Agenda

Combinatorics

Selection with Repetion

Permutations

Combinations

Mathematics of Counting

- Counting with repetion
- Permutation
- Combination
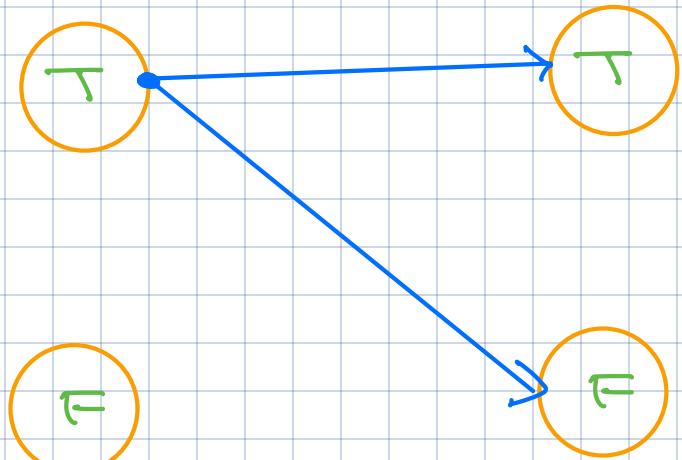
Questions

2 MCQ's, each has two options: $\langle$ T
F

How many different ways you can

submit answers

Answer:

Quiz-1   Quiz-2

$$T T , T F , F T , F F$$

$\dfrac{2}{\text{Quiz1}}$ and $\dfrac{2}{\text{Quiz2}}$

$\begin{array}{c} \dfrac{T}{T} \ \dfrac{T}{F} \end{array}$ $\begin{array}{c} \dfrac{F}{F} \ \dfrac{F}{F} \end{array}$

$\Rightarrow \dfrac{2}{} \times \dfrac{2}{}$

2   ↑ 2 ⟹ 4

Two slots : first
Slot had 2 opt
and 2ⁿᵈ Slot had
2 opt
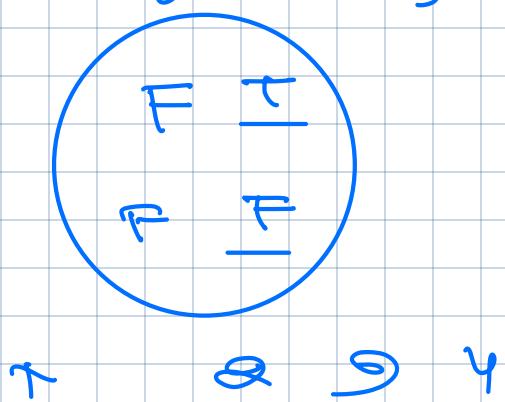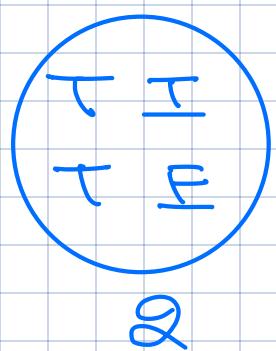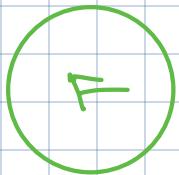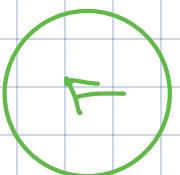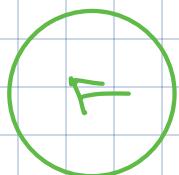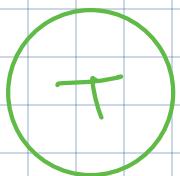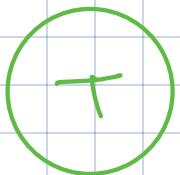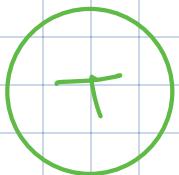
what if there were three Questions?

$$\dfrac{2}{} \times \dfrac{2}{} \times \dfrac{2}{} \Rightarrow 8 \Rightarrow options^{slots}$$

(T) (T) (T)

