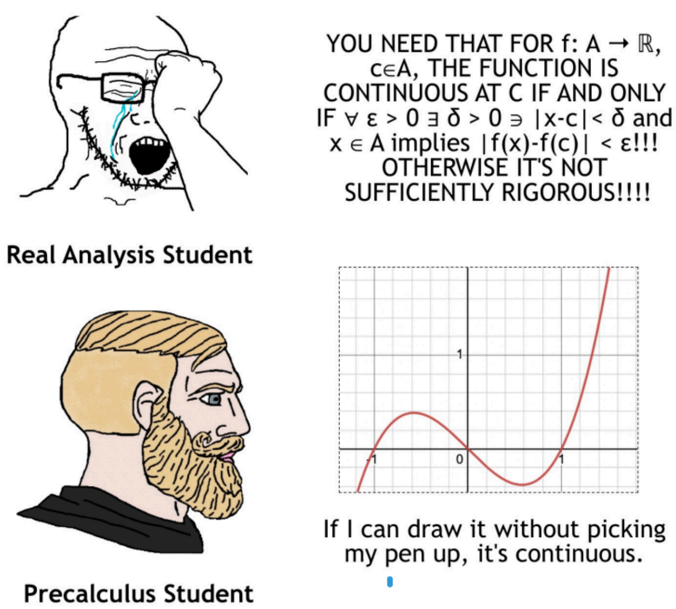
- Recap
- Differentiation
- Check for differentiability
- Commonly Used Derivatives
- Rules for Differiation
- How do we use derivative for optimi-zation?
    - Minima
    - Maxima
    - Saddle point

Recap

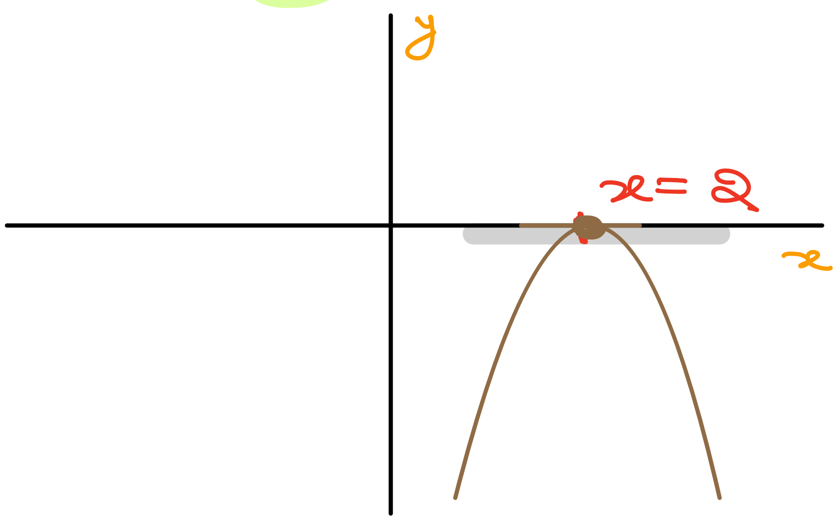


Goal: Maximize the Gain Function

$$\boxed{\bar{w}^*, w_0^* = \underset{\bar{w}\ \ w_0}{\operatorname{argmax}}\ G(D, \bar{w}, w_0)}$$

