Recap

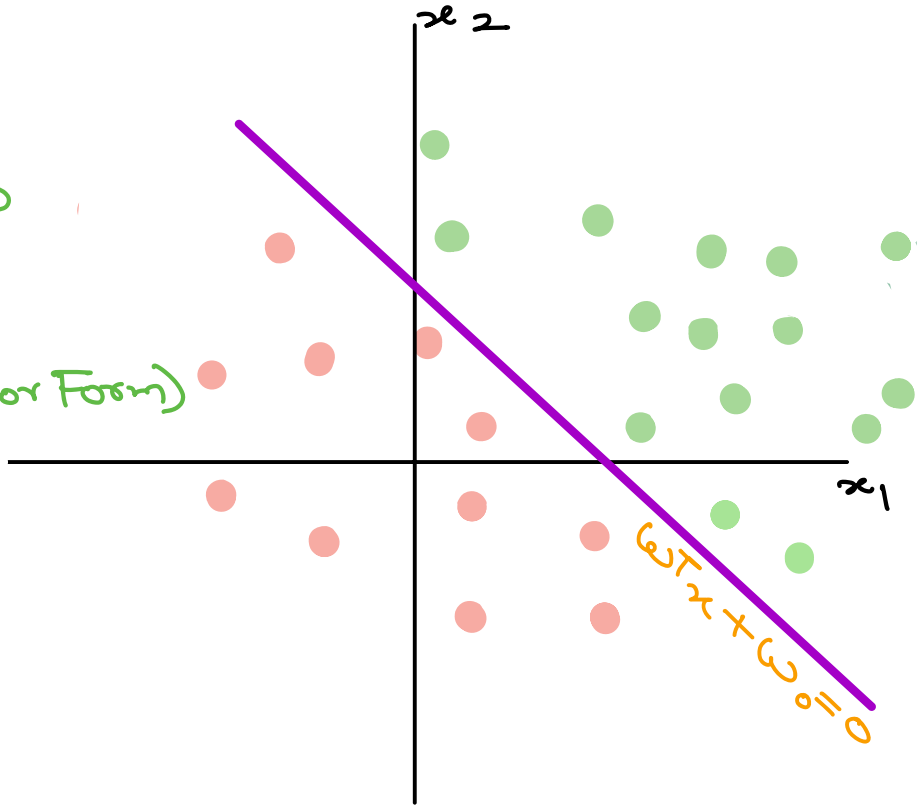
Eqn of Line and HalfSpaceces

x_1	x_2	y
r_1	r_2	$+1$

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$\downarrow$

$w^T x + w_0 = 0$ (Vector Form)



a) Vectors :

$$\bar{x} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_d \end{bmatrix} \qquad \bar{x} \in \mathbb{R}^d$$

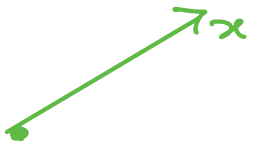
$X_{matrix} \in \mathbb{R}^{n \times m}$

b) Norm : Magnitude = Distance = Length

$||x|| \rightarrow$ L2 Norm $= \sqrt{x_1^2 + x_2^2 \cdots x_d^2}$

$|x| \rightarrow$ L1 Norm $= |x_1| + |x_2| \cdots |x_d|$

Unit Vector $(\hat{x}) = \left|\left|\hat{x}\right|\right| = 1$

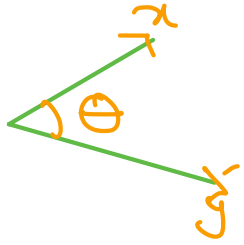


$\frac{x}{||x||}$

① Dot product :

