LECTURE NOTES

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Recommender Systems

Recommender systems are systems that are designed to recommend things to the user based on many different factors. These systems predict the most likely product that the users are most likely to purchase and are of interest to.

Companies like Netflix, Amazon, Instagram, etc. use recommender systems to help their users to identify the correct product or movies for them.

Why recommender systems?

- Increase in revenue based on personalization.
- Better user experience.
- More time spent on the platform
- Help websites improve user engagement.

Recommender systems applications

- Netflix to recommend movies
- E-commerce websites to recommend the products.
- Social media platforms to recommend feeds/ blogs/news/songs…etc. Ex: Instagram, Facebook.
- Food recommendations by Zomato, Swiggy.
- Songs recommendations by Spotify and Wynk music.
- Dating apps recommending people.

Types of recommender systems

1) Apriori Algorithm
2) Content-based filtering system.
3) Collaborative-based filtering system.
4) Similarity-based filtering system.

5) Matrix factorization
6) Popularity-based recommender system.

1. Market-Basket Analysis

- Market basket analysis is used to analyze the combination of products which have been bought together.
- This is a technique used for purchases done by a customer. This identifies the pattern of frequent purchases of items by customers.
- This analysis can help to promote deals, offers, sale by the companies, and data mining techniques helps to achieve this analysis task.

Apriori Algorithm

- Apriori algorithm refers to the algorithm which is used to calculate the association rules between objects. It means how two or more objects are related to one another. we can say that the apriori algorithm is an association rule learning that analyzes that people who bought product A also bought product B.
- It is a Frequency-based algorithm. Generally, the apriori algorithm operates on a database containing a huge number of transactions.
  Ex: People who bought iPhones also bought Airpods.

Examples where the apriori algorithm can be used:

- Telecommunications
- Banking / Insurance

- Medical
- E-Commerce
- Retail

Association rules

- Association Rules are widely used to analyze retail basket or transaction data and are intended to identify strong rules discovered in transaction data using measures of patterns, based on the concept of strong rules.

- There are various metrics in place to help us understand the strength of the association between an antecedent and consequent:

- The **IF component** of an association rule is known as the antecedent. The **THEN component** is known as the consequent.

1. Support:

It is calculated to check how popular a given item is. It is measured by the proportion of transactions in which an item set appears. It is also used to measure abundance or frequency.

$$Support(x) = \frac{Number\ of\ transactions\ with\ x}{Total\ number\ of\ transactions}$$

Drawback: If ‘x’ is very popular then many items will have high support w.r.t to x.

2. Confidence:

It is calculated to check how likely item X is purchased when item Y is purchased. This is measured by the proportion of transactions with item X, in which item Y also appears.

$$Confidence(X -> Y) = \frac{Number\ of\ transactions\ with\ X\ and\ Y}{Number\ of\ transactions\ with\ X}$$

Drawback: If ‘y’ is very popular then it will have high confidence w.r.t to many items.

3. Lift:

It is calculated to measure how likely item Y is purchased when item X is purchased while controlling for how popular item Y is.

$$Lift(X -> Y) = \frac{Confidence\ (X -> Y)}{Support\ (Y)} = \frac{Support\ (X \cap Y)}{Support\ (X) * Support\ (Y)}$$

- lift(X→Y) = 1 if X and Y are independent
- lift(X→Y) < 1, unlikely to be bought together**.**
- lift(X → Y) > 1, likely to be bought together
- Gives Bi-directional recommendation.

4. Leverage or Piatetsky-Shapiro:

Leverage computes the difference between the observed frequency of X and Y appearing together and the frequency that would be expected if X and Y were independent. A leverage value of 0 indicates independence.

$$Leverage(X -> Y) = Support\ (X \cap Y) - Support\ (X) * Support\ (Y)$$

Though it is similar to lift, leverage is easier to interpret.
The leverage value lies in the range of -1 to +1, whereas the lift value ranges from 0 to infinity.

5. Conviction:

Conviction is another way of measuring association, although it is a bit harder to get your head around. It compares the probability that X appears without Y if they were independent of the actual frequency of the appearance of X without Y.

It can be calculated as the ratio of the expected frequency that X occurs without Y if X and Y were independent divided by the observed frequency of incorrect predictions.

A high conviction value means that the consequent is highly dependent on the antecedent.

$$Conviction(X -> Y) = \frac{1 - Support\ (Y)}{1 - Confidence\ (X -> Y)}$$

Steps to implement the apriori algorithm:

1) Create a frequency table of all the items that occur in all transactions.
2) Create a pivot matrix representing 1 if the item is present and 0 if the item is not present.
3) Encode the matrix, return 0 for all the items that have the value less than or equal to 0 and return 1 for all the values that are greater than or equal to 1.
4) Use the library mlxtend.frequent_patterns and import apriori.
5) Calculate frequent itemsets using the metric min_support.

$$min\_support = \frac{Number\ of\ times\ subset\ has\ occurred}{Number\ of\ transactions}$$

6) Use the library mlxtend.frequent_patterns and import association_rules
7) With the help of association_rules select the appropriate metric (ex: support, confidence, lift … default = confidence) to recommend the items.

Advantages:

1) It is the most simple and easy to understand and implement.
2) It doesn't require labeled data as it is completely unsupervised and hence this can be used for many different situations as we can find unlabeled data quite often.
3) It is used to calculate large item sets.

Disadvantages:

1) Apriori algorithm is an expensive method to find support since the calculation has to pass through the whole database.
2) It is computationally expensive.
3) Complexity grows exponentially.
4) Cold start problem.

2. Content-Based recommender System

- Content-based filtering makes recommendations based on the similarity of items. It uses item features to recommend other items similar to what the user likes, based on their previous actions or explicit feedback.
- Content-based filtering does not require other users data during recommendation to one user.

Advantages:

1) The model doesn't need any data about other users, since the recommendations are specific to this user. This makes it easier to scale to a large number of users.
2) No cold start problem.
3) No sparsity problem
4) The model can capture the specific interests of a user, and can recommend niche items that very few other users are interested in.

Disadvantages:

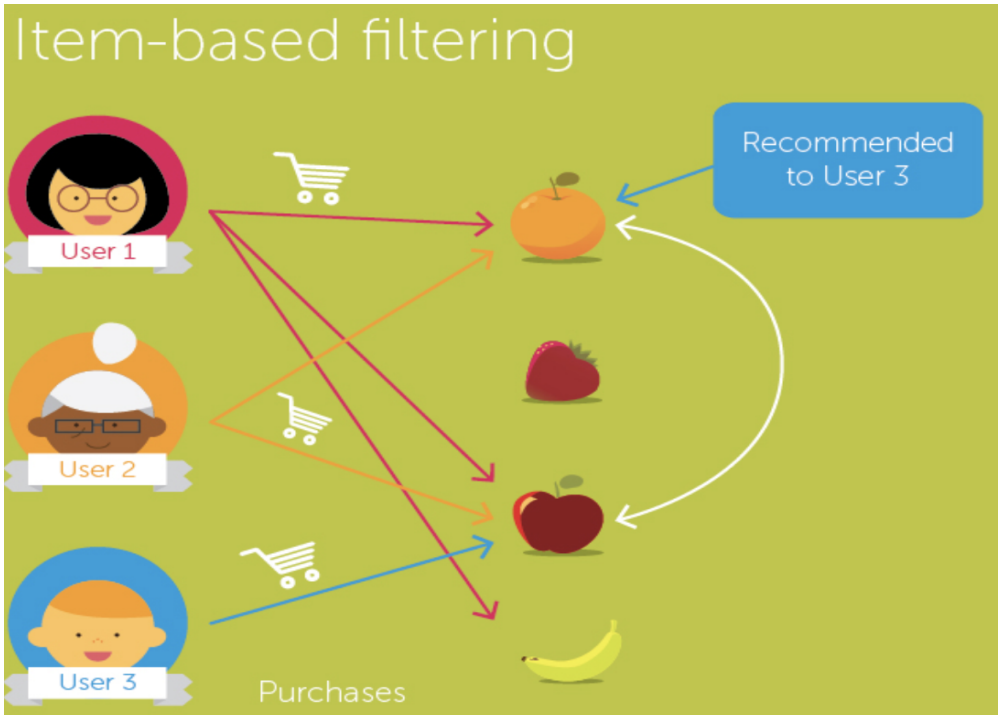
1) Since the feature representation of the items are hand-engineered to some extent, this technique requires a lot of domain knowledge.
2) It always recommends items related to the same categories, and never recommend anything from other categories.

3. Collaborative Filtering System

- To address some of the limitations of content-based filtering, collaborative filtering uses similarities between users and items simultaneously to provide recommendations.
- This system can filter out items that a user might like on the basis of reactions by similar users and makes recommendations based on user interactions.
- This filtering works on the assumption that people who like similar things have similar taste. There are different approaches we can adopt to implement CF recommender systems

3(a). Item - Item Based recommender System

- Item-item collaborative filtering, or item-based, or item-to-item, is a form of collaborative filtering for recommender systems based on the similarity between the items is calculated.
- “Users who liked this item also liked…”



Ex :

- User 1 bought Orange, Strawberry and Apple.
- User 2 bought Orange and Apple.
- User 3 purchases Apple so we calculate similarity between the fruits and recommend Orange.

To calculate similarity between two items, we use hamming distance.

- **Hamming Distance**: Hamming distance is a metric for comparing two binary data strings. While comparing two binary strings of equal length, Hamming distance is the number of bit positions in which the two bits are different.
  The Hamming distance between two strings, a and b is denoted as d(a,b).

  Ex: We have two strings in binary data

  Str1 = [0,0,1,0,0,1]

  Str2 = [0,1,1,0,1,1] .

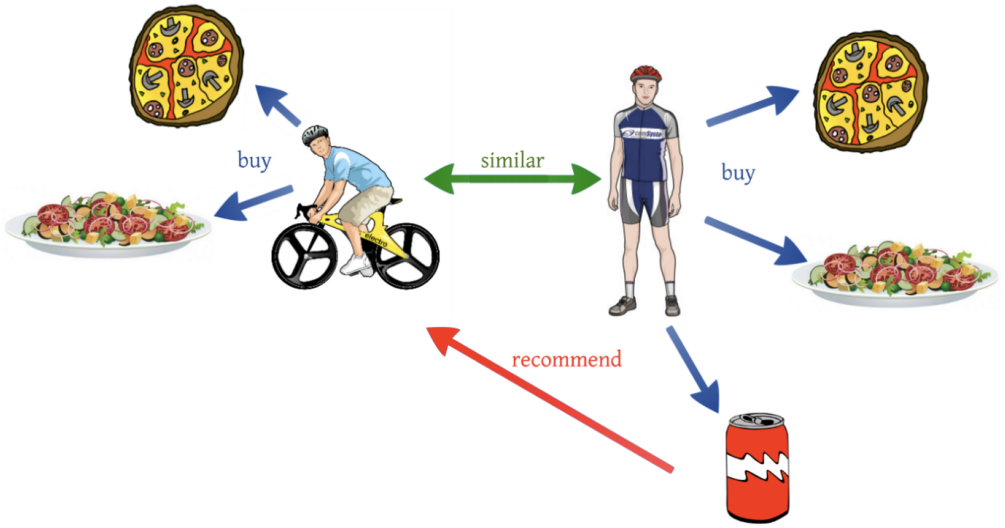
  So, there are two positions in which bits are different hence the distance will be 2.

  How Item-based works?

- Firstly, we compute similarities between items using hamming distance.
- Secondly, based on the computed similarities, items similar to already consumed/rated are looked at and recommended accordingly.

3(b). User - User Based recommender System

- User-user collaborative filtering, or user-based, or user-to-user, is a form of collaborative filtering for recommender systems based on the similarity between the users is calculated.
- “ Users who are similar to you also liked…”



Ex:

- There is user 1 who bought pizza, salad and soft drink and there is user 2 who bought pizza and salad. So, we calculate similarity between two users and recommend soft drink to user 2.

To calculate similarity between two items, we use euclidean distance or cosine-similarity.

- **Euclidean Distance**: The Euclidean distance is already familiar, the Euclidean distance is calculated between two 2-dimensional vectors x = (x1, x2) and y = (y1, y2) is given by:

$$Euclidean\ dist\ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Cosine Similarity**: Cosine similarity is the cosine of the angle between two vectors and it is used as a distance evaluation metric between two points in the plane.

  Cosine similarity ranges from -1 to 1.

  1 indicates the items are the same whereas -1 represents the compared items are dissimilar.

  $$Cosine\ similarity\ = \frac{a \cdot b}{||a|| \cdot ||b||}$$

How User-based works?

- Firstly, we compute similarities between users using euclidean distance or cosine similarity.
- Secondly, based on the computed similarities between the users, the items of similar users are recommended.

Drawbacks:

- Data Sparsity: In case of a large number of items, the number of items a user has rated reduces to a tiny percentage making the correlation coefficient less reliable.
- User profiles change quickly and the entire system model has to be recomputed which is both time and computationally expensive.
- To overcome these drawbacks we use an item - item based recommender system.

3(c). User - Item Based recommender System

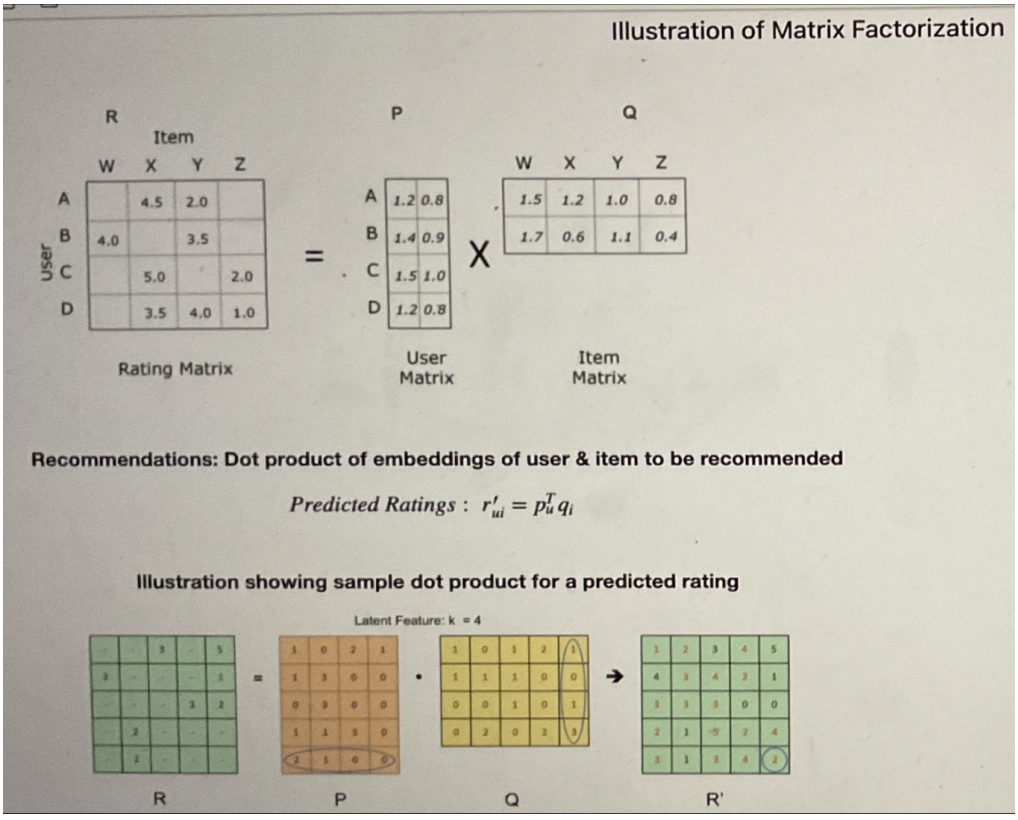
- User-Item collaborative filtering is a form of collaborative filtering for recommender systems based on the similarity between the users and items is

calculated. Data contains a set of users and items and ratings/reactions in the form of a user-item interaction matrix.

- If the user-item interaction matrix is sparse it is not easy to find the users that have rated the same items.
- To deal with this sparsity problem we use matrix factorization.

4. Matrix Factorization

- Matrix factorization is a way to generate latent features when multiplying two different kinds of entities. Collaborative filtering is the application of matrix factorization to identify the relationship between items and user entities.
- With the input of users' ratings on the items, we would like to predict how the users would rate the items so the users can get the recommender based on the prediction.
- Since not every user gives ratings to all the items, there are many missing values in the matrix and it results in a sparse matrix.
- Hence, the null values not given by the users would be filled with 0 such that the filled values are provided for the multiplication.



- Matrix P represents the association between a user and the features while matrix Q represents the association between an item and the features.
- We can get the prediction of a rating of an item by the calculation of the dot product of the two vectors.
- To get two entities of both P and Q, we need to initialize the two matrices and calculate the difference and we minimize the difference through the iterations. The method is called gradient descent, aiming at finding a local minimum of the difference.

$$\min_{U,I} \sum_i \sum_j \left(R_{ij} - u_i \cdot I_j\right)^2$$

- The user-item interaction matrix is very sparse since every user does not rate all items. The missing entries in the matrix would be replaced by the dot product of the factor matrices. Therefore, we know what to recommend to the users with the unseen movies based on the prediction.

