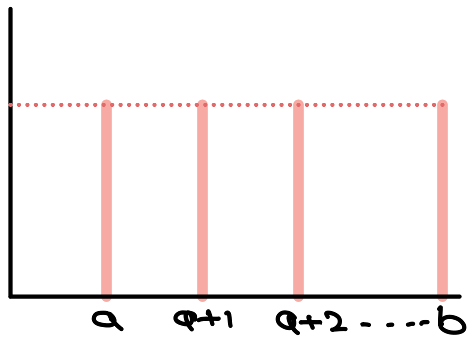
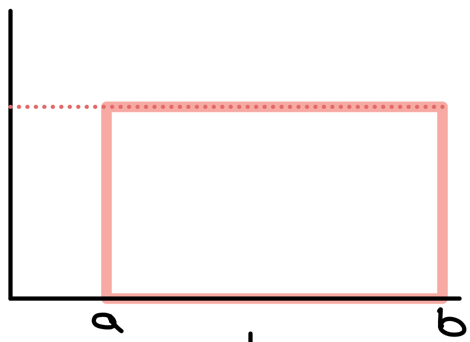
Uniform Distribution

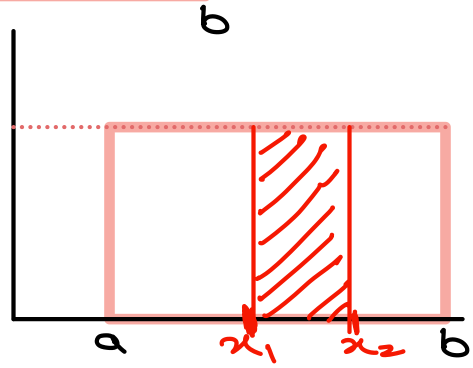
$$pmf(x) = \frac{1}{b-a+1}$$
if $a \le x \le b$
else 0



$$pdf(x) = \frac{1}{b-a}$$
if $a \le x \le b$
else 0



$$P(x_1 < X < x_2) = \left(\frac{x_2 - x_1}{b-a}\right)$$



* Other properties of Uniform Distribution

The uniform distribution has the following properties:

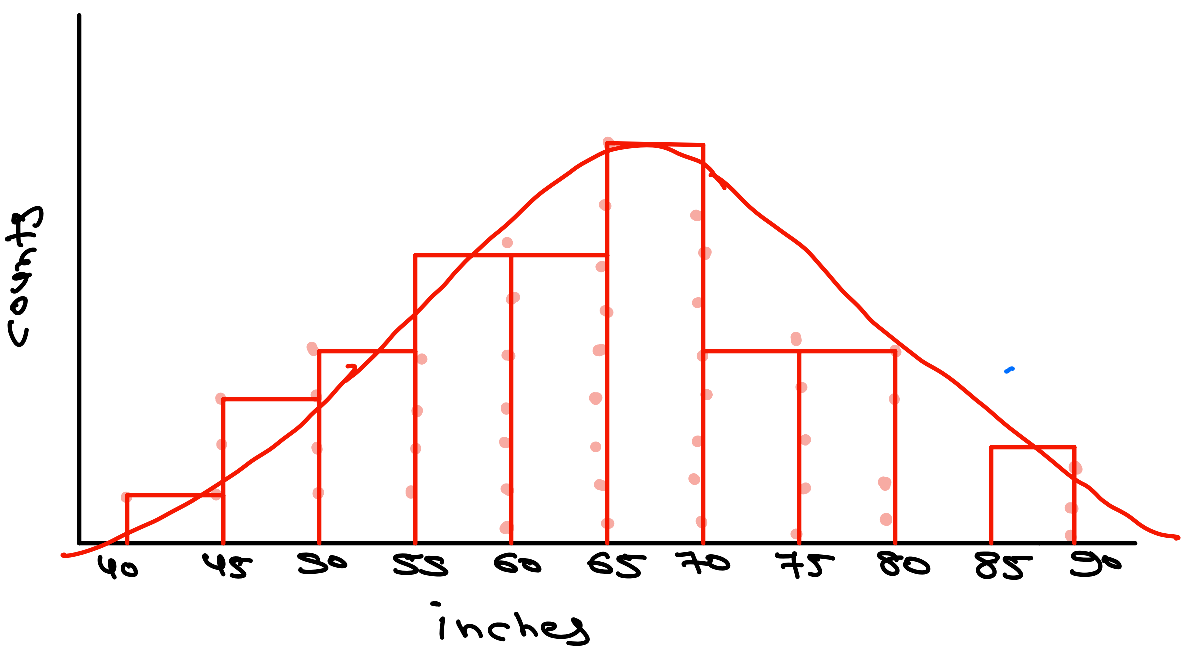
- Mean: $\frac{(a+b)}{2}$
- Variance: $\frac{(b-a)^2}{12}$, for Continuous uniform distributions
- Variance: $\frac{(b-a+1)^2-1}{12}$, for Discrete uniform distribution
- Standard Deviation: $\sqrt{\frac{(b-a)^2}{12}}$, for Continuous uniform distributions.
- Standard Deviation: $\sqrt{\frac{(b-a+1)^2-1}{12}}$, for Discrete uniform distributions.

