Agenda

→ Conditional Probability

→ Multiplication Rule

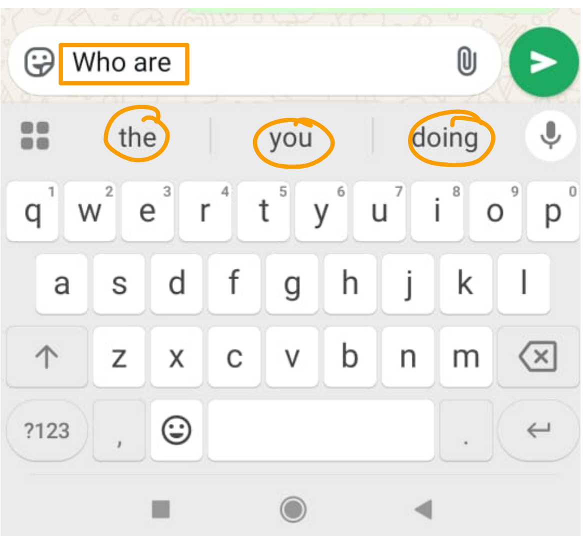
→ Marginal and Joint probability

→ Tree Diagram Approach

→ Law of Total probability

→ Baye's Theorem

Conditional Probability



1) Autocomplete
2) Next word Suggestion

3) ~ 100,000 words

Vocab size

↓

Pick Top 3 words

what words have been typed so far

who, are → Algo → $[w_1, w_2, w_3]$

Mobile Keyboard

who are
→ the
→ you
→ doing

$P[\text{the} \mid \text{who}, \text{are}]$

$P[\text{you} \mid \text{who}, \text{are}]$

$P[\text{doing} \mid \text{who}, \text{are}]$

97

92

All words in Vocab

Selected

Top 3

P Score

Conditional Probability

$$\boxed{P(A \mid B)} = \frac{P(A \cap B)}{P(B)}$$

Experiment

Sum of 2 Dice Throw

Dice 2 (columns), Dice 1 (rows)

D_1+D_2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(D_1 = 2) \Rightarrow \frac{6}{36}$

$P(D_1 + D_2 \leq 5) \Rightarrow \frac{10}{36}$

$P(D_1 = 2 \cap D_1 + D_2 \leq 5) \Rightarrow \frac{3}{36}$

Q What is $P(D_1 = 2)$ given that $D_1 + D_2 \leq 5$

$|SS| \ != 36$

SS → $D_1+D_2 \leq 5$

2	3	4	5	1
3	4	5		2
4	5			3
5				

$P(D_1 = 2 \mid D_1 + D_2 \leq 5) \Rightarrow \frac{3}{10}$

↓

$$\frac{P(D_1 = 2 \cap D_1 + D_2 \leq 5)}{P(D_1 + D_2 \leq 5)} = \frac{\frac{3}{36}}{\frac{10}{36}} \Rightarrow \frac{3}{10}$$

Multiplication Rule

$$P[A/B] \times P[B] = \frac{P[A \cap B]}{\cancel{P[B]}} \times \cancel{P[B]}$$

Multiplication Rule

important

$$P[A \cap B] = P[A/B] \times P[B] \quad (1)$$

$$P[A \cap B] = P[B/A] \times P[A] \quad (2)$$

Questions $P(A/B)$ same as $P(B/A)$?

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$P(D_1 = 2 \cap D_1 + D_2 \leq 5$

$P(D_1 + D_2 \leq 5 \cap D_1 = 2)$

Marginal vs Joint probability

* Marginal probability

$P(\text{Won}) = \frac{184}{360}$

Unconditional or Marginal Probability

$P(\text{Century}) = \frac{46}{360}$

* Joint probability

$P(W \cap C)$

* Conditional =

$P(W \mid C) = \frac{30}{46}$

$P(W^c \mid C)$

$P(W \mid C^c)$

99

Tree Diagram Approach

Questions — Email Spam System

- Let's say 30% of all Emails are Spam
- 70% are Non-Spam
- 80% of all spam Emails contain word 'purchase'
- 10% of Non-spam contains purchase
- Overall what % of Email will have word 'purchase'

