Agenda

Poisson Distribution

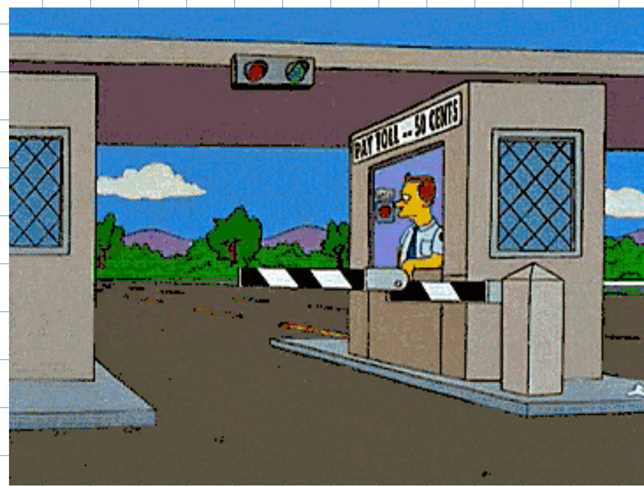
Exponential Distribution

Geometric Distribution

Poisson Distributions

Observation

Count of Vehicles passing through the both in a given Time-period



Questions

num-vehicles

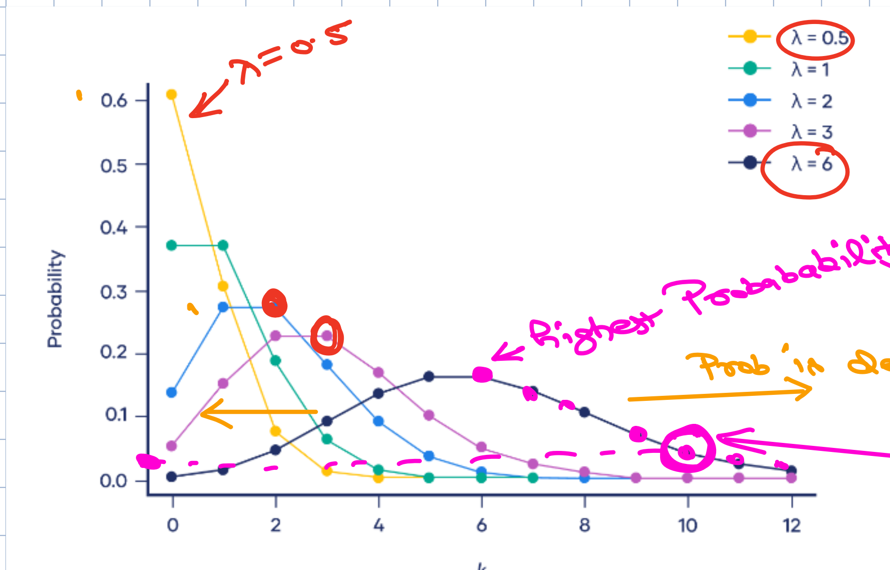
$P(X = K)$ will pass through toll both in next 1 hour

Observation

1. X is discrete
2. Fixed interval is given : 1 hour
3. Avg rate (λ) : Num-vehicle passing per hour

(1) $P(X=25)$ $\lambda = 30\ V/h$

(2) $P(X>40)$ $\lambda = 30\ V/h$



$$\boxed{PMF(X=k) \Rightarrow \frac{\lambda^k * e^{-\lambda}}{k!}}$$

PMF of Poisson Distribution

$k = 10$ $e \Rightarrow 2.7128----$

$\lambda \Rightarrow 6$ per hour

$$\boxed{P(X=10) \Rightarrow \frac{6^{10} * e^{-6}}{10!}}$$

(Probability of observing 10 vehicles in an hour given rate 6 V/h

$\lambda \Rightarrow$ Avg number of Event in the given time interval

Questions

suppose a particular hospital experiences an average of 2 births per hour.
We can use the formula above to determine
the probability of experiencing 0, 1, 2, 3 births, etc. in a given hour:

$P(X=0) \quad \frac{2^0 * e^{-2}}{0!}$

$P(X=1) \quad \frac{2^1 * e^{-2}}{1!}$

$P(X=2)$,,

$P(X=3)$,,

$\lambda = 2/h$

Rules of Poisson Distribution

① Counting :

Poisson distribution is used for P/counting the number of Discrete events happening within Fixed Interval

② Independent Events

③ Rate should be Constant

④ No simultaneous Event

Questions

The shop is open for 8 hours. The average number of customers is 74 - assume Poisson distributed.
(a) What is the probability that in 2 hours, there will be at most 15 customers?
(b)What is the probability that in 2 hours, there will be at least 7 customers?