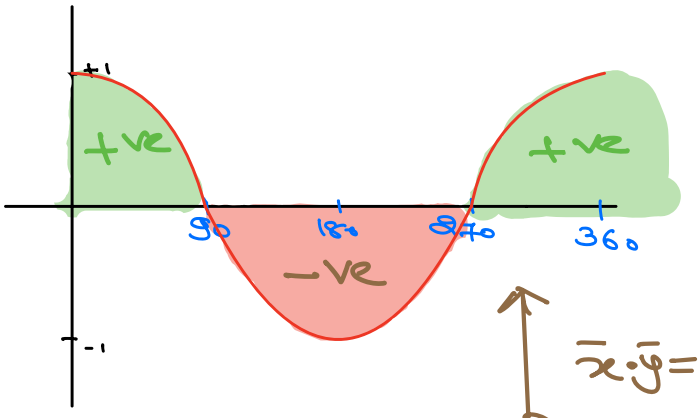
$$\bar{x} \cdot \bar{y} \equiv x^T y \equiv x_1 y_1 + x_2 y_2 \cdots x_d y_d$$

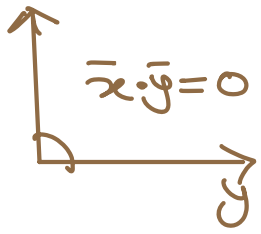
② Angle b/w Vectors : $\bar{x}$ and $\bar{y}$



$$\cos\theta = \frac{\bar{x}^T y}{||x||\,||y||}$$

+1 and −1





③ d dimensional Hyperplane :