Gaussian Distribution

Normal Distribution
Standard Distribution

Case-study: Remember the heights case study

Q If we plot heights of students collected how will the distribution look like?



Q: Is Height(X) continous or discrete?

Yes

Observations

Normal Distribution is Bell shaped curve with following properties:

① The distribution is symmetric around mean as center

② As we move away for mean either in Left or Right, the probability decreases

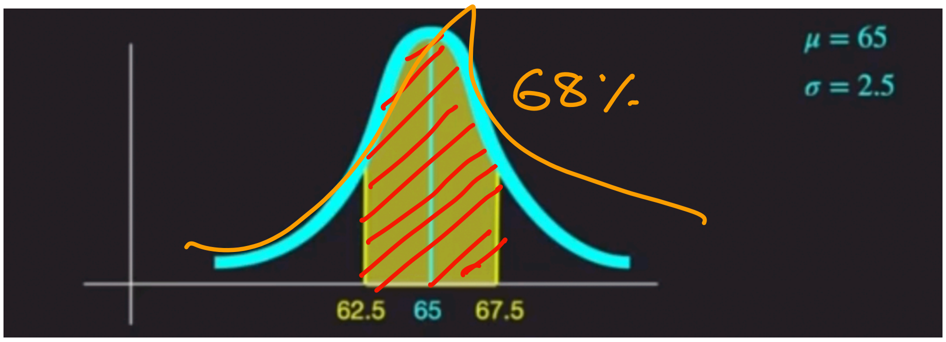
68/95/99 Rule

$\mu = 65$ inches

$\sigma = 2.5$ inches

$\mu + \sigma \Rightarrow 67.5$

$\mu - \sigma = 62.5$



$x - \sigma$ $\quad$ $x + \sigma$

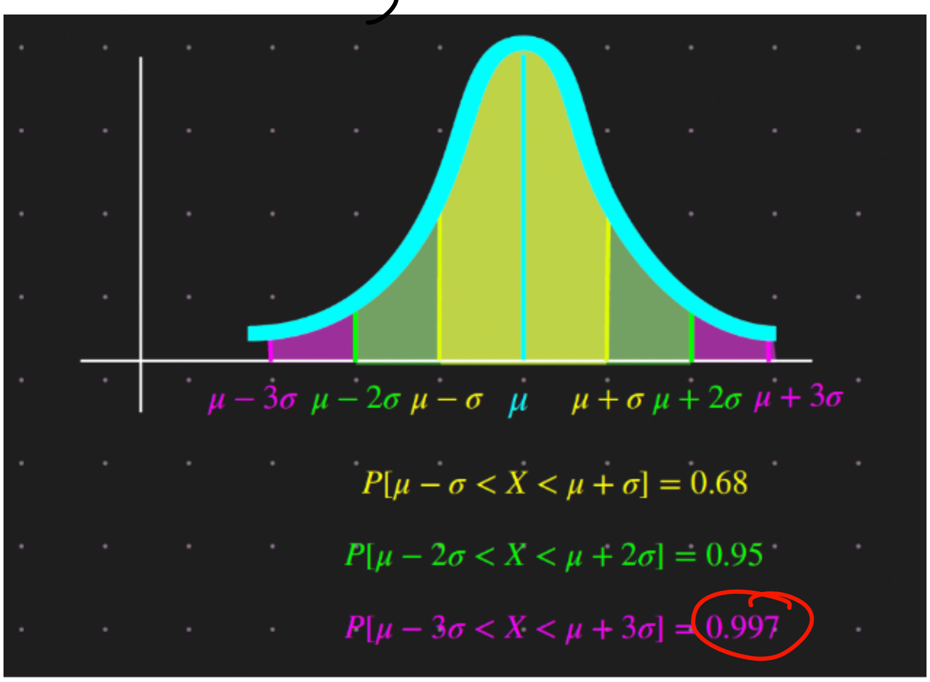
① $P(\mu - \sigma < X < \mu + \sigma) = 0.68$