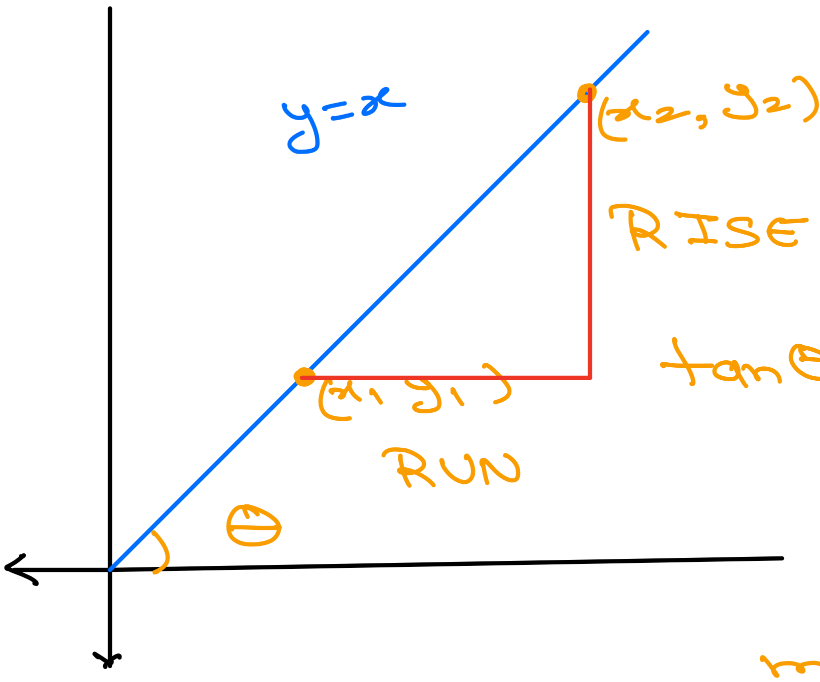
Simple Example:

$$f(x) = \underset{x}{\operatorname{argmax}}\ -(x-2)^2$$



- Domain: Set of All Inputs
- Range: Set of All Outputs
- L.H.L: $\lim\limits_{x \to a^-} f(x)$
- R.H.L: $\lim\limits_{x \to a^+} f(x)$

