⇒ Revision

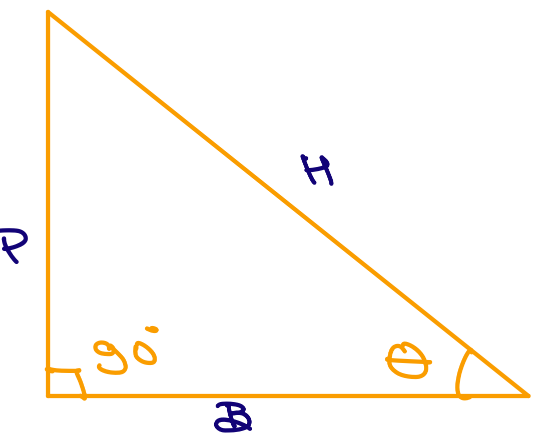
⇒ Relationship b/w $\vec{w}$ and Line

⇒ Trignometric Relation b/w angle and △ Traingle

⇒ Proyection of vector

⇒ Shifting the Line

Trignometry of Angles



$$\tan\theta \Rightarrow \frac{P}{B}$$

$$\sin\theta \Rightarrow \frac{P}{H}$$

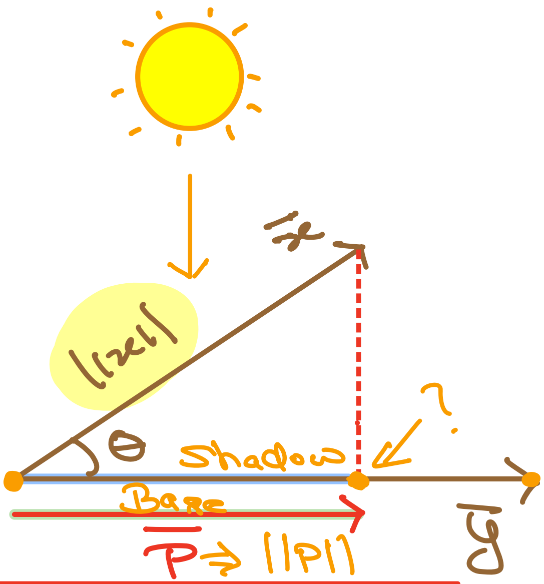
$$\cos\theta \Rightarrow \frac{B}{H}$$

