LAB - XFEM

BY

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1 Introduction

The classical Finite Element Method does not help us simulate efficiently and precisely the propagation of a crack in complex geometries. Continuous re-meshing is required as the discontinuity needs to obey the computationally expensive mesh. This drawback can be overcome by the Extended Finite element method (XFEM) approach, with help of discontinuous functions. Discontinuous basis functions in the form of enrichment are added to classical basis shape functions such that the mesh no longer needs to conform to the discontinuity such as cracks, material interfaces, and voids.

In this lab, in the first part, we are going to study the convergence of error of an XFEM code concerning mesh refinement, polynomial degree, type of tip enrichment function, and the enrichment radius. In the second part, we compute the mode I stress intensity factor for crack opening in a 2D beam model. We compare the obtained results with the analytical model which is based on the classical beam theory.

2 Convergence Analysis

In this section, we are going to study the mode I crack opening for an infinite plane. The exact solution to the problem is known. To emulate the infinite problem, a square-shaped domain is used. On the boundary of the domain, the traction stress of the exact solution is imposed. The below diagram represents the problem. The energy error norm is computed as the difference between the exact solution and the numerical solution. We will study the energy error norm for different parameters such as mesh size, degree of the polynomial, use of the enrichment function, and also the size of the enrichment zone.

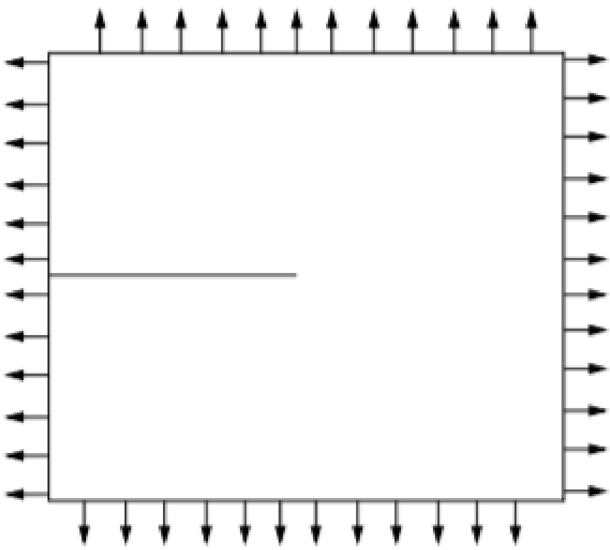


Figure 1: Crack in an infinite plane

2.1 Error Analysis

To the study four different mesh sizes were selected as shown below,

Mesh	Element edge size
1	0.2222
2	0.1111
3	0.7407
4	0.0555

Table 1: Different mesh sizes selected

2.1.1 Enrichment functions and Polynomial degree

The error with respect to mesh size is given by the generalised form as,

$$\varepsilon_m = Ch^{\alpha} \tag{1}$$

where, h is related to the mesh size and m is related to the type of error (0 for displacement error,1 for energy error).α represents the rate of convergence of error. To study the rate of convergence,we are going to consider the energy error (m=1).The rate of convergence for XFEM is given by,

$$\alpha = p+1-m \tag{2}$$

where p represents the degree of the polynomial. It can be observed that the equation is independent of the regularity of the solution. So, we are going to study the effect of different polynomial degrees (p) and tip enrichment functions on the energy norm.

The energy error norm was studied with no tip enrichment for the polynomial order of $p=1$ and $p=2$ for different mesh sizes in particular. The log-log plot below represents the same.