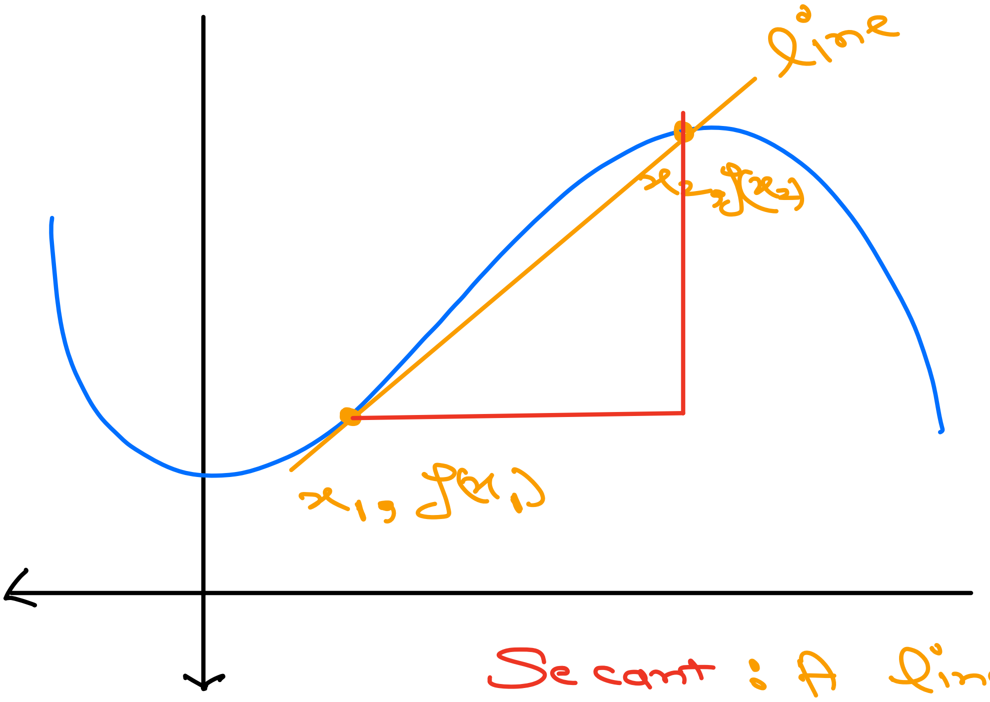
Differentiation: Geometric picture



$$\tan\theta = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

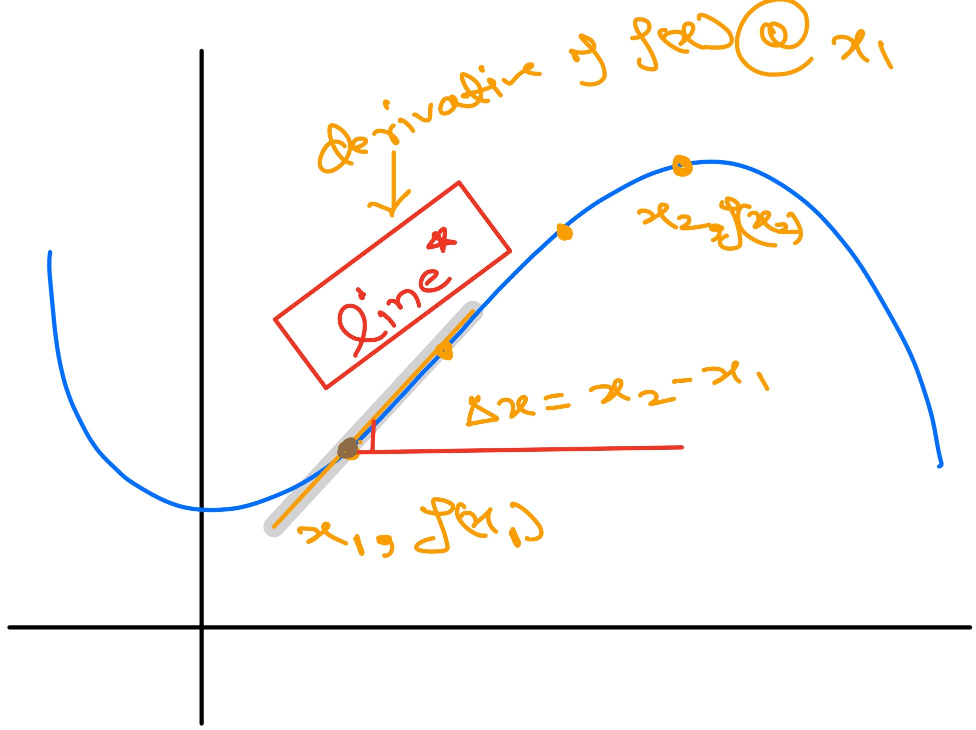
$$m = \text{slope} = \frac{\text{RISE}}{\text{RUN}}$$

$y = f(x)$



$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Secant : A line which crosses the curve at 2 or more points



x_2, y_2 0

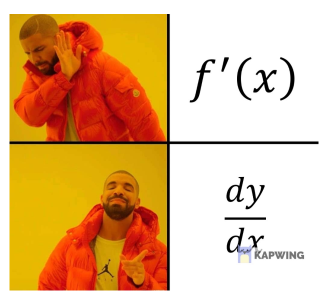
line → Tangent

Tangent → A line that touches the curve at just one point

Derivative of any function @ x is defined as slope of the Tangent @ x

$$f'(x_1) = m = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

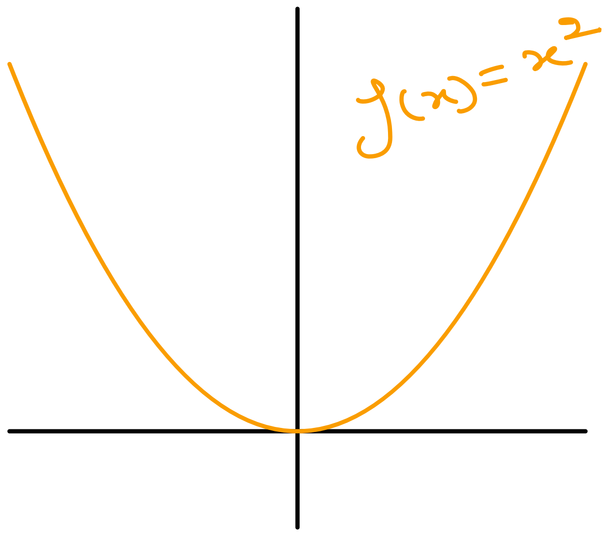
$$\frac{dy}{dx} \qquad \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