② $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.95$

$P(60 < X < 70)$

③ $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.99$

$P(57.5 < X < 72.5)$



$\frac{99.7}{2}$

Quiz-1

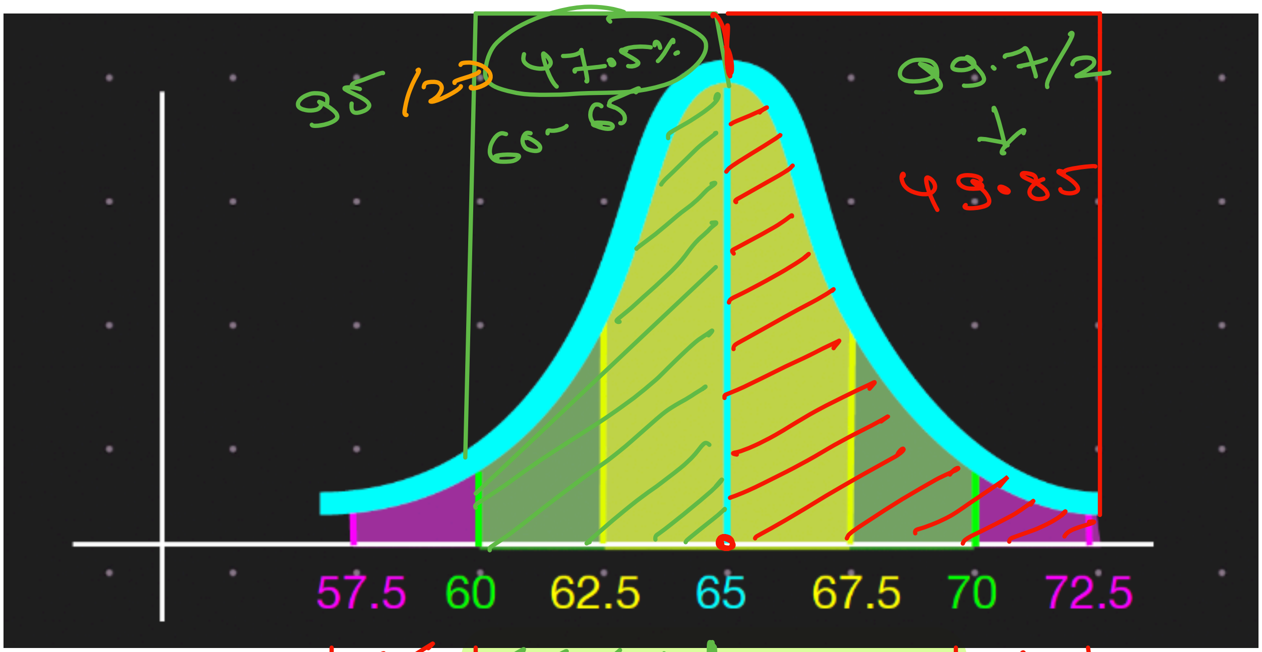
$\bar{\mu} = 65$ $\sigma = 2.5$

47.5 49.85

$P(60 < x < 72.5)$? ⇒ $P(\mu - 2\sigma < x < \mu + 3\sigma)$

99.85

99/2



$\mu - 2\sigma$ $\mu + 2\sigma$

95%

99.7%

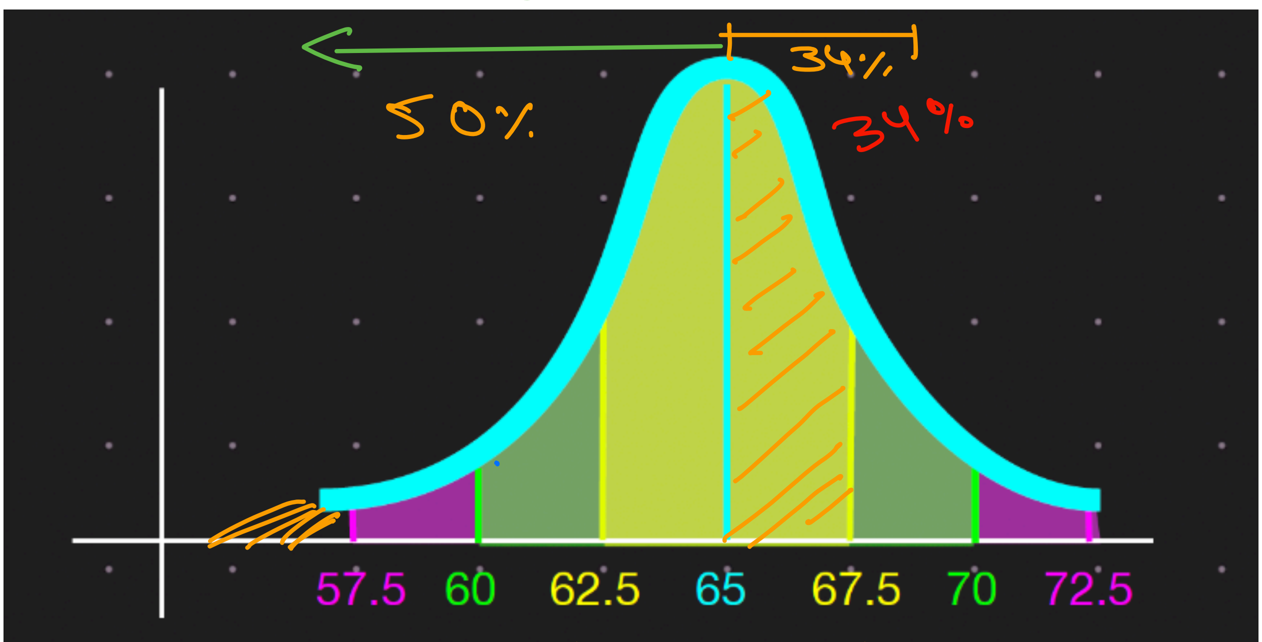
⇒ $47.5 + 49.85 \Rightarrow 97.85$

Quiz-2

$\bar{\mu} = 65$ $\sigma = 2.5$

$P(x < 67.5)$?

$50 + 34 \Rightarrow 84\%$



68%

Z- Score

Q Is the 68/95/99 Rule enough to find P of any Random Value in Gaussian Distribution?

Yes ✗ No ✓

Definition: How far a given value X is from mean in terms of σ (Standard Deviation) as Unit

Consider the following Question:

```
Suppose The height of people is Gaussian with a mean of 65 inches and a standard deviation
of 2.5 inches.
What fraction of people are shorter than 69.1 inches?
```

$\mu + \sigma \Rightarrow 67.5$

$\mu + 2\sigma \Rightarrow 70$