(F) (F) (F)
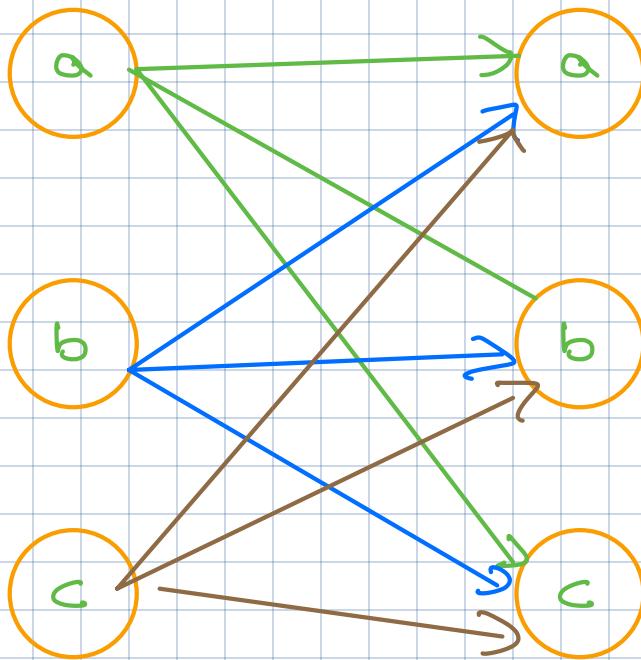
H.w: Can you write all 8 ways
Possible