Advantages of CF:

1) No domain knowledge necessary
    - We don't need domain knowledge because the embeddings are automatically learned.
2) Serendipity
    - The model can help users discover new interests. In isolation, the ML system may not know the user is interested in a given item, but the model might still recommend it because similar users are interested in that item.
3) The system doesn't need contextual features.

Disadvantages of CF:

1) Fails in case of cold start: the model requires enough of other users already in the system to find a good match.
2) Collaborative filtering models are computationally expensive.
3) It's a bit difficult to recommend items to users with unique tastes.

5. How to evaluate the performance of recommender systems?

5(a). Qualitative evaluation through metrics

- There is a common practice to keep a hold out set and evaluate the quality using human evaluators
- Serependity, Diversity, and Novelty are other attributes that are also important

for a recommender system

- Serependity: Ability of the model to help users discover new interests.

    If the ML system treats the user in isolation, it may not know the user is interested in a given item, but the model might still recommend it because similar users are interested in that item when it looks at aggregated preferences.

- Recommender systems that should suggest novel, relevant and unexpected items also help in diversification and popularity bias.

5(b). Quantitative evaluation through metrics

- **Precision at k:** Proportion of recommended items in the top-k set that are relevant

    Precision@k = (# of recommended items @k that are relevant) / (# of recommended items @k)

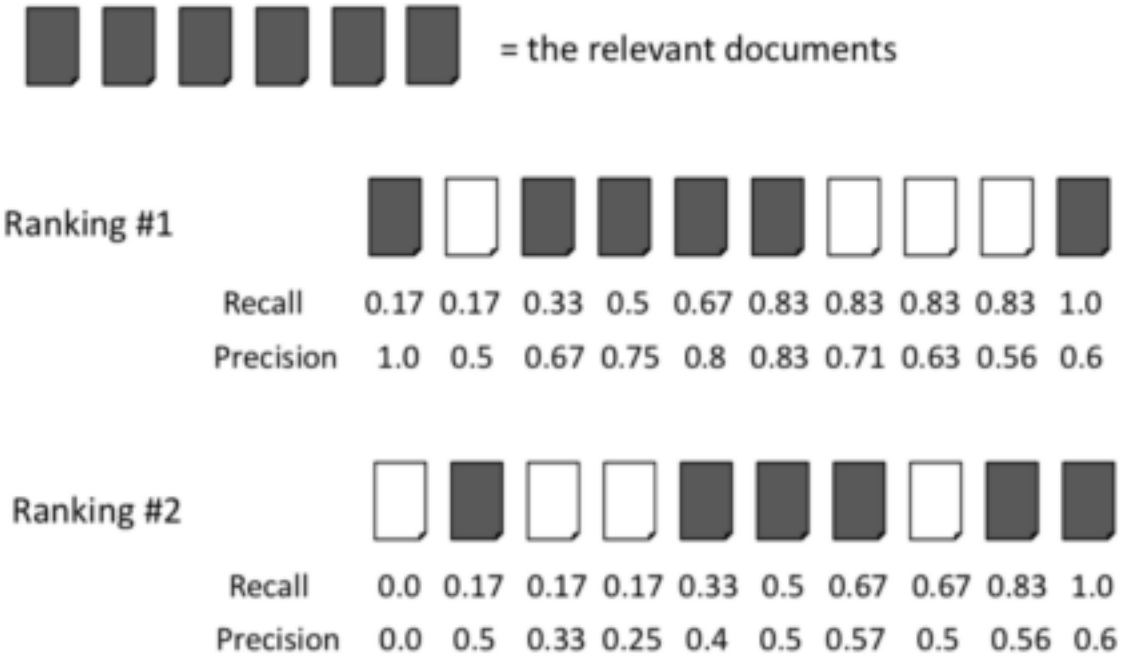
        Ex: If precision at 10 in a top-10 recommender problem is 80% - 80% of the recommender I make are relevant to the user.

- **Recall at k:** Proportion of relevant items found in the top-k recommenders

    Recall@k = (# of recommended items @k that are relevant) / (total # of relevant items)

        Ex: If recall at 10 is 40% in our top-10 recommender system. This means that 40% of the total number of relevant items appear in the top-k results.

- Illustration of precision and recall at 10 for two ranking models



- **Precision vs. Recall**

  Precision as a metric in recommenders optimizes the ability of how good we are in retrieving relevant items (already well rated) but too much precision lowers the ability to give unique and new recommenders (false positives).

  Optimizing for recall makes sense when the number of relevant items is less than recommended items. Here let us look at 100 recommend items & let us look at both precision and recall

- **Metrics for ranking:**

  Coverage, Hit Rate, and F1 are some other accuracy-related metrics for recommender systems MAP, DCG, NDCG, and MRR are some other metrics that also considers order (rank of items) while evaluating recommender systems: Please refer to this article:

  https://towardsdatascience.com/ranking-evaluation-metrics-for-recommender-systems 263d0a66ef54

Time series Analysis

Time series data

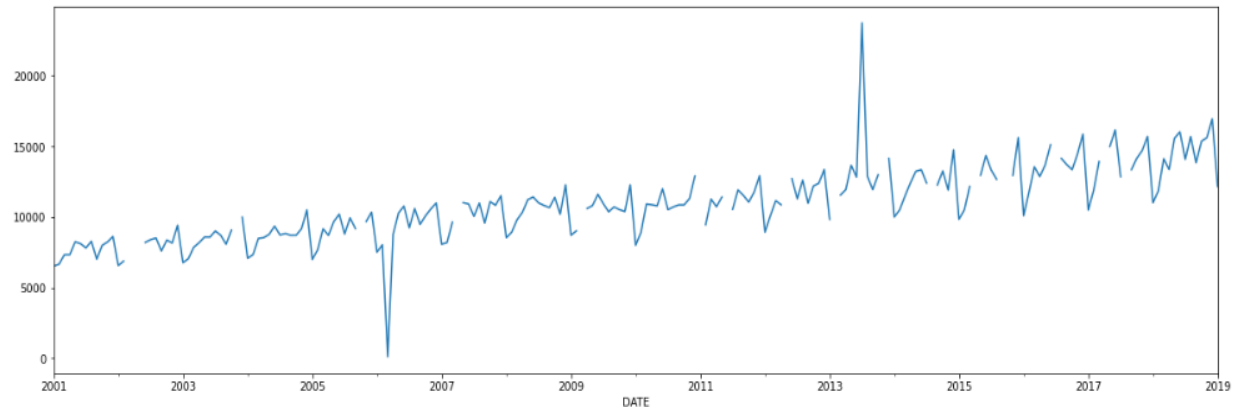
- A signal, indexed by an ordered timestamp is time-series data.
- For our data to be a time series, we need a minimum of two things
    - Date/timestamp (denoted as **t**)
    - One quantity (denoted as **y**)
- The timestamp(t) can be in days, weeks, months, years, or even seconds.
- Data observations like sales, revenues, inventories, etc are commonly expressed as a time series.

Forecasting

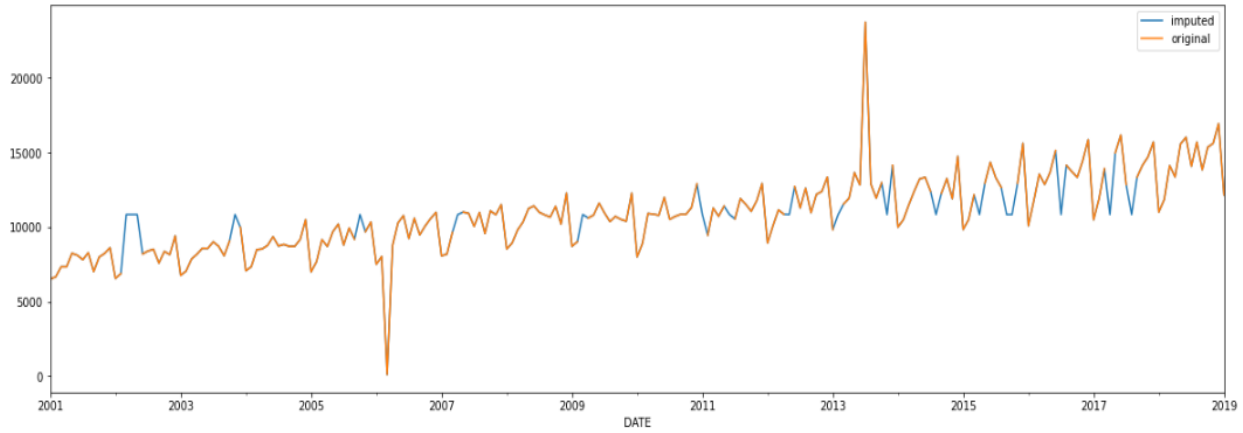
- Forecasting is analyzing historical data to predict future values. It is a supervised learning problem.
- It is more similar to regression in approach since we try to predict real-valued numbers.
- Given the historic values of a feature **x** till the present time **t** (x_1, x_2, x_3 … x_t), we need to predict the value of this feature for the future (x_{t+1}, x_{t+2}, ...).
- In forecasting, Unlike regression, we do not have a set of inputs and one output. Rather, we have a signal and we are looking at some past values to predict some future value.

Handling missing values

Let's consider the sales dataset:



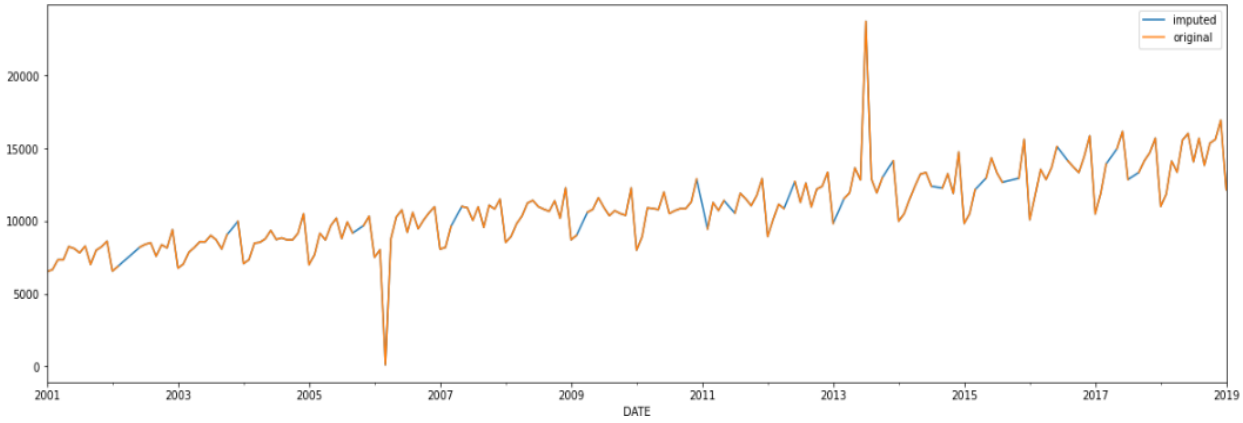
- One basic intuition is filling these missing values with the signal's mean or median. We will get the following plot after the imputation:



- We can observe that there are sharp increments and sharp decrements at some of the imputed places. This is because we imputed the local missing values with the mean of the entire data.
- Replacing with median would've also given the same result, as they have a similar value.

Linear Interpolation

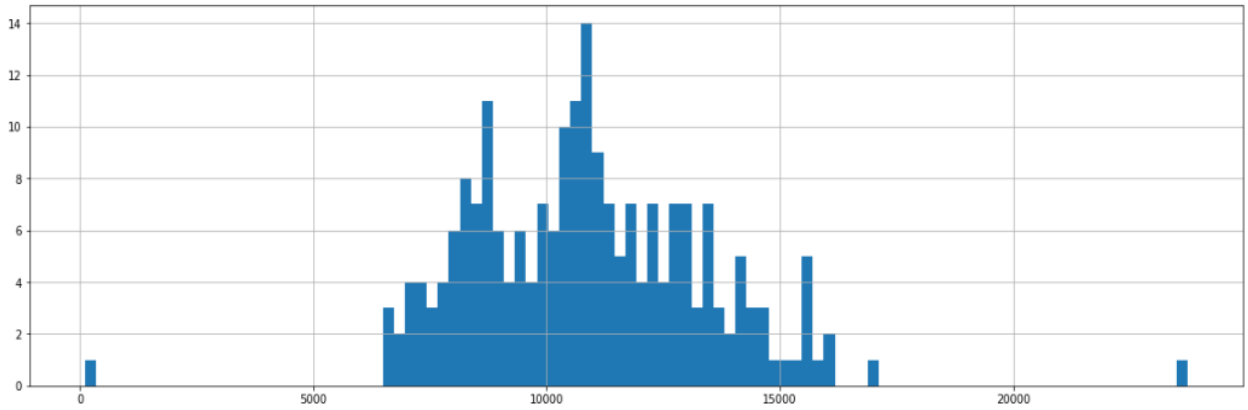
- It is a technique for handling the missing values. We take the average of the first point before and the first point after the missing value and fill the missing value with this average.
- It ensures that the missing values are not under or over-estimated by taking the average of the entire data.
- After using the linear interpolation for the imputation of missing values in the above data, we get the following plot:



- The imputed values fall within two finite points(preceding and succeeding points) because of which the imputed values don't seem to be forced.

Handling Anomalies

- Anomaly/outlier is an abnormal or unusual data point in the data set, which stands out of the data. It can simply be seen as a wrong entry in the data, and that can happen at times.
- Alternatively, it could be the correct entry but for a one-time event, which is not likely to repeat in the future. So, even if it's valid data, we would like to remove it from our training set, because we don't want our model to get biased by that one-time event that is not going to happen again.
- One of the best ways to identify the anomalies is by plotting a **Histogram**.

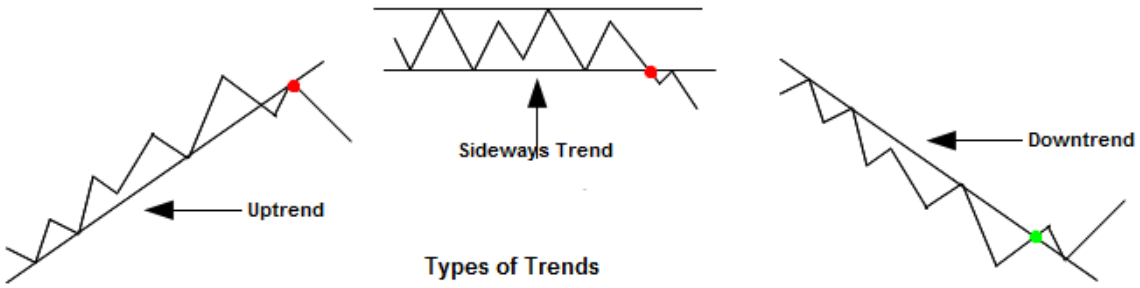


- From the plot, we can see the anomalies and we can easily decide where to cut the data, to get rid of anomalies.
- If the above histogram plot is more continuous, we use the concept of **quantiles** to deal with anomalies. We rule out observation as an anomaly if it is greater than 95 percentile, or less than 5 percentile.

Trend

- Trend can be thought of as the linear increasing or decreasing behavior of the series over a long period. The trend usually happens for some time and then disappears, it does not repeat.
- A trend can be uptrend, downtrend, or can be up and down, need not be a straight line.

- Trend line is a smooth predictable function that traces the trend of a time series, and can help us predict the time series indefinitely in the future.



Types of Trends

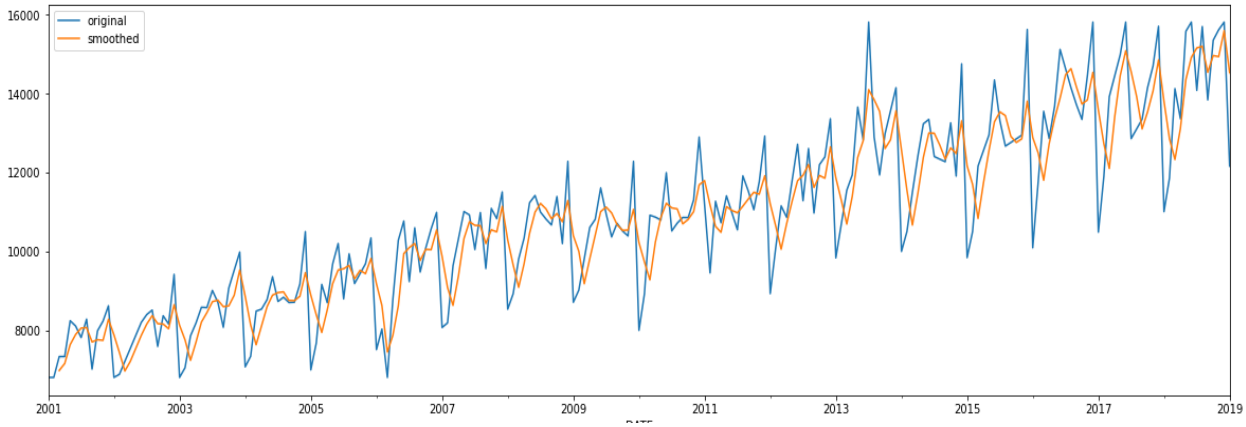
- Trend can be calculated by taking the rolling average (or moving average) over a long period or by just fitting a Linear Regression line on the points.

Moving average

- If we take the average of the last *k* data points in our series and use it to guess the next point at *t=k*. This approach is called the Moving Average.
  i.e.

