

# Agenda

Regularization

Updated Loss Function

Regularization Parameter

L1 Regularization

Elasticnet Regularization

Hyperparameter Tuning

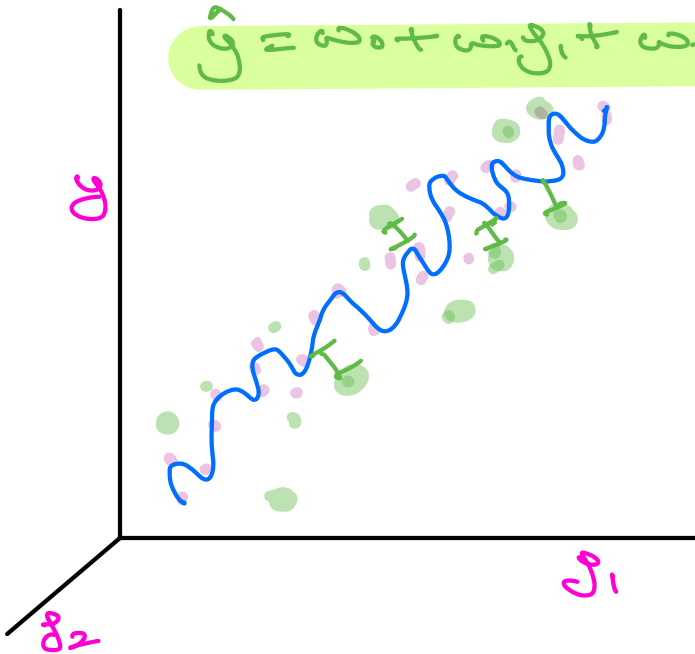
Cross Validation

K-fold Cross Validation

# Regularization

0  $\leftarrow$  0.1 (reduce  
Import  
↑  
10  
↑

$$\hat{y} = w_0 + w_1 f_1 + w_2 f_2 + w_3 f_1^2 + w_4 f_2^2 + w_5 f_1 * f_2$$



if model is  
overfitting? Yes

Complex Model  
(features with high degree)

Fix  
(reduce mag(weight)  
to reduce the  
Importance of  
features)

① idea 1: Reduce features

① Reduce degree

② Manually Remove features  
by setting  $w_j = 0$

② idea 2: Modify the Loss function  
st  $w_j \rightarrow 0$

Loss :

$$\frac{1}{n} \sum_{i=0}^n (y_i - \hat{y}_i)^2 + \text{Regularization Term}$$

Loss

↓  
minimizing  
 $\text{mag}(w_j)$

# Updated Loss Function

$$L \Rightarrow \frac{1}{2n} \sum_{i=0}^n (y_i - \hat{y}_i)^2 + \frac{\lambda}{2n} \sum_{j=1}^p (\omega_j)^2$$