$$\cos\theta = \frac{x^T \cdot y}{\|x\| \|y\|}$$

Proyection of Vector

$$\cancel{\|P\| = \|x\|}$$

P and x == y and x



① (trigo) $\cos\theta = \dfrac{\|P\|}{\|x\|}$

② (L·A) $\cos\theta = \dfrac{x^T y}{\|x\|\,\|y\|}$

$$\boxed{\text{Proyection of } \bar{x} \text{ on } \bar{y}}$$

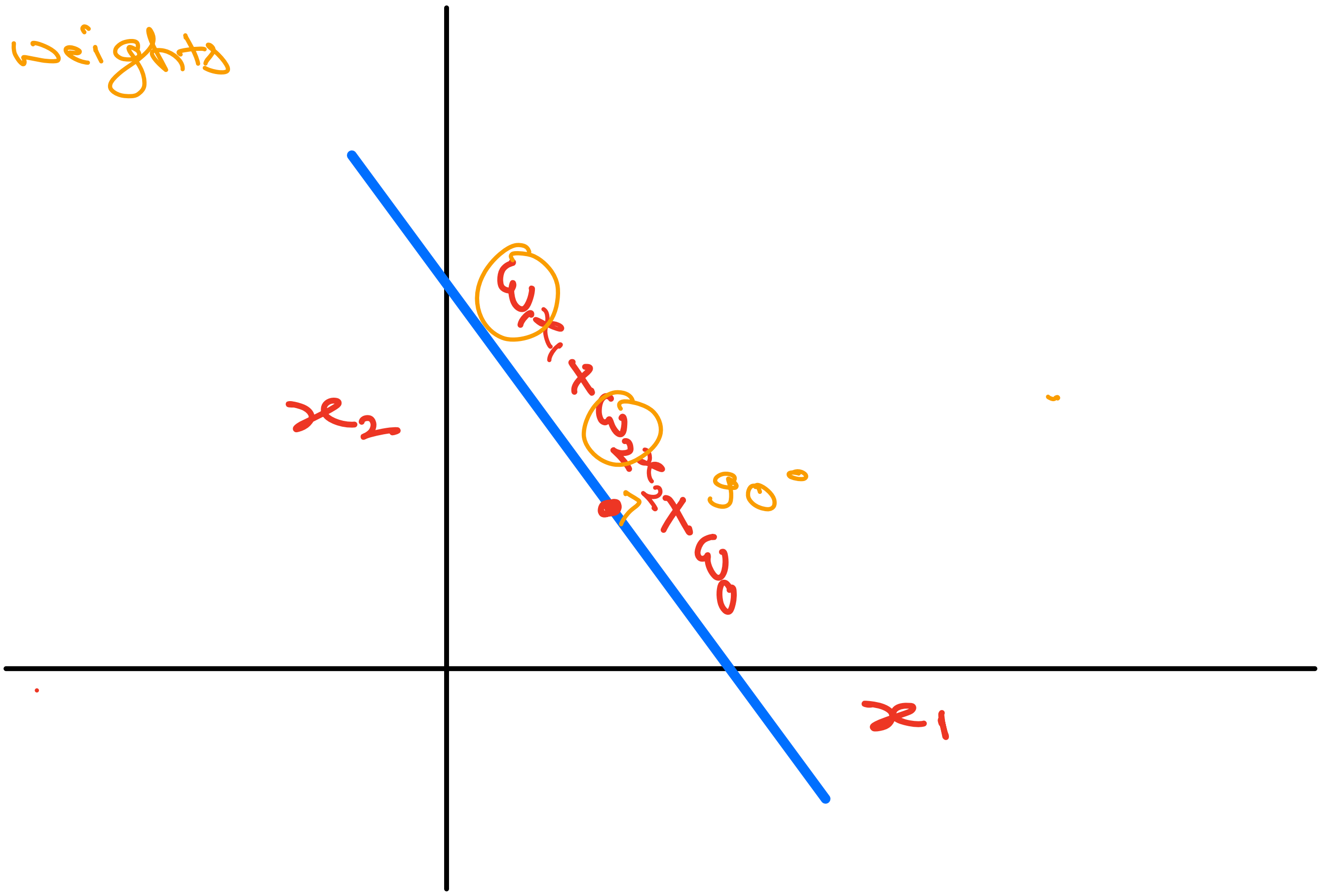
$$\frac{\|P\|}{\cancel{\|x\|}} = \frac{x^T y}{\cancel{\|x\|}\,\|y\|}$$