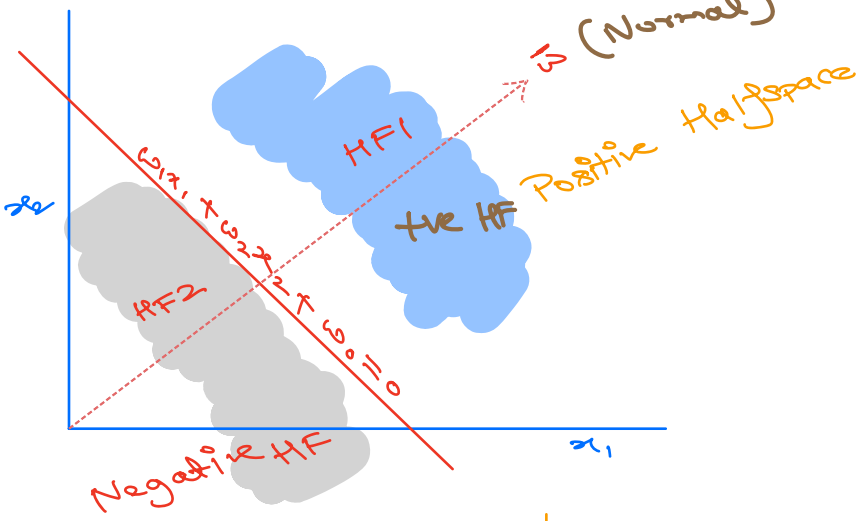
$$w^T x + w_0 = 0$$

where $\bar{x}$ and $\bar{w} \in \mathbb{R}^d$ ↳ Parameters

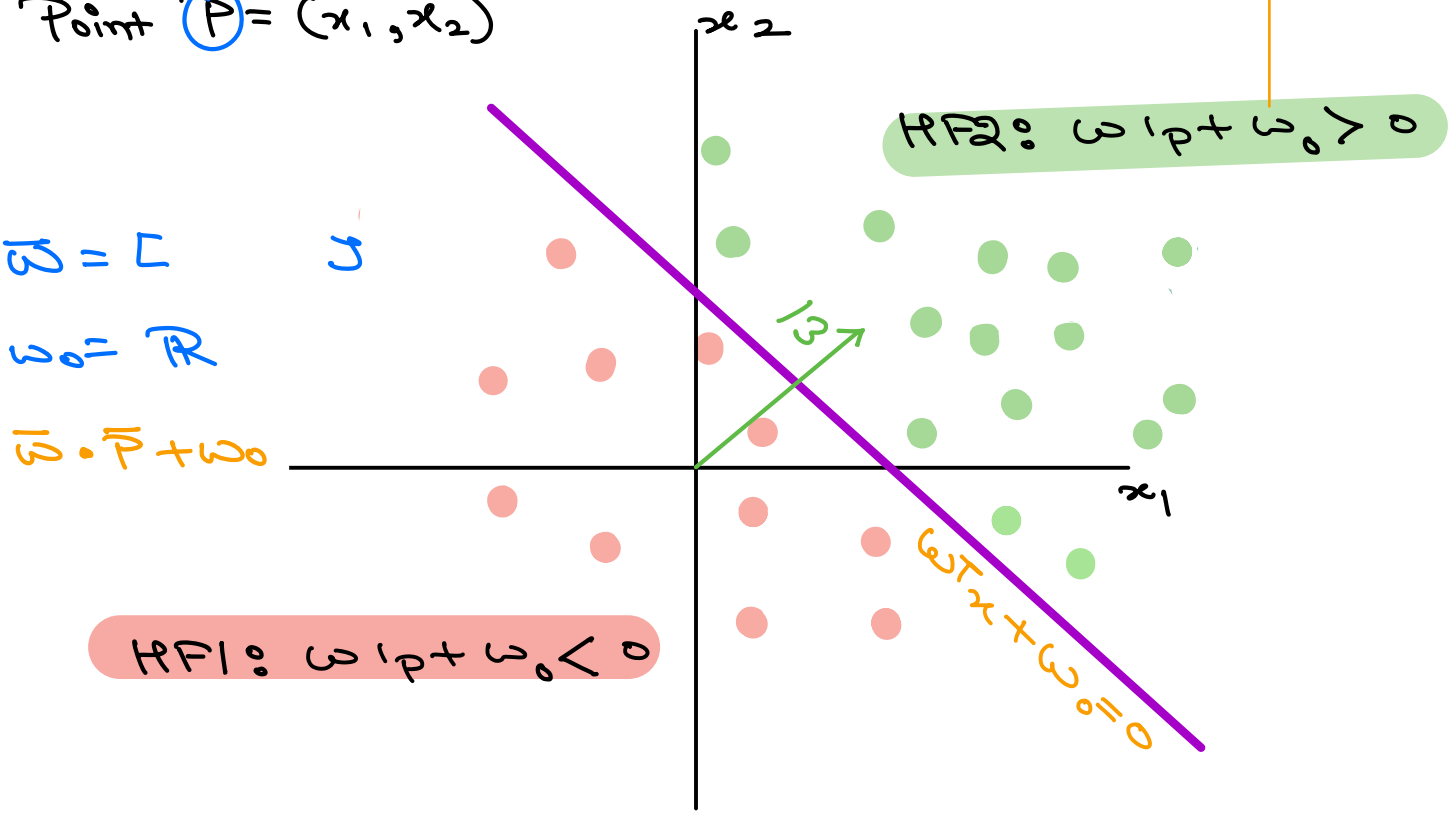
$w_0 \in \mathbb{R}^1$ ↳ bias

$\bar{x} \in \mathbb{R}^2$
$\bar{w} \in \mathbb{R}^2$

④ HalfSpace



Point P = (x_1, x_2)

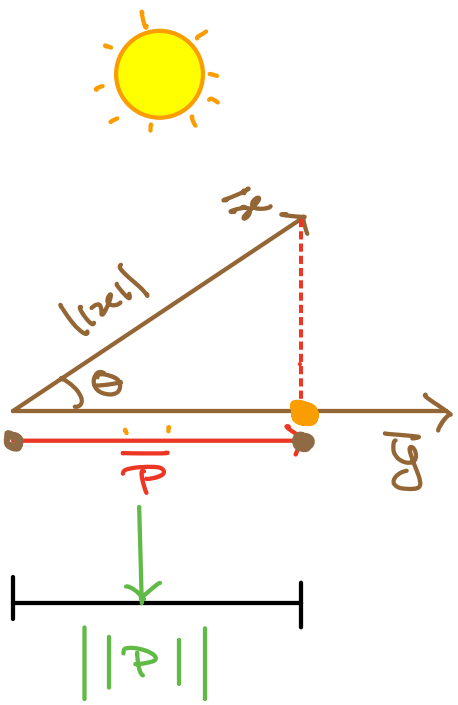


$w = [\quad]$

$w_0 = R$

$w \cdot p + w_0$

ⓐ Projection



Projection of x on w

$$\|p\| = \frac{x^T y}{\|y\|}$$

$$\|p\| = x^T \hat{y}$$

ⓑ Loss function: - Gain Function

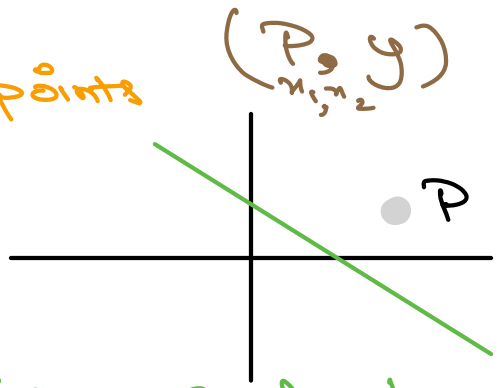
$-$ (Sum of distances b/w CLF and $\vec{x}$)

