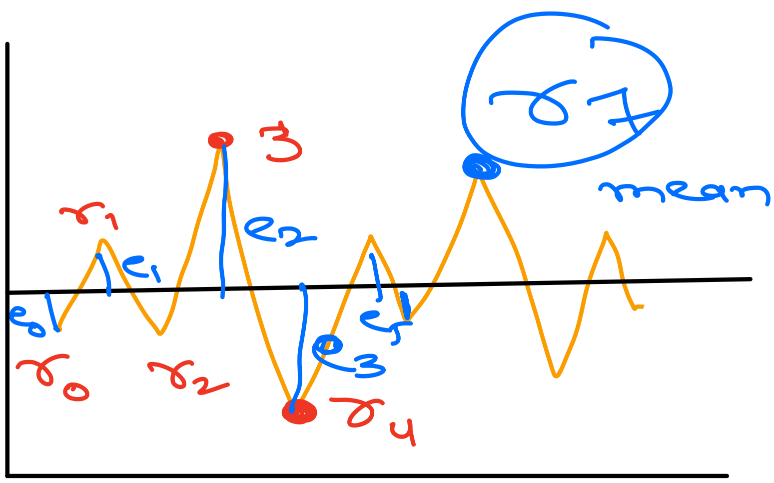
ARMA = AR(p) + MA(q)

p

q

$\hat{y}_t \Rightarrow \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} \dots - \alpha_p r_{t-p}$
$+$
$\beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} - - \beta_p r_{t-p}$

MA $\Rightarrow r_7 \Rightarrow \beta_0 + \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4$



P=3

AR $\Rightarrow r_7 \Rightarrow \alpha_0 + \alpha_1 r_6 + \alpha_2 r_5 + \alpha_3 r$

y's

ARMA $\Rightarrow$ AR(3) + MA(5)

y

e's

$$\boxed{\text{ARIMA}(p, d, q)}$$

Note: It can handle data with trend

ARIMA $\Rightarrow$ $\boxed{\begin{array}{c} AR(p) \\ + \\ \text{Integration}(d) \\ + \\ MA(q) \end{array}}$

$d \Rightarrow$ will be used to convert data into stationary before performing ARMA

- It works with Non-Stationary Time-series as well

Can ARIMA handle seasonality?

$\boxed{SARIMA(p, d, q, S, P, D, Q)}$

From Arima

S ⇒ seasonality window 12

P
D
Q

Extra set of parameter to learn patterns from seasonality

$d \Rightarrow (y_t - y_{t-1}) \Rightarrow y''$

$D_{\to 1} \Rightarrow y_t - y_{t-S}$

P ⇒ AR_seasonality

Q ⇒ MA_seasonality

$e_{t-S} \quad e_{t-2S}$

$\ldots \quad e_{t-QS}$

SARIMA $\Rightarrow$ AR (p) + MA (q)

+ d_{trend}

+ AR-S (P)

+ MA-S (Q)

+ $D_{seasonality}$