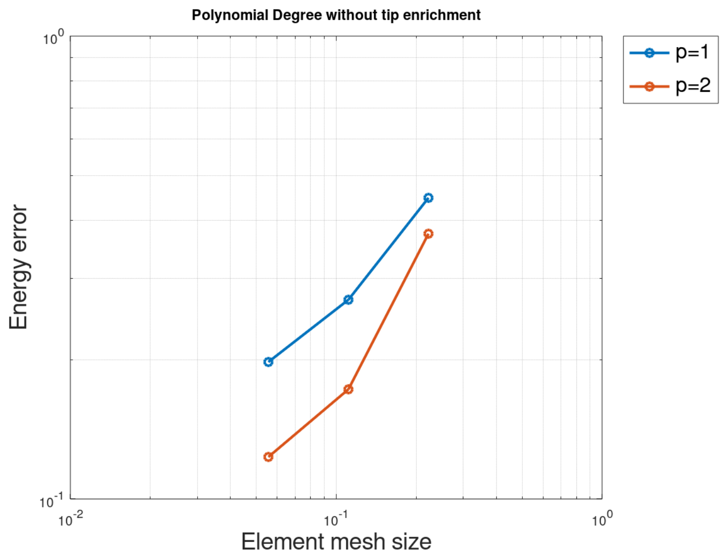


Figure 2: Convergence of error without any tip enrichment for different polynomial degree

It can be observed that as the mesh goes from coarse to fine, The energy error for both the degree of the polynomial (p=1 and p=2) decreases. But polynomial of degree 2 gives lower energy than the polynomial of degree 1. Now, let us study the effect of different enrichment functions i.e, Scalar and Vector Enrichment. The polynomial degree is set to 1 and the enrichment radius is 0.4. The below log-log plot represents the observations made. It can be observed

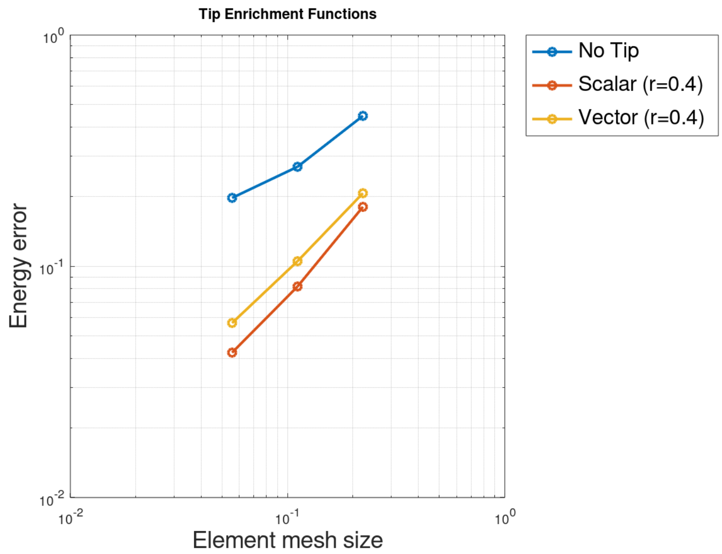


Figure 3: Convergence of error for different enrichment functions

that the scalar enrichment performs better when compared with Vectorial enrichment. Also, the rate of convergence which is the slope of the curve is much better when compared with the no-tip enrichment.

The below plot illustrates the behavior of vectorial enrichment for the different polynomial degrees. The enrichment radius is set to r=0.4. It can be seen that with second-degree

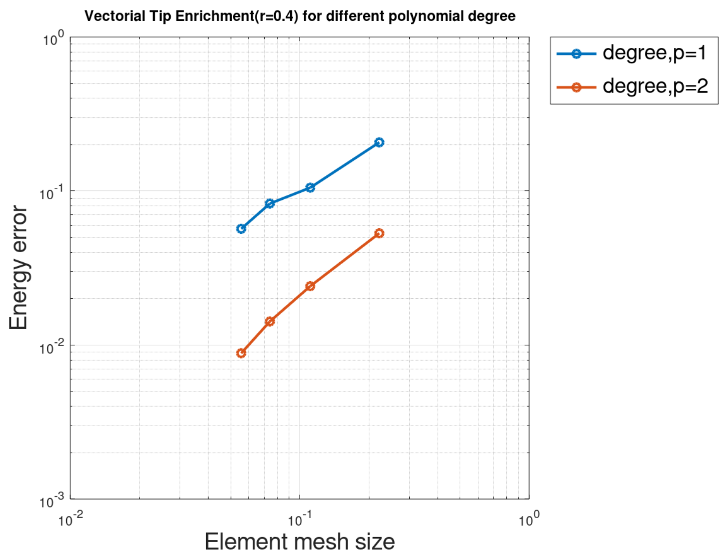


Figure 4: Convergence of error for different polynomial degrees for vectorial tip enrichment

polynomial the error is decreased significantly. The rate of convergence is also increased from polynomial degrees 1 to 2. By increasing the enrichment radius, we can get lower order of error.