How far the points are from line

$$-\frac{1}{r}\sum_{i=1}^{5}\left(\frac{\vec{w}^T\vec{x}_i + w_0}{\|\vec{w}\|}\right) \times y_i$$

① Perceptron : misclassified points

$\text{sign}(w^T p + w_0) \; != \; \text{sign}(\hat{y})$



① Initialize w and w_0 randomly
② Identify misclassified point
③ Update using misclassified points

Case 1 : Actual $= +1$ Predicted $= -1$

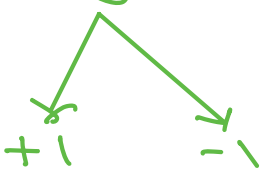
$$w_{new} = w_{old} + p$$

Case 2 : Actual $= -1$ Predicted $= +1$

$$w_{new} = w_{old} - p$$

Combining the two

$$w_{new} = w_{old} + \vec{p} \cdot \vec{y}$$



Problem 1

suppose we have two vectors

vector x =[2,1,-3]

vector y=[5,8,6]

- What is the length of the projection of x onto y?

$$||p|| = \frac{x^T y}{||y||}$$

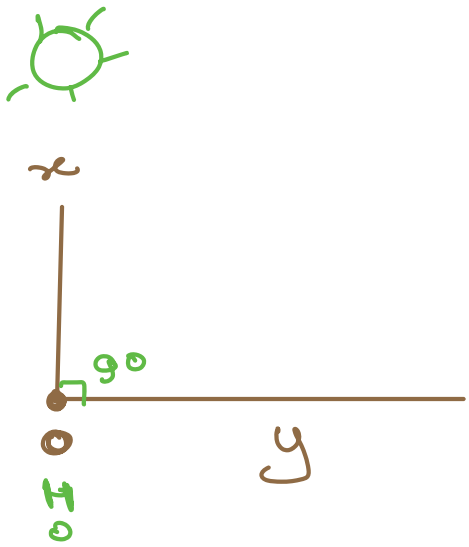
$$||p|| = x^T \hat{y}$$ ← unit vector $\hat{y}$

$x \cdot y$ ⇒ $[2, 1, -3] \cdot [5 \quad 8 \quad 6]$

$x \cdot y$ ⇒ $2 \times 5 + 1 \times 8 + -3 \times 6$

⇒ $10 + 8 - 18$ ⇒ 0

$proj = \frac{x \cdot y}{||y||} \rightarrow 0$ ⇒ 0



$$\cos\theta = \frac{x \cdot y}{\|x\| \|y\|} \quad 90$$

90

Problem 2

given a and b are two unit vectors such that,

c=a+2b

d=5a-4b

these two are perpendicular, then what is the angle between a and b?

$a \rightarrow \|a\| = 1$

$b \rightarrow \|b\| = 1$

$c = a + 2b$

$d = 5a - 4b$

$c \perp d \rightarrow c \cdot d = 0$

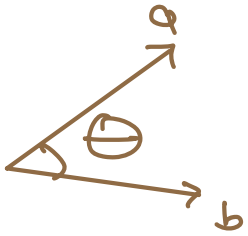
Q: θ b/w a and b?