What if there were three options for
two Questions

aa, ab, ac, ba, bb, bc, ca, cb, cc

$$\frac{3}{Q-1} \times \frac{3}{Q-2} \quad \Rightarrow \quad 9$$
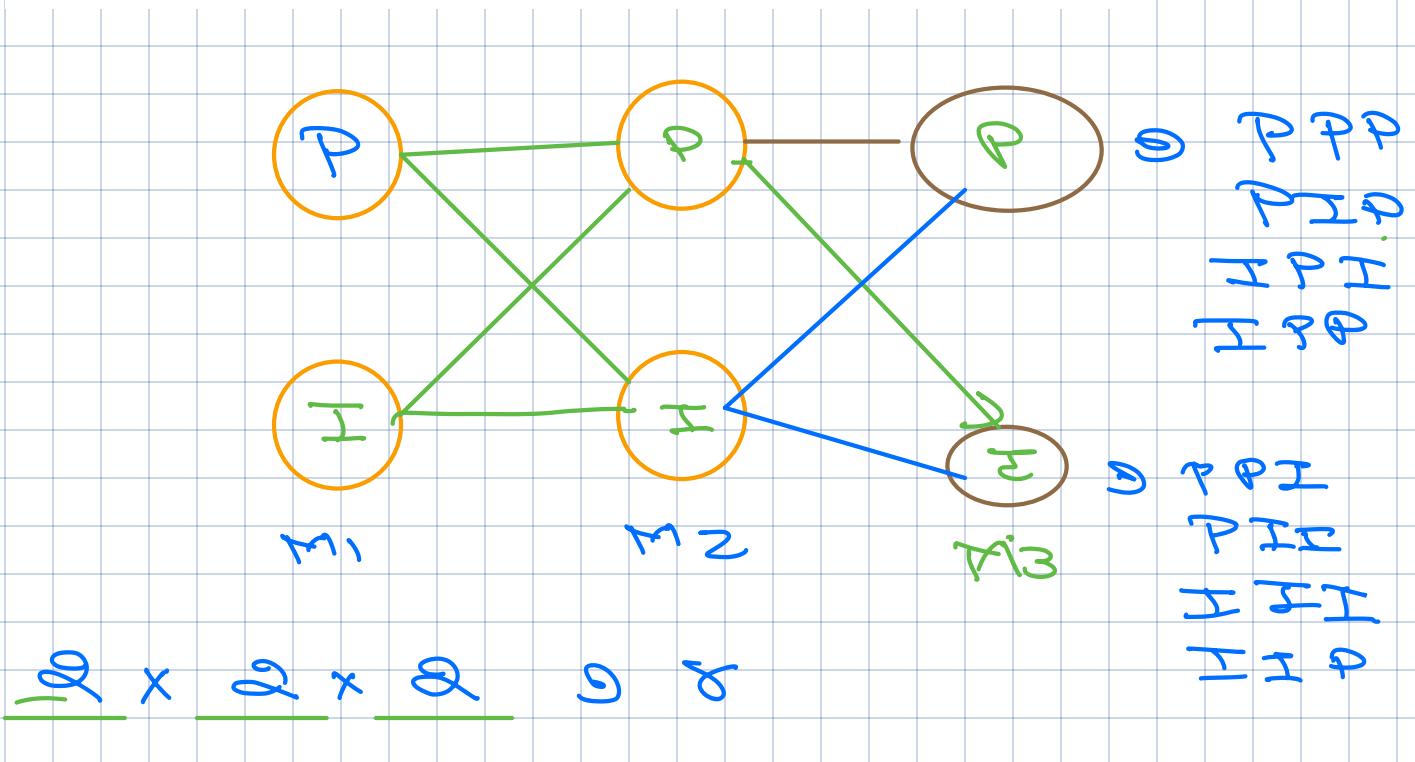
Option ** Slots
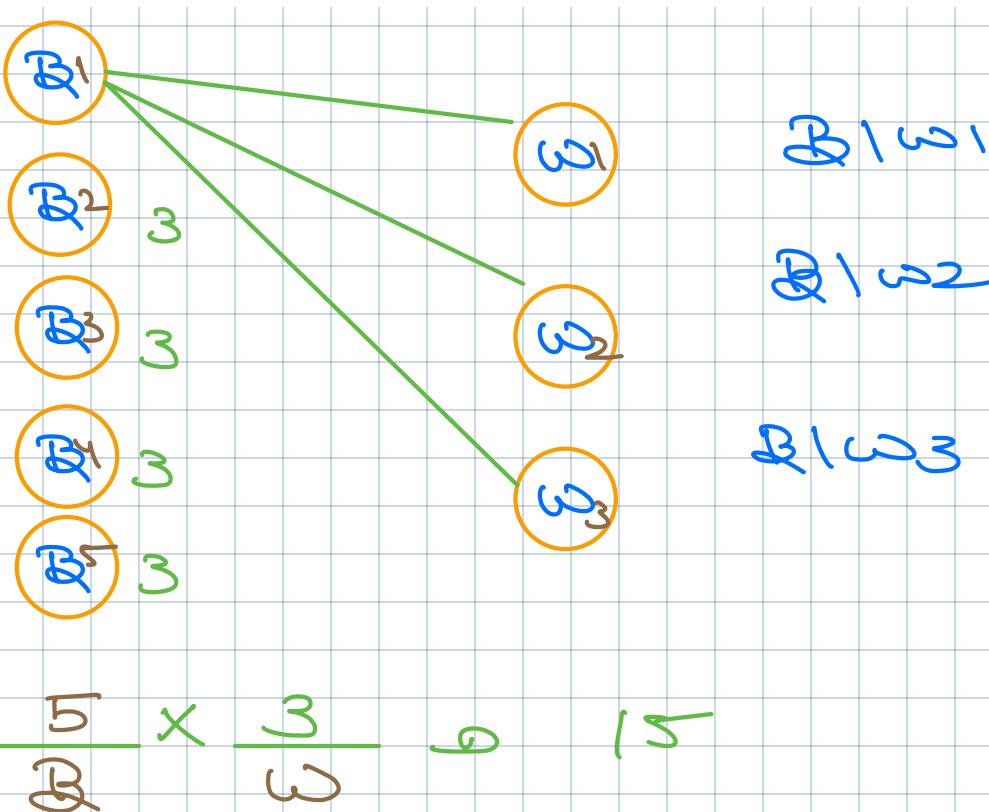
India and Pakistan play a 3-match series. How many results are possible (total number of outcomes)? Note that we consider (Ind, Ind, Pak) different from (Ind, Pak, Ind) etc.

Tie

$3 \times 3 \ a \ 3$

(P) — (P) — (P)

(I) — (I) — (I)