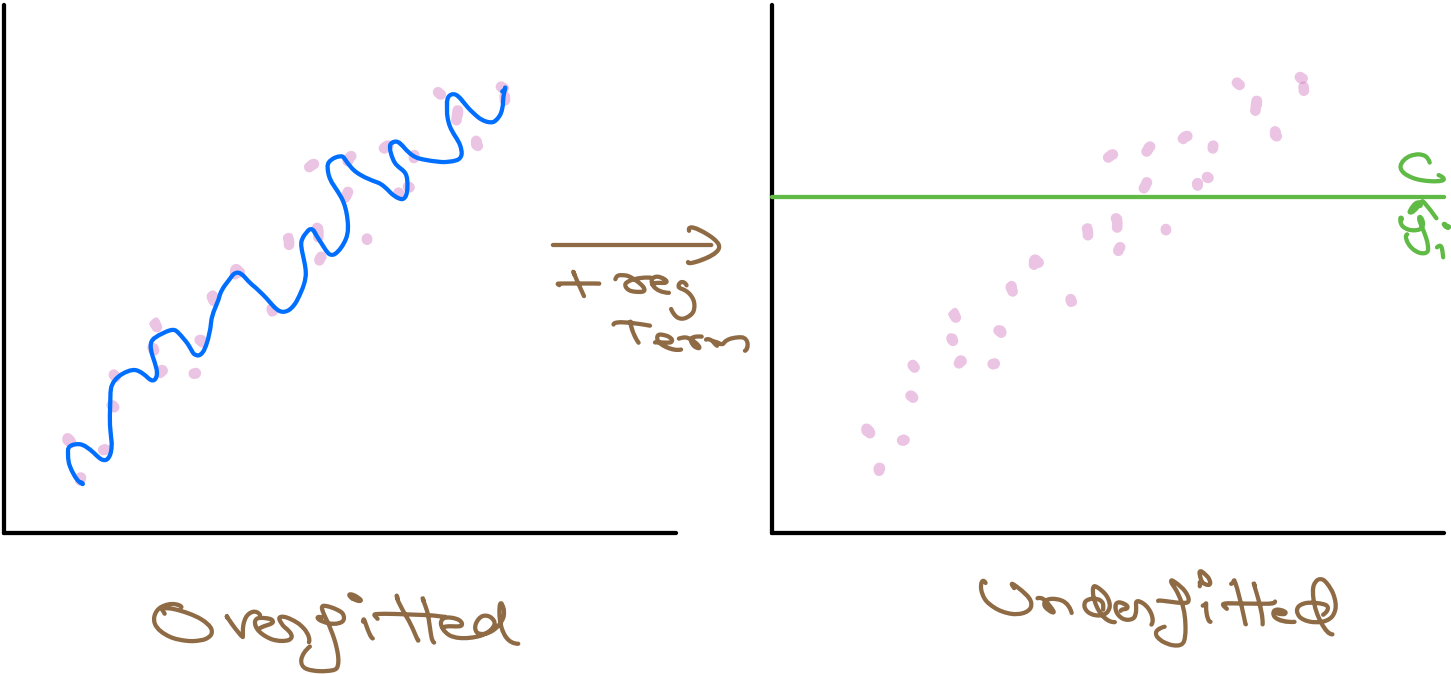
$$\frac{\partial L}{\partial \omega} \Rightarrow$$

$$\hat{y}_i \Rightarrow y_i - \cancel{\left( \sum_{j=1}^p \omega_j x_{ij} \right)} + \omega_0$$

$$\hat{y}_i \Rightarrow y_i - \omega_0 \Rightarrow C$$

$$\text{minimize}$$

$$\text{mag}(\omega_j) \rightarrow 0$$



# Regularization Parameter

$$L \Rightarrow \sum_{i=1}^n \frac{1}{2} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p (\omega_j)^2$$

Loss Term (Variance)      Regularization Term (Bias)

Hyperparameter

Bias and Variance

Tradeoff

Act as Tuner  
To find Optimal Balance

↑  
λ increase  
updates  $\hat{y}$  cur  
more on  
mag  $(\omega_j)$   
model Underfits

↓  
λ decrease  
model Overfits

$$\sum_{i=1}^n \frac{1}{2} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^p (\omega_j)^2$$