$$\|P\| = x^T \frac{y}{\|y\|} \rightarrow \hat{y}$$

$$\|P\| = x^T \hat{y}$$

Shifting lines

Weight vectors are co-ordinates of weights



Shifting Origin

line → $\boxed{w_1 x_1 + w_2 x_2 + w_0 = 0}$

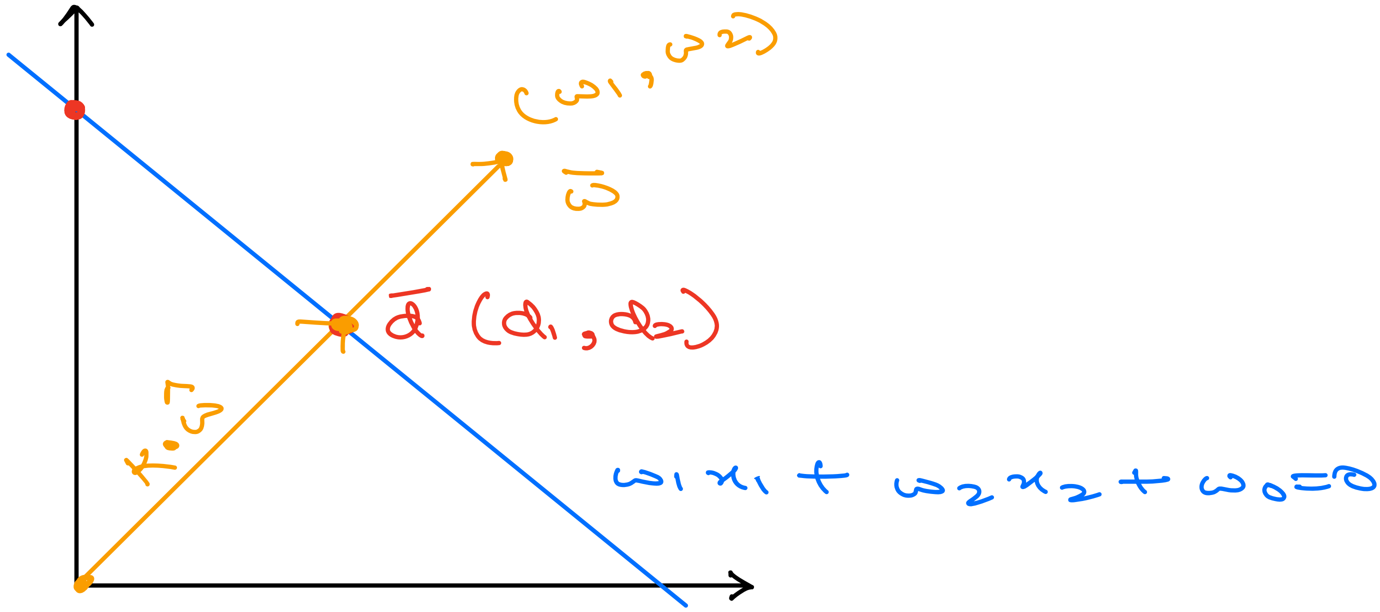
Shift line by a units to → $\boxed{w_1(x_1 - a) + w_2 x_2 + w_0}$

Shift line by a units to ← $\boxed{w_1(x_1 + a) + w_2 x_2 + w_0}$

Shift line by a units to ↑ $\boxed{w_1 x_1 + w_2(x_2 - a) + w_0}$

Shift line by a units to ↓ $\boxed{w_1 x_1 + w_2(x_2 + a) + w_0}$

Weight Vector is $\perp$ Line



$$\bar{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad ||\omega|| = \sqrt{\omega_1^2 + \omega_2^2}$$

① lets say point d is the intersection point of line and $\bar{\omega}$

$$\bar{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

Hence it should satisfy the equation of line $\omega_1 d_1 + \omega_2 d_2 + \omega_0 = 0$

ⓑ If a point lies on line, it must satisfy eqn of line

Q. What would be unit vector of $\bar{\omega}$?

$$\hat{\omega} = \frac{\bar{\omega}}{||\omega||}$$

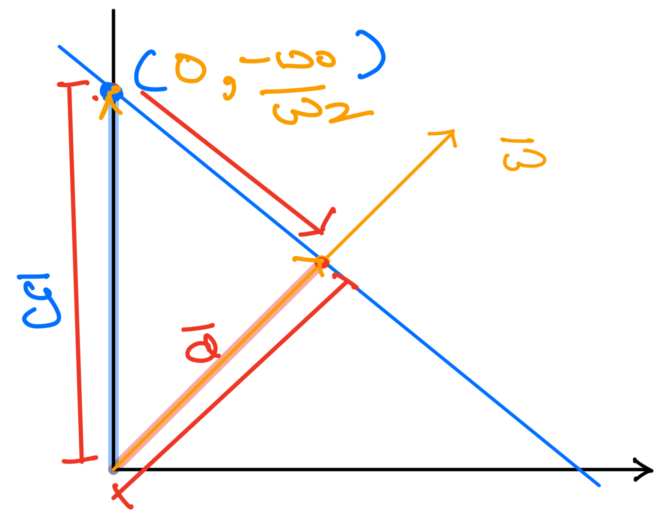
Q. Can we say that $\bar{a}$ is in same direction of $\bar{\omega}$?