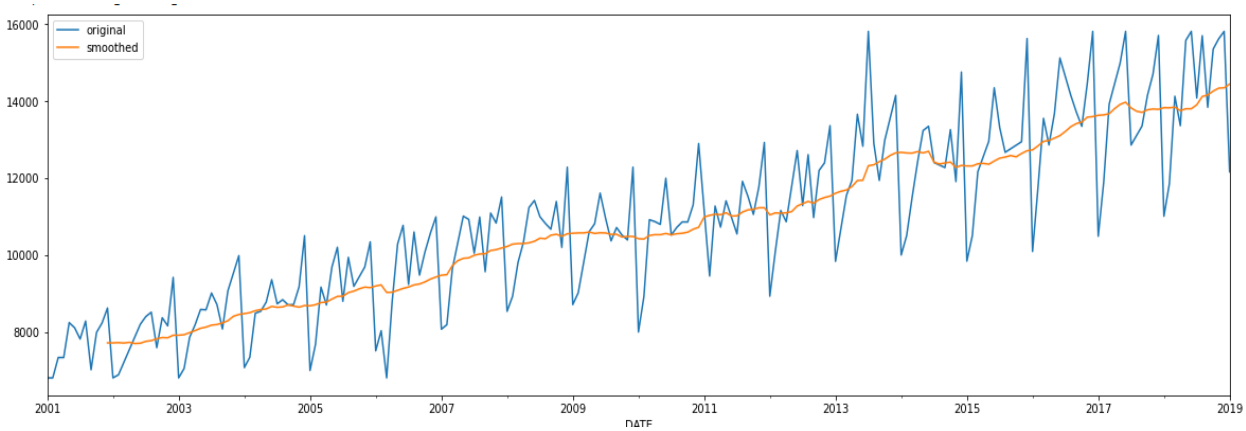
$$y_t = \frac{1}{k}\sum_{i=1}^{k} y_{t-k}$$

- The value of k acts as a hyperparameter, which we can be set based on what works best for us. It is also called the **window size**.
- The moving average line has been shown below.



Moving averages with k=3

- If we increase the window, the moving average curve gets smoother.



Moving averages with k=12

Centered moving average

- The average of n points before the current point, and n points after the current point is called the Centered moving average.
- We can't use this in forecasting as the future values are not available.

Seasonality

- Seasonality in time-series data refers to a pattern that occurs at a regular interval.
- Making copies of seasonality can fetch us our time series.
- A seasonal pattern occurs when a time series is affected by seasonal factors such as the time of the year or the day of the week.
- The steps for calculating the seasonality are:
    - Calculate the trend and subtract it from the original time series.
    - From the result, take the average across the period. If it is a monthly time series for 4 years, then you have 4 Jan, 4 Feb, 4 Dec, etc. Take the avg of all Jans, Febs, etc.

Time series decomposition

- There are three important components which we want to decompose our signal into.

- Trend - general movement over time
- Seasonality - behaviors captured in individual seasonal periods
- Residual - everything not captured by trend and seasonal components

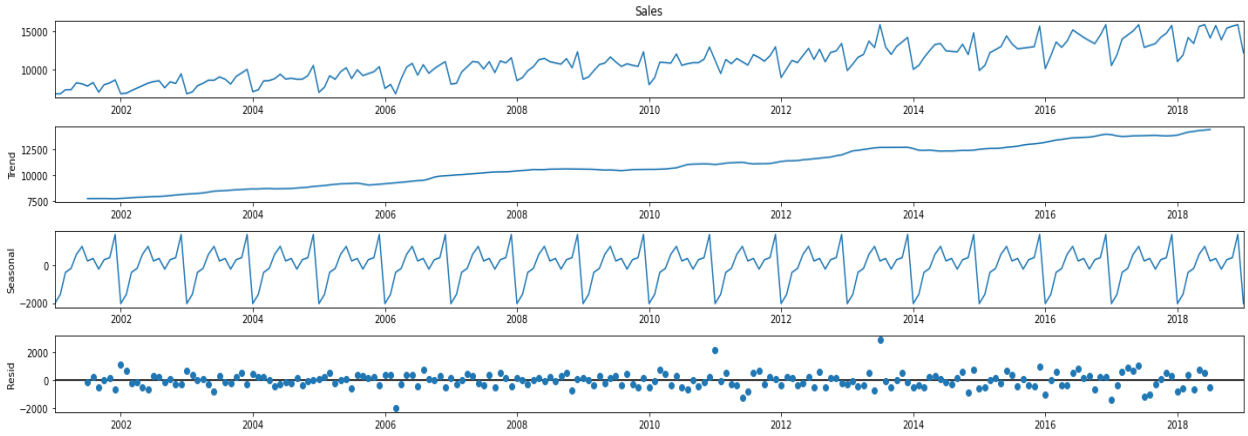
- Time series decomposition is formulated in two different techniques combining these three different components:

Additive

- $$y(t) = b(t) + s(t) + e(t)$$

  Where, b(t) -> trend of signal
  s(t) -> seasonality
  e(t) -> error term

- For the sales data, decomposition would be in the following way:



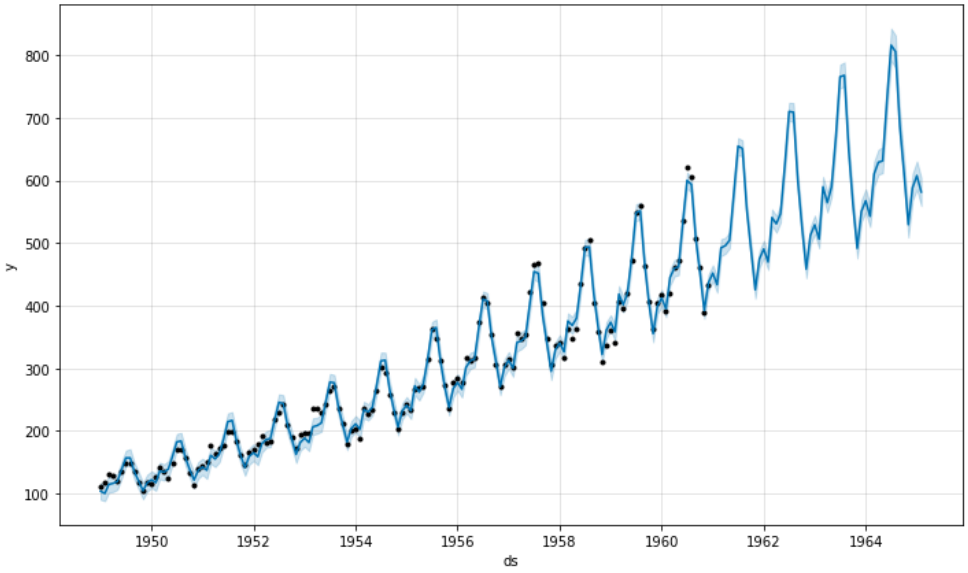
- The error term e(t) cannot be estimated since it is based on the real values that we have but can be computed using the same formulation.

$$e(t) = y(t) - b(t) - s(t)$$

- If the trend and seasonality components obtained are a good estimate, then the errors would be small and would be scattered around zero.

Multiplicative

- It is a time series in which the amplitude of the seasonal component is increasing with an increasing trend.



Multiplicative seasonality Example

- In multiplicative seasonality, we do not obtain the time series by adding the trend and seasonality components with each other. Instead, if we multiply them, then we obtain a time series in which the amplitude of the seasonal component is increasing with an increasing trend.
- Therefore, the **multiplicative seasonality decomposition** is written as

$$y(t) = b(t) * s(t) * e(t)$$

$$e(t) = \frac{y(t)}{b(t) * s(t)}$$

Train test split

- For time series data, we do not perform a random shuffle for train and test data split.
- Random shuffling on time series data would be meaningless since it is a collection of data over time and we look at the past data to predict future values.

- Therefore, we do a **time-based splitting**. For example, for our sales dataset, out of the 18 years of data, we decide to train on 17 years of data and use the last year, i.e. the last 12 values for test data.

DATE	Sales
2018-02-01	11852.0
2018-03-01	14123.0
2018-04-01	13360.0
2018-05-01	15576.0
2018-06-01	15809.4
2018-07-01	14080.0
2018-08-01	15697.0
2018-09-01	13838.0
2018-10-01	15351.0
2018-11-01	15615.0
2018-12-01	15809.4
2019-01-01	12160.0

Test data

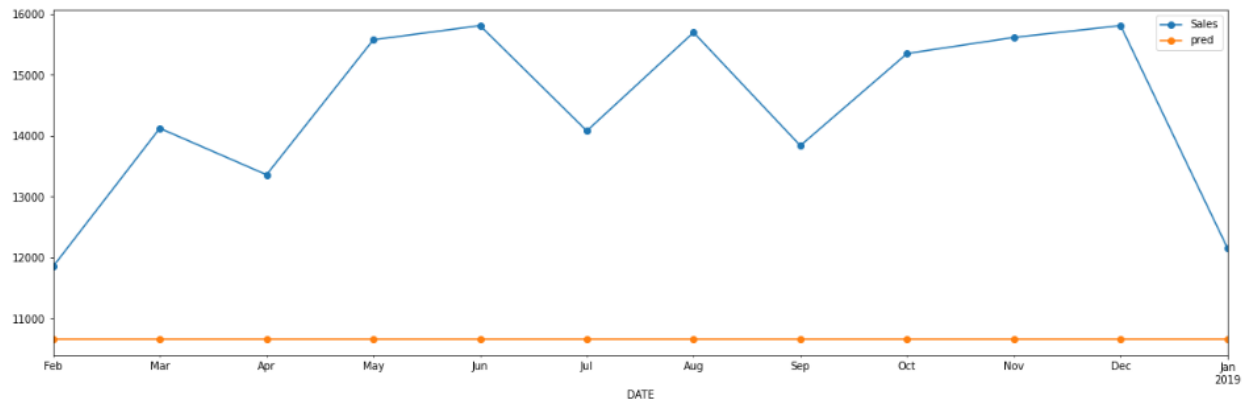
Simple Forecast Methods (not-so-intelligent approaches)

1. Mean forecast

- We take the average of all past *k* values of the time series to forecast the value at time *t=k+1*.

$$x_{k+1} = \frac{x_1 + x_2 + ... + x_k}{k}$$

```
MAE : 3786.031
RMSE : 4025.906
MAPE: 0.255
```

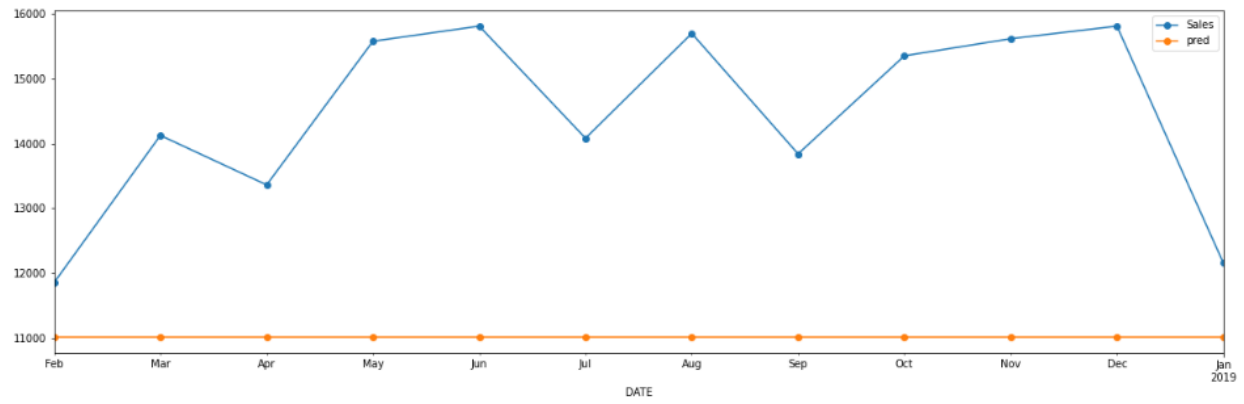


- The MAPE metric is showing that we have a 25.5% error, concerning its value. Thus, it is not a good model.
- A similar argument can be made for the median. So, we can rule out using the median of the entire data as future forecast values.

2. Naive Approach

In this approach, we simply take the value of the series at time $t=k$ and forecast that for the future.

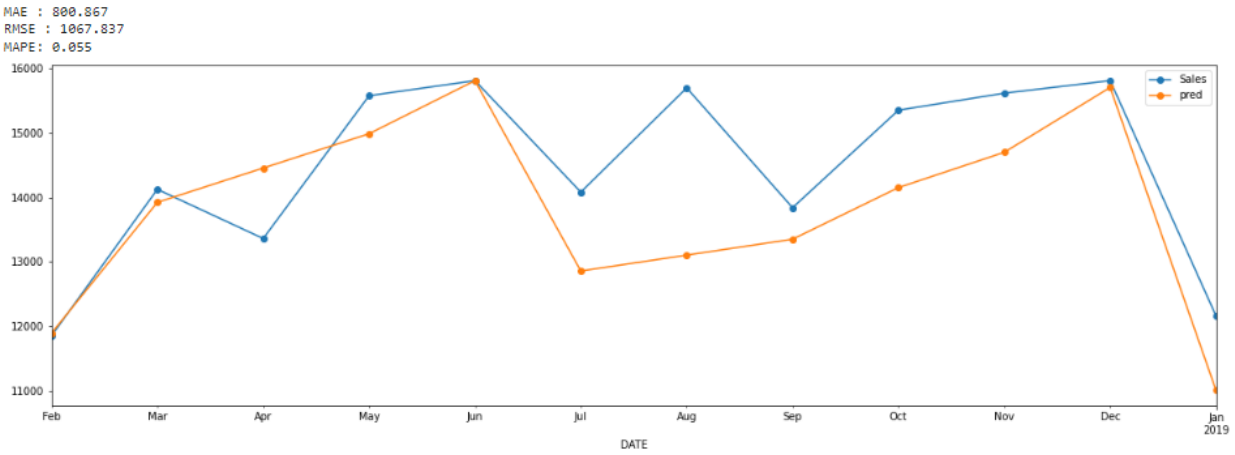
```
MAE : 3434.233
RMSE : 3697.005
MAPE: 0.23
```



- This model's performance is slightly better(23%), but we can see it's still really bad.
- But if we get a different value for series at t=k, this would change all the future forecasts we had made earlier. Hence, this is not a very intelligent approach.

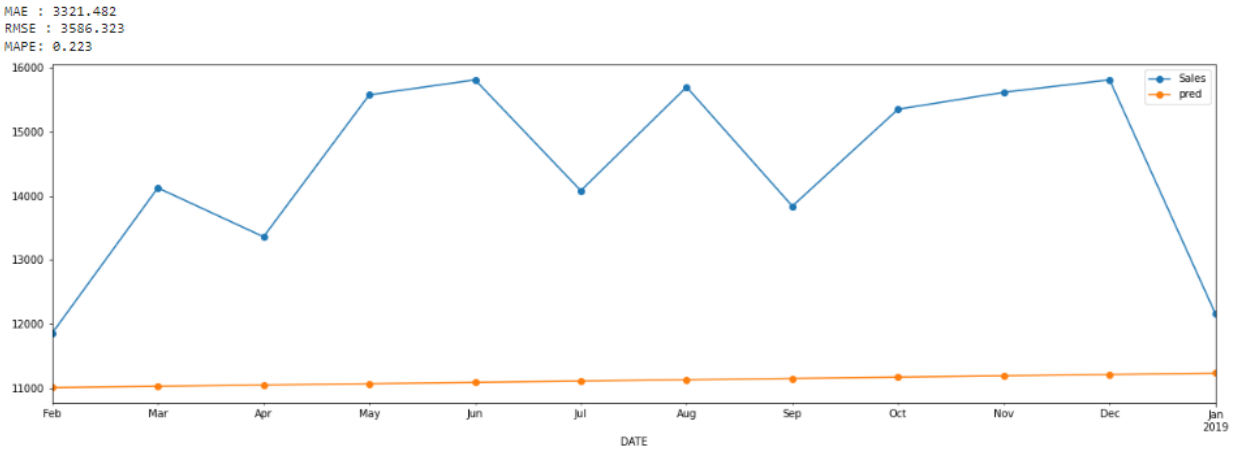
3. Seasonal Naive Forecast

- It is a smarter approach to optimize the Naive approach.
- We set each forecast to be equal to the last observed value from the same season (e.g., the same month of the previous year).
- This way, we essentially forecast the future values to be the same as last season.



