Agenda

- Distribution Functions
    - Histogram
    - Probability Mass Function (PMF)
    - Probability Density Function (PDF)
    - Cumulative Distribution Function (CDF)
- Empirical vs Theoretical Probability
- Binomial Distribution + Expectations
- Case Study Conclusion + BD Conditions
- Bernoulli Distribution

Random Variable

1) Continous RV
   - Temperature Forecast
   - Rain in MM
   - Price Prediction

2) Discrete RV
   - Outcome of A Toss / Dice
   - Win vs Loss Prediction

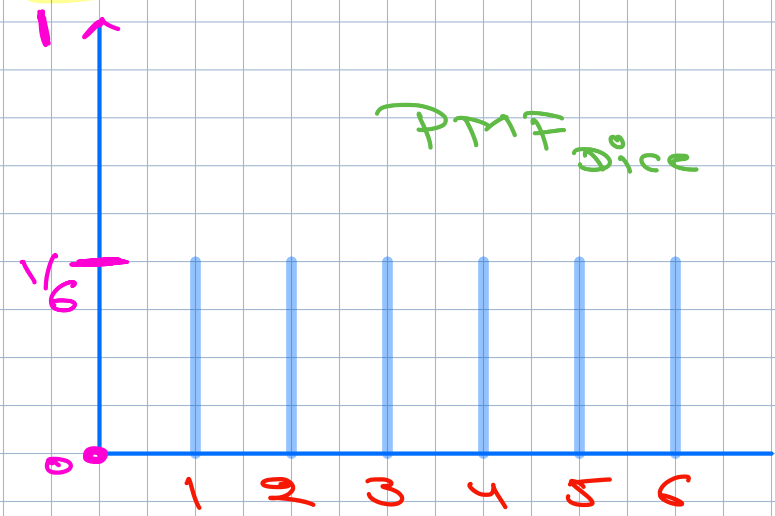
Distributions Functions

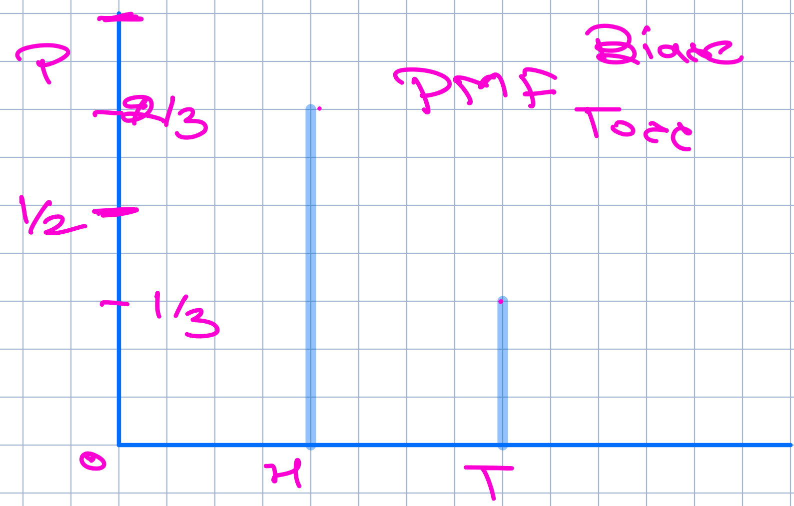
1. PMF (Probability Mass Function)
2. PDF (Probability Density Function)
3. CDF (Cumulative Density Function)

PMF

- PMF is a Function that describe probability of a discrete Random Var

$X_{Dice} \rightarrow \{1, 2, 3, 4, 5, 6\}$





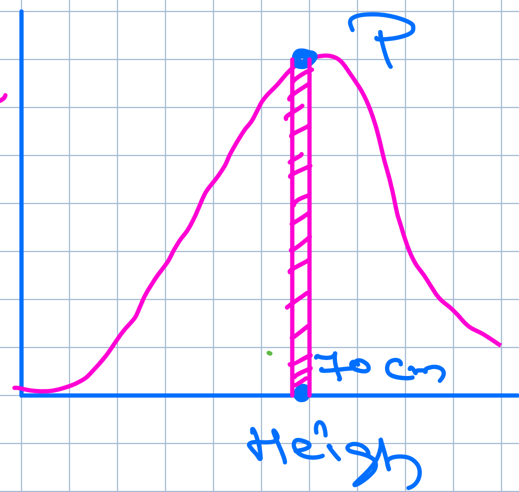
Biased Coin

↓

H	2/3
T	1/3

PDF

- PDF is a Function that describe Probability Density of a Continous Random Variable