67.5 ⇒ 1 SD away

70 ⇒ 2 SD away

70 ⇒ 5 inch from mean

62.5 ⇒ −1 SD away

$\frac{5 \text{ inch}}{2.5 (1 SD)} \Rightarrow 2 SD$

69.1 SD away? ⇒ $\boxed{\frac{4.1}{2.5} \Rightarrow 1.64}$

$$\boxed{Z = \frac{x-\mu}{\sigma}}$$

Link to Z-score for CDF Table:

https://www.math.arizona.edu/~rsims/ma464/standardnormaltable.pdf

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169

$$\frac{69.1 - 65}{2.5}$$

$\Rightarrow$ 1.64 SD away $\Rightarrow$ 94.95 %

In python?

```python
u = 65
s = 2.5

x = 69.1

z = round((x - u) / s, 2)

print(z)
```

```
1.64
```

```python
from scipy.stats import norm

norm.cdf(z) # 94.94 %of people have height less than 69.1 inches
```

```
0.9494974165258963
```

Q) z-score ⇒ 1.5, $\mu = 50$, $\sigma = 2$

$$z = \frac{x - \mu}{\sigma}$$

$$1.5 \Rightarrow \frac{x - 50}{2} \Rightarrow x = 1.5 \times 2 + 50 \Rightarrow 53 \text{ mm}$$

Quiz

$\mu = 50$, $\sigma = 2$

$X \Rightarrow 53$ mm

$P(X < 53)$

① Calc Z-score ⇒ $\frac{x - \mu}{\sigma} \Rightarrow \frac{53 - 50}{2} \Rightarrow 1.5$