Observation ⇒ 74 customers in 8 hours

$P(X \le 15)$

↓

2 hour Time

$$\frac{\lambda^{k} * e^{-\lambda}}{k!}$$

⇒ Bring the rate to same Scale

$\lambda_{rate\_2hr} \rightarrow \boxed{74/8 * 2} \Rightarrow 18.5 / 2hr$

$P(X \le 15) \longrightarrow cdf(k=15, 18.5)$

Q-2

ⓐ $1 - cdf(7)$

ⓑ $1 - PMF(7)$

ⓒ $1 - cdf(6)$ ✓

ⓓ $1 - PMF(6)$

$$1 - \sum_{k=0}^{6} \frac{18.5^{k} * e^{-18.5}}{k!}$$

atleast 7

$P(X \ge 7)$

Questions

It is known that a certain website makes 10 sales per hour.
In a given hour, what is the probability that the site makes exactly 8 sales?

$\lambda = 10 / h$

$P(X = 8)$

$\downarrow$

interval 1 hour

Additional Question for HW

Let “X” be the number of typos in a page in a printed book, with mean of 3 typos per page
What is the probability that a randomly selected page has atmost 1 typo?

It is known that a certain hospital experience 4 births per hour.
In a given hour, what is the probability that 4 or less births occur?

Poisson Distribution Approximation to Binomial Distribution

```
There are 80 students in a kinder garden class.
Each one of them has 0.015 probability of forgetting their lunch on any given day.

(a) What is the average or expected number of students who forgot lunch in the class?
(b) What is the probability that exactly 3 of them will forget their lunch today?
```

Q=1

$P_i \times X_i = 0.015 \times 80 =$

Expected_Value = 1.2 ← rate

(Avg number of students forgetting Lunch on a given Day)

Q-2

$P = 0.015$

$X = 3$

$\lambda = n \cdot p = 1.2$

Poisson(k=3, mu = 1.2) ≈ 8.6%

* Binomial Distribution

$$\boxed{P(X=k) = {}^{n}C_{k}\, P^{k} (1-P)^{n-k}}$$

$n = 80$

$P_s = 0.015$ $\quad$ $P_F = 1 - 0.015$

$X=3$

$$\boxed{Bin = {}^{80}C_{3} (0.015)^{3} (1-0.015)^{80-3}}$$

* Poisson Approximation of Binomial Distribution

For a reasonable approximation:

- This approximation is good if $n \geq 20$ and $p \leq 0.05$ such that $np \leq 1$,
- or if $n > 50$ and $p < 0.1$ such that $np < 5$,
- or if $n \geq 100$ and $np \leq 10$.

Exponential Distribution

Example: Q. You receive 240 messages per hour on average - assume Poisson distributed. Rate of messages arriving per second is $\frac{1}{15}$.

> **Q1) What is the probability of having no message in 10 seconds?**

Poisson $\frac{\lambda^k * e^{-\lambda}}{k!}$

$\lambda = 10/15$

$k = 0$

$$P(X=0) = \frac{(10/15)^0 * e^{-10/15}}{0!} = e^{-10/15}$$

$$\boxed{P(X=0) \quad e^{-10\lambda}}$$

$\to \lambda$ rate per second

P of getting 0 message in 10 second given λ = m/sec

Q2. What is the probability of waiting for more than 10 seconds for the next message?

$P(T > 10) \longrightarrow$ to get a message

Probability of getting 0 message in 10 seconds

$P(T > 10) = e^{-10\lambda}$

Q3. What is the probability of waiting less than or equal to 10 seconds?