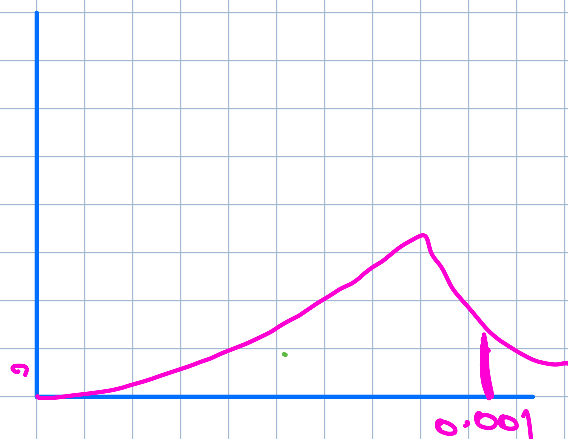
P

70.00

70.5

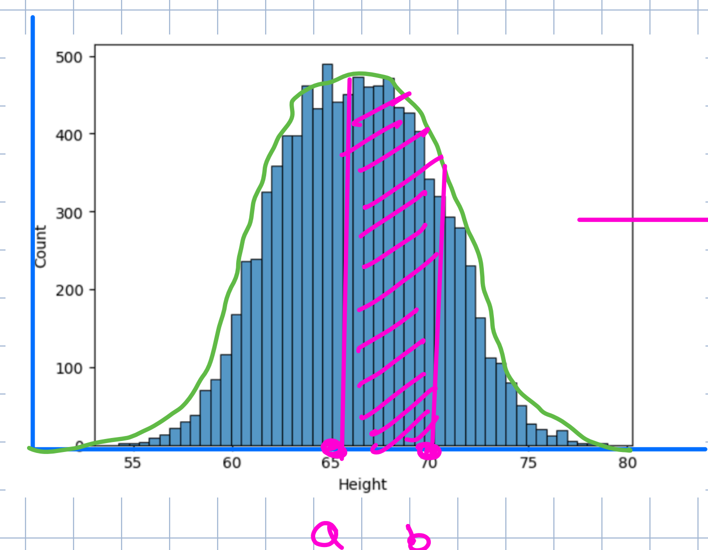
~~70.0000000000 -- -- cm~~

~~70.000000001 cm~~



$\mathbb{R} \in 70$ cm

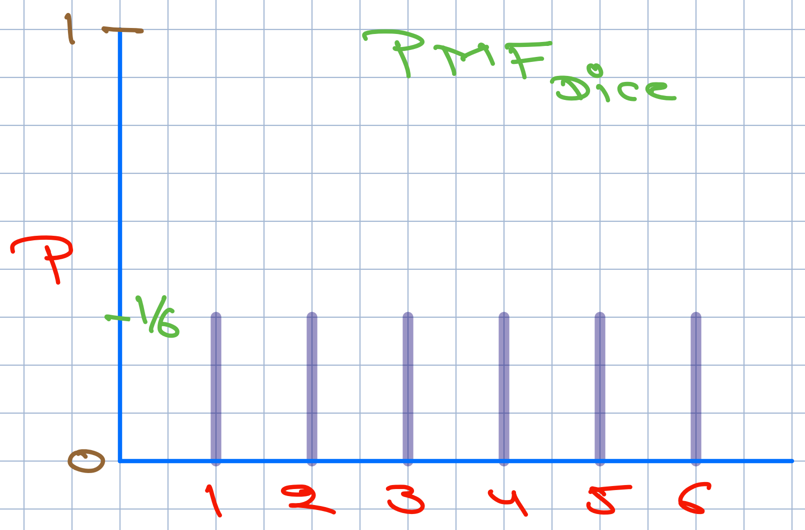
$$\int_{x\to a}^{x\to b} f(x)$$

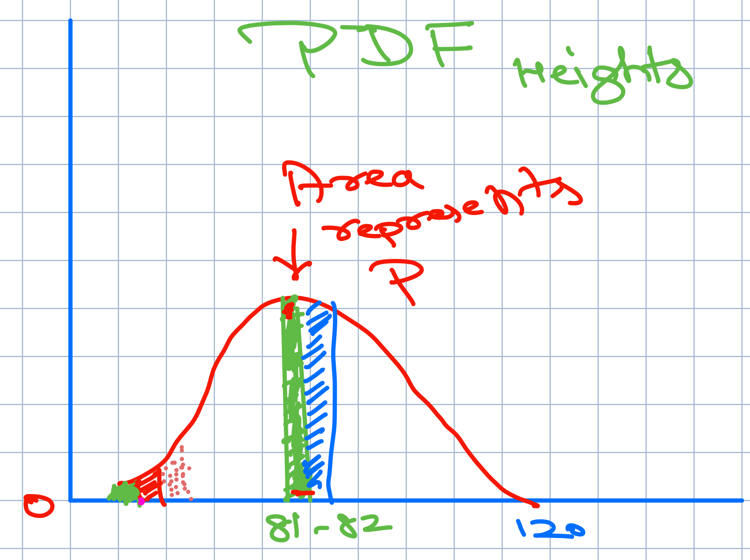


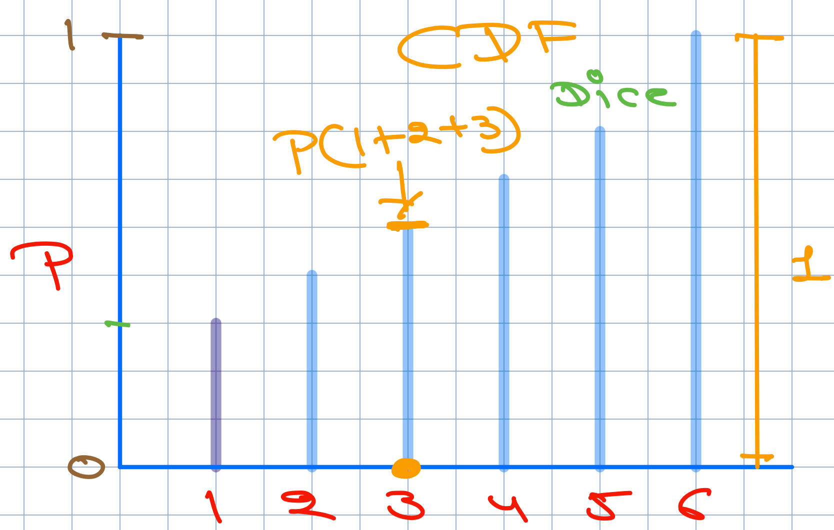
$$\longrightarrow \int_a^b f(x)$$