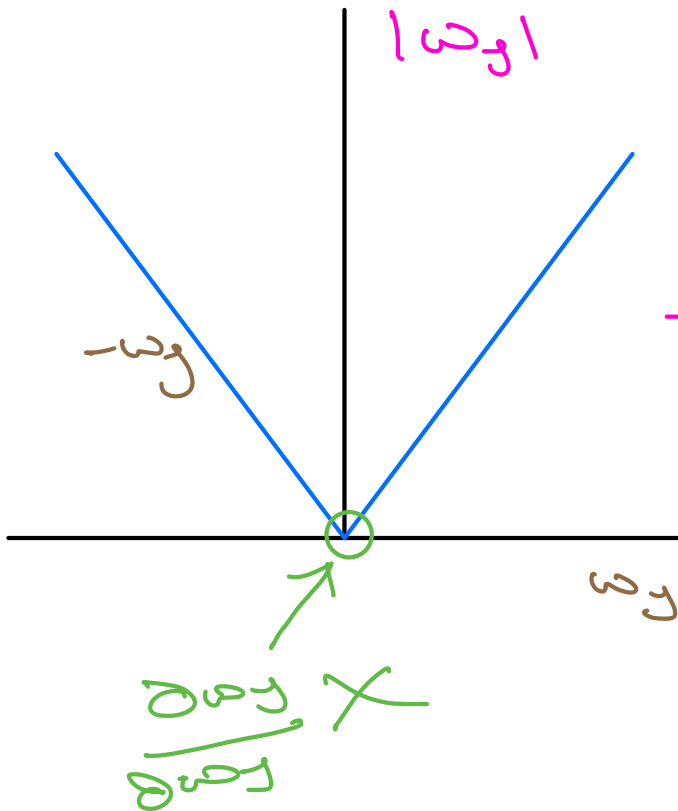
L2-Regularized MSE

↓  
ridge

# L1 Regularization

$L_{\text{app}}$

$$\frac{1}{2} \sum_{i=0}^n (y_i - \hat{y}_i)^2 + \lambda \sum_{j=0}^n |w_j|$$



$$\frac{\partial |w_j|}{\partial w_j} = \begin{cases} 1 & : w_j > 0 \\ 0 & : w_j = 0 \\ -1 & : w_j < 0 \end{cases}$$

$$\frac{\partial}{\partial w_j} \left( \frac{1}{2} \sum_{i=0}^n (y_i - \hat{y}_i)^2 \right)$$

## L1 Reg Term

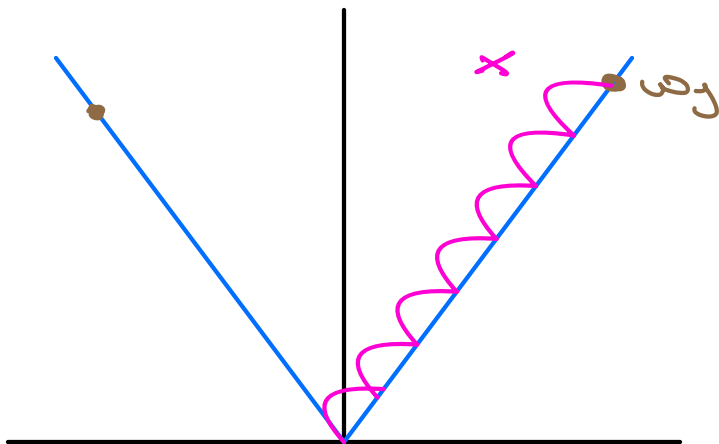
Can  $w_j = 0$   
 (Can be used for  
 auto feature elim)  
 ↓  
 Can create Sparse  
 $w_j$

## L2 Reg Term

① Update to  $w_j$   
 will be smoother  
 ② Will get impacted  
 by outliers in  
 weight

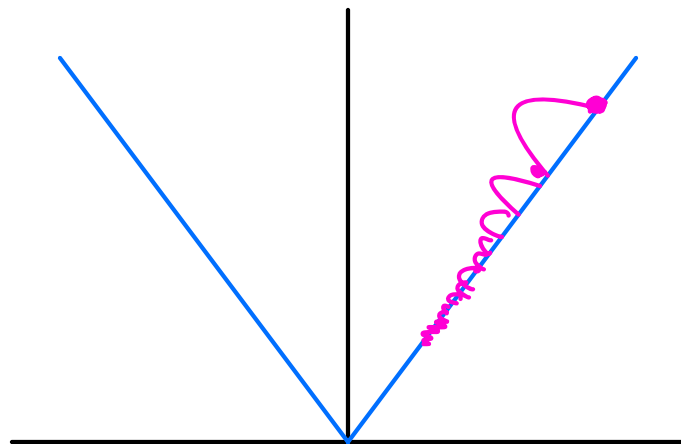
L1 Reg

$$\sum_{j=1}^n |\omega_j|$$



L2 Reg

$$\sum_{j=1}^n \omega_j^2$$



Update step :

$$\omega_j = \omega_j - \eta \frac{\partial L}{\partial \omega_j}$$

L1 Reg Term

$$\frac{\partial |\omega_j|}{\partial \omega_j} = \begin{cases} +1 \\ -1 \end{cases}$$