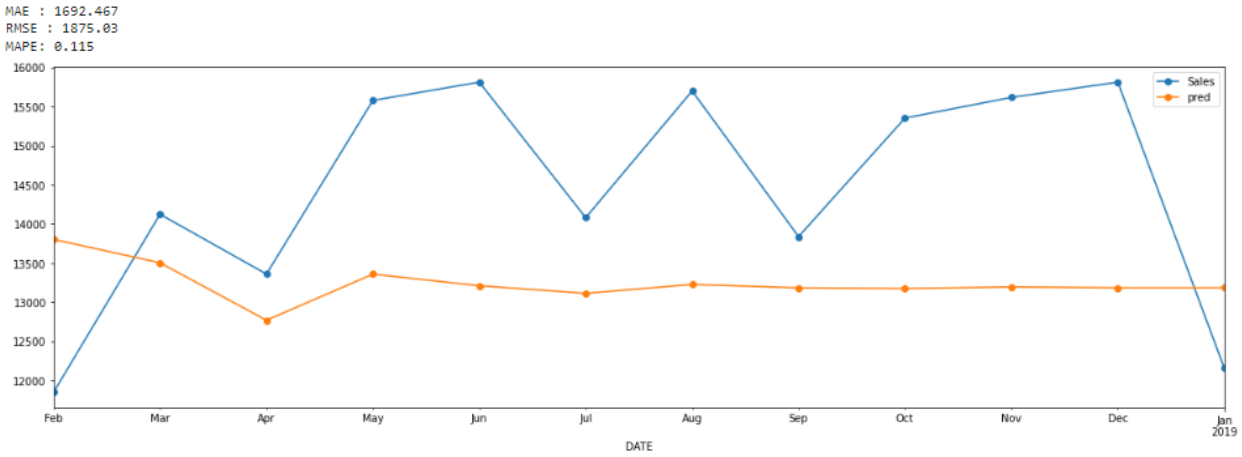
4. Drift Method

- In this method of forecasting, we take the first and last point of the data and draw a straight line between those points, and then extend this line into the future to get a forecast (**Linear extrapolation**).
- So, instead of just picking up some value from the past, we let our values increase or decrease over time.
- Depending on what the last point is, the drift (slope) may change significantly. thus, this is highly sensitive to the last value available.



5. Moving average

- The average of the last k data points in our series is used to forecast the next point in this approach.
- For our sales data, our model has an 11.5 % error when using this model.
  This is much better than the other simple models we saw.



- These were some simple models that do not give any smart forecasts.

Smoothing-based methods

1. Simple Exponential Smoothing (SES)

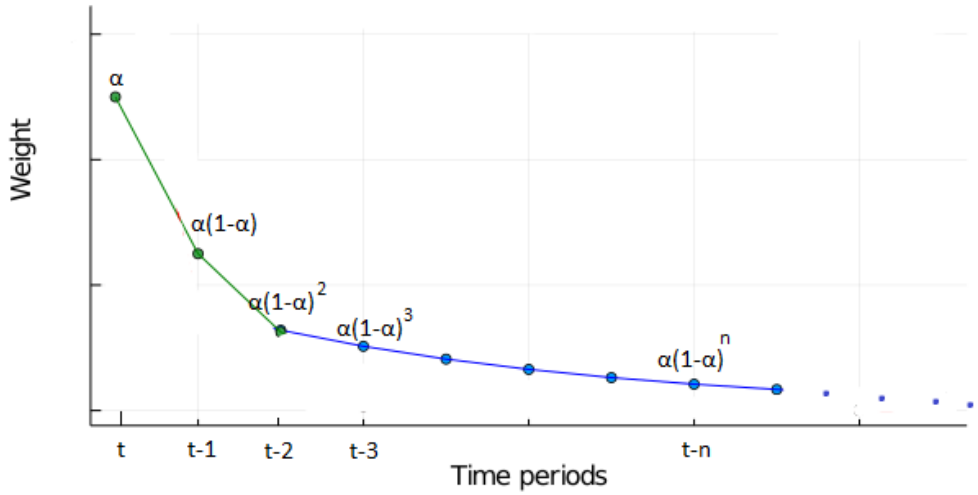
- The key idea of this method is to keep some memory of the entire time series, but also, we want to give more value to the recent data and less value to the past value. This forms a decaying trend.
- Let's consider the weight we assign to the recent most value as α.
  α is called the **smoothing parameter**.
  So, our forecast at time *t* for the time t+1 is:

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t$$

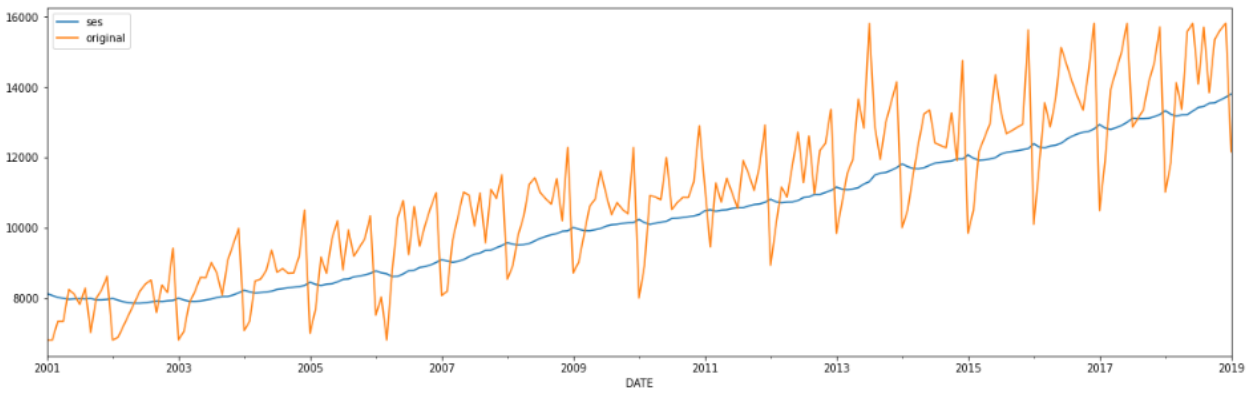
- The above formulation is recursive and expands in the following form:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2\hat{y}_{t-2} + \dots$$

- We can observe from the formulation that the weights are exponentially decaying. Therefore, we give more weightage to the most recent values, and this weightage keeps decreasing for earlier values.



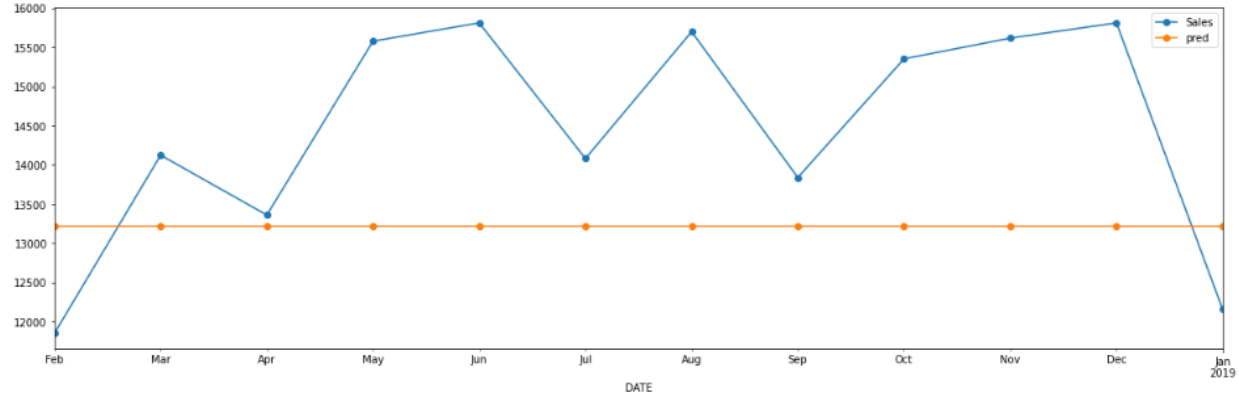
- Let's fit the SES model on the sales data:



Unlike the moving averages, it does not have the offset at the beginning and end, because this method is initialized properly.

- The forecast plot of this model on the sales test data is given below:

```
MAE : 1621.347
RMSE : 1830.599
MAPE: 0.109
```



- The advantage of the above forecast is that the **level** of the forecasted values is right.
- However, the forecast is a completely straight line This is because we don't have the previous actual value available for horizon > 1. So the current forecast is used for all the next values.
- The prediction is a straight line, but the error is 10% which is less than the error of the moving average.
- Disadvantage of this model is that it is missing both trend and seasonality
- Now, we have the right levels, if we can predict the trend and seasonality right, we should get a good forecast.

2. Double Experimental Smoothing (DES)

- In this method, we incorporate the trend of the entire time series in the SES formulation to forecast future values.
- The weights are assigned to the trend value also and this forms an exponentially decaying series. Hence this is called Double Exponential smoothing (**aka Holt's method**). So basically, we are doing exponential smoothing on trend too.
- The formulation of DES is as follows:

$$\hat{y}_{t+h} = l_t + hb_t$$

where

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$$

l_t is called the **level** of time series at time t.

- Calculation of the trend value:
    - Slope of a curve for Δt=1 is given as

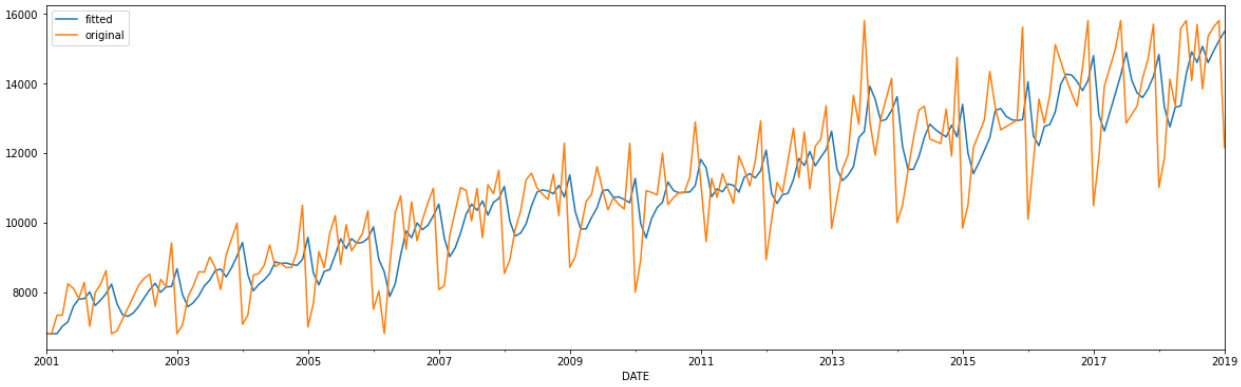
$$\frac{\Delta y}{\Delta t} = y_t - y_{t-1}$$

This slope value is equal to the trend of the series.
By plugging this in, we get,

$$b_t = \beta * (l_t - l_{t-1}) + (1 - \beta) * b_{t-1}$$

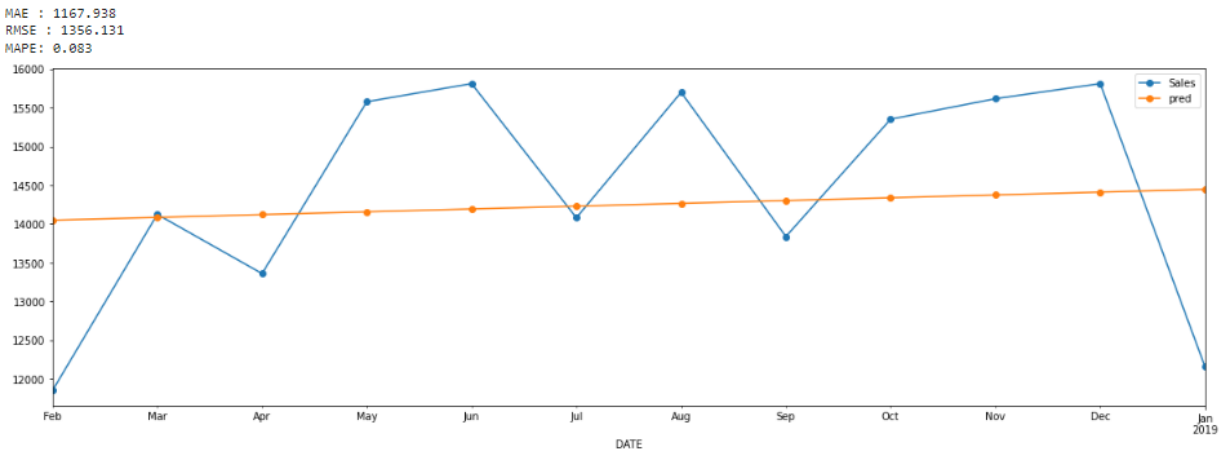
where, (l_t - l_{t-1}) is representing the current slope of the curve and b_{t-1} represents the previous slope.

- β is a parameter that needs to be tuned while training the model.
- Let's fit the DES model on sales data:



It is a better fit than Simple exponential smoothing.

- The performance of the DES model on the test set:



- The error (8.3%) is less as compared to the SES model (10%).
- Disadvantage of this model is that it does not consider the seasonality of time series.

3. Triple Exponential smoothing (aka Holt-Winters Method)

- Triple Exponential Smoothing is an extension of Double Exponential Smoothing that explicitly adds support for **seasonality** to the univariate time series. The seasonality value of the entire time series is also incorporated in this model.
- Upon incorporating the seasonality, our equation becomes,

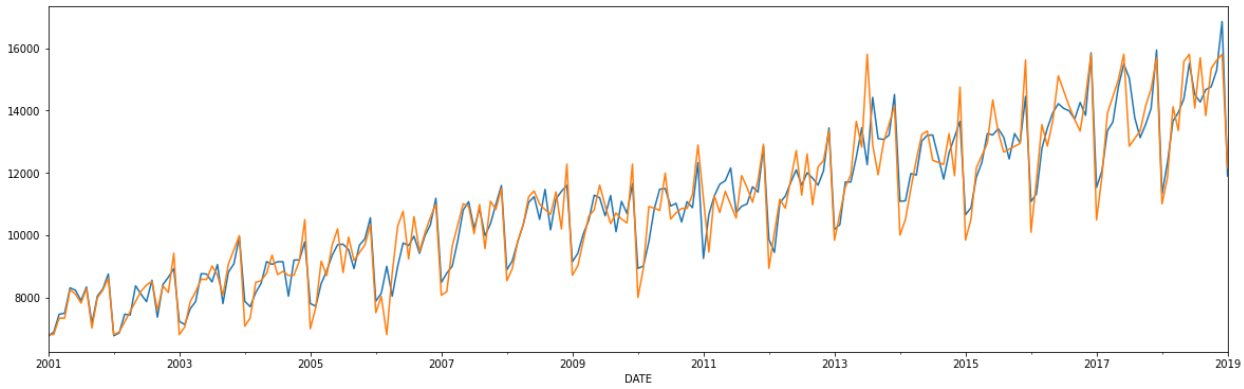
$$\hat{y}_{t+h} = l_t + hb_t + s_{t+h-m}$$

where, **m** -> frequency of the seasonality
Therefore, s_{t+h-m} is representing the smoothed seasonality.

Also,

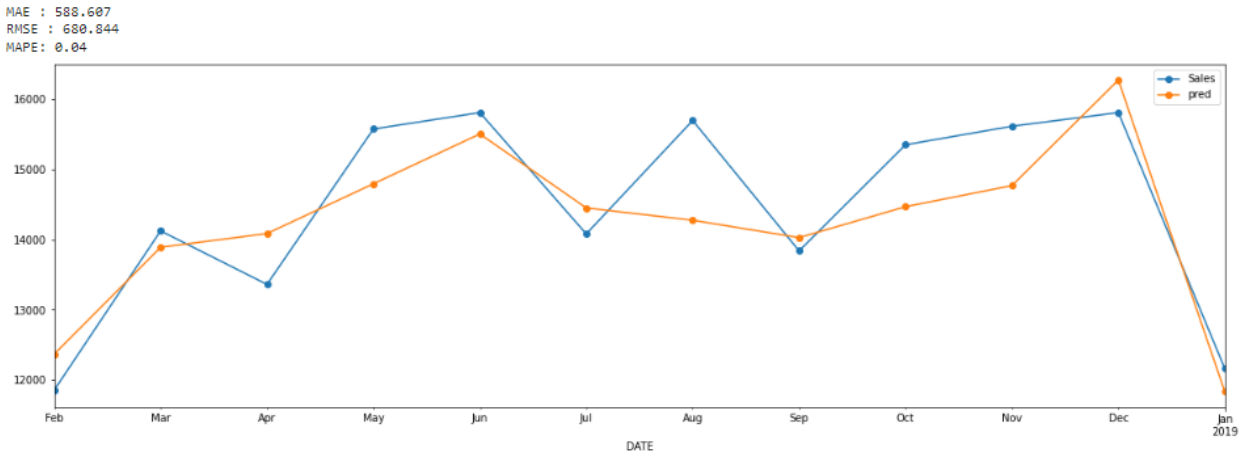
$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$$
$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$
$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$$

- Let's fit the TES model on sales train data:



It can be clearly observed that this model captures more information than Double Exponential smoothing.

- Let's look at the forecasts of the TES model on the sales test data:



We can clearly see that this model has a better performance as compared to SES and DES.
The MAPE error is only 4% now.

- We can take a mixture of additive and multiplicative models. There is no rule for which model (multiplicative/additive) to use when. We need to try and see which performs better.

Stationarity

- For a time-series model to be stationary, the parameters (like mean, variance, amplitude, and frequency) of the models should **not be dependent on time**.
- Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times.

- **For example** Heartbeats (mean=0; standard deviation=1) are stationary — it does not matter when you observe it, it should look much the same at any point in time.
- In general, a stationary time series will have no predictable patterns in the long term.

- If a model's parameter varies over time, then there is a complex relationship that needs to be modeled, which all models are not able to account for.
- Many models assume the series to be stationary to be able to give useful results.
- So, either we want to have a stationary time series, or convert to it. Categorizing a time series by just looking at it can be a little subjective.