② for z-score, get CDF

93.31% of Balls are less than 53 mm

PPF

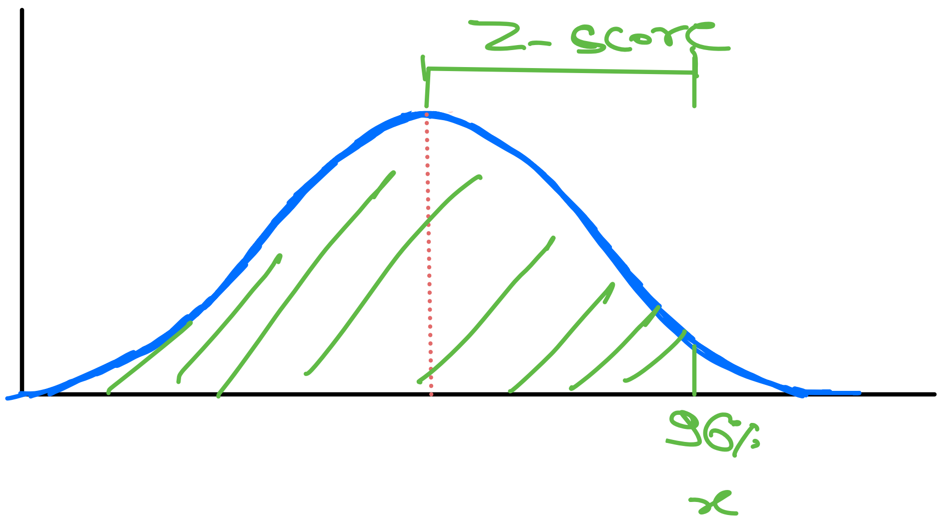
Point Percentile Function

⇒ Given a CDF value, it gives Z-score

Consider the following Question:

```
The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches.

One person says:
96% people are shorter than me. What is my height?
```



⇒ Norm.ppf can be used in python to give Z-score for any give prob value

⇒ ppf is inverse of cdf

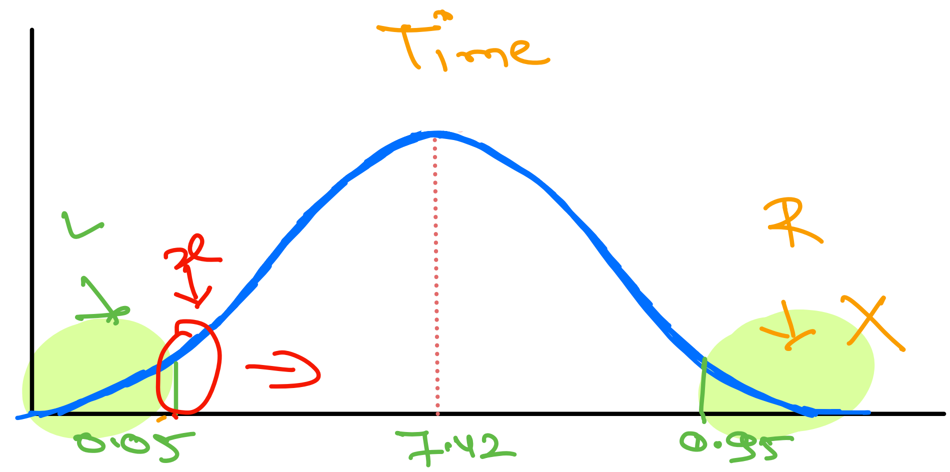
Quiz

Skaters take a mean of 7.42 seconds and std dev of 0.34 seconds for 500 meters.
What should his speed be such that he is faster than 95% of his competitors?

Top $\Rightarrow$ 0.05

$\mu = 7.42$

$\sigma = 0.34$



-1.64 Sd

$X = ?$

6.86 Seconds

$S \Rightarrow \dfrac{500}{6.86} \Rightarrow$

What should his speed be such that he is faster than 95% of his competitors?