Hence we can write

$$\bar{a} = k \cdot \hat{\omega} \leftarrow \text{unit vector of } \omega$$

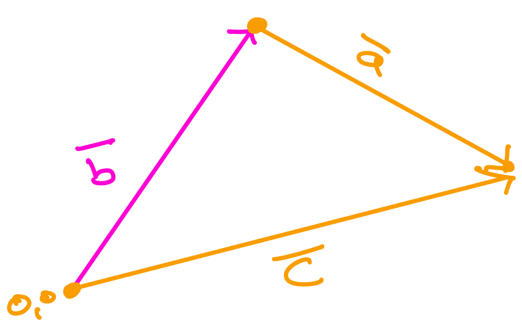
$$\bar{a} = \begin{bmatrix} k \frac{\omega_1}{||\omega||} \\ k \frac{\omega_2}{||\omega||} \end{bmatrix}$$

Q. What will be value of y intercept?

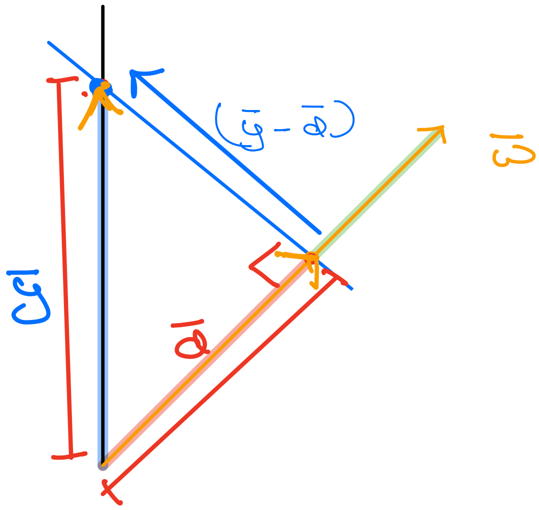


$\omega_1 x_1 + \omega_2 x_2 + \omega_0 = 0$

Law of Vector Addition



$$\bar{c} = \bar{a} + \bar{b}$$



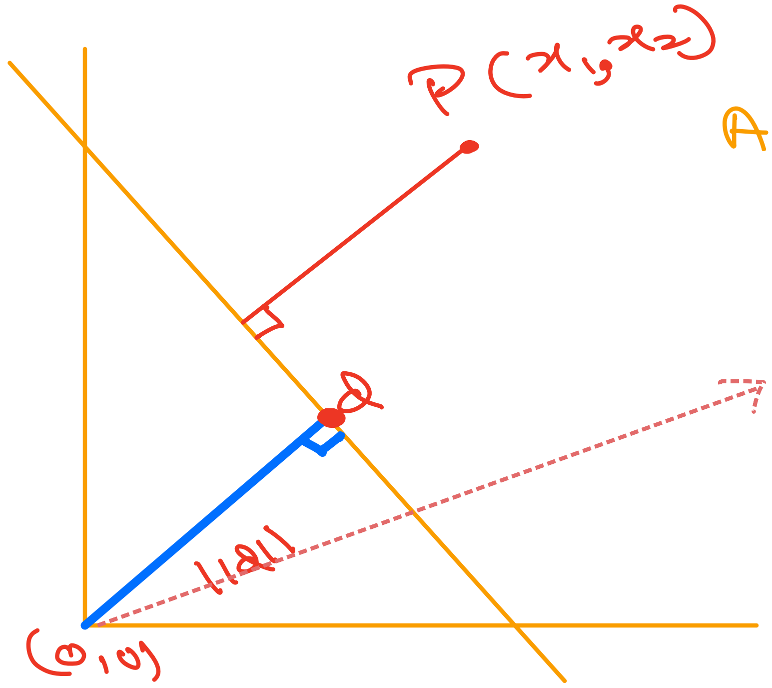
if you calculate

$$\bar{a}^T \cdot (\bar{y} - \bar{a}) \Rightarrow 0$$

$$\bar{y} = \left(0, -\frac{w_0}{w_2}\right)$$

$$\bar{a} = \begin{bmatrix} k\dfrac{w_1}{\sqrt{w_1^2+w_2^2}} \\ \\ k\dfrac{w_2}{\sqrt{w_1^2+w_2^2}} \end{bmatrix}$$