Dickey-Fuller Test

- There is a **statistical** method called the **Dickey-Fuller test**, which is designed for testing for stationarity.
- It fits an auto-regressive model and checks if it worked or not. If it did, then that means it was a stationary time series.
- There is a complicated mechanism to it. We don't need to know how it works. Just need to be aware of this test, as it can be handy.

- **How to implement Dickey-Fuller Test?**
    - We can find this as a built-in function under the **statsmodels** library as sm.tsa.stattools.adfuller().

- **How do we interpret the result of the Dickey-Fuller test?**
    - This test returns the **p-value**. For a time series to be stationary, the **p-value** should be less than 0.05

Converting a Non-stationary Time Series to a Stationary Time Series

- As per basic intuition, if we remove trend and seasonality from our time series, it should become stationary.
- If it's still not stationary, then maybe there's still some seasonality or trend component left in the series.
- Ideally, the trend gets removed in one step. However, seasonality can take multiple steps to be removed.
- These processes are called **Detrending** and **Deseasonalising** respectively.

De-Trending

- The trend-seasonality decomposition that a time series can be written as:

$$\hat{y}_t = m * x_t + c + s(t) + r(t)$$

  Where; $m * x_t + c$ represents the trend component.

- To remove this, we can **differentiate** the above equation concerning time t. This way, we'll get a stationary time series. This gives us:

$$\frac{d\hat{y}_t}{dt} = m,$$

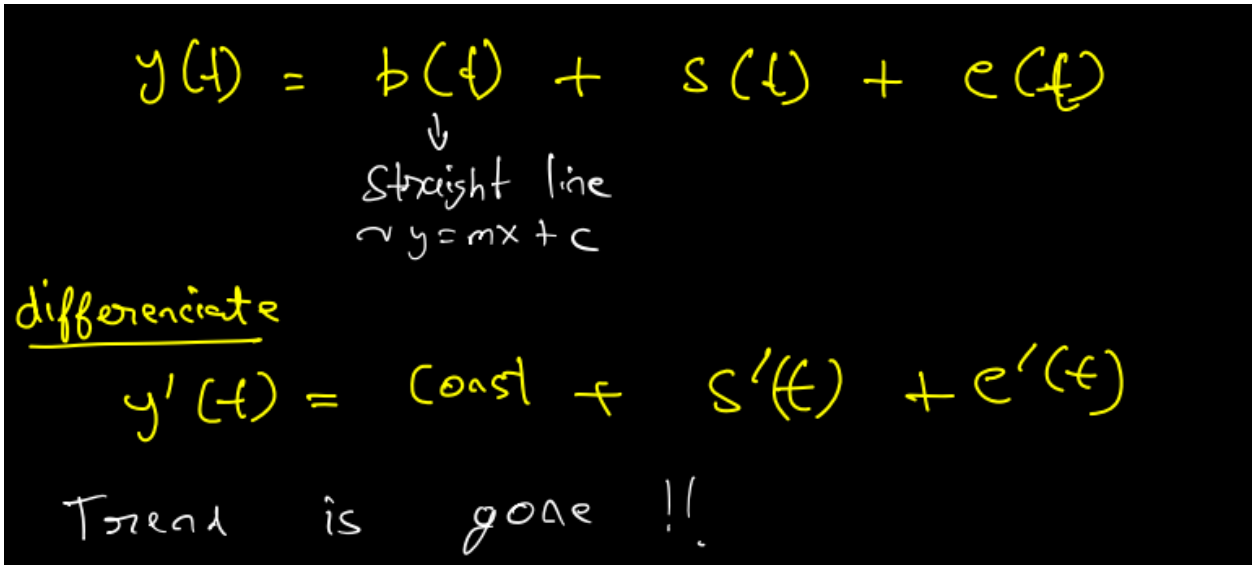
$$\frac{d\hat{y}_t}{dt} \approx lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}$$

- The minimum value of Δt that we have is 1, since we're talking in terms of time, and our minimum step is 1 month. So the equation becomes:

$$\frac{d\hat{y}_t}{dt} \approx \frac{y(t+1) - y(t)}{1}$$

- This process is called **differencing**.
- **NOTE:**
  - Differencing gives a good approximation of differentiating

- Differentiating gives us a Detrended time series.

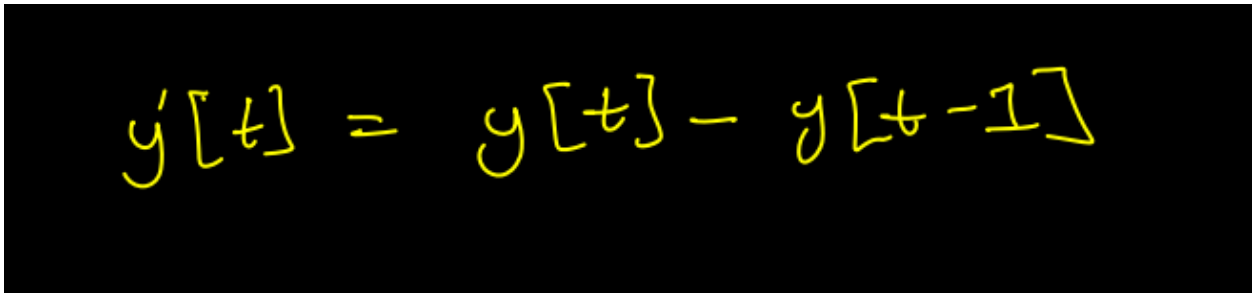


- If the time series has a non-linear trend, then we'll have to differentiate it multiple times, in order to finally achieve a stationary series.

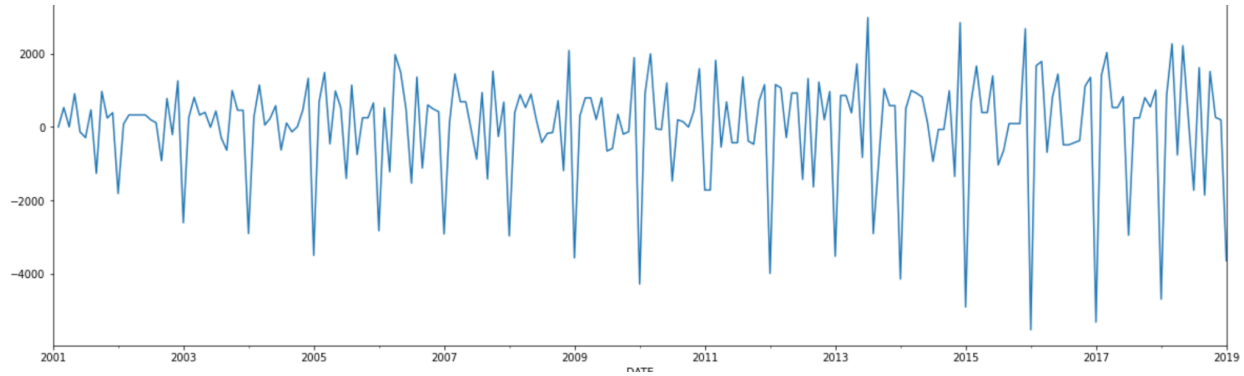
- **NOTE:**
  - If the trend is an exponential function, then we'll not be able to convert it into stationary.
  - This is a very rare case.
  - Some exponential functions can be approximated by polynomials. In that case, we differentiate this polynomial to obtain a stationary time series.

- For applying differencing, a new series is constructed where the value at the current time step is calculated as the difference between the original observation and the observation at the previous time step.

$$value(t) = observation(t) - observation(t-1)$$



- For this, We use the **diff()** method of **pandas**.
- This method calculates the difference between a Dataframe element compared with another element in the Dataframe (default is an element in the previous row, as the default value is 1).

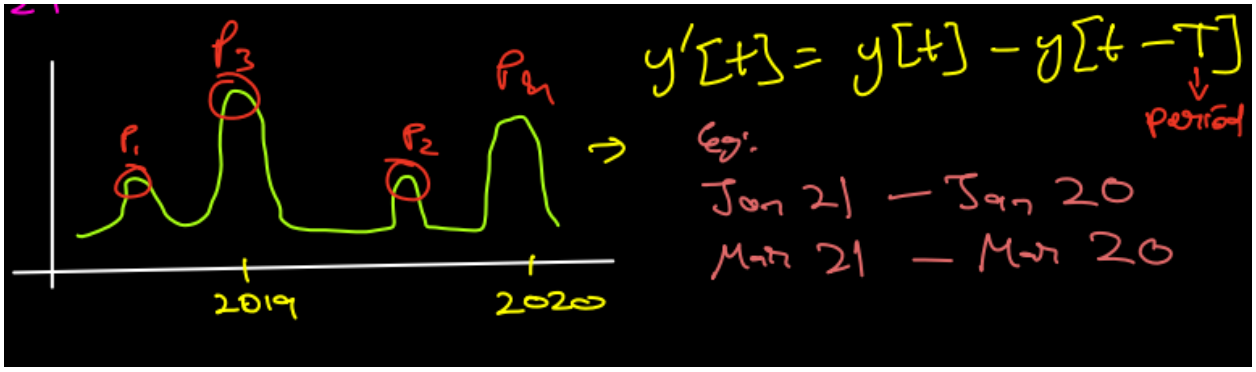


De-Seasonalising

- We can use differencing, but instead of subtracting from the last point, we need to take a difference from the last mth point, where m is the period of seasonality of the series.

$$\Delta y = y_t - y_{t-m}$$

- This is called **m-differencing**.
- If there is a seasonal component at the level of one week, then we can remove it on an observation today by subtracting the value from last week.

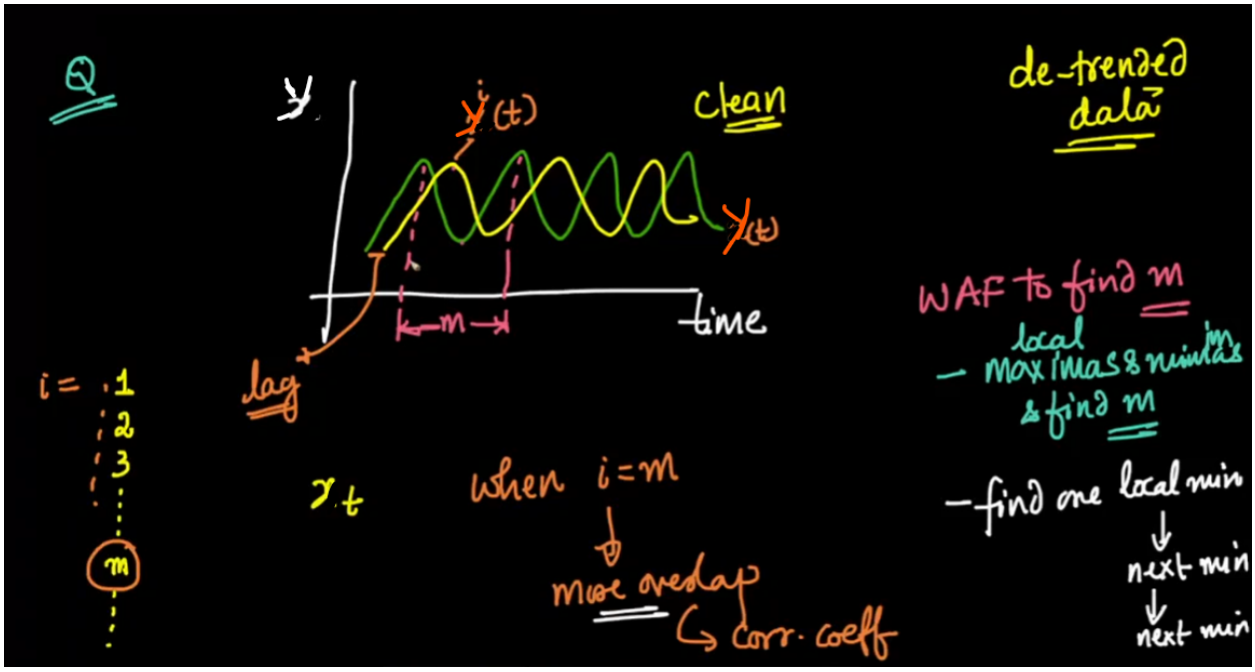


Autocorrelation

- One approach to find the value **m** is that we find local maxima and minima in the time series, and try to analyze the intervals at which they are observed.
    - This approach makes sense.
- Another approach would be; given a time series $y(t)$,
    - What if we consider another time series where we introduce a **lag** of 1, i.e. **shift the series** by 1 unit of time: $y^{`}(t)$
    - We can then find the **correlation coefficient** between these two time series: $y(t)$ and $y^{`}(t)$.
    - Similarly, we find the correlation between the original time series $y(t)$ and a time series lagged by i units: $y^{i}(t)$, where;

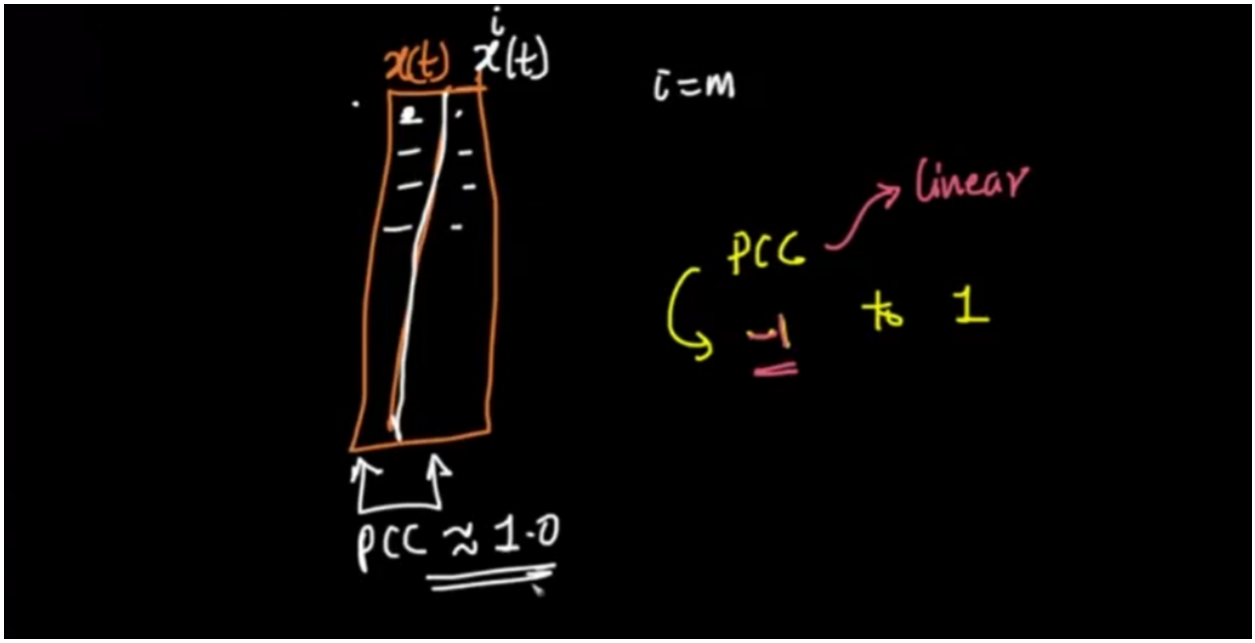
      i=1,2,3,... (i represents lag)

    - In doing so, we would find a value of i, where the lagged time series **roughly overlaps** over the original series.



- Then, we create a table containing: $y(t)$ and $y^{i}(t)$ time series values
- When ***i=m,*** (the lagged time series roughly overlaps over the original series), the Pearson correlation coefficient would be very close to 1.