M₁   M₂   A₃

$\underline{2} \times \underline{2} \times \underline{2} \ \ \ni \ 8$

① P P P
   P I P
   I P I
   I P P

② P P I
   P I I
   I I I
   I I P
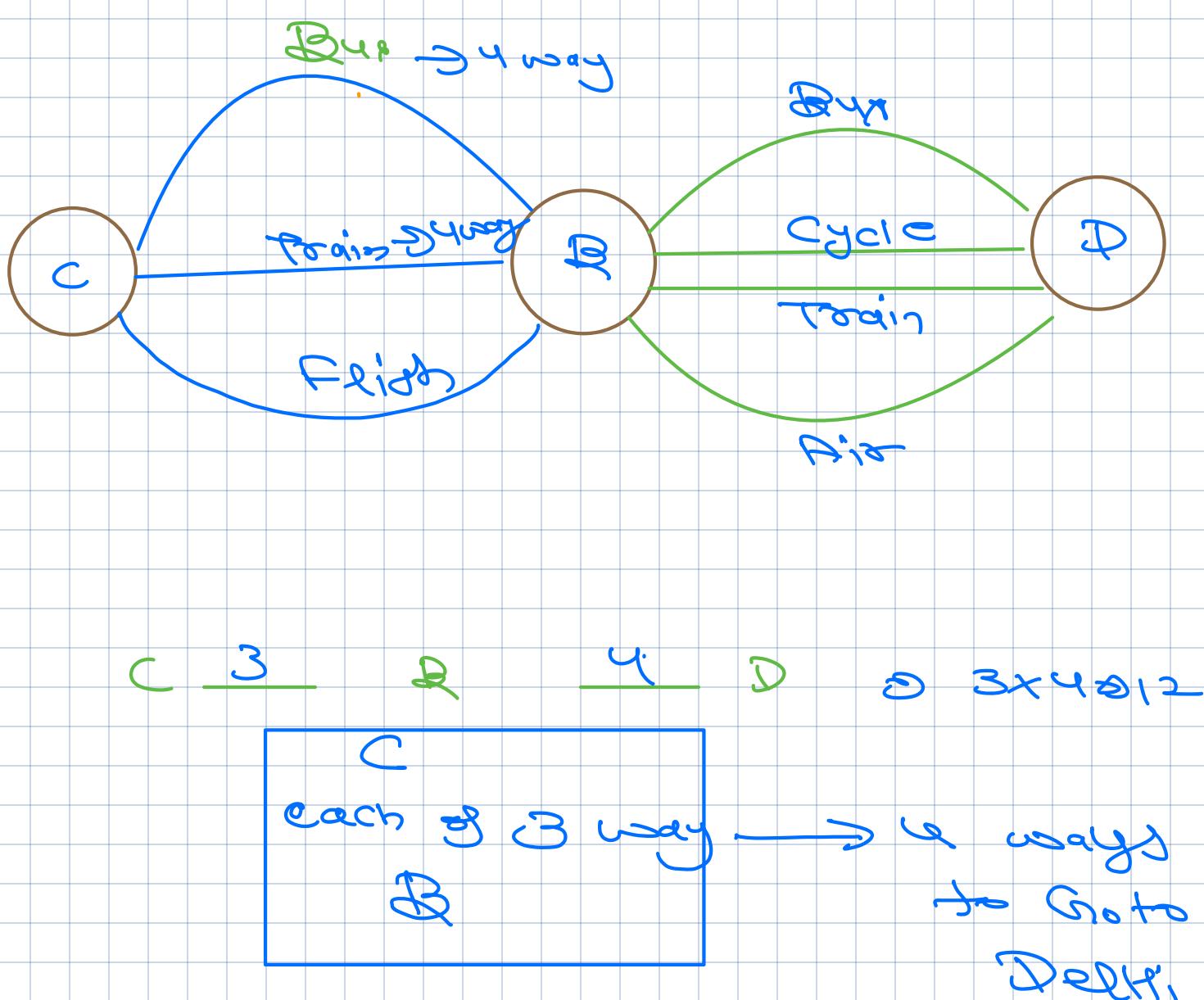