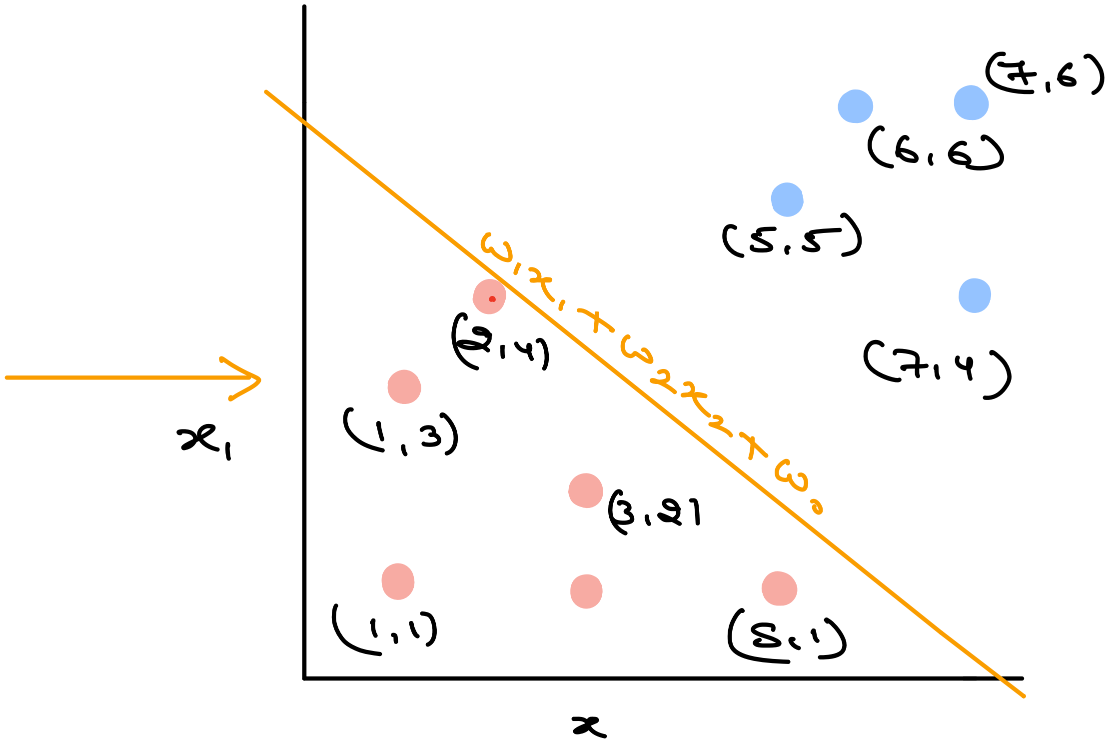
Agenda

- Revision
- Goal
- Vectors
  - intro to Vectors
  - Representation of Vectors
  - Visualization of Vectors
  - Magnitude of Vectors
    - Norms : L1 and L2
  - Dot product of Vectors
    - Matrix Multiplication
  - Angle b/w 2 vectors
  - Connection b/w geometry and LA
  - Unit Vector
  - Vector projection