Distance between origin and Line



$$Ax + By + c = 0$$

$$\downarrow$$

$$\frac{Ax_1 + Bx_2 + c}{\sqrt{A^2 + B^2}}$$

$$\frac{C}{\sqrt{A^2+B^2}}$$

For eq^n $ax + by + c = 0$

$$\text{distance} = \frac{c}{\sqrt{a^2+b^2}}$$

$(\omega_1, \omega_2, \omega_0)$

$$\|Q\| = \frac{\omega_0}{\left(\sqrt{\omega_1^2+\omega_2^2}\right)} = \frac{\omega_0}{\|\omega\|}$$

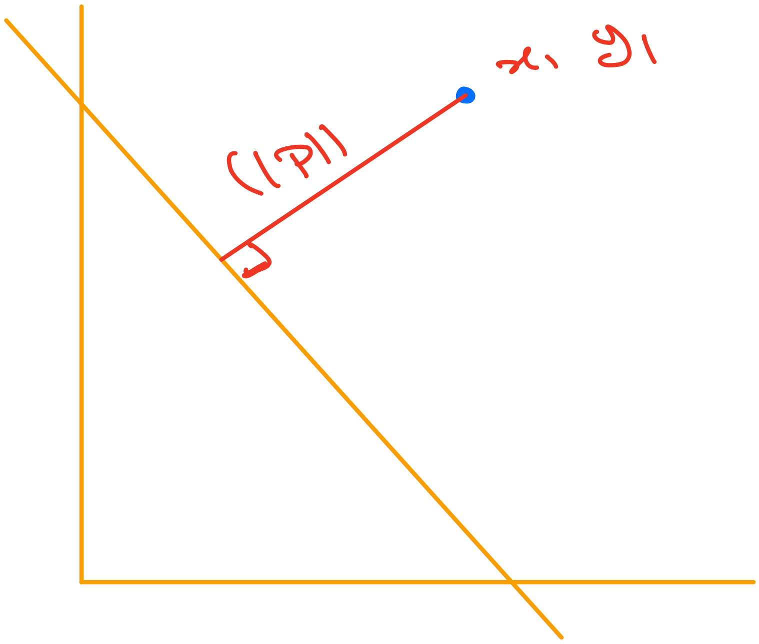
$$\bar{Q} = \begin{bmatrix} \dfrac{-\omega_0 \omega_1}{\omega_1^2+\omega_2^2} \\ \\ \dfrac{-\omega_0 \omega_2}{\omega_1^2+\omega_2^2} \end{bmatrix}$$

A	The absolute difference between the intercept term and the norm of the weight vector
B	The product of the intercept term and the norm of the weight vector
C	The intercept term divided by the norm of the weight vector ✓
D	None of the above