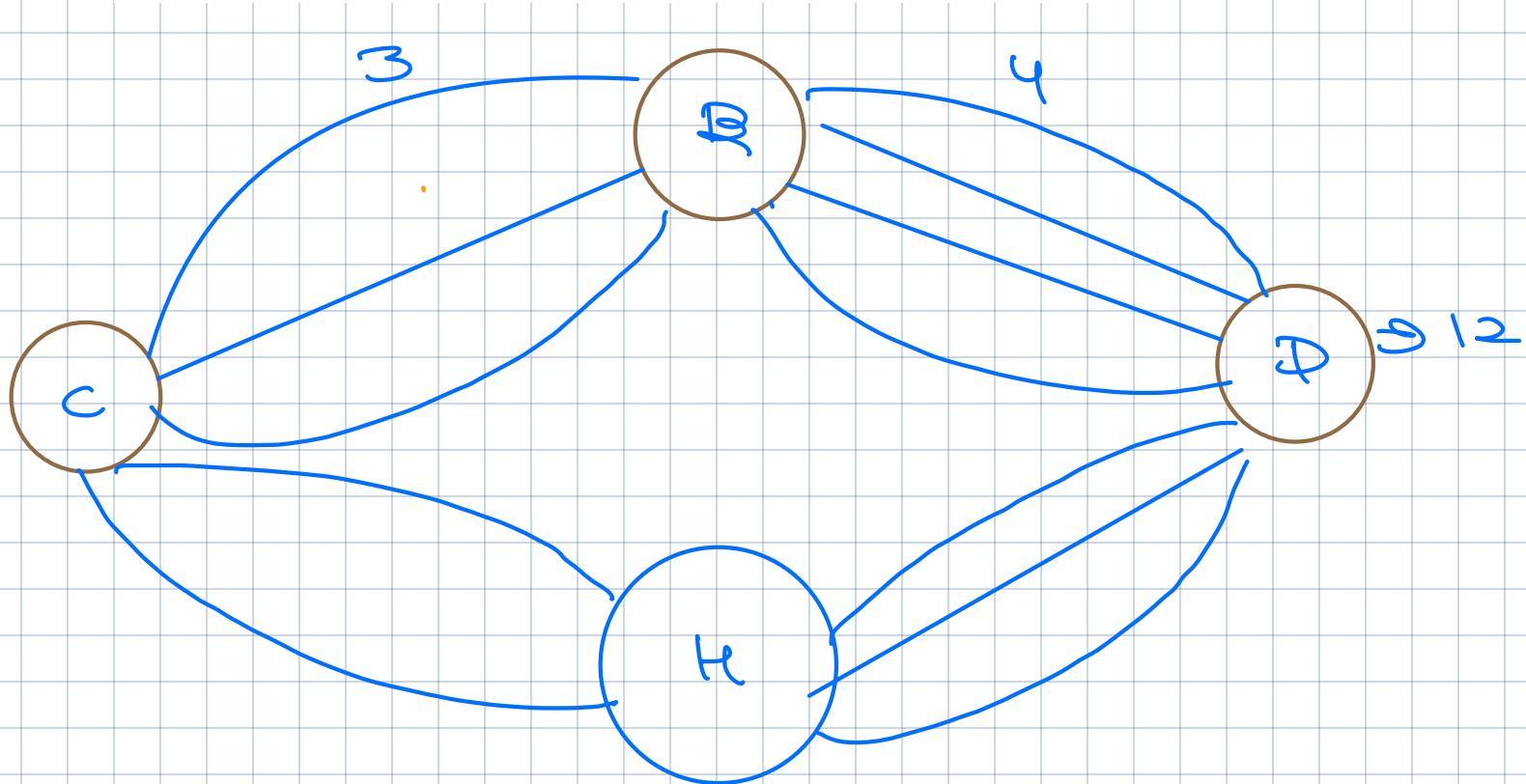
In a bowl-out, for a specific ball you have to choose a bowler and a wicket keeper. Suppose you have 5 bowlers and 3 wicket keepers. How many ways can you select for a ball?