4 options

Active Duration(Most preferred: 30 seconds)

Appears for	60 Secs

A	68.6 m/s
B	62.6 m/s
C	72.8 m/s
D	83.7 m/s

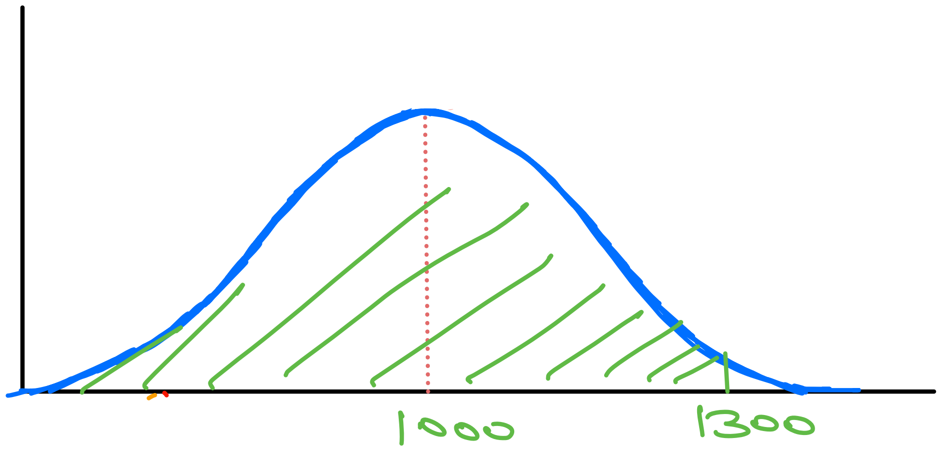
Quiz

$\mu = 1000$

$\sigma = 200$

$X = 1300$

$P = ?$ of Not OOS



$\frac{1300 - 1000}{200}$

$\frac{x - \mu}{\sigma} = 1.5$

cdf → 93.31%

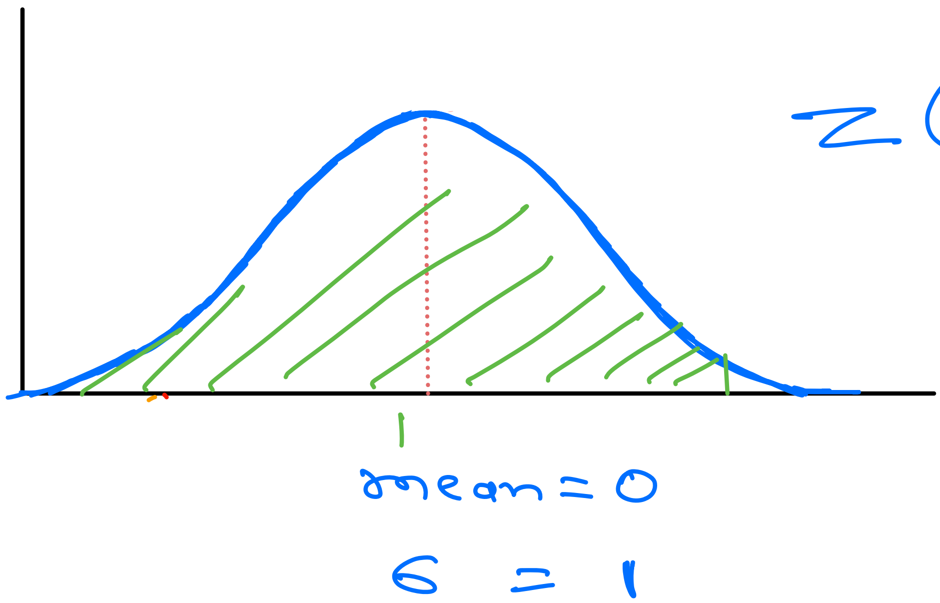
A retail outlet sells around 1000 toothpastes a week, with std dev = 200. If we have 1300 stock units as our inventory, then what is the probability you'd need to replenish stocks within the week?

$1 - 0.9331 \Rightarrow 0.0669$

Q :

How much inventory should you have, such that there is only a 3% chance of running out of stock