Answers

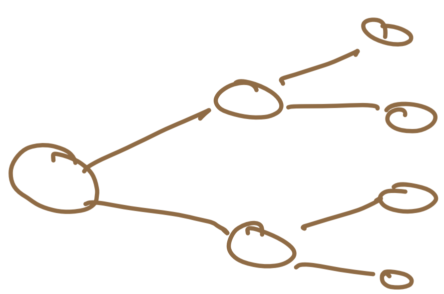
P(Spam) = 0.3

P(Non-Spam) = 0.7

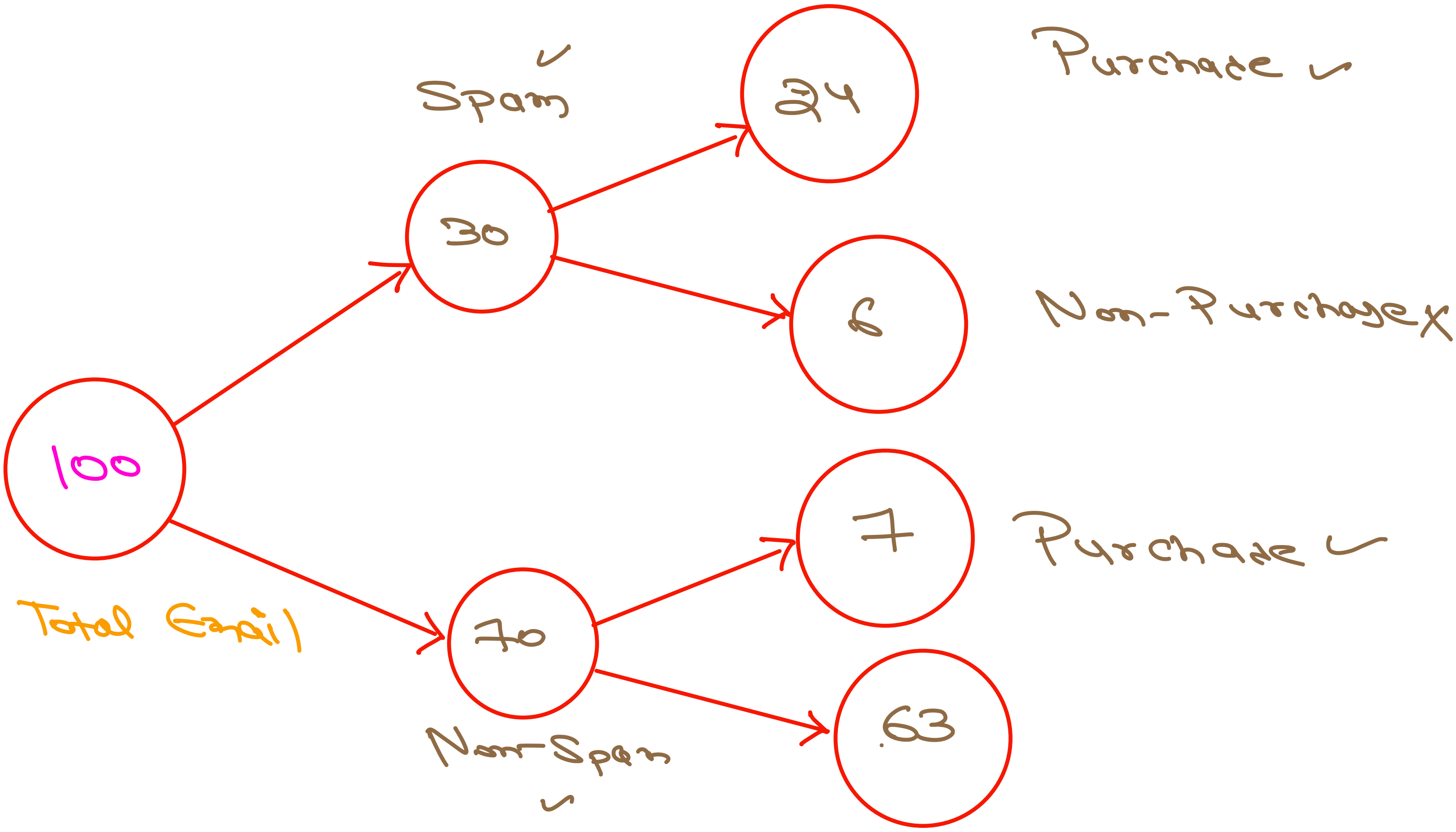
P(Purchase | Spam) = 0.8

P(Purchase | Non-Spam) = 0.1

P(Purchase) = ?



* Tree Diagram Approach



Purchases $\Rightarrow \dfrac{24+7}{100} \Rightarrow 0.31$

Formula to calculate Margin P given condition P's
(Law of Total Probability)

$$\boxed{P(A \cap B) = P(A/B) \times P(B)}$$

P(Purchase) = P(Purchase ∩ Spam) +

P(Purchase ∩ Non Spam)