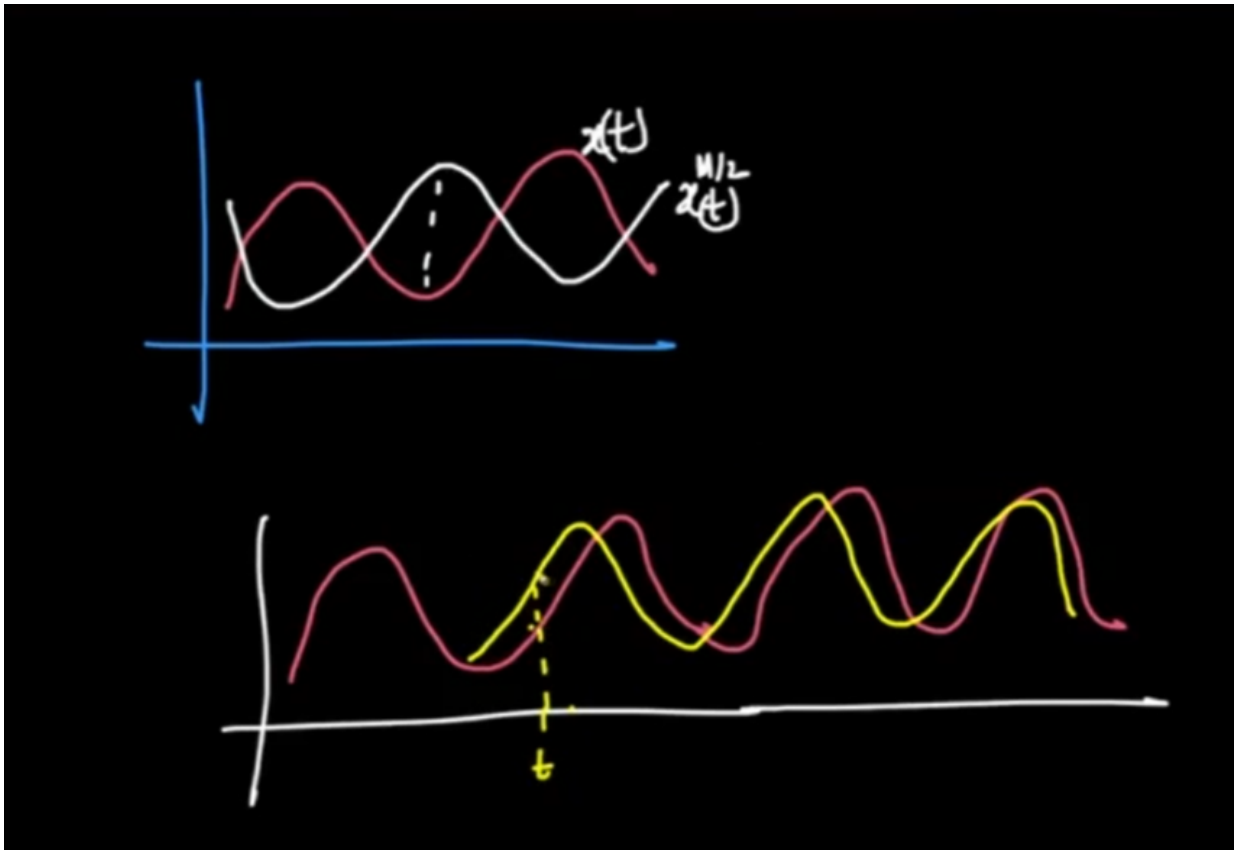
- This value of i would indicate the optimal value of m.



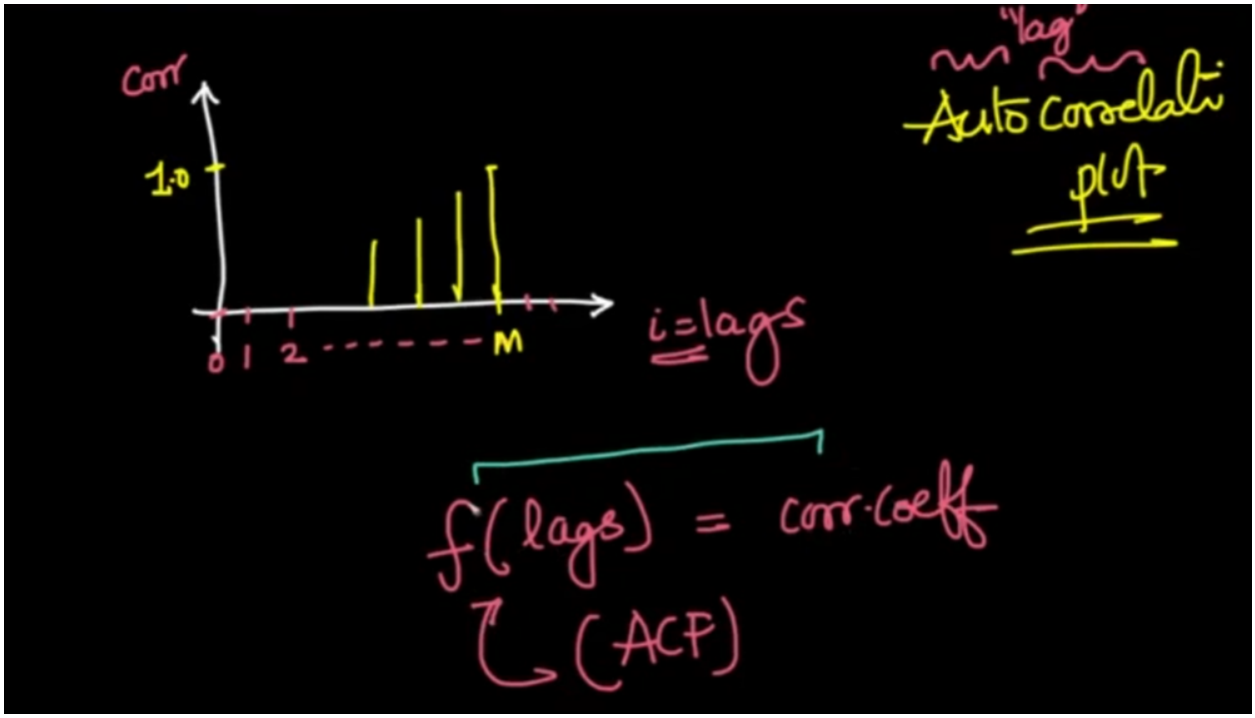
- **What are the advantages of using the concept of correlation for finding the optimal value of m?**
  - Easily interpretable
  - Value ranges from -1 to +1
  - Captures linear relationship

Q. What will the plot between the correlation coefficient and lag (i) look like?

- At $i=m$, we would get a correlation value very close to 1.
- At $i=m/2$, as the value of lagged time series increases, the value of the original series decreases, giving us a **strong correlation.**
- For a value of i that is even close to m, though the correlation value would not be as strong as at m, it would be relatively strong.

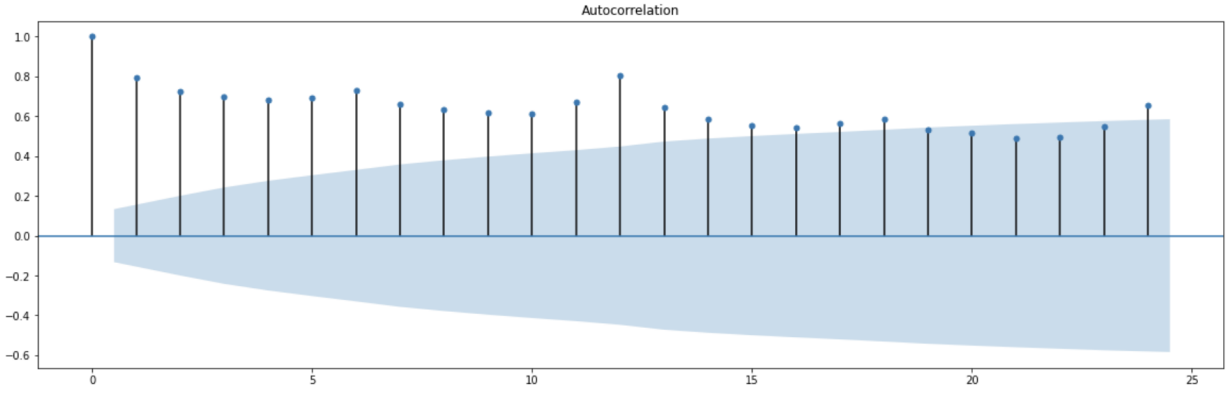


- So, the final plot between lag value (i) and correlation coefficient would look something like this:-

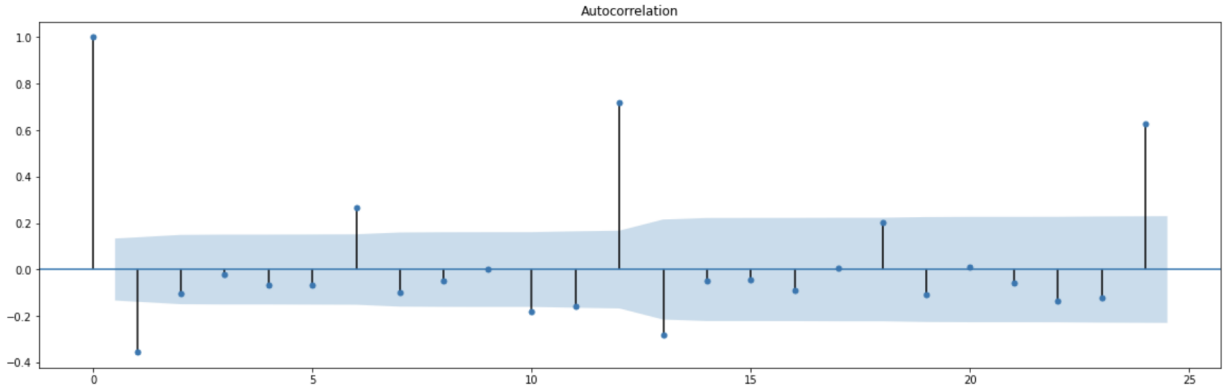


- This plot is called an **Autocorrelation Plot** because we are computing the correlation of the series with itself with various values of lag.
- This can be written as a function also. That is called **Autocorrelation Function (ACF)**.

ACF Plot of Trended Series:



ACF Plot of De-Trended Series:



- After detrending the series, we can see the correlation of the series is high at the period of 12 and 6. This indicates there is some seasonality.
- Besides that, the correlation values are random and small.
- On removing the trend, we're **able to capture negative correlations** between the original series and lagged series with more ease.

- The blue color highlighted part is the **confidence interval** which gives the significance level of the correlation
- If the correlation is higher or outside of the blue highlighted area, that can be considered as a highly significant value.

- So the major values of lag we consider to study the seasonality pattern here are:-
    - Correlation at lag=1
    - Correlation at lag=6
    - Correlation at lag=12
    - Correlation at lag=24

- Other points lie within or very close to the blue shaded region of confidence intervals.
- This means that we are not very confident about these correlation values
- The confidence interval increases as we consider time series that lagged more

Partial Autocorrelation

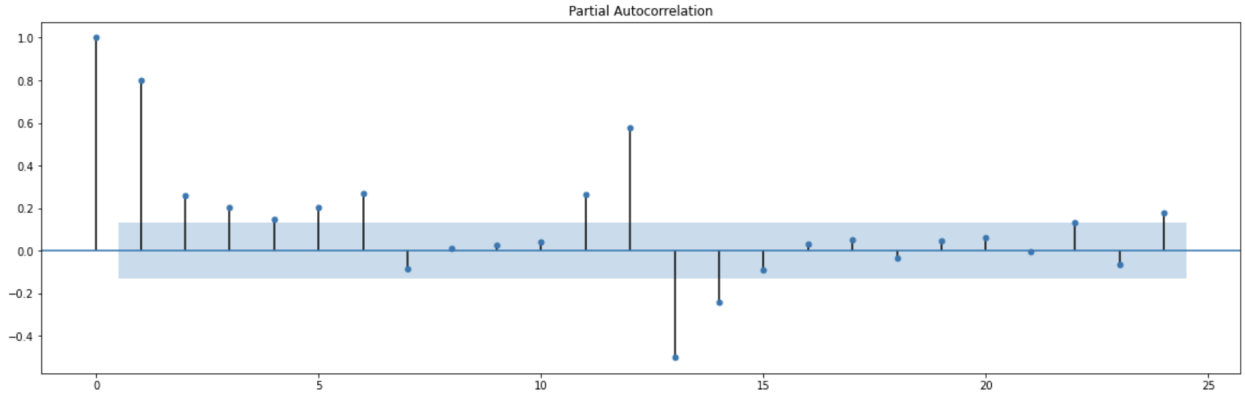
- This is similar to AutoCorrelation with only a small difference.
- We try to find a relationship between the original time series $y(t)$ and time series lagged with i steps $y^i(t)$, where i=1,2,3,..., to find the optimal value of m.
- The difference is that: **All intermediate/indirect correlations are removed**.
    - The correlation between observations at successive time steps is a linear function of the indirect correlations.
    - These indirect connections are eliminated using the **partial autocorrelation function (PACF)**.

For example

- When we're considering the correlation between $y(t)$ and say, $y^{12}(t)$,
    - Then, we do not want this correlation value to get corrupted by the correlations when i=1,2,3,...,11 (i.e. the intermediate correlations)

- The partial correlation for each lag is the **unique correlation** between the two observations after the intermediate correlations have been removed.
- This is also known as **Conditional AutoCorrelation**.

PACF Plot:



- Here, one can see:
    - Value at i=1 is high: This means that given the values of time series $y(t)$, we can compute the values of a time series with lag 1, $y^1(t)$ with ease.
    - There is a high value at i=12

- This means that even if we ignore the information given by all the time series with $lags = 1,2,...,11$, the information carried in time series $y^{12}(t)$ alone is very high.
- This shows strong seasonality

ARIMA Family of Forecasting Techniques

Auto Regression (AR)

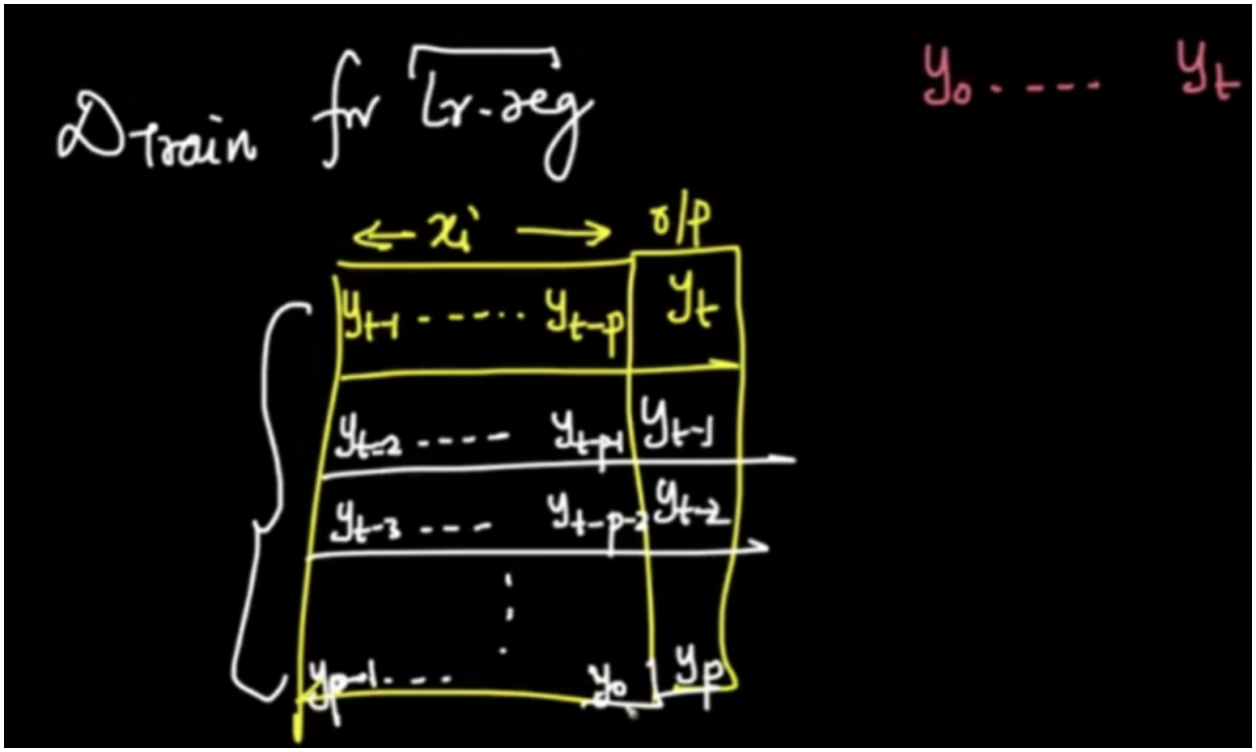
- For stationary time series, one is helpless because of the absence of pattern in the series.
- Any trend or seasonality that was present has been removed from the series, and it will be added back later, in the final prediction.
- For such cases we study a new family of models called **ARIMA**.
- Though such series look like they are completely random, there is still some extent of forecastability here, there is still information left to be extracted from stationary series.

What if we had a feature in our time series, besides the value to be predicted?

- In that case, one could just utilize Linear Regression, by mapping this feature's values with the value to be forecasted.
- For creating a new feature, we can map the value of the stationary time series at time t with the value of series at time $t-1, t-2, t-3, ..., t-p$, where p could be a **hyperparameter** we set.

- This way, now our data becomes as shown.
    - It contains date as an index,
    - Past values y_{t-1}, y_{t-2}, ,..., y_{t-p} as features and
    - value at time t as the value to be predicted ($\hat{y}_t$)

Now we can successfully implement Linear Regression using these features.



- Since the aim is to convert the forecasting problem to Linear Regression, what we're doing is;
    - Future value $\hat{y_t}$ = LinearRegression(Past p values)

- Thus, the forecasting problem is converted into the following form:

$$\hat{y_t} = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} \ldots + \alpha_p y_{t-p} + \epsilon_t$$

where;

- α_0: intercept (parameter)
- ϵ_t: error term
- $\alpha_1, \alpha_2, \ldots, \alpha_p$: weights (parameters)
- p: No of past values to be considered (hyperparameter)

- This seems similar to Simple Exponential Smoothing (SES).