$P(T \leq 10) = 1 - e^{-10\lambda}$

$P(T \leq 10.5) = 1 - e^{(-10.5)\times\lambda}$

$X \rightarrow$ time (Continuous RV)

$$\boxed{P(T \leq x) = 1 - e^{-\lambda x}}$$

CDF of Exponential Distribution

Poisson (λ) : Calculates P of certain number of Events in Time

Expon (λ) : Calculate P of time b/w Events

PDF :

$\lambda \Rightarrow$ rate

$\beta \rightarrow$ Scale $= 1/\lambda$

$$\boxed{P(T \le x) = 1 - e^{-\frac{x}{\beta}}}$$

CDF of Exponential Dist

Questions

```
You are working as a data engineer who has to resolve any bugs/
failures of machine learning models in production.
The time taken to debug is exponentially distributed with mean of 5 minutes
```

Q1 Find the probability of debugging in 4 to 5 minutes

1 bug $\Rightarrow$ 5 minute

$\lambda = 1/5$

$\beta = 5$

$$\downarrow$$

$$P(4 < T < 5)$$

$$\text{cdf}(5, \beta) - \text{cdf}(4, \beta)$$

Q2. Given that you have already spent 3 minutes, what is the probability of needing more than 9 minutes

$$P(T > 9 \mid T > 3)$$

$$\downarrow$$

$$\frac{P(T > 9) \cap P(T > 3)}{P(T > 3)}$$

* Memory Less Property

Time already spent Does not impact any additional Time Taken

$$\boxed{P(T > 9 \mid (T > 3)) = P(T > 6)}$$

additional 6 Seconds

Exponential Distribution treats each moment as if you are starting new, Past time does not Matter

Poisson Distribution vs Exponential Distribution

Poisson Distribution:

- **Use Case:** Models the number of events in a fixed interval of time or space.
- **Example Question:**
  - "How many customers will enter a store in the next hour?"
  - "How many messages will you receive in next 15 mins?"
  - "How many calls can the call center expect in the next 30 minutes?"
- **Parameter:** Rate (λ) represents the average number of events in the specified interval.

Exponential Distribution:

- **Use Case:** Models the time between consecutive events.
- **Example Question:**
  - "How long do I have to wait for the next message?"
  - "On average, how much time will a customer spend waiting for service in a queue?"
  - "How long, on average, will passengers wait between consecutive bus arrivals?"
- **Parameter:** Scale represents the average time between events. It's the reciprocal of the rate.

Geometric Distribution

Questions

```
Imagine you're in a job search, and you're giving interviews until you land your first job.
```

> **Q. What are the possible outcomes in this situation?**

P = 0.1

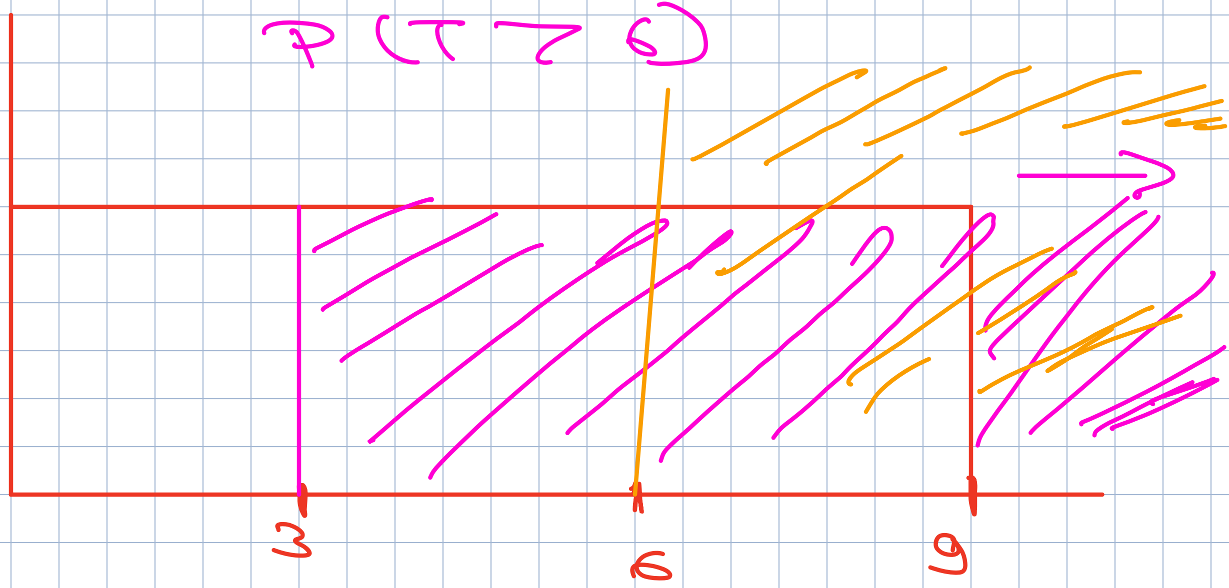
Q:

After K interviews you will pass your first one

You are flipping a biased coin with a 30% chance of getting heads until you succeed.
What is the probability of getting heads on the 2nd flip?

$P(T > 9 \mid T > 3)$

$P(T > 6)$



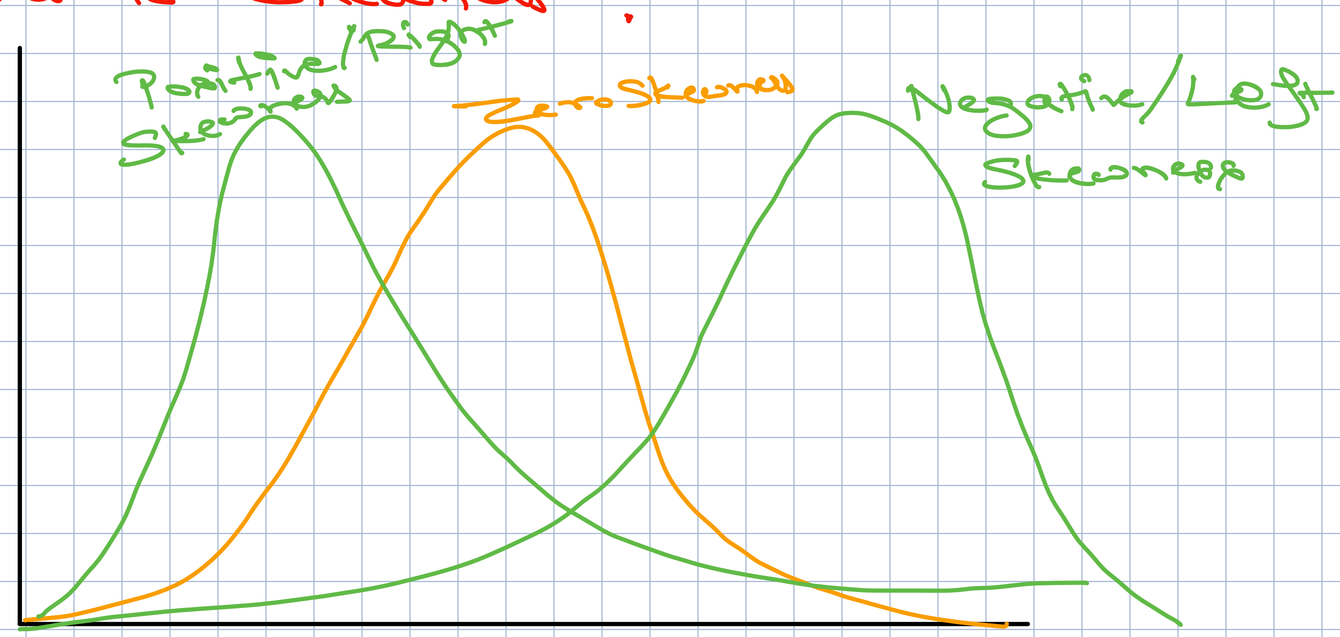
Post Read

Log Normal Distributions

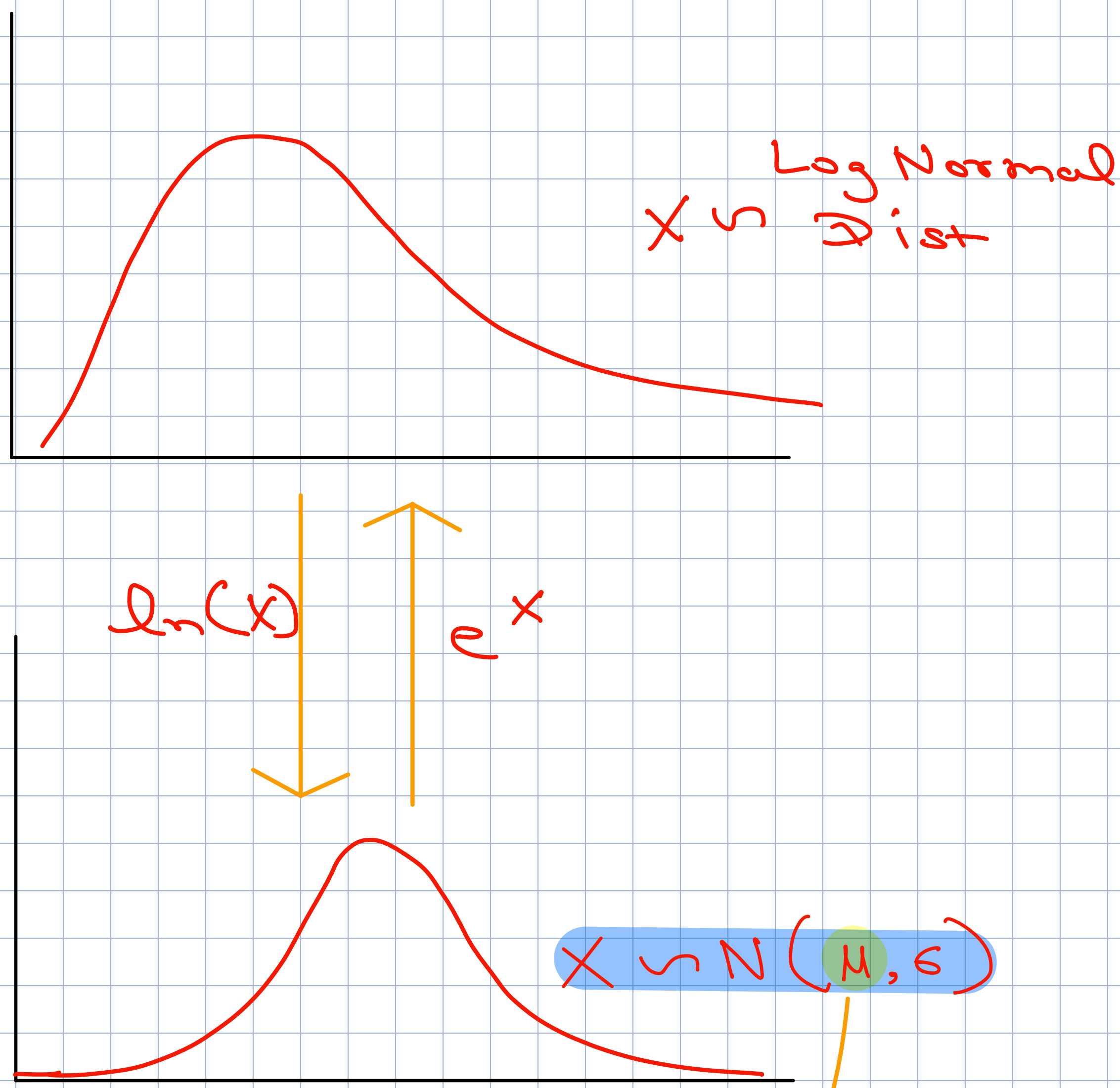
Case: You are collecting wait-time before the food parcel gets delivered to customer.

The Log Normal Distribution is a continuous probability Distribution that models the Right Skewed Data

Q: What is Skewness?



Log Normal $\rightleftarrows$ Normal Dist



Log Normal Parameters

mean $= e^{\left(\mu + \frac{\sigma^2}{2}\right)}$

Var $= \left(e^{\sigma^2} - 1\right) e^{2\mu + \sigma^2}$

Key properties of Log Normal:

① Positive

② Right Skewed

③ Multiplicative process

$X \rightarrow$ depends on multiple factors

(Traffic, availability, parcel and prep-time)