B¹
B²   3
B³   3
B⁴   3
B⁵   3

3₁
3₂
3₃

B1 3 1

B1 3 2

B1 3 3

$\underline{5} \times \underline{3} \ \ni \ 15$
  B   3
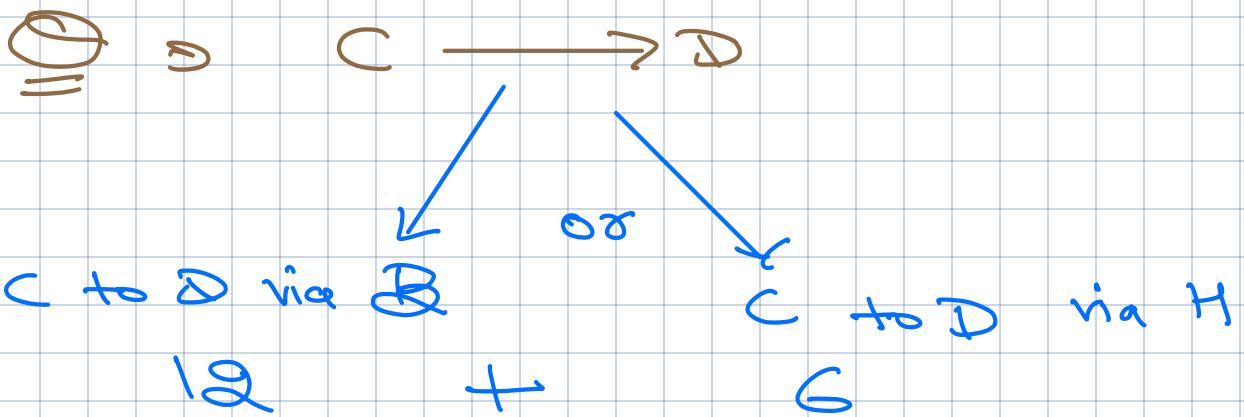
There are 3 ways to move from Chennai to Bangalore. There are 4 ways to move from Bangalore to Delhi. In how many ways can one reach from Chennai to Delhi via BLR?

Bus →4 way

Bus

Train →4 way

Cycle

B

Flight

Train

Air

C __3__   B   __4__ D ⇒ 3×4=12

C
each of 3 way ⟶ 4 ways
B   to Goto Delhi

There are 3 ways to move from Chennai to Bangalore, and 4 ways to move from Bangalore to Delhi.
There are 2 ways to move from Chennai to Hyderabad, and 3 ways to move from Hyderabad to
Delhi. In how many ways can we move from Chennai to Delhi?

3   B   4

C   D → 12

H

Q   How many ways to travel C
to   D via H → 6

Q → C ———→ D

C to D via B   C to D via H
     12   +   6

A fast food outlet has the following types of items in their menu:

- **Burgers: 3**
- **Pizzas: 3**
- **Drinks: 3**
- **Sandwiches: 5**
- **Fruits: 7**

From these items, you can choose one of the followi combos:

- **1 Buger and 1 Sandwhich**
- **1 Fruit and 1 Drink**
- **1 Pizza**

How many different combos can you order ?

Combo 1 : $\dfrac{3}{B} \times \dfrac{5}{S} = 15$

Combo 2 : $\dfrac{7}{F} \times \dfrac{3}{D} = 21$

Combo 3 : $\dfrac{3}{P}$ =

$15 + 21 + 3 = 39$

Permutation	vs	Combination

arrangement of objects

Order Matters

$$ab \neq ba$$
2 ways

Selection of object

Order Does
Not Matter

$$ab == ba$$
1 way

What does order mean

Ind var series (2-)

① I P I

② I I P

③ P I I

⟶ Selected of
2 I's
and
I P

⟶ Combin

↓
Arrangement

What are the number of ways of ARRANGING three characters A, B and C, such that there is no repitition?

$$\underline{\quad 3 \quad} \quad \underline{\quad 2 \quad} \quad \underline{\quad 1 \quad} \quad \Rightarrow 3 \times 2 \times 2 = 6$$

$$\underline{3} \times \underline{3}^3 \Rightarrow 27 \quad \text{if rep allowed}$$

In how many ways can the letters of the word "COMPUTE" be arranged such that the vowels always come together?

Vowel Group + Consonant $\nearrow$ 4 Cons + 1 Vowel Group

$$\underline{\quad 5 \quad} \quad \underline{\quad 4 \quad} \quad \underline{\quad 3 \quad} \quad \underline{\quad 2 \quad} \quad \underline{\boxed{1}} \Rightarrow \boxed{120}$$

Vowel Arrangement : $\boxed{O \;\; E \;\; U}$

$$\underline{\quad 3 \quad} \quad \underline{\quad 2 \quad} \quad \underline{\quad 1 \quad} \Rightarrow 6$$

$$120 \times 6 \Rightarrow$$

Given 5 different characters, in how many ways can we arrange them in 2 places, without repitition?

→ 5 item availabe

Select → 2

n → 5

k → 2

→ 20

5	4
5	5

with Rep

5×4 → $\dfrac{5!}{3!}$ → $\dfrac{5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}}$

Permutation

$^{n}P_{k}$ → $\dfrac{n!}{(n-k)!}$ → $\dfrac{5!}{(5-2)!}$ → $\dfrac{5!}{3!}$

→ 5×4

→ Condition

→ Select k items among

N total items

→ No repetition

→ Order matters

$$^nP_k = \frac{n!}{(n-k)!}$$

There are 4 players P1, P2, P3, and P4 who can play in the top-order batting positions of 1, 2, and 3.
How many arrangements of top-order can we make from 3 of these 4 players, keeping in mind the order in which these batsmen come?