Reaches to

0  
Quickly

L2 Reg Term

$$\frac{\partial (\omega_j)^2}{\partial \omega_j} = 2\omega_j$$

Initially Fast  
but  
Slow Downy  
never  $= 0$

# Elasticnet Regularization

$$\frac{1}{2} \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \lambda_1 \sum_{j=1}^p |\omega_j| + \lambda_2 \sum_{j=1}^p (\omega_j)^2$$

Find Best Value,  
Using  
Hyperparameter  
Tuning

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L2 Regularization  $\Rightarrow \lambda (\omega_j)^2$

→ Balance use of  
all features

L1 Regularization  $\Rightarrow \lambda |\omega_j|$

→ Automatic  
feature scaling

ElasticNet  $\rightarrow \lambda_1 L1 + \lambda_2 L2$

# Hyperparameter

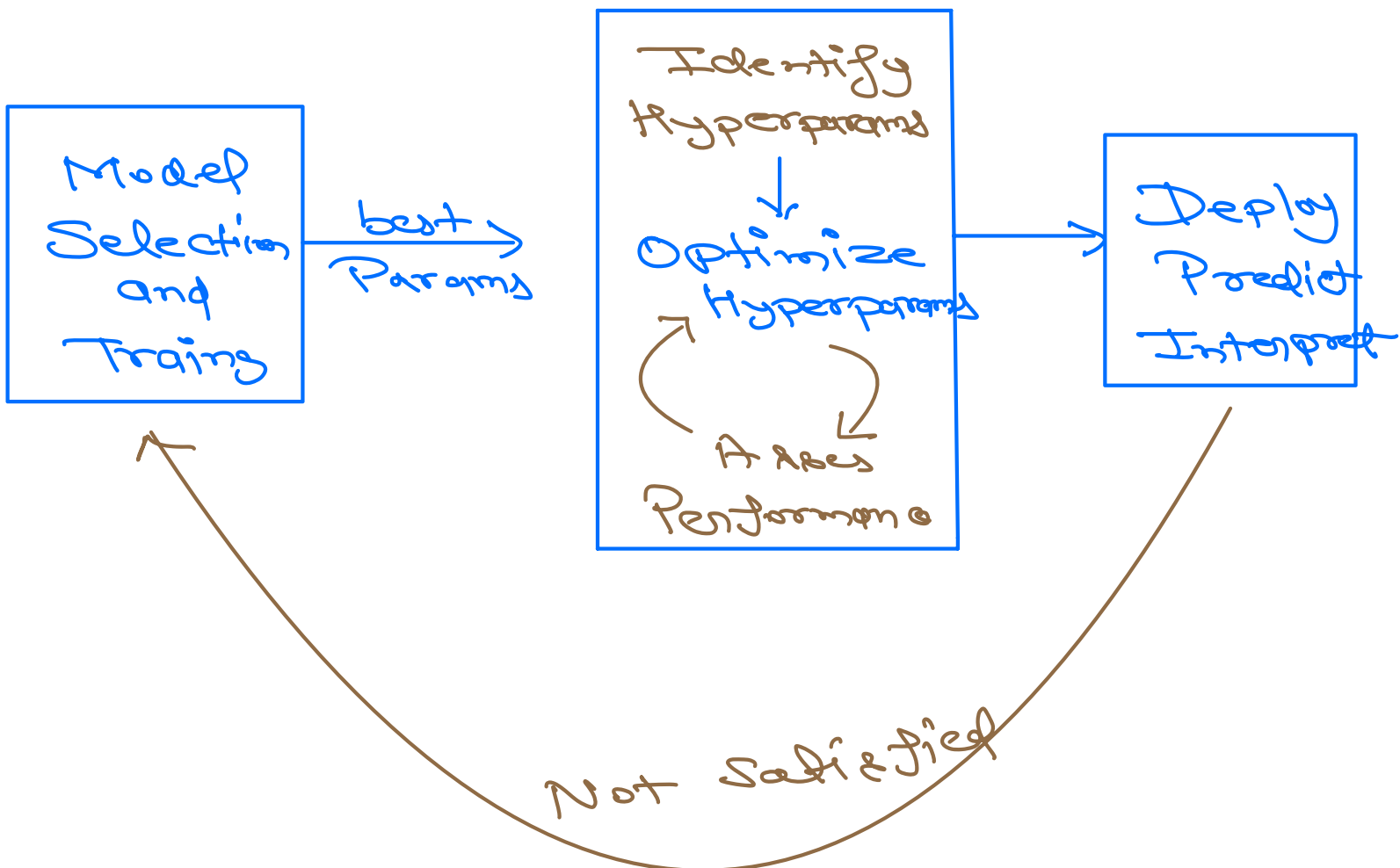
Parameter

vs

Hyperparameter

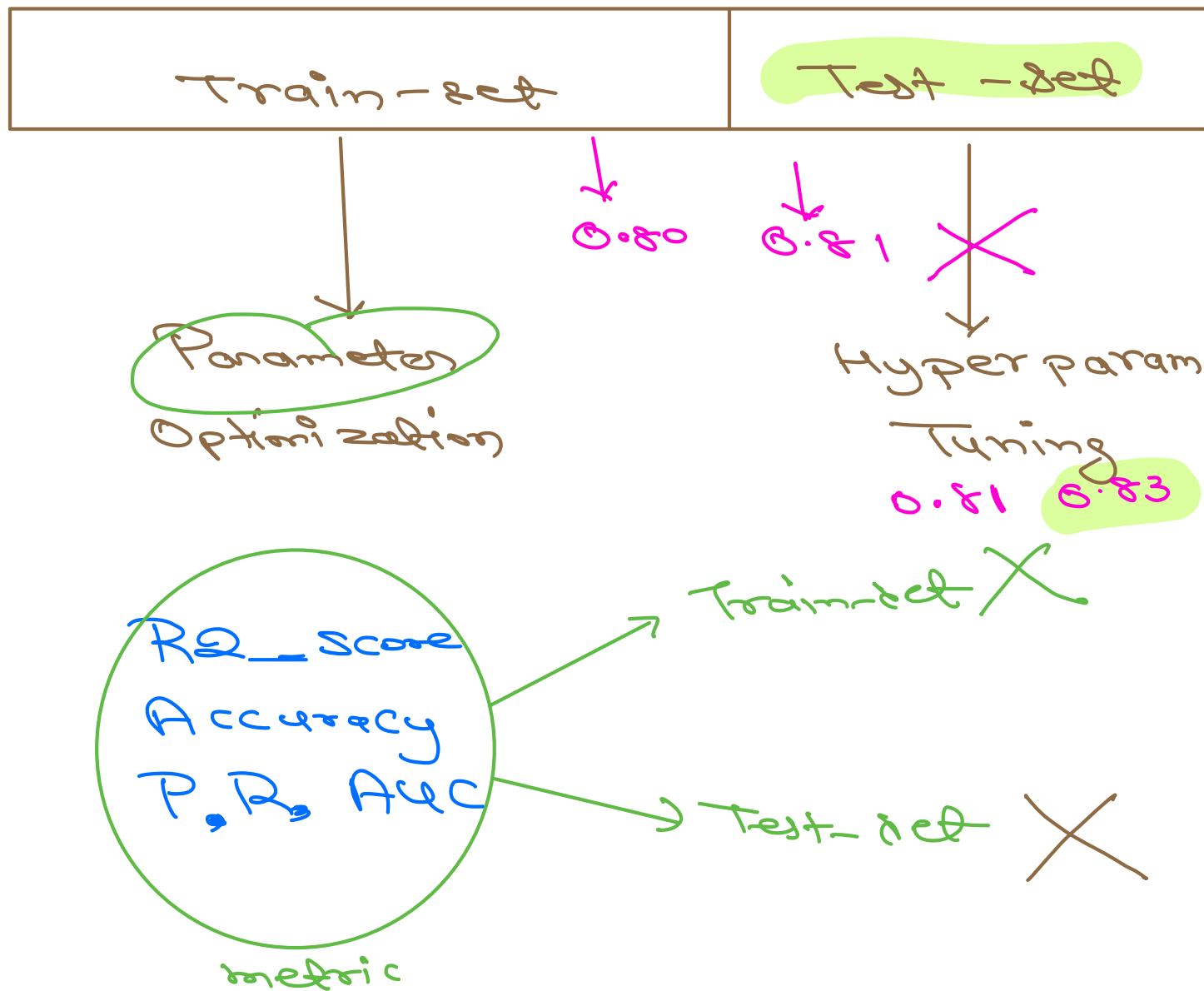
Weights Learnt  
during model  
training using  
the Train-set  
 $\bar{w}$  and  $w_0$