The below plot represents the convergence of error for all the cases stated. The vectorial tip enrichment with degree 2 gives the better convergence of error.

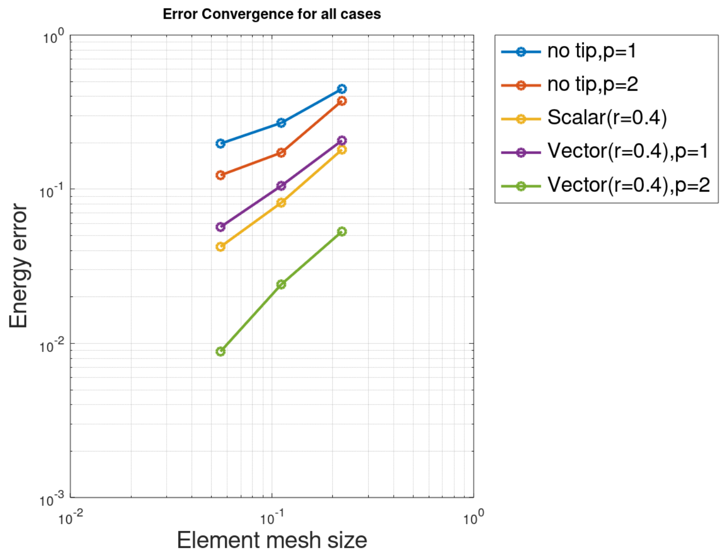


Figure 5: Convergence of error for all cases

The geometrical tip enrichment strategy which is studied here doesn't change with the mesh refinement. It depends on the enrichment radius. So, we get a much accurate solution than the topological enrichment strategy. Now, The fine mesh, the element mesh size 0.0555 is considered, The polynomial degree is set to 1 and the vectorial tip enrichment function is considered. The enrichment radius is varied and the convergence error is considered. It can be observed that as the size of the enrichment radius increases the error also decreases.

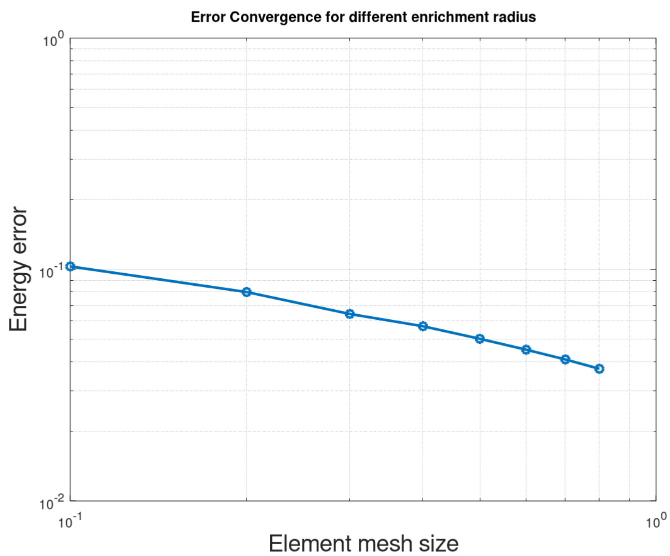
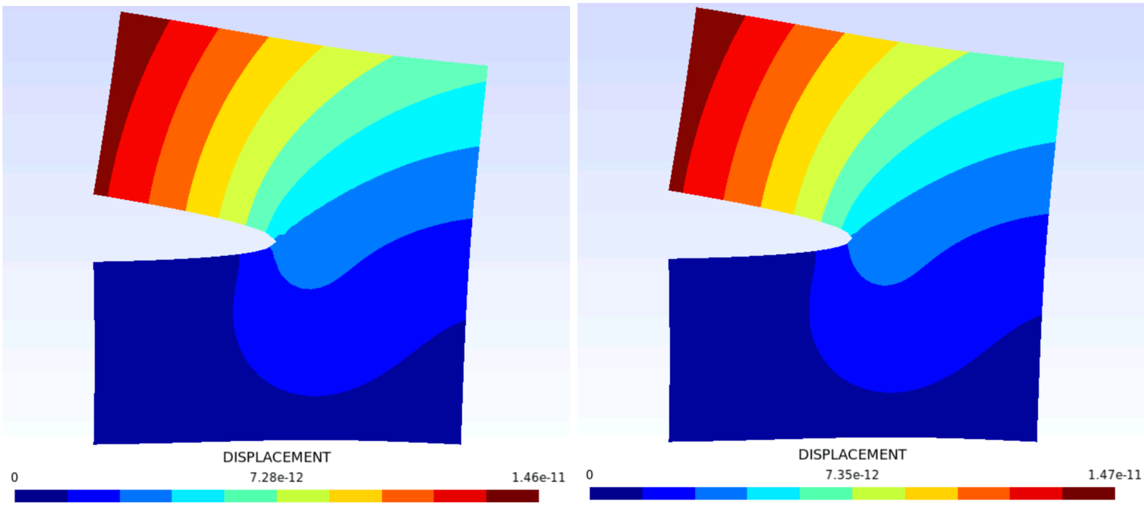