Standard Normal Distribution

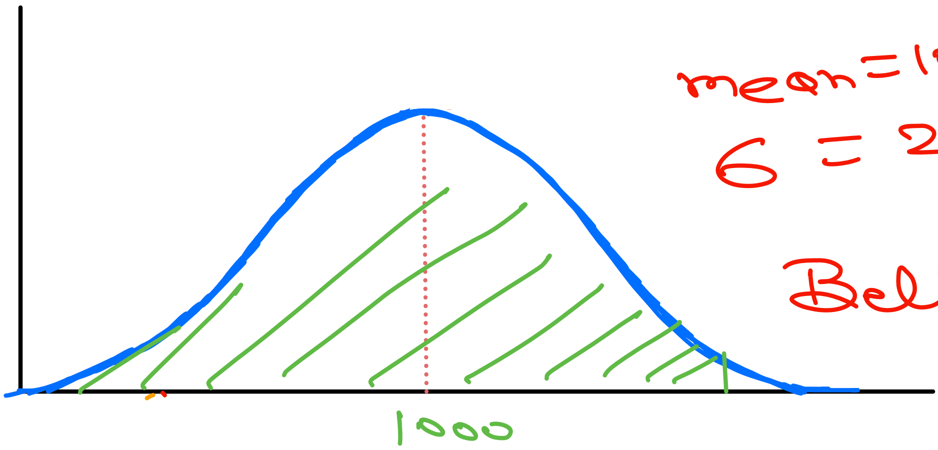


μ, σ

$Z(0, 1)$

Optional

Q Can we convert any Normal Distribution to $Z(0,1)$

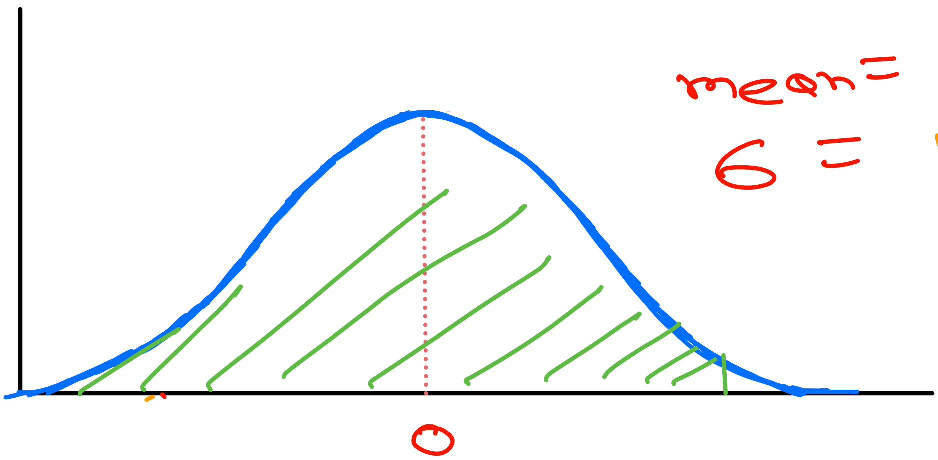


mean = 1000

$\sigma = 200$

Bell shaped

Standardization



mean = 0

$\sigma = 1$

we shifted the mean
and scale SD

Q → Bin Dist

T ⇒ 1/7 ⇒ (No T) for 3 Times

(P1) (No T) ⇒ 6/7 ⇒ 3 times ⇒ $\left(\frac{6}{7}\right)^3$

(P2) P(of at least One T) ⇒ $1 - \left(\frac{6}{7}\right)^3$

⇒ 1st 3 times No T and Next 3 times at least One T

$$\Rightarrow \left(P_1 \times P_2\right) \Rightarrow \left(\frac{5}{7}\right)^3 \times \left(1-\left(\frac{5}{7}\right)^3\right)$$

Coin Toss

$H \Rightarrow 1/2$

2 times $p(HH)$

$p(HH) \Rightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

1

HH

HT

TH

TT

P(No Head in any Trial)

$\Rightarrow \frac{1}{4}$

$^nC_2(P)^2(1-$

$1-P$

Prob of atleast 1 Head

$\Rightarrow 1 - p(\text{No Head})$

$\Rightarrow 1 - \frac{1}{4} \Rightarrow 3/4$

Q-4

$P = \frac{1}{20}$

$1-P = 1 - \frac{1}{20} = \frac{19}{20}$

$K = 10$ times

P(atleast One Coupon of Type i)

${}^{10}C_1 P^1 (1-P)^{10-1} + {}^{10}C_2 P^2 (1-P)^{10-2} \ldots$

${}^{10}C_{10} P^{10} (1-P)^{10-10}$

$1 - {}^{10}C_0 (P)^0 (1-P)^{10-0}$

1 − (Getting 0 of Type i in 10 trials)

$1 - 1 \times \left(\frac{1}{20}\right)^0 \times \left(\frac{19}{20}\right)^{10}$

$1 - \left(\frac{19}{20}\right)^{10}$

Expected Value = $\sum_{i=1}^{n} n_i \times p_i$

Uniform Dist

$$\sum_{i=1}^{20} 1 \times \left(1 - \frac{19}{20}^{10}\right)$$

$$20 \times \left(1 - \frac{19}{20}^{10}\right)$$