End Goal

x_1	x_2
5	5
2	4
6	6
3	2
7	4
5	1



Find out the classifier

$$w_1x_1 + w_2x_2 + w_0 = 0$$

$w_1, w_2, w_0 = ?$

using x_1's and x_2's

⇒ w_1, w_2, w_0 ⇒ weights / parameters

⇒ x_1's and x_2's ⇒ features / data points

Vectors

1) magnitude

2) Direction

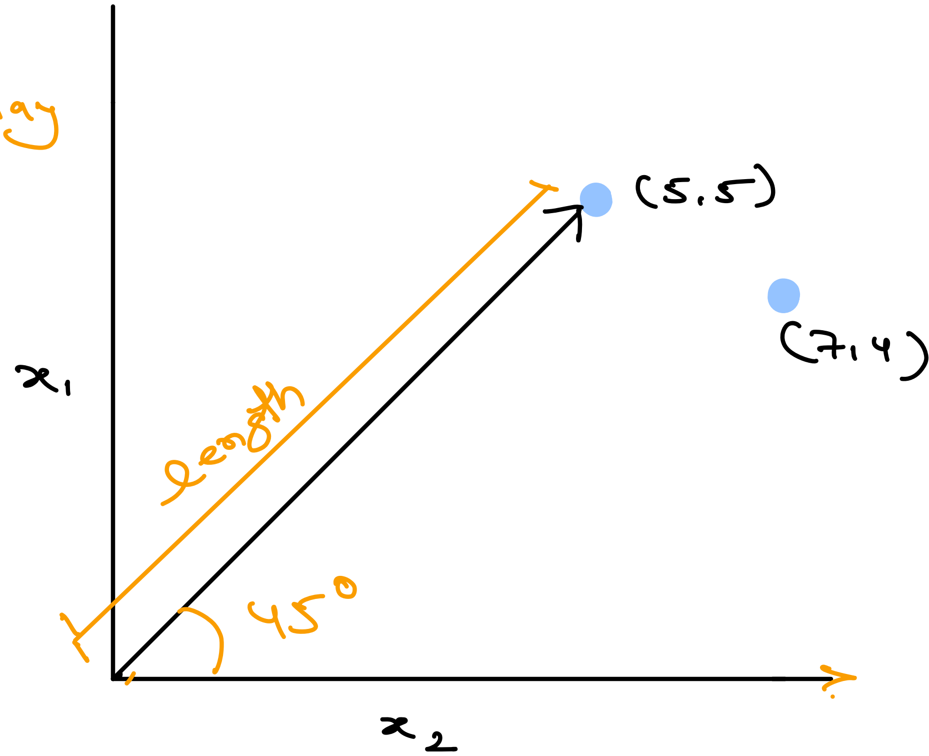
$\vec{x}$ or $\bar{x}$

$(5, 5)$
$\vec{x}$

$$\begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

3) list, tuple, np-array

$\vec{x} = (5, 5)$



$\bar{x} \supset 2, 6, 10, 12$

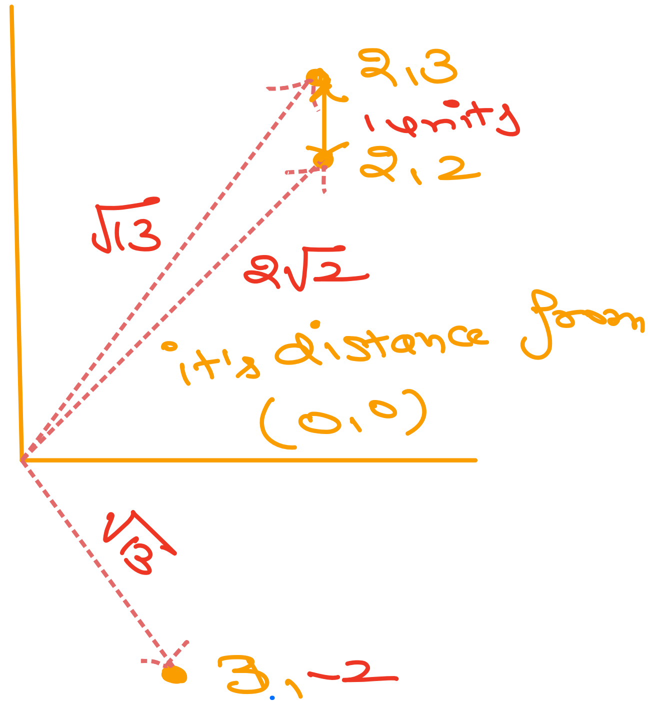
$$\begin{bmatrix} 2 \\ 6 \\ 10 \\ 12 \end{bmatrix} \longleftarrow \text{Column Vector}$$

$\|\vec{x}\|$

$$\begin{bmatrix} 2 & 6 & 10 & 12 \end{bmatrix} \longleftarrow \text{row Vector}$$

Norm of Vector

$$\bar{x}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \bar{x}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \bar{x}_3 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$



$$mag(\bar{x}_1) > mag(\bar{x}_2)$$

* Distance b/w two points

Euclidean Distance Formula

$x_1\, y_1$ and $x_2\, y_2$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$x_1 = 0$, $y_1 = 0$

(2, 2)

(2, 3)