How to find Derivative of any Function

① Ab initio Method

② Rule Based Calculations



Ab initio Method

$$\frac{d f(x_1)}{dx} = f'(x_1) =$$

$$\boxed{\lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}}$$

* Smaller Δx tends to more accurate Results

Rule Based Calculations

Rule : $\frac{d\, x^n}{dx} \Rightarrow n\, x^{n-1} = f'(x)$

$$\frac{d(x^2)}{dx} \Rightarrow 2\, x^{2-1} \Rightarrow 2x \text{ @ } x = 2$$

$$\Rightarrow 2 \times 2 = 4$$

Check for Differentiability

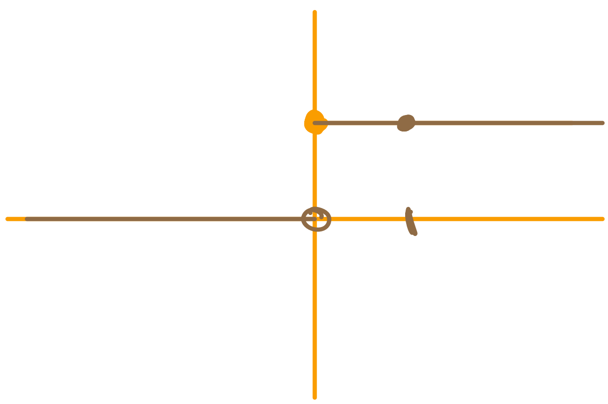
To check if a function is differentiable at point x

Step1 : $f(x)$ must be continous at x

Step2 : $f'(x)$ must be continous at x

$$L.H.L\ f'(x) = R.H.L\ f'(x) = f'(x)$$

Ex-1 $\quad f(x) = \begin{cases} 1 : x \geq 0 \\ 0 : x < 0 \end{cases}$



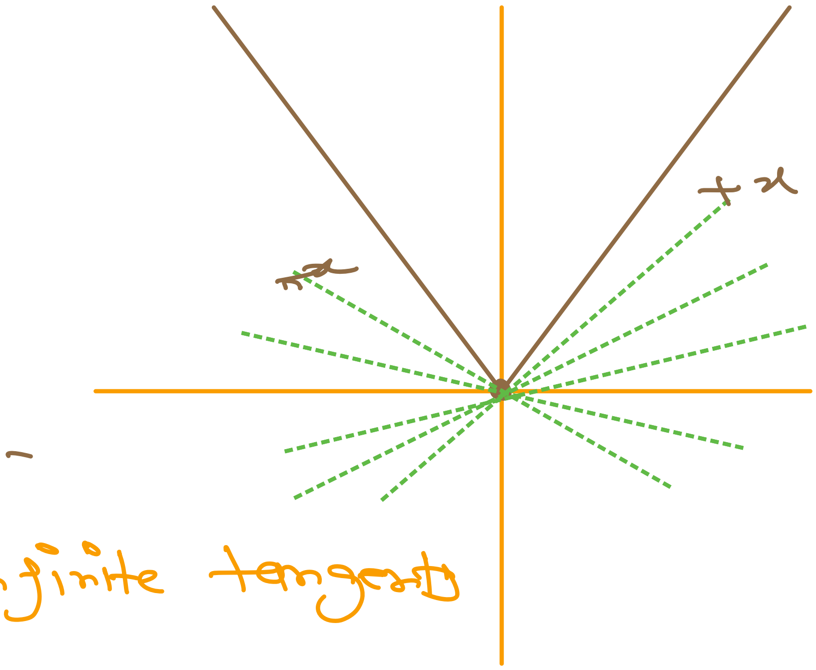
@ $x = 1$ ✓

@ $x = 0$

Rule 1 X

Ex-2 $f(x) = |x|$

Diff at $x = 0$?