$P_1 \quad P_2 \quad P_3 \quad P_4 \quad \Rightarrow n=4$

$k=3$

$^4P_3 \quad \Rightarrow \quad \frac{4!}{(4-3)!} \quad \Rightarrow \quad \frac{4!}{1!} \quad \Rightarrow \quad 24$

$4 \quad \times \quad 3 \quad \times \quad 2 \quad \Rightarrow \quad 24$
___   ___   ___

3 batman   3 Slot

① G, D, J   J G D   ⇒ $\frac{3 \times 2 \times 1}{\_\_\_\ \_\_\_\ \_\_}$
   G, J, D   J D G
   D G J   ⇒ 6 permutation
   D J G   ⇒

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2

↓ 1 Selection   ↓ 1 Select   ↓ 1 Selection   ↓ 1 Selection

$$^4 P_3 \longrightarrow \text{Order Matters}$$

$$\frac{^4 P_3}{3!} \Rightarrow \text{Selection}$$

$$^n C_k \Rightarrow \qquad \frac{^n P_k}{k!} \Rightarrow \frac{n!}{(n-k)!\,(k)!}$$
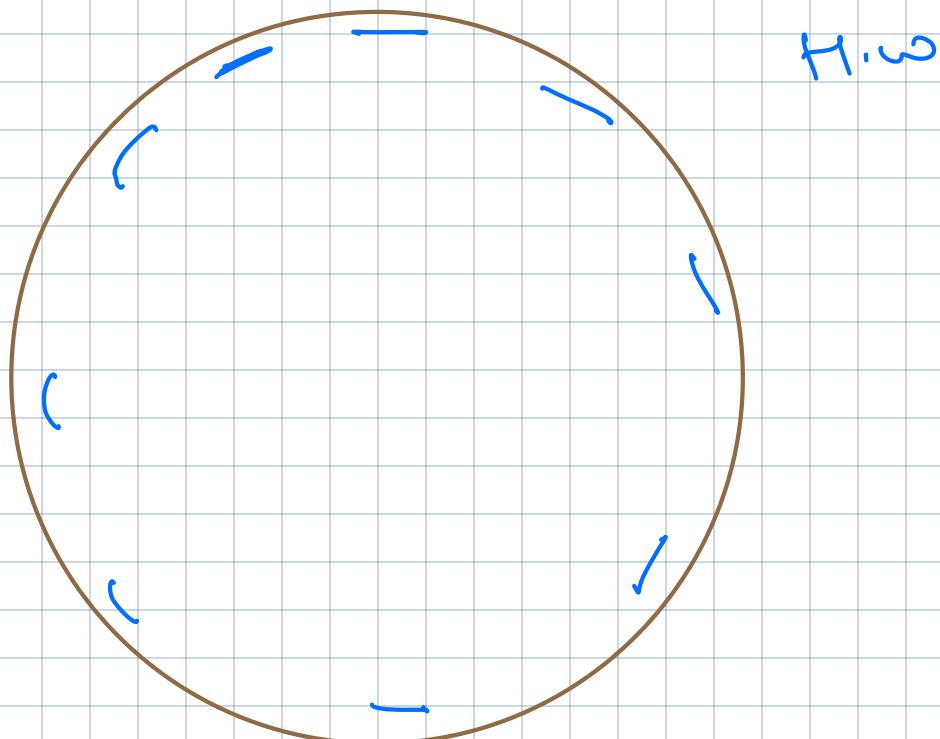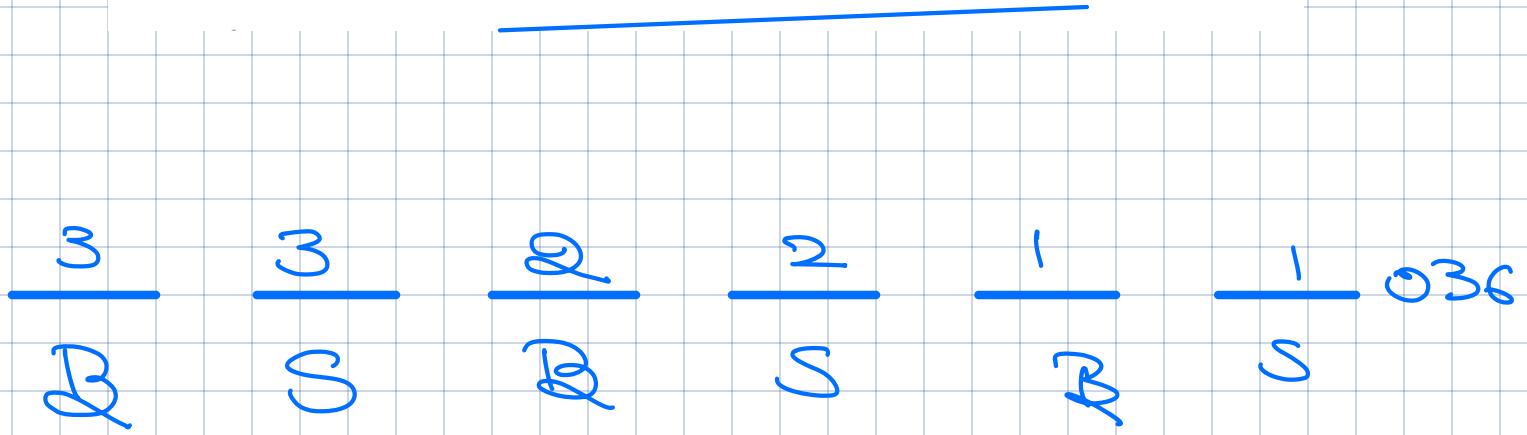
$$\boxed{^n C_k \Rightarrow \frac{^n P_k}{k!} \Rightarrow \frac{n!}{(n-k)!\,(k)!}}$$