Figure 6: Convergence of error with respect to enrichment radius

2.1.2 Displacement and Stress field

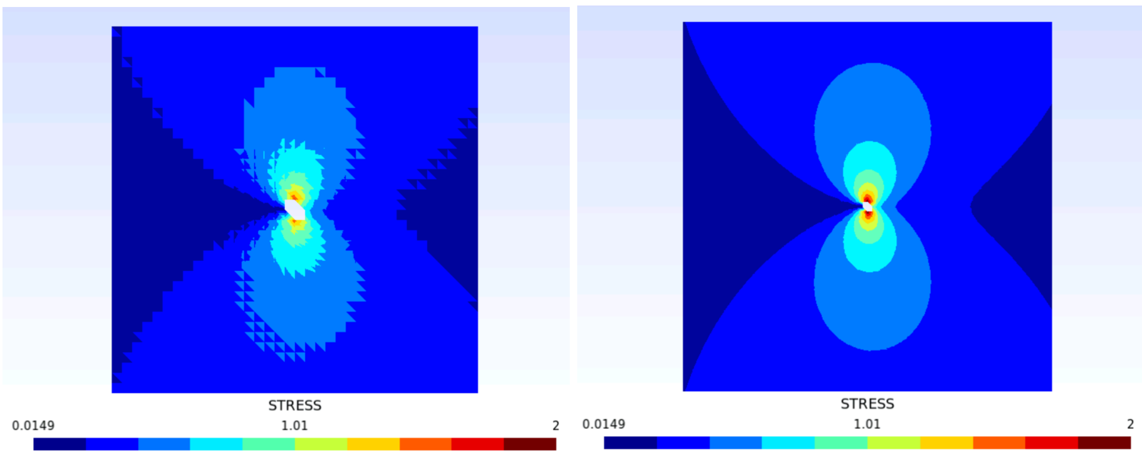
In this section,the Displacement and Stress field plots obtained for Vectorial tip Enrichment of r=0.4 is presented below.The degree of polynomial are $p = 1$ and $p = 2$.



(a) Displacement field (p=1)

(b) Displacement field (p=2)

Figure 7: Displacement field for different polynomial degree



(a) Stress field (p=1)

(b) Stress field (p=2)

Figure 8: Stress field for different polynomial degree

3 Crack in a Beam

In this section,We are going to study the Stress intensity factor as function of the crack length.We are going to compute this factor at the crack tip for mode I.

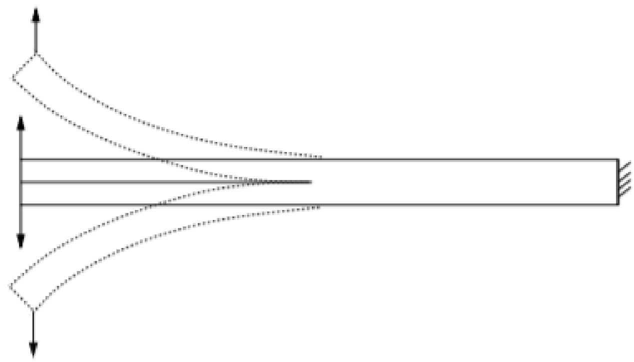


Figure 9: Crack in a Beam

3.1 Analytical Solution of the Problem

The analytical solution of the problem presented above is shown in this section.