$$\|Q\| = \frac{\omega_0}{\|\vec{\omega}\|}$$

$$\frac{\text{bias}}{\|\bar{\omega}\|}$$

Distance between point and Line



$$ax + by + c = 0$$

$$(x_1, y_1)$$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

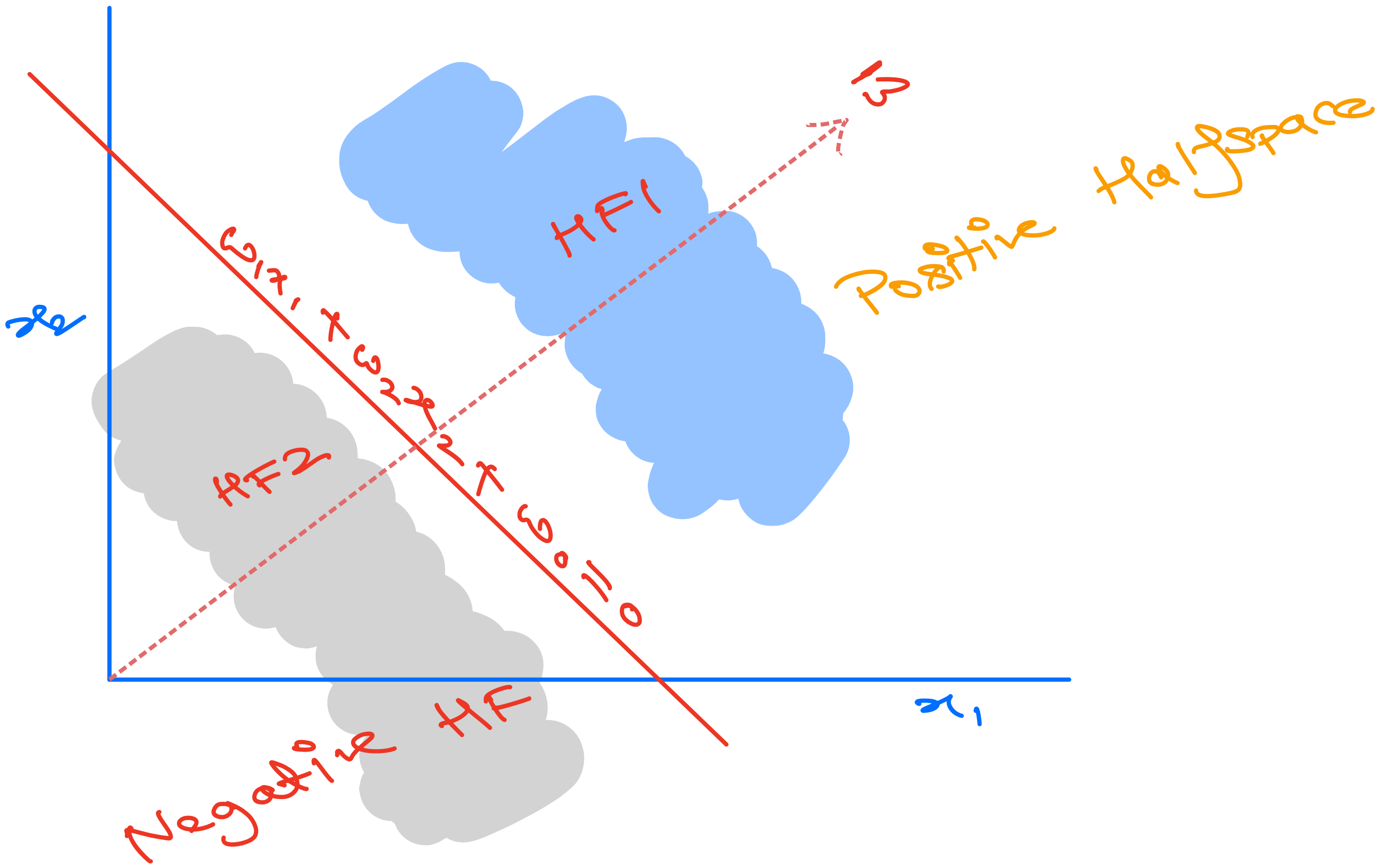
$$\bar{P} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

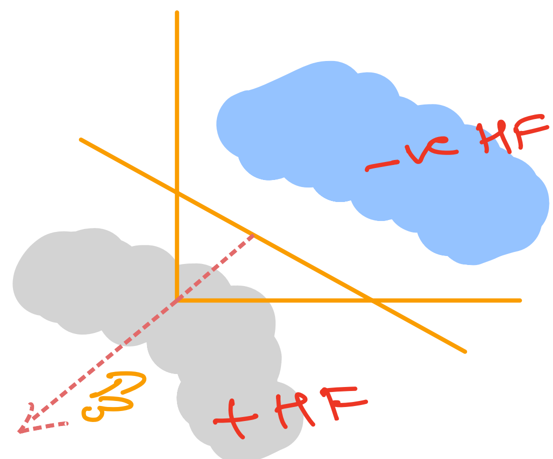
$$||P|| = \frac{\overset{\bar{w}^T \cdot \bar{P}}{\overline{w_1 x_1 + w_2 x_2 + w_0}}}{||\bar{w}||}$$