infinite tangents

Rule-1 $\Rightarrow$ ✓

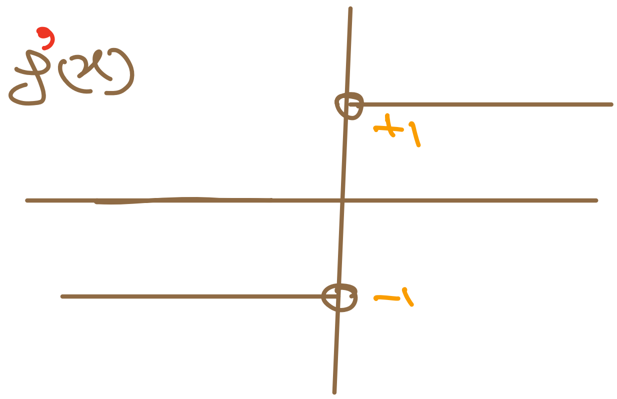
$f(x) \Rightarrow$ Continous

Rule-2 $\Rightarrow$

$f'(x) \Rightarrow$ Continous

$x \Rightarrow f'(x) \Rightarrow +1$

$-x \Rightarrow f'(x) \Rightarrow -1$



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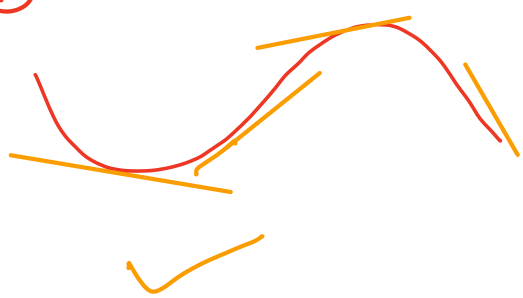
①



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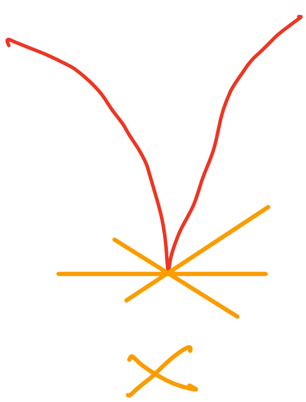
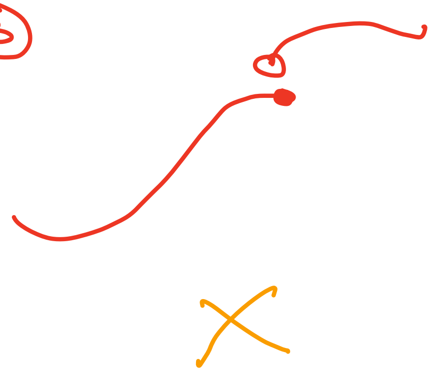


③



Commonly Used Derivatives

① $\frac{d}{dx} x^n = n x^{n-1}$

② $\frac{d}{dx} C = 0$

③ $\frac{d}{dx} \log(x) = \frac{1}{x}$

④ $\frac{d}{dx} e^x = e^x$

⑤ $\frac{d}{dx} \sin(x) = \cos x$

⑥ $\frac{d}{dx} \cos(x) = -\sin x$

⑦ $\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d(x)}{dx}$ and $\frac{\partial(-x)}{\partial x}$

$1 \times x^0$ $\quad -1 \times x^0$

$\frac{\partial(10)}{\partial x} = 0$

Rules of Differentiation

① Linearity Rule

$$h(x) = g(x) + f(x)$$

$$h'(x) = g'(x) + f'(x)$$

Q $h(x) = x^3 + \log(x)$

$f(x) + g(x)$

$x^n \rightarrow n x^{n-1}$

$$h'(x) \Rightarrow \boxed{3x^2 \cdot + \frac{1}{x}}$$

② Product Rule

$$h(x) = f(x) \cdot g(x)$$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Q: $h(x) = x \sin(x)$

$f(x)$ $\quad$ $g(x)$

$f'(x) \Rightarrow 1$

$g'(x) \Rightarrow \cos x$

$\Rightarrow 1 \times \sin x + x \cos x$

$\downarrow$

$\cos 2x$

③ Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x) \times f'(x) - g'(x) \times f(x)}{g(x)^2}$$