A Maruti Showroom has 3 colours in their "Baleno" model and 3 different colours in the "Swift" model. In how many ways can they place these 6 cars, such that Baleno and Swift are kept in alternate slots?

$$\frac{3}{B} \quad \frac{3}{S} \quad \frac{2}{B} \quad \frac{2}{S} \quad \frac{1}{B} \quad \frac{1}{S} \quad 036$$

$$+$$

$$\frac{3}{S} \quad \frac{3}{B} \quad \frac{2}{S} \quad \frac{2}{B} \quad \frac{1}{S} \quad \frac{1}{B} \quad 036$$

3.M

Suppose you're making a list of **three digit** numbers.

Answer the following questions:

i) How many three digit numbers are there that **do not contain 5**?
ii) Which contain 5 **at least** once?
iii) Which contains 5 **at most** once?

\# Total 3 digit

$$\frac{1-9}{\underset{9}{\downarrow}} \qquad \frac{0-10}{10} \qquad \frac{0-10}{10} \qquad \Rightarrow \quad 900$$

① No 5's allowed

$$\underline{\quad 8 \quad} \qquad \underline{\quad 9 \quad} \qquad \underline{\quad 9 \quad} \quad \Rightarrow \quad 648$$

② 5 atleast once
$$\downarrow$$

one 5 + two 5 + three 5

$$\underline{\ }\ 5\ \underline{\ } \qquad\qquad 5\ .\ 5\ \underline{\ } \qquad\qquad \downarrow$$

$$5\ \underline{\ }\ \underline{\ } \quad + \qquad 5\ \underline{\ }\ 5 \quad + \quad 1\ way$$

$$\underline{\ }\ \underline{\ }\ 5 \qquad\qquad \underline{\ }\ 5\ 5$$

M2 : Total - No 5

③ Contain 5 at most once
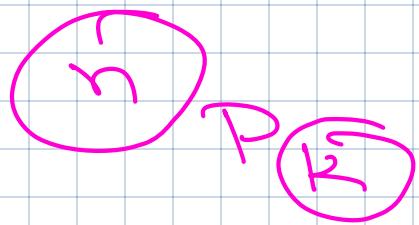
One 5 + Zero Five

Priority

⓵ Probability ⟹ Conditional

⓶ Normal Dist : Most important

⓷ Combination ⟹ Binomial Dist

$$\left(n \atop k \right) P \Rightarrow \text{Pick } k$$

data point

from n total