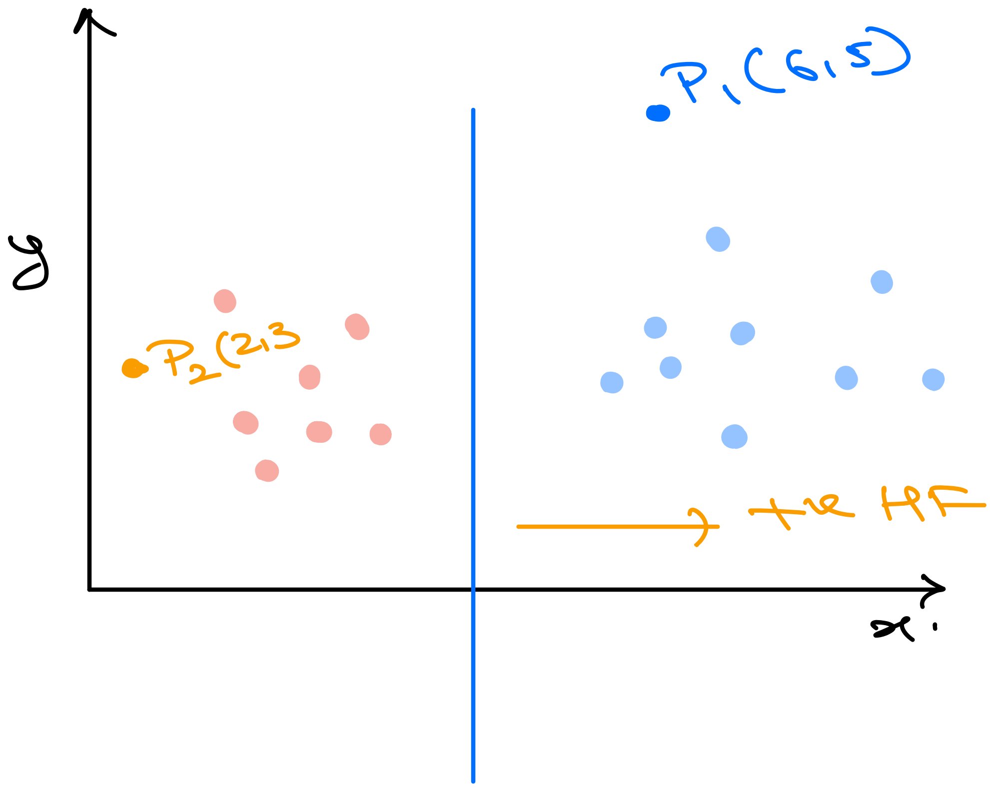
A	Calculate the ratio of the point's coordinates
B	Subtract point's coordinates from the line's equation
C	Take the square of the point's coordinates and add them
D	Substitute point in line equation, take absolute value, divide by weight vector's norm

$$||P|| = \frac{\bar{w} \cdot \bar{P} + w_0}{||\bar{w}||} \Rightarrow \frac{\text{np.dot}(w, P)}{\text{np.linalgNorm}(w)}$$