Value which  
we tune  
Manually  
Based on  
Experiment

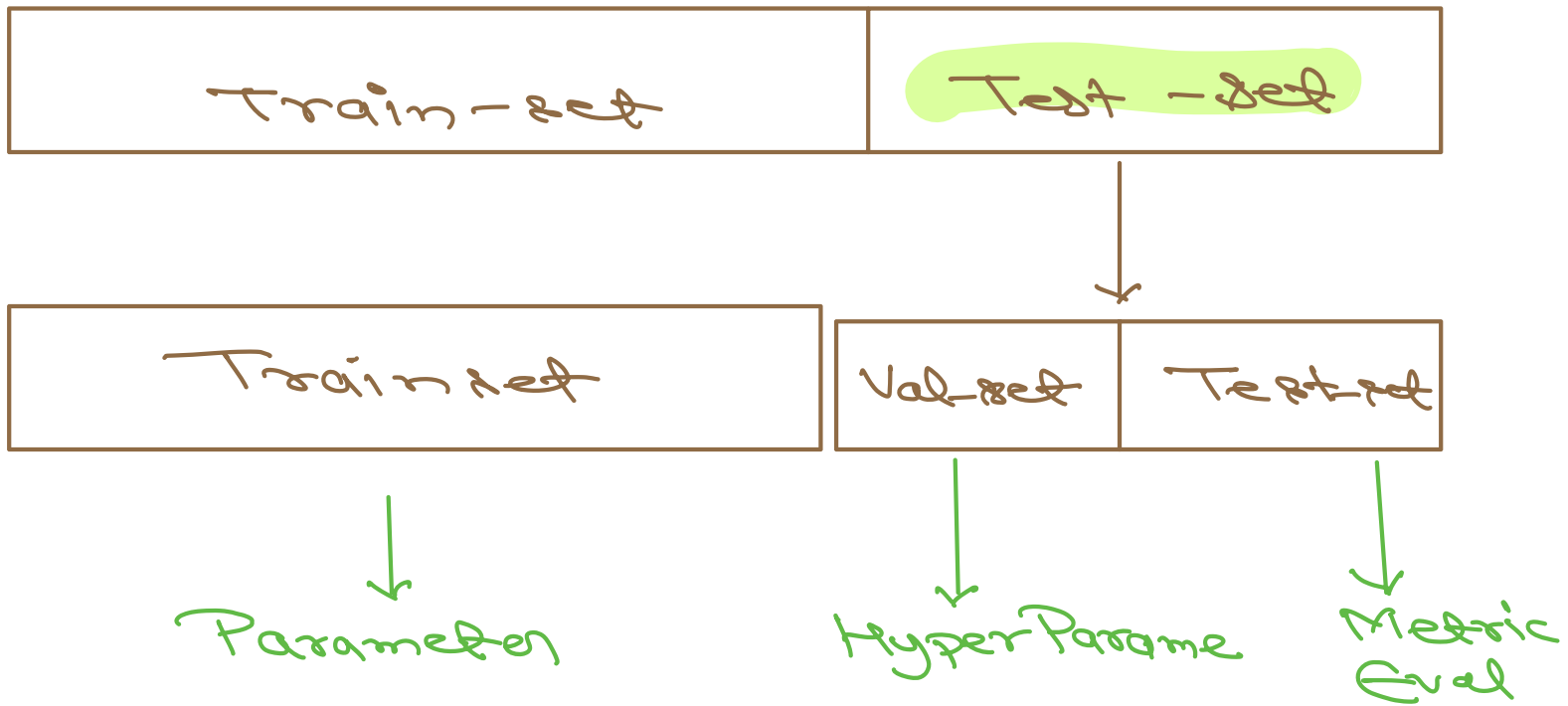




# Hyperparameter Tuning



# Cross Validation



H.W: Explore

Grid Search  
Random Search } ✓

Optional: Bayesian Search

# K-fold Cross Validation

Full  
Data  
Set