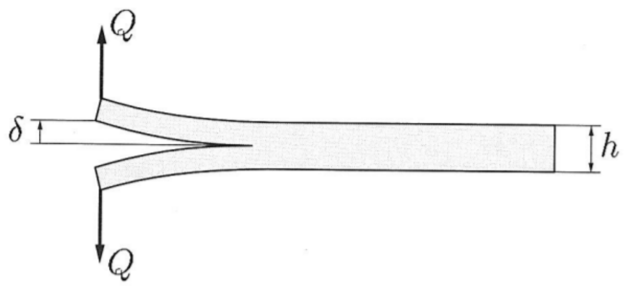


Figure 10: Cracked Structure in the shape of a double cantilever,subjected to mode I loading

From Elementary beam theory,The deflection at the tip is given by,

$$\delta = \frac{Qa^3}{3EI}$$

where, E is young's modulus,a is the crack length,δ is half the crack opening under the forces Q.I is the moment of inertia which is given by,

$$I = \frac{bh^3}{12}$$

where h is the height of the each cantilever,b is the width of the beam.

The expression for J equals the energy flux and there is an energy release from the cantilever during a crack advance da under the fixed deflection δ,We get

$$bJ = \frac{d(Q\delta)}{da}$$

$$K_I^2 = EJ$$

$$\sigma_b = \frac{Qa}{W}$$

$$W = \frac{bh^2}{6}$$

where σ_b is the bending stress on the crack faces,and K_I is the 1st mode Stress Intensity factor.

Thus,we have,

$$\sigma_b = \frac{6Qa}{bh^2}$$

$$K_I = \frac{2\sqrt{3}Qa}{bh\sqrt{h}}$$

The 1st mode stress intensity factor in terms of deflection is ,

$$K_I = \frac{\sqrt{3}\delta E h^{3/2}}{2a^2}$$

The analytical model based on the beam theory will hold well ,if the ratio $\frac{h}{a} << 1$.

3.2 Computed Stress Intensity Factor

The young's modulus E is assumed to be 1. For all the cases studied, vectorial tip enrichment is set and the degree of the polynomial is set to 1. Then, the mesh for different geometries is generated, but the mesh size is kept the same for all the different geometries. This ensures that the same number of elements with the same size are present in the integration zone to compute the K_I for the different geometries considered and also the crack lengths.

The stress intensity factor is computed at the post-processing step. To ensure the precision of the stress intensity factors, a fine mesh is considered. After fixing the mesh size. The enrichment radius is fixed based on the observations made. It is set to 0.4. Then, the integration zone radius is set such that the 1st mode stress intensity factor is precise. The below plot shows the convergence of SIF's with respect to different integration zone radius. The Integration zone radius is set to 0.4 based on the observations and also while respecting the boundaries of all the geometries considered.

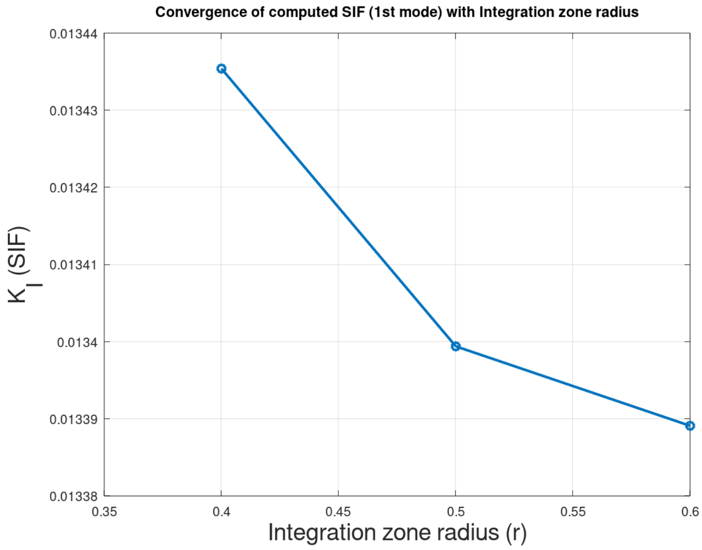


Figure 11: Convergence of computed SIF(1st mode) with Integration zone radius

The different lengths of the beam which is considered for this study are 10,30,60. First,lets consider the beam length 10.The different crack lengths are considered and the analytical and computed 1st mode stress intensity factor are compared.The below plot represents the same.