Q $h(x) = \frac{\sin(x)}{\cos(x)}$ ⟶ $f'(x) = \cos(x)$

$g'(x) = -\sin(x)$

$$\frac{\cos x \times \cos(x) - (\sin(x) \times (-\sin(x)))}{(\cos x)^2}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} \Rightarrow \frac{1}{\cos^2 x}$$

$\Rightarrow \sec^2 x$

④ Chain Rule

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

Q) $h(x) = \log(x^2)$

$f(g(x)) \longrightarrow \log(x^2)$

$f(g(x))$

$g(x) \Rightarrow x^2 \Rightarrow$ ✓

$f'(g(x)) \Rightarrow \frac{1}{x^2}$

$g'(x)$

$2x$

$f' \; g(x) = \frac{1}{x^2}$

$\frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} \Rightarrow \frac{2}{x}$ ✓

Q2: $f(x) = e^{-x}$

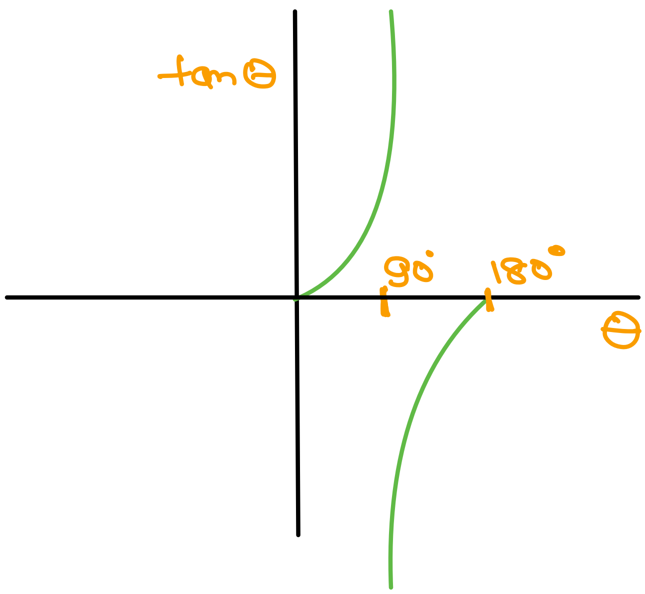
$e^{-x} * -1$

$-e^{-x}$

$e^{x} \rightarrow e^{-x}$