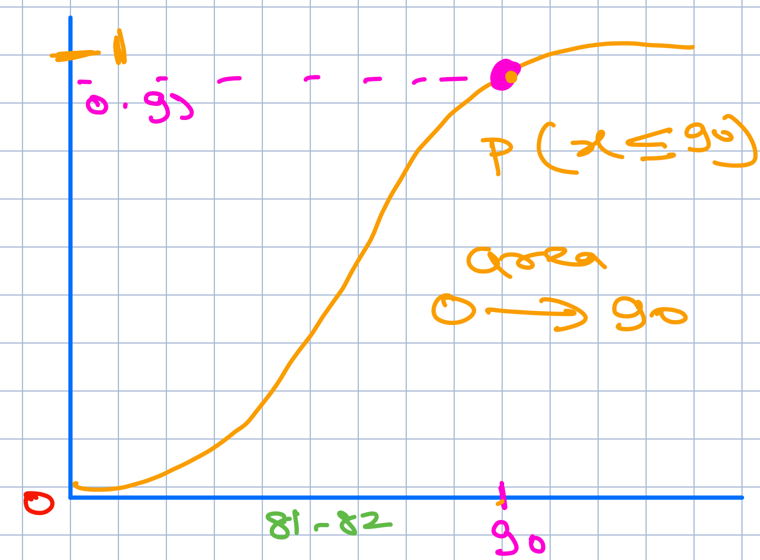
CDF

Cumulative Distribution Function









Precentile (x, 50%) → Value

CDF (x, Value) → 50%

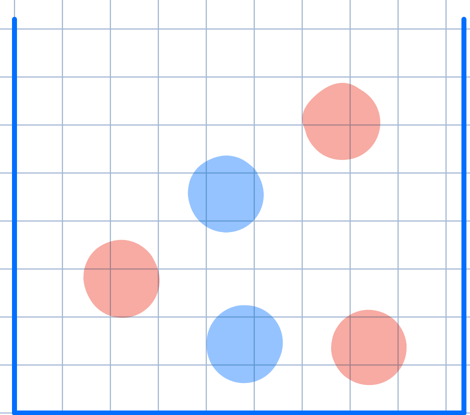
* Conclusion

1. PMF → P(Discrete Random Vars)
2. PDF → P(Continous Random Vars) * Range
3. CDF → Cumulative for Both

Emprical vs Theoretical Probability

Case Study :