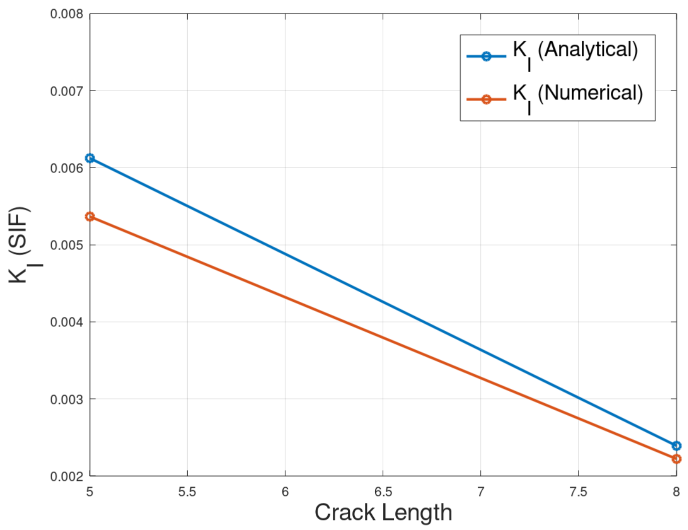


Figure 12: Convergence of computed SIF(1st mode) with respect to Crack length for beam length,10

Now,lets increase the Beam length to 30.The different crack lengths are considered and the analytical and computed 1st mode stress intensity factor are compared.The below plot represents the same.It can be observed that as the crack length is increased.The computed 1st mode SIF gets closer to the analytical.

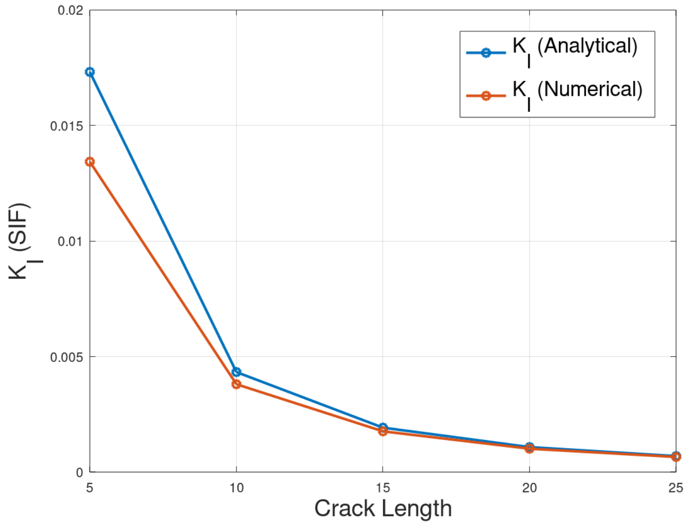


Figure 13: Convergence of computed SIF(1st mode) with respect to Crack length for beam length,30

Let's further increase the beam length to 60. The different crack lengths are considered and the analytical and computed 1st mode stress intensity factor are compared. The below plot represents the same. It can be like the crack length is increased, the analytical and the numerical solution matches with each other in lower-order also.

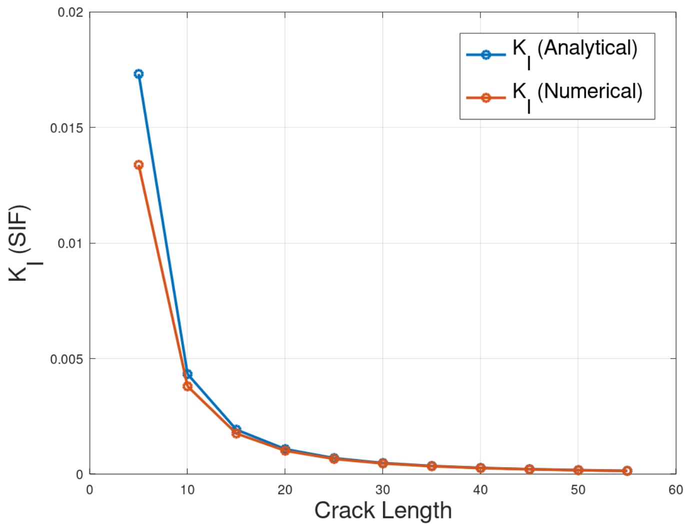


Figure 14: Convergence of computed SIF(1st mode) with respect to Crack length for beam length,60

The below table represents the values of 1st mode SIF for both analytical and computed/numerical for different crack length considered.