$(-x)' = -1$

How do we use derivative for optimization

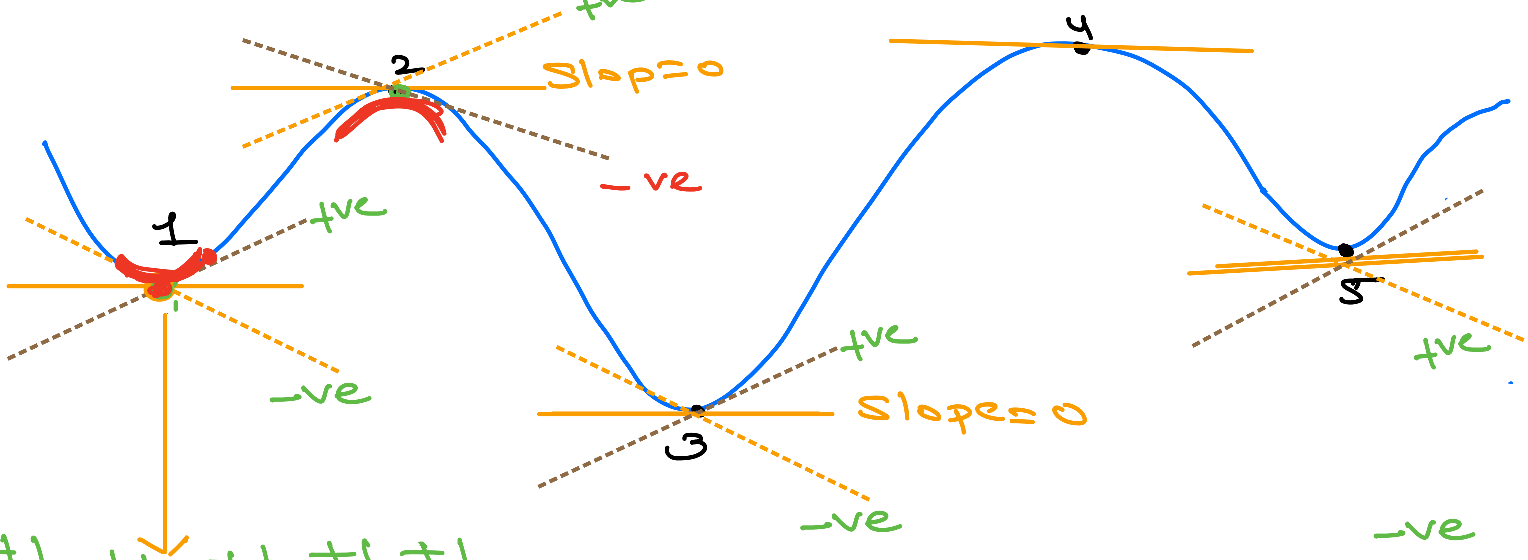


① 0 θ < 90 ⟶ tan θ > 0 (+ve)

② 90 θ 180 ⟶ tan θ < 0 (−ve)

maxima $\quad f'' \Rightarrow -ve$

$(\underbrace{+3, +2}_{-1}, \underbrace{+1}_{-1}, 0, -1, -2, -3)$



+1 +1 +1 +1 +1

$\boxed{-3, -2, -1, 0, 1, 2, 3}$ Slope

minima

$f''(x)$

↓

+ve

minima-points ⇒ (1, 3, 5)

maxima-points ⇒ (2, 4)

Q:- How to Find the Candidate points for maxima and Minima?

$\boxed{f'(x) \Rightarrow 0}$

↓

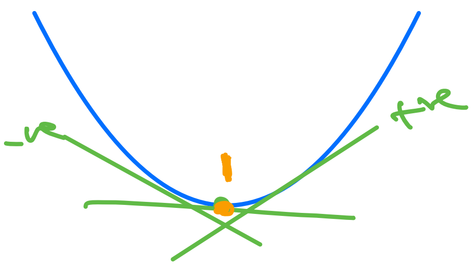
1, 2, 3, 4, 5

Q Maxima, Minima ?

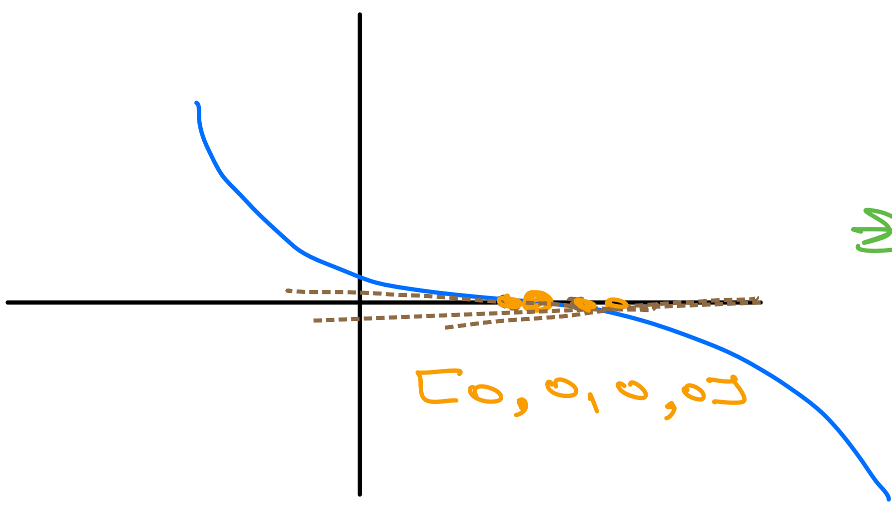
⇒ Calculate $f''(x)$

$f''(x)$ →
- $f''(x) > 0$ → minima
- $f''(x) < 0$ → maxima
- $f''(x) = 0$ ⇒ Saddle point