$$\sqrt{(2-2)^2 + (3-2)^2} \Rightarrow \sqrt{1^2} \Rightarrow 1$$

---

magnitude $\Rightarrow \sqrt{x_1^2 + y_1^2}$

magnitude of Vector is also known as **Norm** of Vector

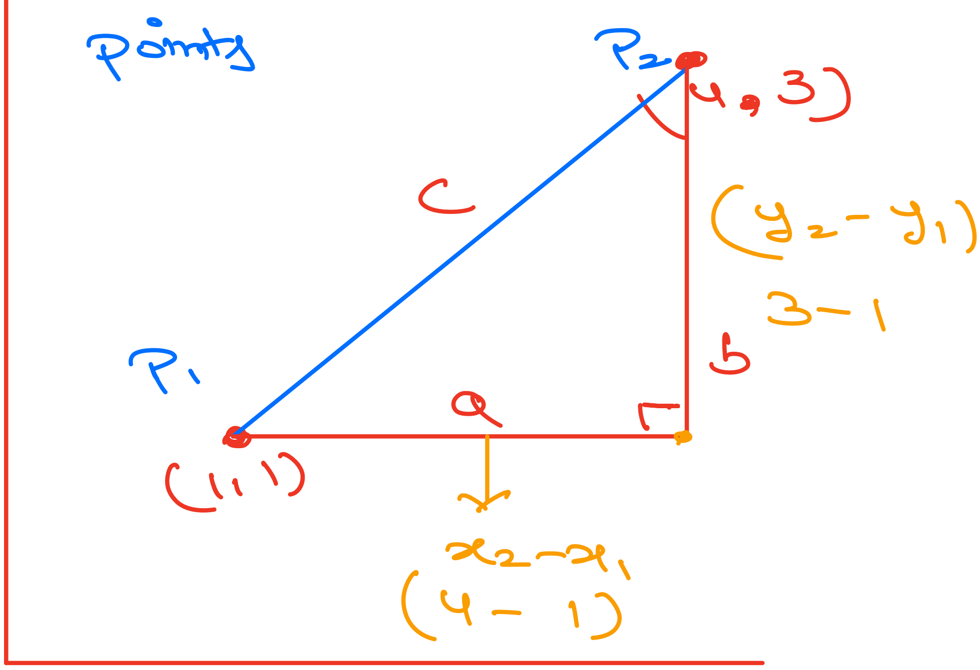
ℓ_2 Norm $= \sqrt{x_1^2 + y_1^2}$ (Euclidean Norm)

ℓ_1 Norm $\Rightarrow |x_1| + |x_2|$ (Manhattan Norm)

Q $(2, 4, 1, 3) \Rightarrow \ell_1 \Rightarrow 10$

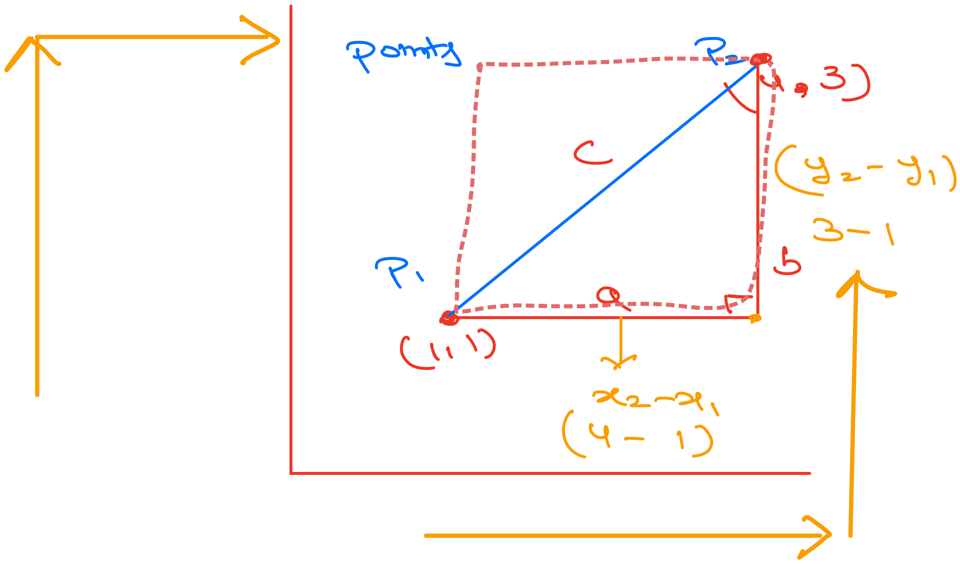
$\ell_2 = \sqrt{30}$

ℓ_2 Norm = Shortest Distance b/w points



$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$



l1 dist ⇒ a + b = manhattan Distance

Matrix Multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1\times5+2*7 & 1\times6+2\times3 \\ 3\times5+4\times7 & 3\times6+4\times3 \end{bmatrix}$$

m1

2×2

Validity Condition

A × B	M × N
B == M ✓	
B != M ✗	

Output ⇒ A × N

Dot product of Vector

$\bar{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \bar{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$2 \times 1 \qquad\qquad 2 \times 1$

① $\bar{y}^T = \begin{bmatrix} 3 & 4 \end{bmatrix} \longrightarrow \bar{x} * \bar{y}^T$

1×2

$2 \times 1 * 1 \times 2$

$\Rightarrow 2 \times 2$

or

② $\bar{x} = \begin{bmatrix} 1 & 2 \end{bmatrix} \longrightarrow \bar{x}^T * \bar{y}$