1) A Bag has 3 red Balls and 2 Blue Balls
2) You pick a ball, write its color and put it Back into Bag (With Replacement)
3) This is done 4 Times
4) If all 4 times Red Ball is drawn, you win RS 150, else Lose Rs 10



Expected Value ?

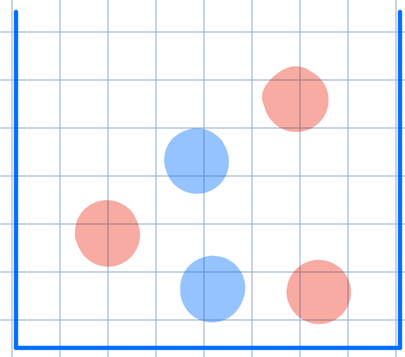
of Money

+ve ✓

-ve ✗

Questions Would playing this game result in profit or Loss?

Can we calculate Expected Value of Red Balls →



① Empirical Experiment

$$X_{\text{Red Count}} \quad \{0 \quad 1 \quad 2 \quad 3 \quad 4\}$$

$$E(X) = \sum_{i=0}^{n} \frac{X_i * P(X_i)}{\sum_{i=0}^{n} X_i}$$

② Theoretical Experiment

Empirical Approach

```
rolls = np.random.choice(bag, size = 4, replace = True)
print(rolls)
sum(rolls == 'R')
```

```
['R' 'B' 'B' 'R']
2
```

Higher the Number Better Estimate

[12]
```
red_vals = []
for _ in range(10000):
    rolls = np.random.choice(bag, size = 4, replace = True)
    count_r = sum(rolls == 'R')
    red_vals.append(count_r)
```

[13]
```
len(red_vals)
```

```
10000
```

```
pd.Series(red_vals).value_counts()
```

	count
3	3472
2	3358
1	1579
4	1334
0	257

Biased coin

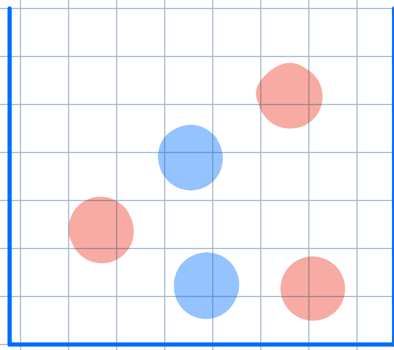
① $\boxed{10 \Rightarrow 7H}$

② $10000 \Rightarrow 7000H$

$E(x) \Rightarrow$ is 2.4 Red balls

Theoretical Approach

Questions



1) $P(\text{Red Ball once})$

$P(R) = \frac{3}{5}$ $\quad P(B) = \frac{2}{5}$

2) $P(\text{Red Ball Twice})$

$P(RR) = \frac{3}{5} * \frac{3}{5}$

3) $(P \text{ Red Ball followed by Blue Ball})$

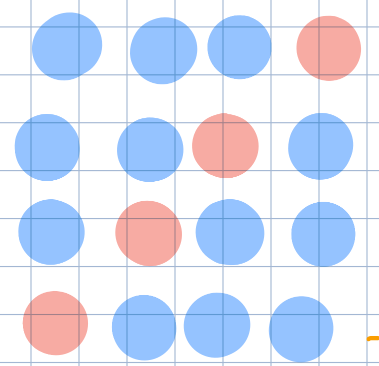
$P(RB) = \frac{3}{5} * \frac{2}{5}$

$P(BR) = \frac{2}{5} * \frac{3}{5}$

4) $P(RBBB) = \frac{3}{5} * \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$