Crack Length	K_I**(Analytical)**	K_I **(Numerical)**
5	0.017320508	0.0133852
10	0.004330127	0.0038006
15	0.001924501	0.00175711
20	0.001082532	0.00101311
25	0.00069282	0.000657501
30	0.000481125	0.000460309
35	0.00035348	0.000340106
40	0.000270633	0.000261942
45	0.000213833	0.000207202
50	0.000173205	0.000168519
55	0.000143145	0.000139835

Table 2: Analytical and Computed values of 1st SIF for different crack length

The below plot represents the k_{II} with respect to the crack length of the beam. It can be seen that the value of k_{II} is very small. This represents the 1st mode pure crack model which is as per the analytical model assumptions.

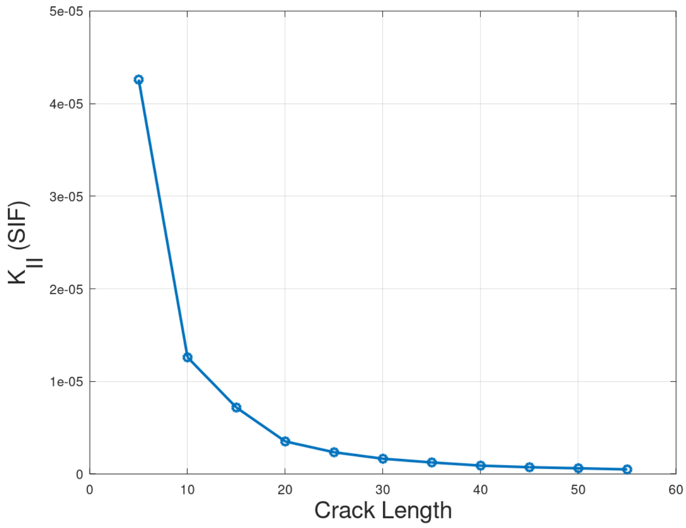


Figure 15: Evolution of the K_{II} with respect to crack length

The below figures shows the Displacement and Stress solution field for a particular geometry.The maximum stress at the crack tip can be observed.

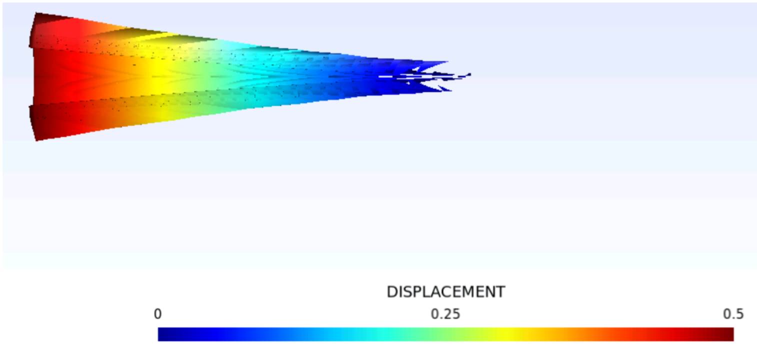


Figure 16: Displacement field for a particular geometry

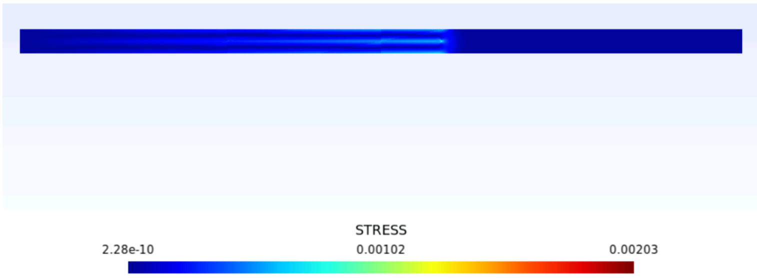


Figure 17: Stress field for a particular geometry