$1 \times 2 \qquad 2 \times 1 \Rightarrow 1 \times 1$

$\bar{x}^T y \Rightarrow$ Dot Product of Vectors

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow 11$

$10 \times 1 \quad 10 \times 1 \Rightarrow$ Dot product $= 1 \times 1$

Angle b/w 2 Vectors

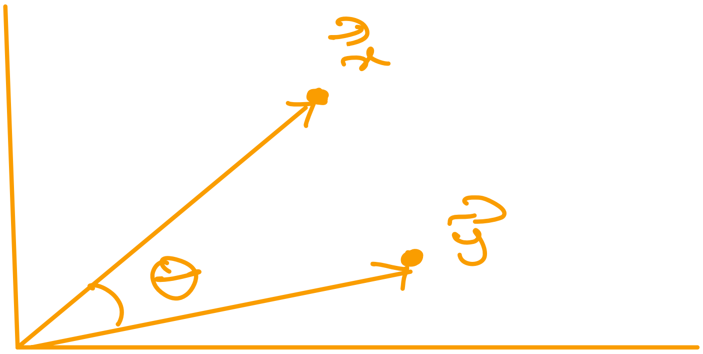
* Just by looking at dot product

$$\cos\theta = \frac{\vec{x}^T \cdot \vec{y}}{||\vec{x}|| \, ||\vec{y}||} \quad \text{+ve}$$

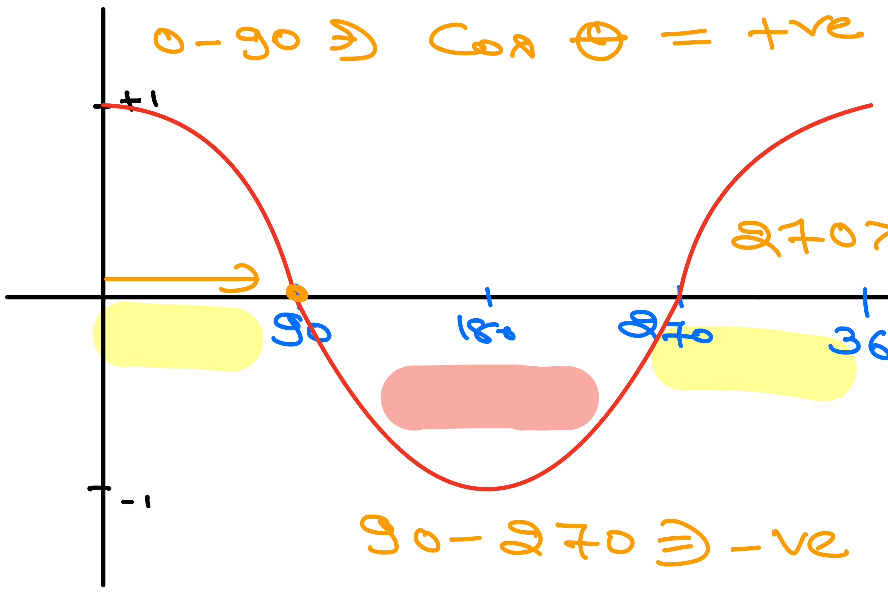
$\downarrow$ l2 Norm

$|\bar{x}|$ l1 Norm

$||\bar{x}||$ l2 Norm

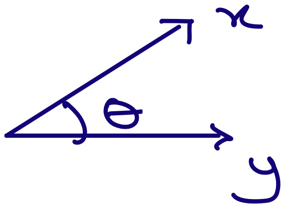


0-90 ⇒ Cos θ = +ve

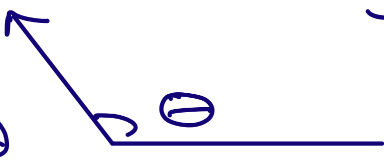


270 > +ve

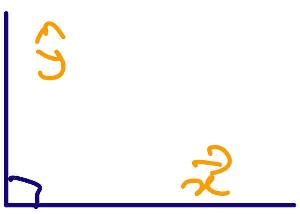
90-270 ⇒ -ve

1) 

θ < 90 then $x^T y$ ⇒ +ve ?

2) 

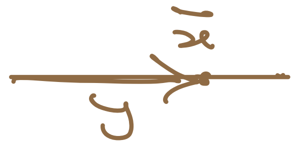
180 > θ > 90 then $x^T y$ -ve ?

3) 

θ = 90 then $x^T y$?