- Though we are essentially doing a **weighted average** of the past time series values in both SES and AR models, there is a fundamental difference.
    - In the case of SES,

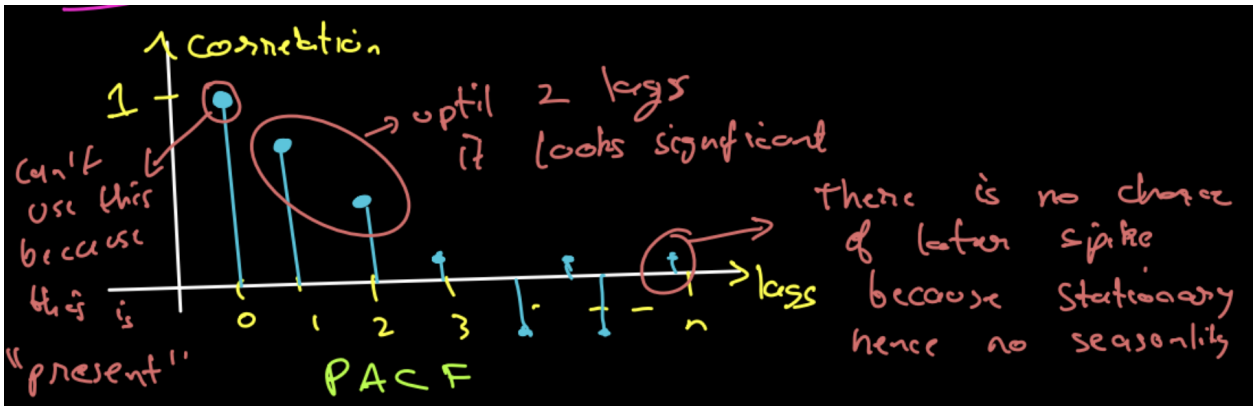
        - The weights are **exponentially decaying**
        - The hyperparameter is **α**

    - In the case of AR,
        - The weights are **learned** by multiple iterations.
        - The hyperparameter is *p*

- **Pre-requisites of the AR model:**
    - The idea is that this assumption will be true if the Partial Auto Correlation Plot has a high value at lag = k.
    - This way, from the plot we know that the future value is highly correlated with one value in the past, which means that it makes sense for us to compute linear regression on past 1 value.
    - We are using PACF because we don't want features to be correlated with each other in LR. PACF can help to identify that

Deciding the value of p

- We look at the PACF plot, and based on the values there, we decide how many lag values we can consider.
- For example, in the given plot,
    - we consider 2 lags only,
    - as for the third lag, the PACF value is 0.1, so considering it will not give as promising results.



Moving Averages (MA)

- The idea here is to use the value of the series at time t-1, we use the error of value at t-1 from the mean of data in the regression setup
- The error of each data point in the series from the mean/average, would be different.
- This should also work, as we are able to successfully create a new feature that is unique for each point.
- In fact, this idea is called the **Moving Averages (MA)** technique.
- Though the name is the same as the smoothing technique, this has nothing to do with that and is a completely different concept.

- The formulation of MA is as follows:

$$\hat{y_t} = \mu + me_{t-1}$$

where,

- μ -> mean of entire data
- e_{t-1} -> error of the data point at time t-1, with respect to the mean
- m -> constant

- This idea can be extended to the order of **q.** In that case, the formulation becomes:

$$\hat{y_t} = \mu + m_1 e_{t-1} + m_2 e_{t-2} \ldots + m_q e_{t-q} + \epsilon_t$$

Here, **ε** represents the final error remaining that is actually truly random, which we cannot help. This is also added for representation.

NOTE:

- q becomes the **hyperparameter** for the MA(q) model
- In the case of MA, there is a fixed way to determine the value of q, we need to try a bunch of different values to find the best fit.

Auto Regression - Moving Averages (ARMA)

- The combined technique of Auto Regression (AR) and Moving Averages (MA) is called the ARMA model.
- While combining the two ideas, **p**: order of AR and **q**: order of MA, **p** may or may not be equal to **q**
- $\alpha_1, \alpha_2, \ldots, \alpha_p$: coefficients of AR
- $m_1, m_2, \ldots, m_q$: coefficients of MA
- Hence the formulation becomes:

$$\hat{y_t} = c + \alpha_1 y_{t-1} \ldots \alpha_p y_{t-p} + m_1 e_{t-1} + \ldots m_d e_{t-q} + \epsilon_t$$

$$\text{i.e. } \hat{y_t} = c + \epsilon_t + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} m_j e_{t-j}$$

- Here, **p** and **q** are hyperparameters. Thus it is also called **ARMA(p,q)** model.
- The major limitation of this technique is that it cannot handle non-stationary time series because, if we're training a Linear Regression, the variables can not be dependent on each other.

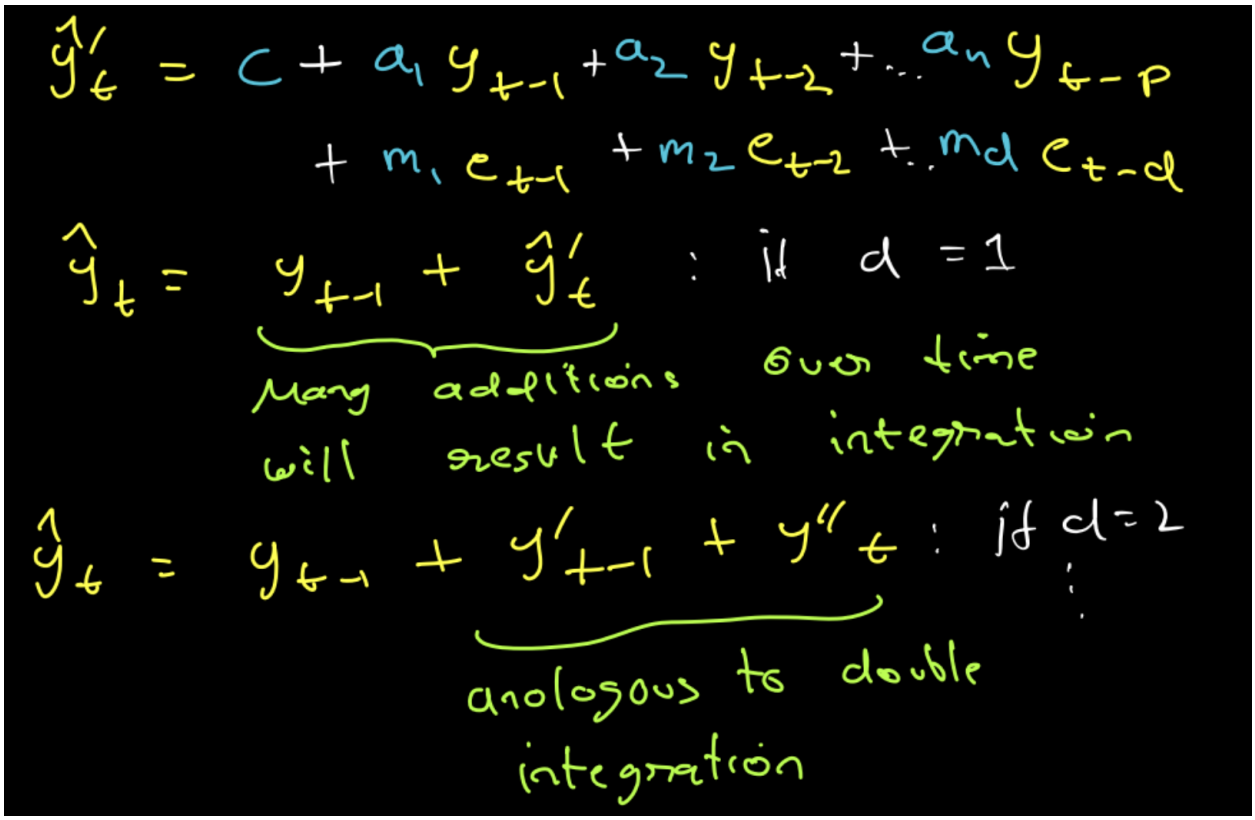
ARIMA

- If a time series is not stationary,
  - We perform **differencing** to de-trend
  - Then we apply the ARMA technique, to get an approximation.
  - Now to get a good forecast, we need to **integrate** the trend **back** to get the final result
- This technique is called **ARIMA**
- Instead of us manually doing integrations (and note that it can get very hard when you need to double / triple differentiate), we can simply use the ARIMA model, which does this job for us.

Formulation of ARIMA

- ARIMA can be formulated as a summation of
  - Differencing of order d
    - d=1 for linear trends, d=2 for quadratic trends, ...
  - Auto Regression of order p
  - Moving Averages of order q
  - Getting the resulting forecasts

- Integrating d times to restore the trend back



SARIMA

- If we wish to account for seasonality, the process becomes:-
    1. differentiating $(x[\, i\,] - x[\, i-T\,])$, to remove seasonality
    2. perform AR and MA
    3. integrating the seasonality back
- This is very tiring.
- Instead of doing so much work to utilize the ARIMA model, we can just apply another model called the **SARIMA model** directly.
- There are 7 parameters for SARIMA: **P, Q, D,p,q,d,s**

Hyperparameter s

- The parameter s represents **seasonality**
    - This value we can find using **ACF and PACF plots**.

    - Alternately, we can treat s as a **hyperparameter** and tune it to get the best value.
- In case of lag of order 1, at time t, translates to y_{t-1}

- For example,
    - For a normal time series, a lag of order of 2 means y_{t-1} and y_{t-2}
        - And this makes sense as if we're in March 2022, then order of 2 means, lag of 1, i.e. February 2022 and lag of 2, i.e. January 2022

    - In case of yearly seasonality, if we're in March 2022, a lag of order 2, means March 2021, and March 2020.

    - Therefore, here, an order of m gets translated as y_{t-s}, $y_{t-2s},,,\ldots$, y_{t-ms}.

- There is one **problem** with the SARIMA Model in terms of capturing seasonality.
- **We can only use one value of seasonality.**

Hyperparameter P

- When we set the order of AR as p, the formulation becomes:

$$\hat{y_t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}$$

- **The effect of setting the `P` hyperparameter for SARIMA is:** $\sum_{i=1}^{Q} c_i \epsilon_{t-is}$
    - Since our seasonality is 12
    - Essentially, P enables in creating of an AR Model on data that is 12 months old, 24 months old, ..., 12P months old: y_{t-12}, $y_{t-24},\ldots,y_{t-12P}$.
    - So this is exactly like an AutoRegression, but with seasonality.

- Suppose we wish to forecast $\hat{y}_{100}$ (i.e. t=100) with the following hyperparameter values:-
- p=4
    - Contribution to final forecast:

$$\alpha_1 y_{99} + \alpha_2 y_{98} + \alpha_3 y_{97} + \alpha_4 y_{96}$$

- s=12
- P=3
    - Contribution to final forecast:

$$\beta_1 y_{100-12=88} + \beta_2 y_{88-12=76} + \beta_3 y_{76-12=64}$$

Hyperparameter Q

- Similarly, the effect of setting the Q hyperparameter for SARIMA is:

$$\sum_{i=1}^{Q} c_i \epsilon_{t-is}$$

- Suppose if seasonality in a certain time series data is 12.
- Essentially, Q enables the creation of an MA Model on data that is 12 mon old, 24 months old, ..., 12Q months old: y_{t-12}, y_{t-24},…, y_{t-12Q}.
- So this is exactly like a Moving Average, but with seasonality.

- Suppose we wish to forecast $\hat{y_{100}}$ with the following hyperparameter values:-
    - q = 4
        - Contribution to final forecast:

$$m_1 \epsilon_{99} + m_2 \epsilon_{98} + m_3 \epsilon_{97} + m_4 \epsilon_{96}$$

    - s = 12
    - Q = 3
        - Contribution to final forecast:

$$c_1 \epsilon_{100-12=88} + c_2 \epsilon_{88-12=76} + c_3 \epsilon_{76-12=64}$$

Hyperparameter D

- Recall that hyperparameter d performs differencing on the time series d times, before applying the model.
    - For example if d=1: $y'_t = y_t - y_{t-1}$

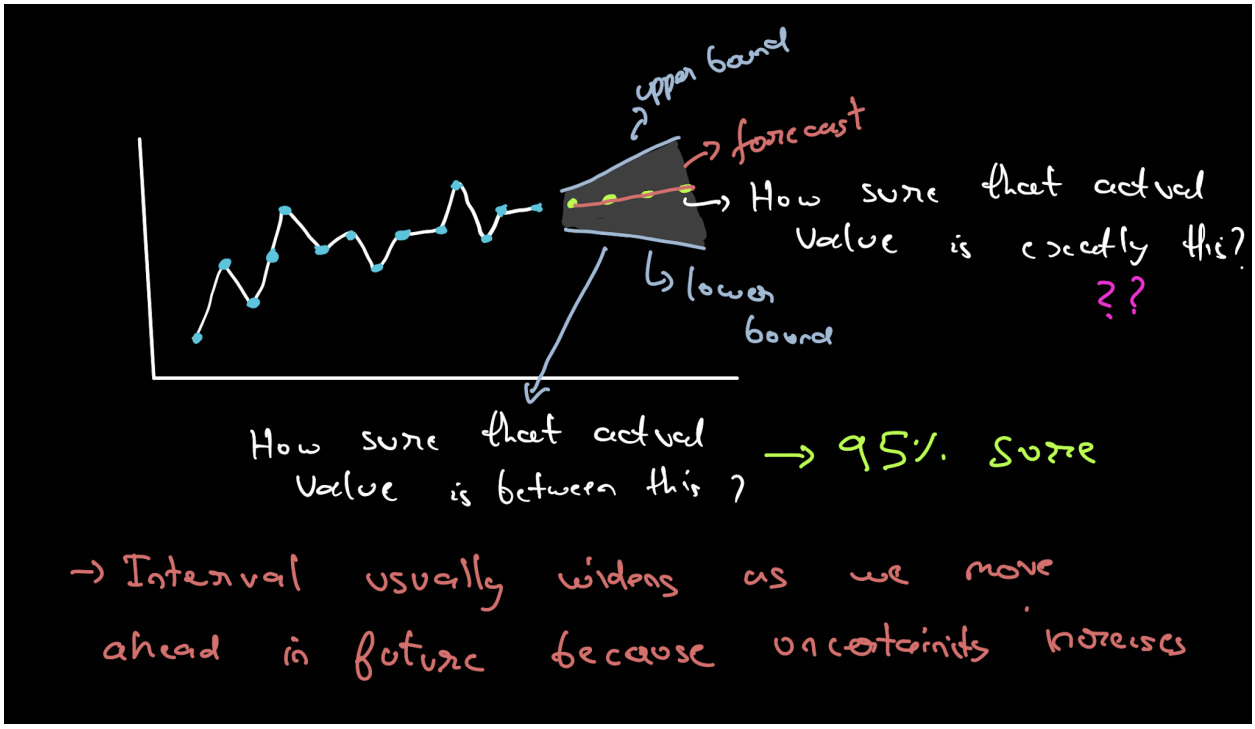
- Similarly, D helps in doing **seasonal differencing** on the time series.
    - For example if `D=1` : $y'_t = y_t - y_{t-s}$
    - Since $s = 12$: $y'_t = y_t - y_{t-12}$

So, to sum it up SARIMA can be formulated as

$\hat{y}_{t+1}$ = p(AR term) + q(MA term) + d(differencing) + D(differencing) + P(AR-Seasonality) + Q(MA - Seasonality)

Confidence Intervals

- In addition to making predictions, the models can also provide Confidence Intervals.
- In addition to making predictions, time series models can also provide Confidence Intervals.
- Owing to the **potential error** of our forecast, and the **unpredictability of the future**, we cannot claim to be a hundred percent sure of our forecasts.
- But we can still, based on our study of time series, say that the future value would lie in the **range** of [A, B].
- Naturally A becomes the minimum possible value of our target variable at time t, and B becomes the maximum possible value at time t.
- This [A, B] becomes a range for determining the **real possible value** for a certain prediction.
- It is called the **confidence interval.**
- It can also be used for performance evaluation.

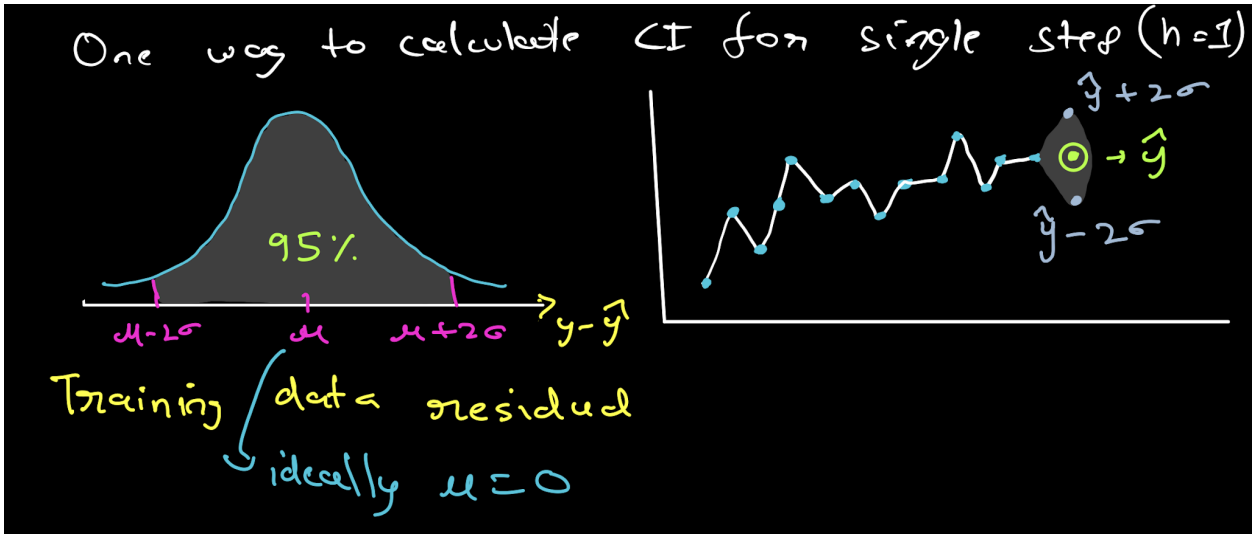


- Confidence Intervals are required for determining the uncertainty of your predictions, after making forecasts.
- Confidence Intervals provide an expected range for the real observation. When making decisions, one can utilize the range of information better, than just a single point.