Revisit Halfspace







$x = 5$

$$w_1x + w_2y + w_0 = 0$$

$$w_1x + w_0 = 0$$

$$\boxed{x - 5}$$ classifier/model

$P_1 = (6,5)$

$$w^T \cdot P + w_0 > 0$$

$$6 - 5$$

$$1 > 0$$

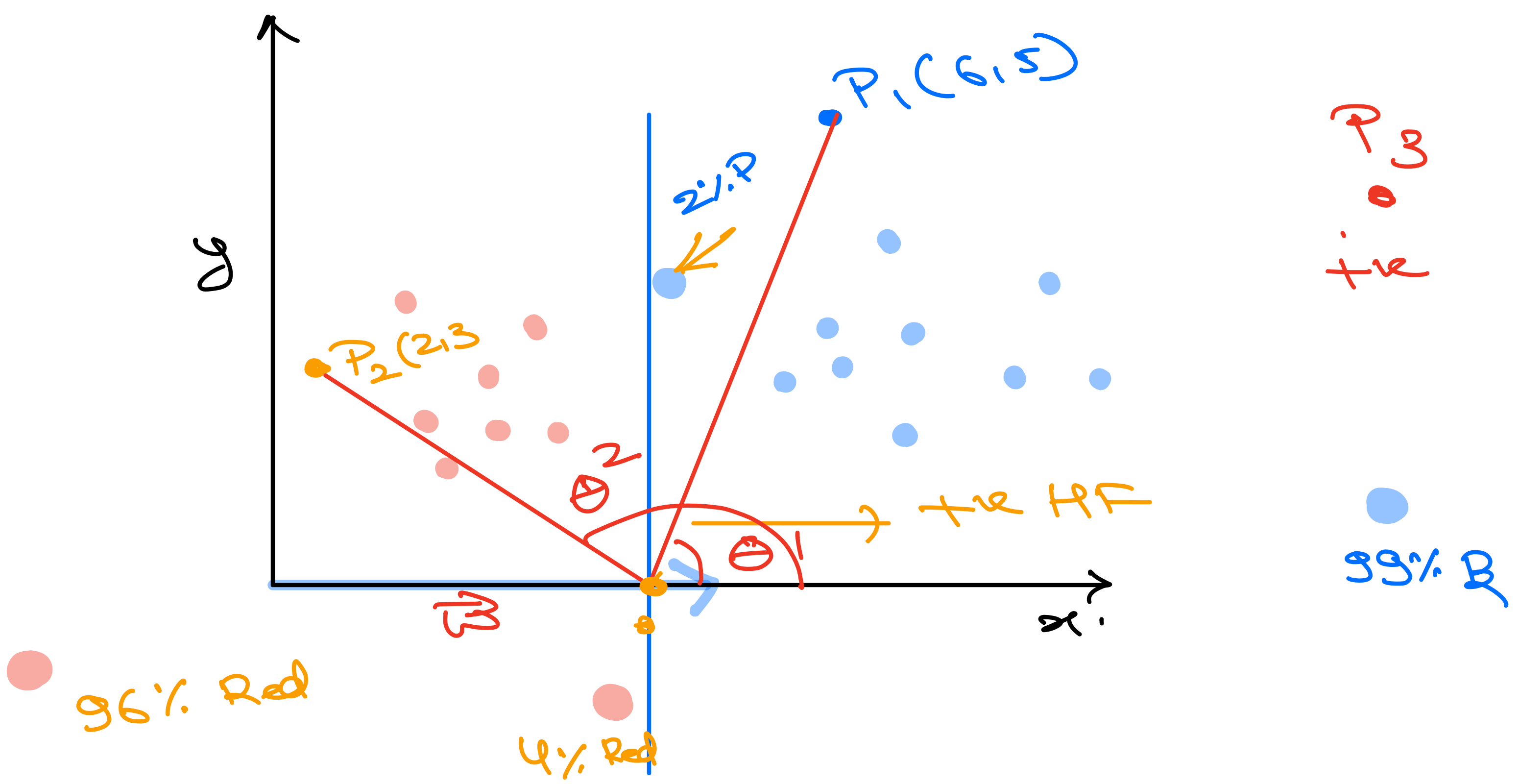
$P_2 = (2, 3)$

$x - 5 = 0$

$$\bar{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\bar{p} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$w^T \cdot P + w_0 \Rightarrow 2 - 5 = -3 < 0$$



$$\cos\theta = \frac{\overset{+ve}{w^T \cdot p}}{\|w\|\|p\|}$$

$0-90 \Rightarrow +ve$

$90-270 \Rightarrow -ve$

$270-360 \Rightarrow +ve$

P is $w^T \cdot p$

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{a \cdot b}{1 \times 2} \Rightarrow \frac{1}{2}$$

$\cos\theta = \frac{1}{2}$ $\qquad \theta = \cos^{-1}\frac{1}{2}$