⇒ 0

* Just by looking at dot product we can tell which Quadrant the angle lies in

* Case 4 : $\theta = 0^\circ \Rightarrow$ 

Equation of Classifier

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$\downarrow$

Desmos Classifier

$$x_1 + x_2 - 98 = 0$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

weight Vector   feature Vector

$$x_1 + x_2 - 98 = 0$$

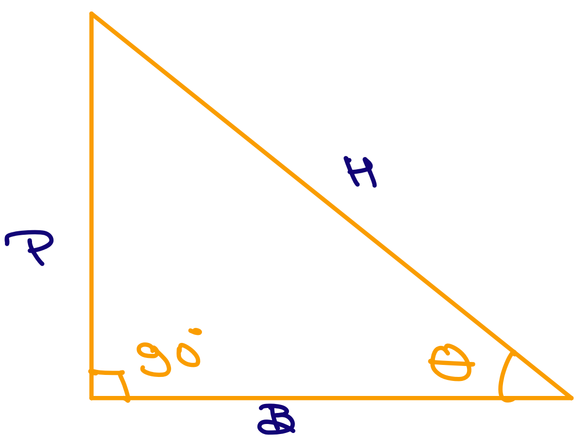
$$\begin{bmatrix} 1 \\ 1 \\ -98 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$w^T x + w_0 = 0$$

$$\bar{w}^T \bar{X} = 0$$

Note: Weight Vector is alway $\perp$ to the classifier

Trignometry of Angles

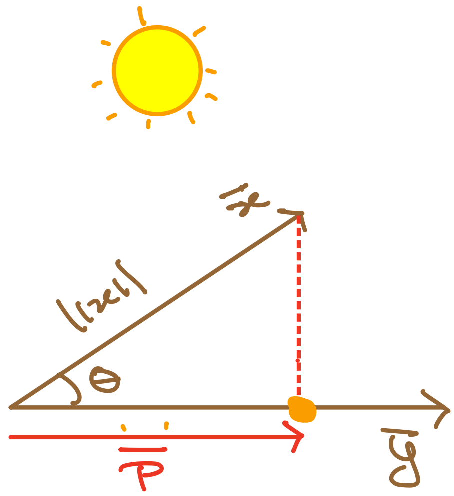


$\tan\theta \Rightarrow \frac{P}{B}$

$\sin\theta \Rightarrow \frac{P}{H}$

$\cos\theta \Rightarrow \frac{B}{H}$

Projection of Vector



Projection of Vector $\bar{x}$ on $\bar{y}$ is the shadow x casts on y

$$\boxed{||P|| \Rightarrow x^T \hat{y}}$$

Length of Projection is equal to Dot product b/w $\bar{x}$ and $\hat{y}$

Projection of $\bar{x}$ on $\bar{y}$

Derivation of projection

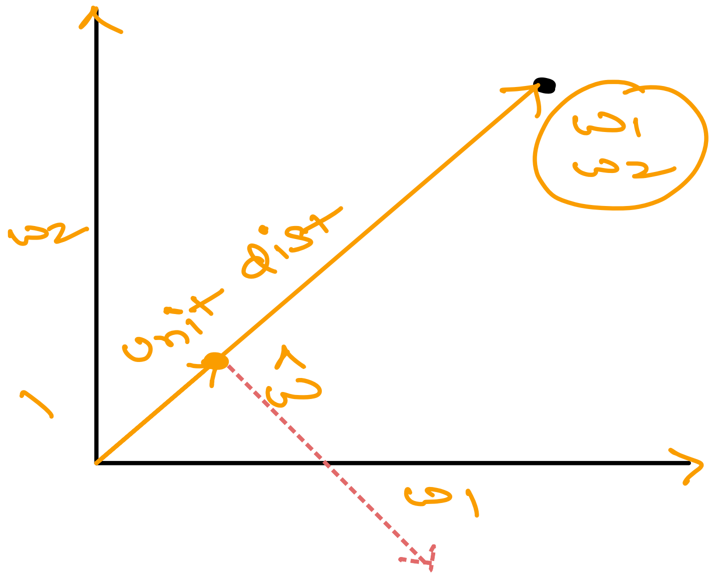
Hints

① (trigo) $\cos\theta =$

② (L·A) $\cos\theta =$

Unit Vector

Vector with Magnitude of 1



$$mag = \sqrt{w_1^2 + w_2^2}$$

$$\hat{w} \Rightarrow \left(\frac{w_1}{\sqrt{w_1^2 + w_2^2}} \, , \, \frac{w_2}{\sqrt{w_1^2 + w_2^2}} \right)$$

H.W 1

$$\|\hat{w}\| \Rightarrow ?$$

1

⊙ A vector with magnitude and Same direction as Original Vector