$c \perp d \rightarrow c \cdot d = 0$

c $\quad$ d

$\Rightarrow (a + 2b) \cdot (5a - 4b) = 0$

$5a^2 + 10a \cdot b - 4ab - 8b^2 = 0$



$\Rightarrow 5a^2 + 6a \cdot b - 8b^2 = 0$

$5 \times 1 + 6ab - 8(1) = 0$

$-3 + 6ab = 0$

$ab = \frac{3}{6} \Rightarrow \frac{1}{2}$

$a \cdot b = \frac{1}{2}$

$a \rightarrow \|a\| = 1$

$b \rightarrow \|b\| = 1$

$a = \sqrt{i^2 + j^2} = 1$

$a \cdot a = i^2 + j^2 = \|a\|^2$

$(i,j)\ (i,j)$

$\Rightarrow (1)^2$

$\Rightarrow 1$

$$\cos\theta = \frac{\bar{x}^T y}{\|x\| \|y\|}$$

$\cos\theta = \frac{a \cdot b}{\|a\| \|b\|}$

$\cos\theta = \frac{1/2}{1 \times 1}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) \rightarrow 60^\circ$

If vector a=[3,4] and vector b=[5,0], what is the cos inverse of the angle between a and b?

1 user has participated

A	cos inverse of 4 by 5	0%
✓ B	cos inverse of 3 by 5	100%
C	cos inverse of 2 by 5	0%
D	cos inverse of 1 by 5	0%

End Quiz Now

$$\cos\theta = \frac{x \cdot y}{\|x\|\|y\|}$$

$$\frac{[3,4] \cdot [5,0]}{\sqrt{3^2+4^2}\ \sqrt{5^2+0^2}}$$

$$\frac{3\times5 + 4\times0}{\sqrt{25}\ \sqrt{25}} = \frac{15}{25} = \frac{3}{5}$$

$$\cos\theta = \frac{3}{5}$$

Given 2 lines which are parallel to each other find the distance between them

- $L1 = w_1^T \bar{x} + w_{01}$
- $L2 = w_2^T \bar{x} + w_{02}$

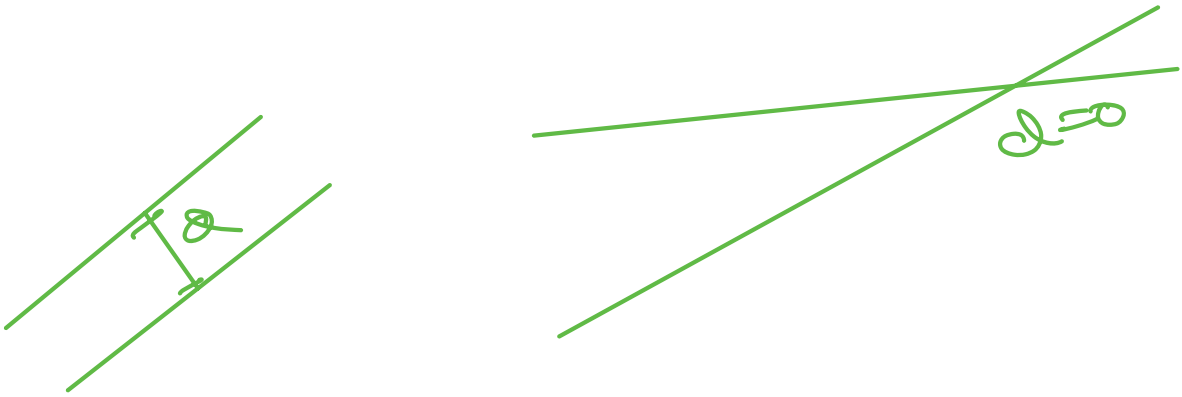
$$-\frac{3}{4} \Rightarrow -\frac{12}{16}\ \frac{3}{4}$$

Where:

$$m = -\frac{w_2}{w_1}$$

$$\frac{w_{01} - w_{02}}{\|w\|}$$

- w_1=$[4,3]^T$
- w_2=$[16,12]^T$
- w_{01}=3
- w_{02}=7



$L_1 : 4x + 3y + 3 = 0 \quad \times 4$

$16x + 12y + 12 = 0 \qquad d = ?$

$L_2 : 16x + 12y + 7 = 0$

$$\frac{c_{01} - c_{02}}{\|w\|} = \frac{(12-7)}{\sqrt{16^2+12^2}}$$

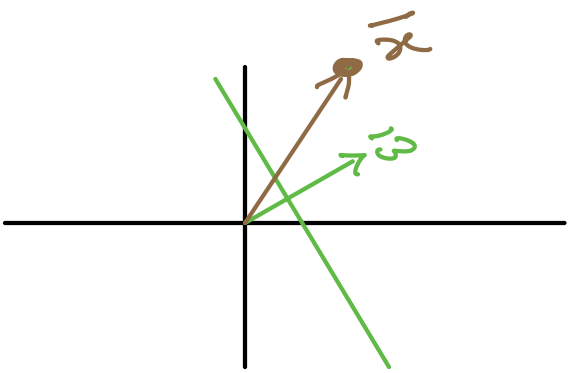
$$= \frac{5}{\sqrt{400}}$$

$$= \frac{5}{20} = \frac{1}{4}$$

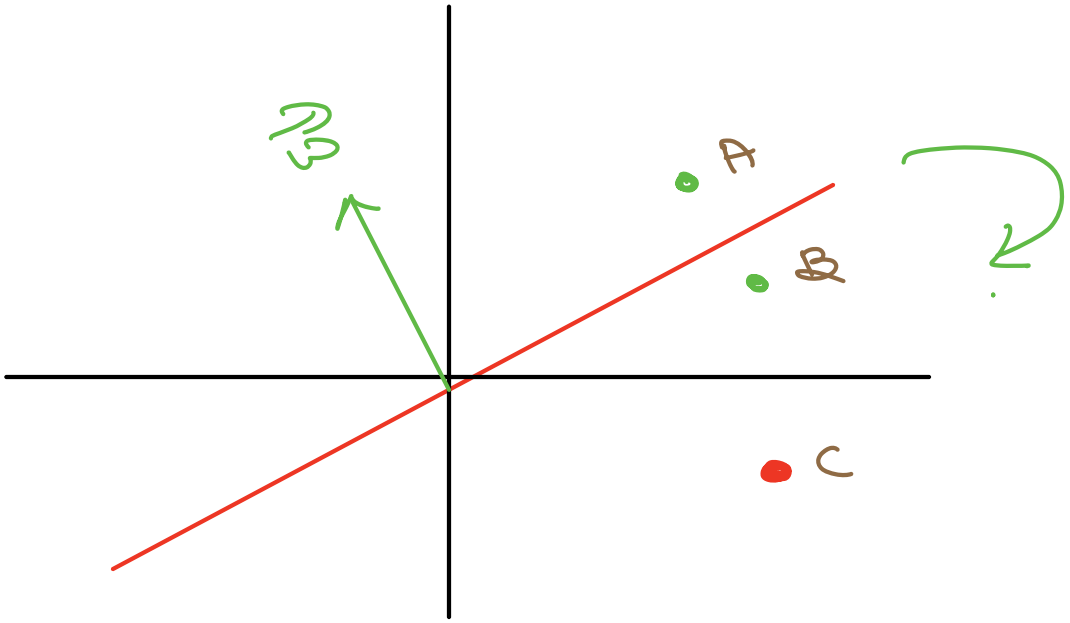
In the classification context we have two vectors x and w Define v=x+w

In which direction should we move w to reach v ,options are:

- clockwise
- anti-clockwise



$V = w - x$



new $\vec{w}$ + $\vec{B}$

What is the distance from the origin to the line 3x+4y=12?

6 users have participated

A	7/5	0%
B	3/5	0%
✓ C	12/5	100%
D	4/5	0%

End Quiz Now

$\frac{w_0}{\|w\|}$ $\frac{12}{\sqrt{3^2+4^2}}$

$\frac{12}{5}$

$w_1 x_1 + w_2 x_2 = w_0$

$w_1 x_1 + w_2 x_2 - w_0 = 0$

What is the distance between the point (2,−3) and the line 3x−4y=5?

0 users have participated

A	2/5	0%
B	3/5	0%
✓ C	13/5	0%
D	4/5	0%

End Quiz Now

$\vec{w} = (3, -4)$

$\vec{p} = (2, -3)$

$w_0 = -5$

$\frac{\vec{w} \cdot \vec{p} + w_0}{\|w\|}$

$\frac{3 \times 2 + (-4 \times -3) - 5}{5}$

$(6 + 12 - 5)/5$

$y = mx + c$