Ex:



① ⇒ minima



Saddle point

⇒ 0

Global Minima ⇒ minimum of

minimum

0   60

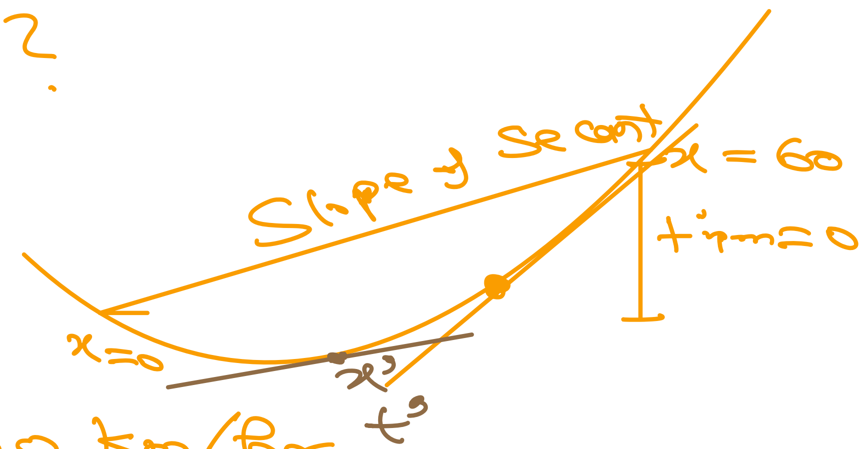
x_1 x_2

t units

60 km

2 hr

30 km hr ?



Avg time = 30 km/hr

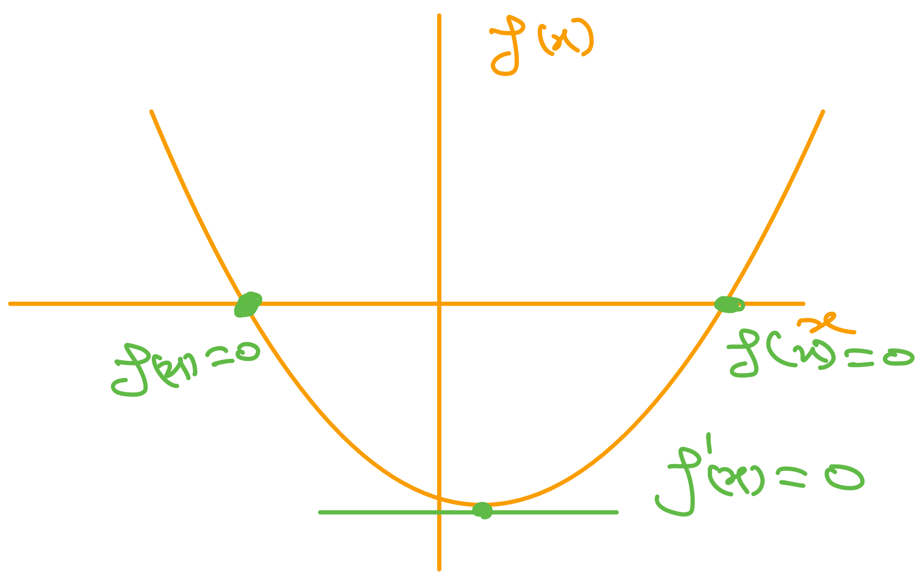
$\frac{d(x)}{d(t)}$ @ x' = Velocity @ time t'

Time Left: 0s

How do we identify critical points in a function?

20 users have participated

A	By finding the points where the function equals zero.	15%
B	By identifying the points where the function intersects the y-axis.	0%
C ✓	By finding the points where the derivative of the function is equal to zero	85%
D	By identifying the points where the function intersects the x-axis.	0%



Slope $\Rightarrow$ 0

Slope $\Rightarrow$ 0