$= \frac{24}{100} + \frac{7}{100} = \frac{31}{100}$

P(Purchase) = P(Purchase/Spam) * P Spam

P(Purchase/Non-spam) * P non-spam

$$\boxed{\text{Law of Total probability}}$$

$$\boxed{P(A) = \sum_{i=1}^{n} P(A \mid B_i) * P_{B_i}}$$

n = 2

B_1 = Spam

B_2 = Non-Spam

```
It is known that -
5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities.
Overall, what percentage of people are actively seeking new job opportunitie
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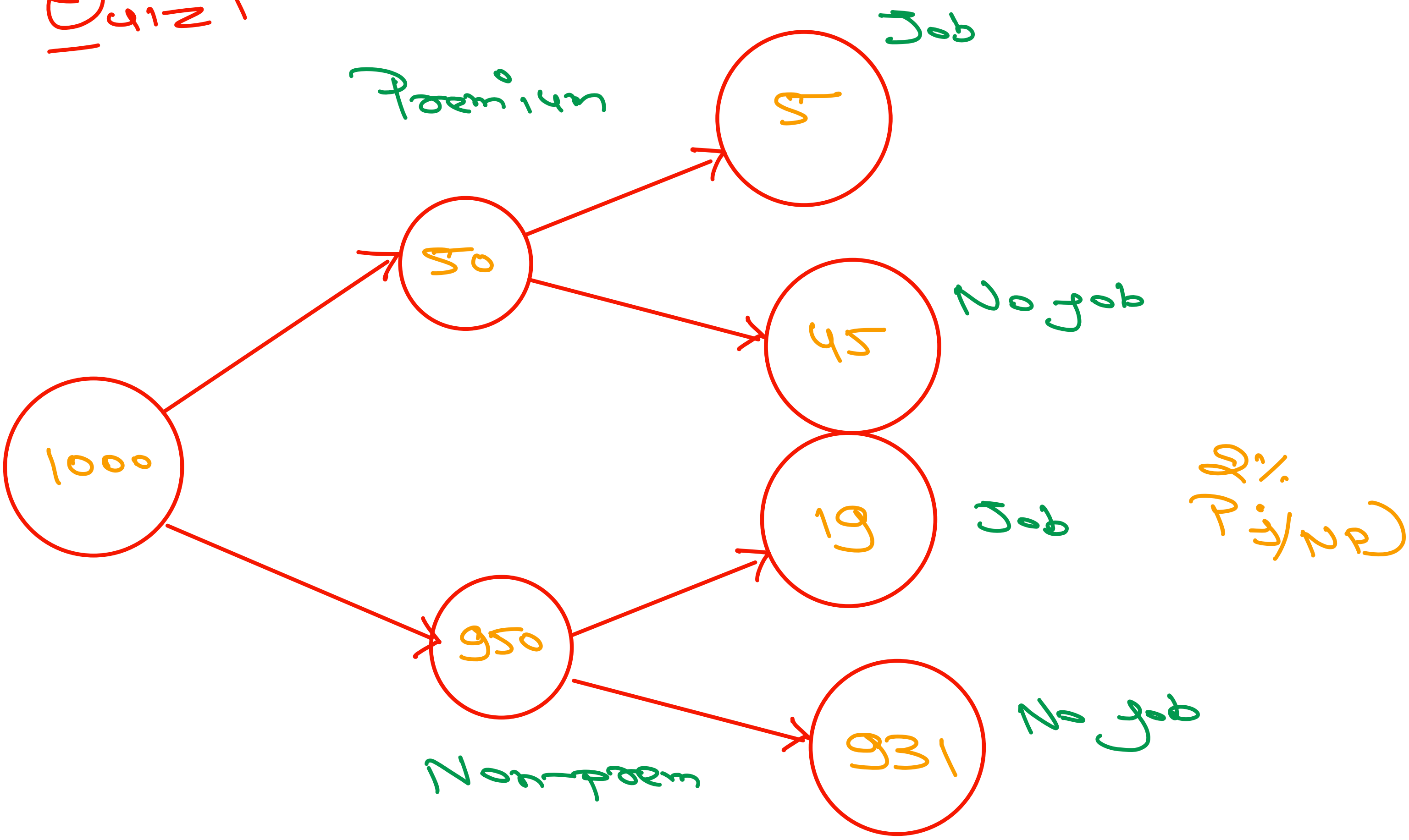
$P(J|\text{Prem})$

Quiz 1



$$P(\text{job}) = \frac{19+5}{1000} = \frac{24}{1000} = 2.4\%$$

H.W.

Questions : Solve above Questions with using Law of Total probability

Summary

1. Conditional Probability:

- $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

2. Multiplication Rule:

- $P(A \cap B) = P(A \mid B) \cdot P(B)$

3) Law of Total Probability: $\longrightarrow$ Tree based Method