- Different algorithms have different formulations for CI.
- These can be complicated to derive and are out of scope for this lecture.
- We can get the upper and lower values according to the models available in the statsmodel library.
- It takes a value of a parameter: **alpha**, which is a measure of how much confidence we want in our forecasts.
- One example of a 1-step ahead forecast CI is to just take the residual distribution.
- For example, the 95% confidence interval will be

$$\hat{y_{t+1}} = \hat{y_t} + u_{resid} \pm 2\sigma_{resid}$$

- Note that for a good forecast U_{resid} should be close to 0



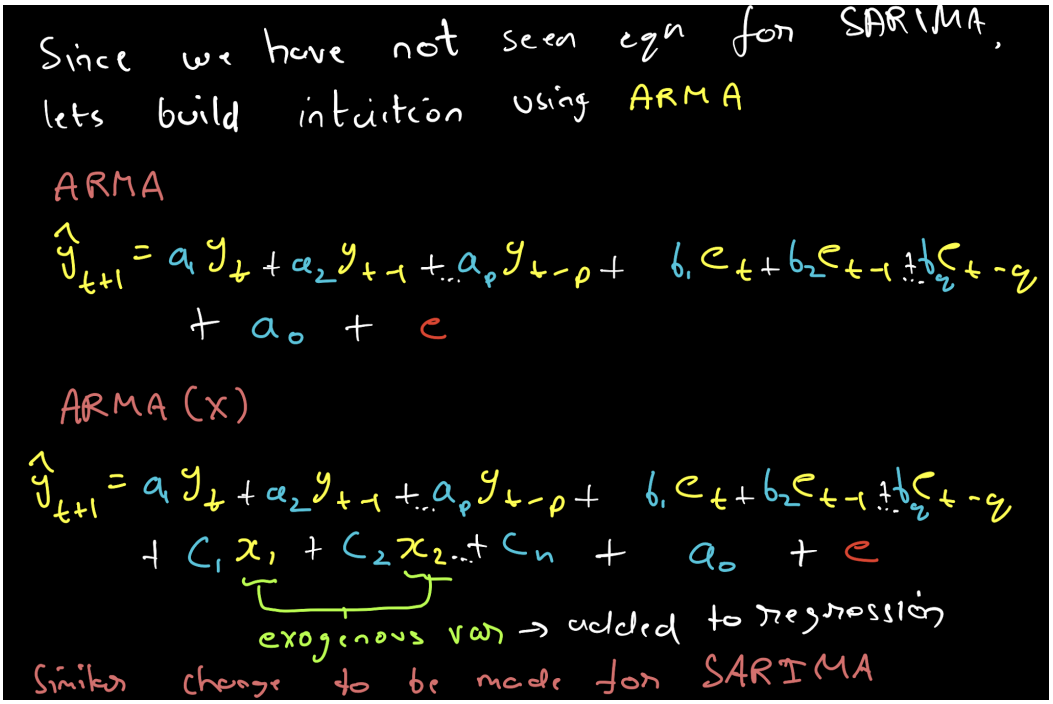
- To interpret confidence intervals, let our forecasted value is $\hat{y}$. And you pass a value of **alpha = 0.05** for which you get the confidence interval as (m,n).
- It means that the time series model will estimate the upper(n) and lower(m) bound of values around the forecast, where there is only a 5% chance that the real value will **not be** in that range.
- That is, 95% that our forecast will fall within the range (m,n).

SARIMAX Model

- We can utilize the exogenous variable and incorporate it in our SARIMA Model.
- This idea of incorporating exogenous variables into SARIMA gave rise to a new model: **SARIMAX Model**
- Here, the X represents an exogenous variable
- The only difference between SARIMA and SARIMAX is that here, we can incorporate exogenous variables into the calculations of our forecasts.
- Exogenous variables are assigned a weight, say w_i.
- We don't need to initialize this, it is learned and trained by SARIMAX, and taken care of under the hood.
- This is done in addition to the SARIMAX operations.

- SARIMAX model takes exogenous variables into account
    - i.e. variables measured at time t that influence the value of our time series at time t, but that is not auto regressed on.

- To do this, we simply add the terms on the right-hand side of our ARIMA and SARIMA equations.



Facebook's Prophet

- One of the drawbacks of the SARIMAX Model is that one cannot have multiple seasonality and can only select one value.
- To overcome this disability, we're studying an open-source tool/library called prophet which is developed by Facebook

Following are the features of Facebook's Prophet:

- Provides intuitive parameters which can be **easily tuned**
- It is **robust to missing data and shifts in the trend** and typically **handles outliers** well.
- It can account for **multiple seasonalities**. This is possible because, under the hood, the math of seasonalities is based on **Fourier transforms**, which help incorporate this.
- The Prophet uses a decomposable time series model with three main model components
- They are combined in the following equation:

$$\mathbf{y(t) = g(t) + s(t) + h(t) + \varepsilon t}$$

- **g(t):** piecewise linear or logistic growth curve for modeling non-periodic changes in time series (**trend**)
- **s(t):** periodic changes (e.g. weekly/yearly **seasonality**)
- **h(t):** effects of **holidays** (user provided) with irregular schedules
- **εt: error term** accounts for any unusual changes not accommodated by the model.

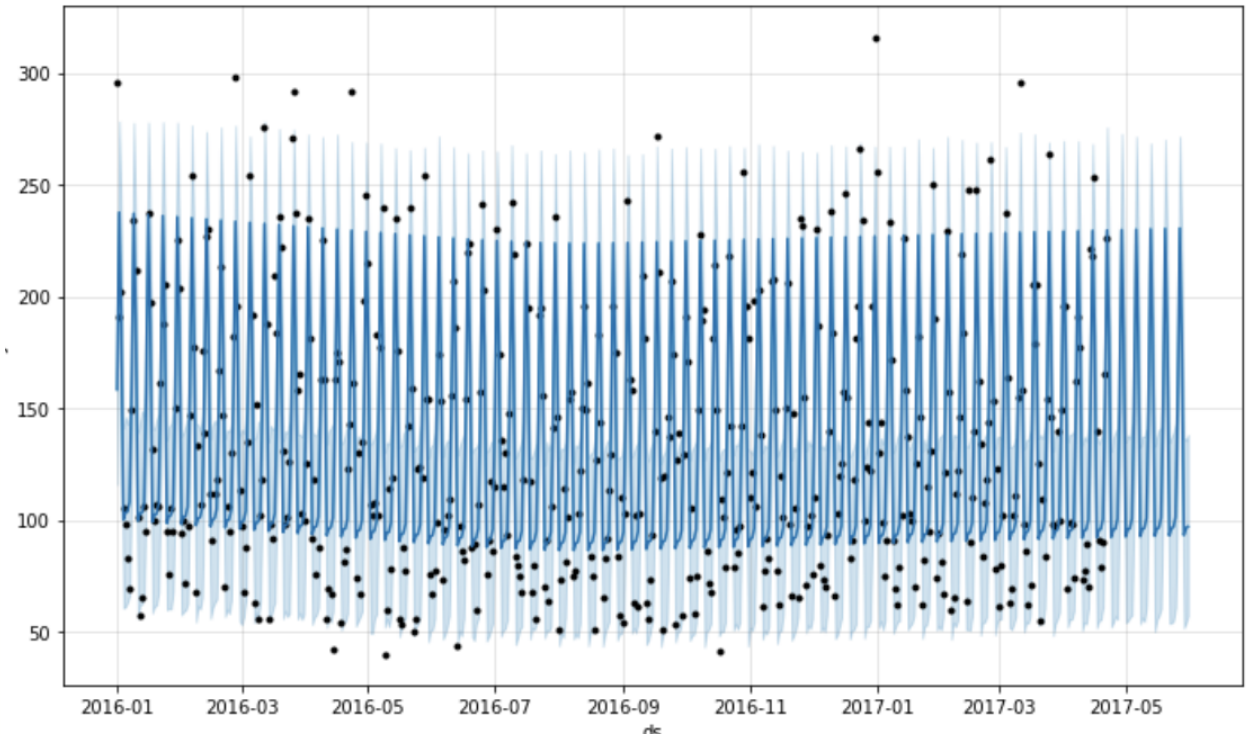
- For using Prophet, the data set should contain only two columns with column names as 'ds' and 'y'
- 'ds' should always be in 'date-time' format. And 'y' represents a feature that you want to forecast.

- To install prophet to your environment, you can use the following environments:

```
!pip install pystan~=2.14
!pip install fbprophet
```

→

```
from fbprophet import Prophet
m = Prophet()
m.fit(df[['ds', 'y']][:-39]) #here we are leaving
last 39 observations because we will predict it in
'future'
future = m.make_future_dataframe(periods=39,freq="D")
forecast = m.predict(future)
fig = m.plot(forecast)
```

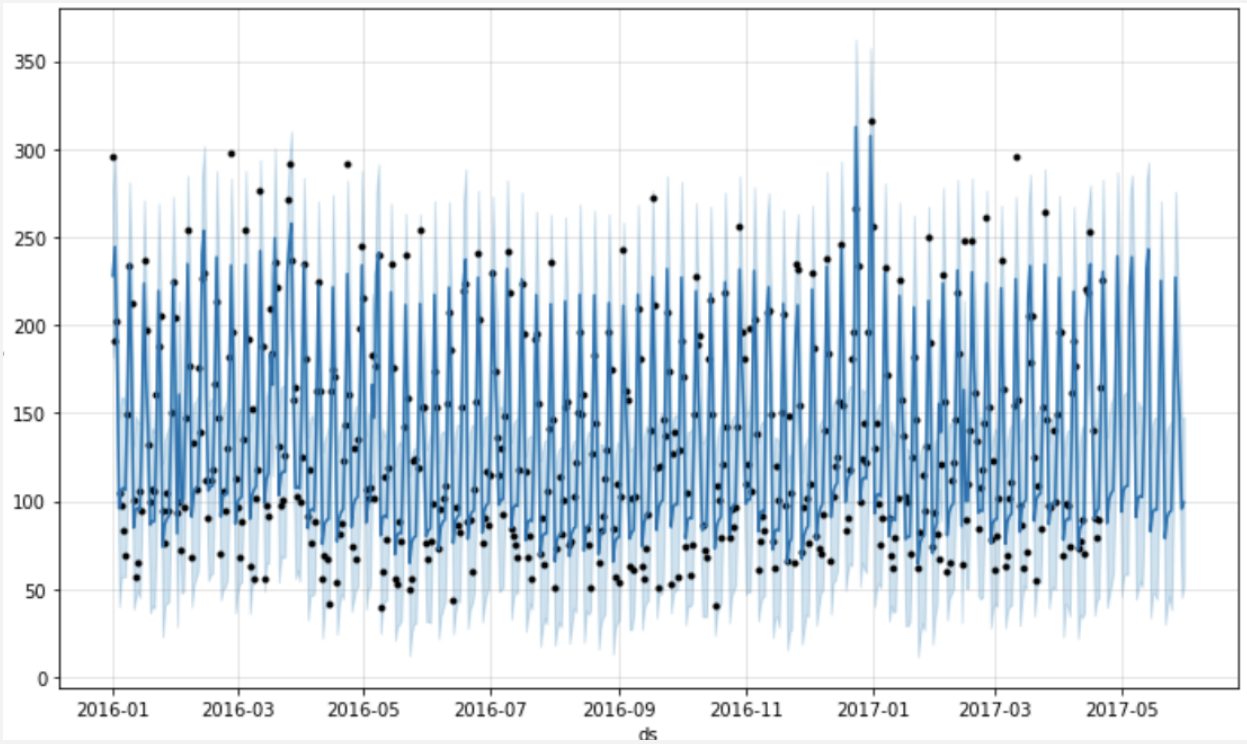


- In the plot, black dots are actual visits, deep blue lines are the predicted visits and light blue lines are the 95% confidence interval around the prediction.
- You can see that the lines are flat and the model is not able to capture the seasonality properly so it is not a good fit.
- Here light blue lines are 95% confidence intervals around the predictions.
- Here we also didn't do anything explicitly for Nan values it was handled by the prophet.

- We can use **`'add_regressor'`** for adding features to the prophet model.
    - There is another interesting parameter here: **`'changepoint_prior_scale'`**
- If the trend changes are overfitting (too much flexibility) or underfitting (very less flexibility)
- We can adjust the strength of sparse prior using this parameter.
- Default value: 0.05
- Increasing this will make the trend more flexible.

```
→    model2=Prophet(interval_width=0.95,
yearly_seasonality=True,
weekly_seasonality=True,changepoint_prior_scale=4)

     model2.add_regressor('holiday') #adding holidays data in
the model3
     model2.fit(df[:-39])
     forecast2 = model2.predict(df)
     fig = model2.plot(forecast2)
```

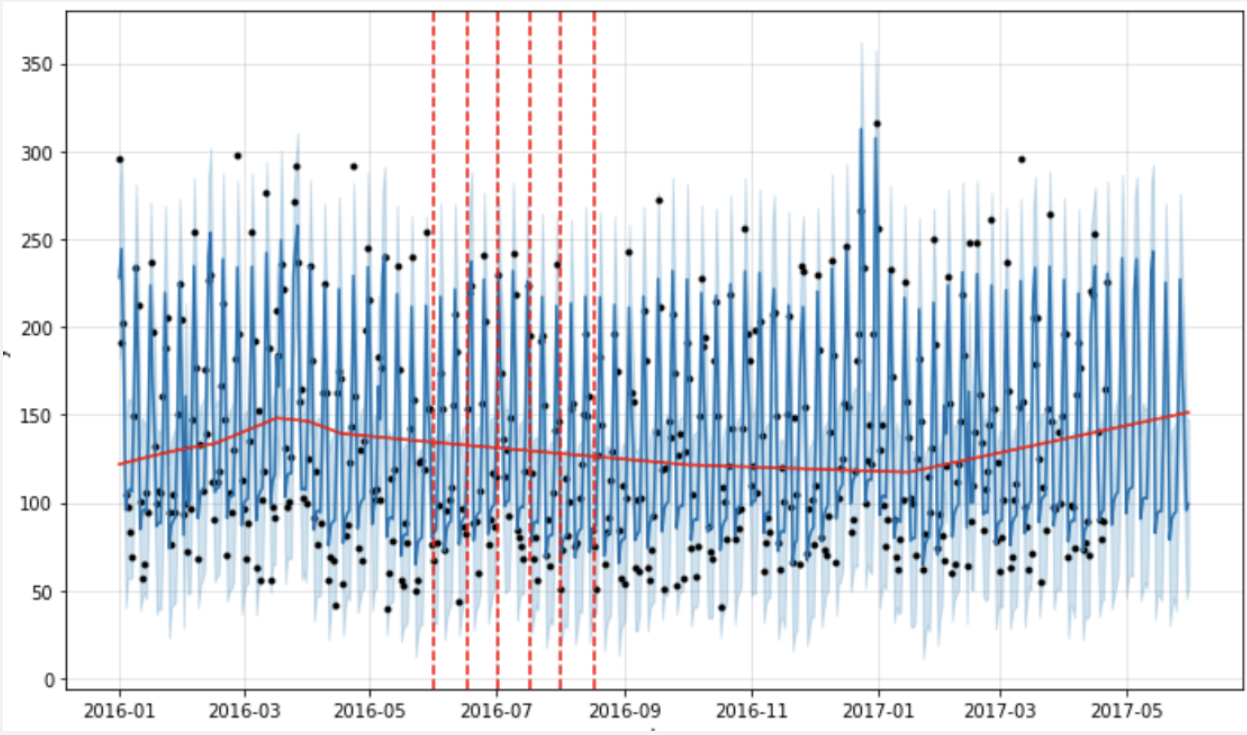


Changing Trends

- Prophet automatically detects the changepoints and will allow the trend to adapt appropriately.
- Prophet detects changepoints by first specifying a large number of potential changepoints at which the rate is allowed to change.

```
→    from fbprophet.plot import add_changepoints_to_plot
```

```
fig = m.plot(forecast2)
a = add_changepoints_to_plot(fig.gca(), m, forecast2)
```



- The vertical lines in this figure indicate where the potential change-points were placed.
- Facebook prophet provides automated methods to forecast.
- The model has easily interpretable parameters that can be changed by the analyst to impose assumptions on the forecast.

Benefits of Prophet

- Prophet is a simple library and is great for beginners.
- It works best with time series that have strong seasonal effects.
- Prophet is robust to missing data and shifts in the trend, and typically handles outliers well.
- We can add multiple regressors or exogenous variables.
- It forms a very good baseline model because almost no feature engineering is required.
- Interpretability is one of the key advantages of the Prophet.

- If your time series follows some business cycles, you can obtain very decent performance quickly.
- It can also be helpful while detecting change points.