$\left(\frac{3}{5}\right)^1 \times \left(\frac{3}{5}\right)^3$

④ P (1 Red Ball in 4 Trials)



$$4 \times \left(\frac{3}{5}\right)^1 * \left(\frac{2}{5}\right)^3$$

$$\downarrow$$

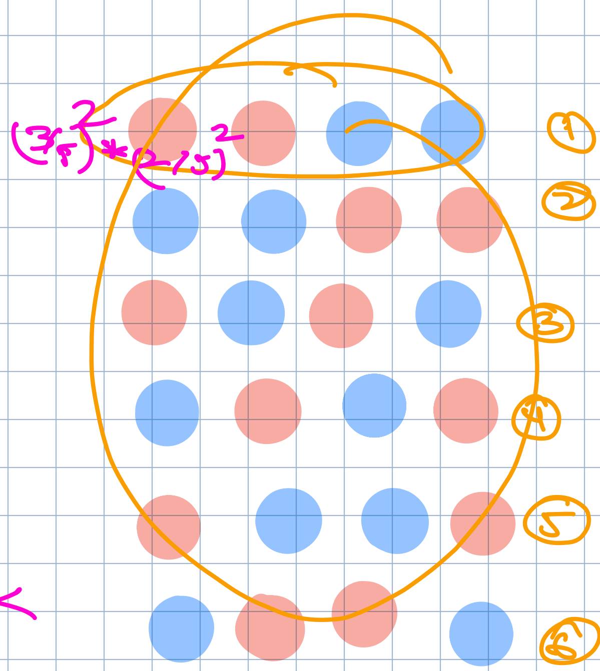
$$^4C_1 \left(\frac{3}{5}\right)^1 * \left(\frac{2}{5}\right)^{4-1}$$

⑤ P (2 Red Ball in 4 Trials)

$$6 \times \left(\frac{3}{5}\right)^2 * \left(\frac{2}{5}\right)^2$$

$$\downarrow$$

$$^4C_2 \left(\frac{3}{5}\right)^2 * \left(\frac{2}{5}\right)^{4-2}$$



$$^nC_k \left(\frac{3}{5}\right)^k * \left(\frac{2}{5}\right)^{n-k}$$

6 P (3 Red Ball in 4 Trials)

7 P (4 Red Ball in 4 Trials)

- $P(X=0) = {}^4C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^4$
- $P(X=1) = {}^4C_1 \left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right)^3$
- $P(X=2) = {}^4C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2$
- $P(X=3) = {}^4C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^1$
- $P(X=4) = {}^4C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^0$

Binomial Distribution

Binomial Distribution help us in calculating K success from n trials

$$\boxed{P(x=K) \rightarrow {}^{n}C_{K}(P)^{K}(1-P)^{n-K}}$$

$P_{success} \rightarrow$ Given

$P_{failure} \rightarrow (1-P)$

Q)

n

K

$P_{success}$

$P(x=K)$

Condition of Binomial Distribution:

1. Independent Trials

2. Trails should have two possible

Variance of Binomial Distribution

$$\sigma^2 = n * p * (1-p)$$

$$n * p * q$$

Original Question

Should we play the game

Expected Money $= 150 * P(x=4) + -10 \times P(x \le 3)$

$= (150 * 0.1263) + -10(1-0.1263)$

Bernoulli Trial

Special Case of Binomial Distribution

$$P(x=k) \rightarrow {}^{n=1}C_1 (P)^k (1-P)^{n-k}$$

$$P * (1-P)$$

Success, failure

Binomial Distribution is a Collection of Bernoulli Trials

Questions

Suppose that we float 10 quizzes, with 4 options each.
Only 1 option is correct. What is the probability that we will get exactly 2 answers correct?

$n = 10$

$P = 1/4 = 0.25$

$k = 2$

P of Getting Exactly two right Answer

$^{n}C_{k} (P)^{**k} (1-P)^{(n-k)}$

$^{10}C_{2} (1/4)^{**2} * (3/4)^{**8}$

Questions

Suppose that we float 10 quizzes, with 4 options each. Only 1 option is correct, and we are guessing the answers. What is the probability that we will get at least 4 answers correct?