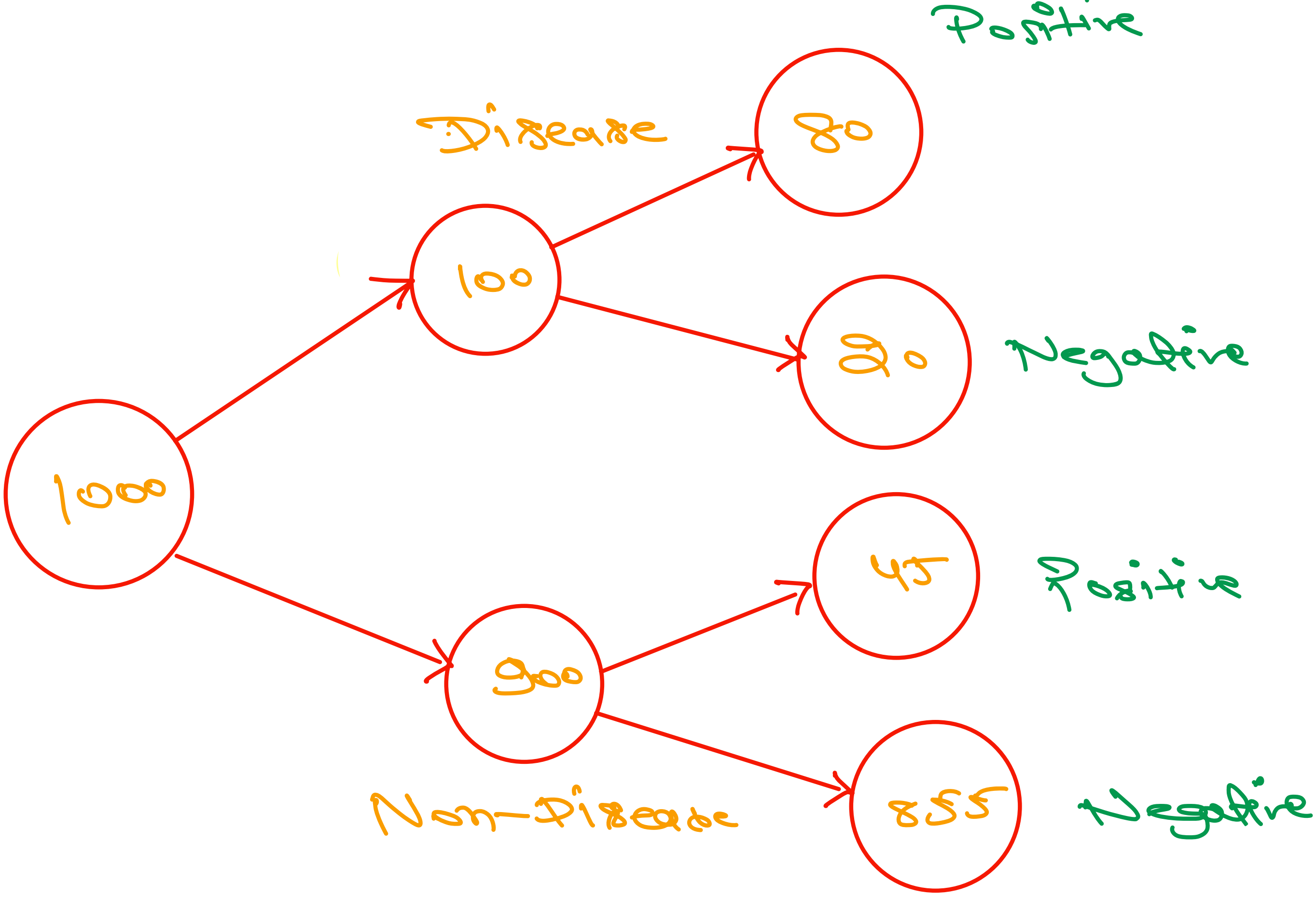
- $P(A) = \sum_{i=1}^{n} P(A \mid B_i) P(B_i)$

Questions

```
A disease affects 10% of the population.
Among those who have the disease, 80% get "positive" test result ⇒ P(+ve|D)
Among those who don't have the disease, 5% get "positive" test result.
What is P(+ve | Disease) ?

a) 0.1
b) 0.8
c) 0.05
d) 0.85
```

P(+ve / ND) = 5%



$P_{+ve} \cap \text{disease}$

$\frac{80}{1000} \Rightarrow 0.08$

or

$P[A \cap B] \Rightarrow P_{A/B} \times P_B$

$\Downarrow$

$0.8 * 0.1$

$\Rightarrow 0.08$

important

Baye's Theorem

$$\boxed{P[A|B] \Rightarrow \frac{P[B|A] \times P[A]}{P[B]}}$$

Questions : Derive Above formula

Hint :

$$P(A \cap B) = P(A/B) \times P(B)$$

$$P(B \cap A) = P(B/A) \times P(A)$$

Likelihood

Marginal

$$P[A \mid B] \Rightarrow \frac{P[B \mid A] \times P[A]}{P[B]}$$

Marginal

Posterior Probability