$P(x \geq 4) =$

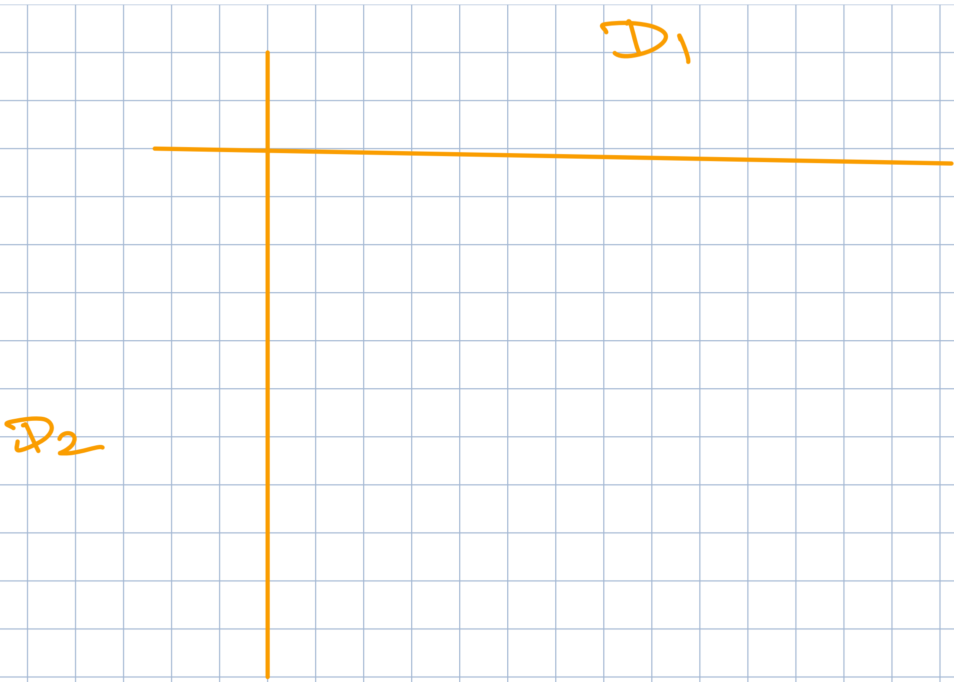
① $P(x=4) - - - - P(x=10)$

② $1 - P(x=0) - - - P(x \leq 3)$

↓ Same

③ $1 - CDF(x=3)$

You toss 2 dice. If both dice are 6, you get Rs 2. Else, if one dice is 6, you get Rs 1. Otherwise, you do not get anything.



$$X_{count-6} \in \{0, 1, 2\}$$

Question = Expected Money you can Make.

6 Speakers 2 identical twins
(can share the seat)

4 Seats to be filled
Order Matters

① Case 1 : No twin Selected

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} \Rightarrow 24$

② Case 2 : One twin $\Rightarrow 2 \times 4 \times 24$

t_1, t_2

4 → Speake

Lets start with t_1

$\underline{T_1}\ \underline{4}\ \underline{3}\ \underline{2} \Rightarrow 24$

$\underline{4}\ \underline{T_1}\ \underline{3}\ \underline{2} \Rightarrow 24$

$\underline{4}\ \underline{3}\ \underline{T_1}\ \underline{2} \Rightarrow 24$

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{T_1} \Rightarrow 24$

$T_1 \Rightarrow 4 \times 24$

$+$

$T_2 \Rightarrow 4 \times 24$

$\boxed{192}$ T_1 or T_2

0 Twin $\Rightarrow$ 24

1 Twin $\Rightarrow$ 192

③ Both twins

(a) Share Seat

$\underline{T_1/T_2}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\Rightarrow 24$

$\underline{4}$ $\underline{T_1/T_2}$ $\underline{3}$ $\underline{2}$ $\Rightarrow 24$

$\underline{4}$ $\underline{3}$ $\underline{T_1/T_2}$ $\underline{2}$ $\Rightarrow 24$

$\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{T_1/T_2}$ $\Rightarrow 24$

$216 + 96$

$\Rightarrow 312$

5 Not Same Seat

$\underline{T_1}$ $\underline{T_2}$ $\underline{4}$ $\underline{3}$ 12×6

$\underline{4}$ $\underline{T_1}$ $\underline{T_2}$ $\underline{3}$

$\underline{4}$ $\underline{3}$ $\underline{T_1}$ $\underline{T_2}$ 1

$$\boxed{312 + 12 \times 6}$$

$\underline{T_2}\ \underline{T_1}$ _ _ $=$ T_1 T_2 _ _ _ _

$\underline{6} \times 5 \,\_\, \underline{4} * 3$ $\quad {}^6P_4$

30 ⇒ 120 ⇒ 360