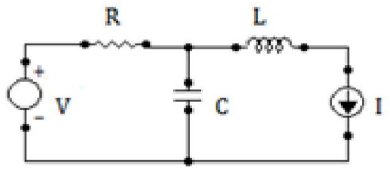
Unit-1 [DC CIRCUIT ANALYSIS & NETWORK THEOREM]

Concept of Network: A combination of various electric elements connected in any manner what so ever is called an Electrical Network. The purpose of electric circuit is to convert electrical energy into some other forms.



Classification of Electrical Elements:

The circuit elements are classified as:

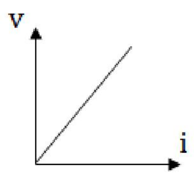
- ❖ Active and passive elements
- ❖ Linear and Non-linear elements
- ❖ Unilateral and bilateral elements

❖ **Active and passive elements:**

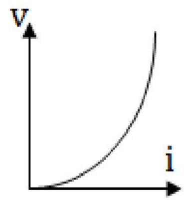
- ➢ **Active Elements:** Active elements are those elements, which supplies the electric power or control the electrical power to the electric circuit. Example: Diode, Transistor, Voltage source, Current source etc.
- ➢ **Passive Elements:** Passive elements can't deliver or control the power, it only consumes the power, and converts into heat or stored in it. Example: Resister R, Inductor L and Capacitor C.

❖ **Linear and Non-linear elements:**

- ➢ **Linear element:** A linear circuit is one whose parameters are constant with the time and, for which volt-ampere characteristics is a straight line or linear, is known as Linear element.



- ➢ **Nonlinear element:** A Nonlinear element is one, whose parameter changes with voltage or current. A nonlinear element has nonlinear volt ampere characteristics.



❖ **Unilateral and bilateral elements:**

- ➢ **Bilateral element:** Conduction of current in both directions in an element with same magnitude is termed as bilateral element. Example: Resister, Inductor and capacitor.

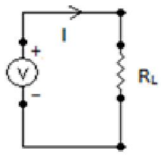
- ➢ **Unilateral element:** Conduction of current in one direction is termed as unilateral element. Example: Diode and transistor

Types of Sources: Sources are the circuit elements that supply energy. Sources are of two types:

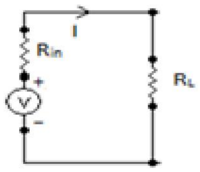
- ❖ Voltage Source
- ❖ Current Source

Both are classified in **Ideal** and **Practical** Source.

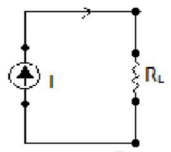
- ➢ **Ideal voltage source:** An Ideal voltage source, produces a constant voltage across its terminal. The terminal voltage is independent of the load connected across it. **The Ideal voltage source has zero internal resistance.**



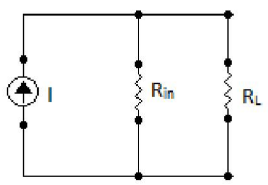
- **Practical voltage source:** A practical voltage source, produces a variable voltage across its terminals. The terminal voltage decreases with the increase in load and vice-versa. **The Practical voltage source has some internal resistance R_{in} in series**.



- **Ideal current source:** An Ideal Current source, supplies a constant current. The current supplied is independent of the load connected across it. **The Ideal current source has infinite internal resistance**.



- **Practical current source:** A practical current source, supplies a variable current. The current supplied is decreases with the increase in load and vice-versa. **The Practical current source has some internal resistance R_{in} in parallel**.

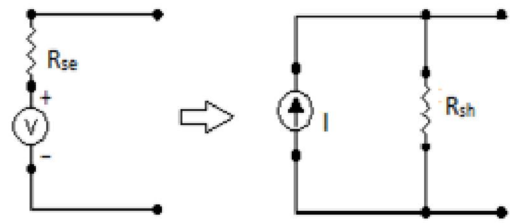


Source Conversion: A Practical source can be interchange i.e. a voltage source can be changed into current source and current source can be change into voltage source.

- **Voltage source to Current source:** A Voltage source of V volt with series resistance R_{se} can be transformed into an equivalent current source of I with parallel resistance R_{sh}. The value of I will be,

$$I = \frac{V}{R_{se}}$$

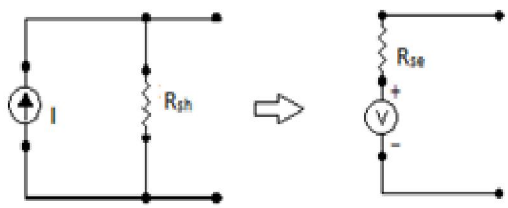
& Value of R_{sh} will be equal to the value of R_{se}.



- **Current source to Voltage source:** A current source of I with parallel resistance R_{sh} can be transformed into an equivalent Voltage source of V volt with series resistance R_{se}. The value of V will be,

$$V = IR_{sh}$$

& Value of R_{se} will be equal to the value of R_{sh}.

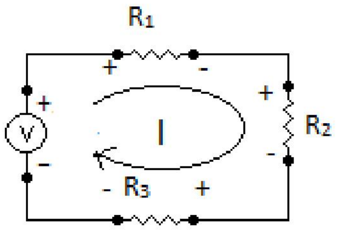


Kirchhoff's Laws: Kirchhoff postulated two basic laws:

(i) Kirchhoff's voltage law
(ii) Kirchhoff's current law

(i) Kirchhoff's Voltage Law: Kirchhoff's voltage law usually abbreviated as KVL, states that "The algebraic sum of all branch voltages around any closed loop of a network is zero at all instants of time"

For the circuit shown in fig. below,



Let current I flowing through closed path. Then,

Voltage across voltage source = V

Voltage across $R_1 = IR_1$

Voltage across $R_2 = IR_2$

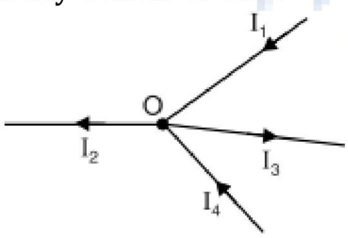
Voltage across $R_3 = IR_3$

On applying KVL in the clockwise direction,

$$-V + IR_1 + IR_2 + IR_3 = 0$$

* *First sign across each element has been considered.*

(ii) Kirchhoff's Current Law: Kirchhoff's current law usually abbreviated as KCL, states that the algebraic sum of all currents terminating at a node equals zero at any instant of time.



At node O, on applying Kirchhoffs current law abbreviated by KCL we get

$$I_1 - I_2 - I_3 + I_4 = 0$$

$$I_1 + I_4 = I_2 + I_3$$

Incoming current = Outgoing current

Network Analysis:

(i) Mesh (Loop) Analysis

(ii) Nodal Analysis

Mesh (Loop) Analysis: Mesh analysis is based on Kirchhoff's voltage law.

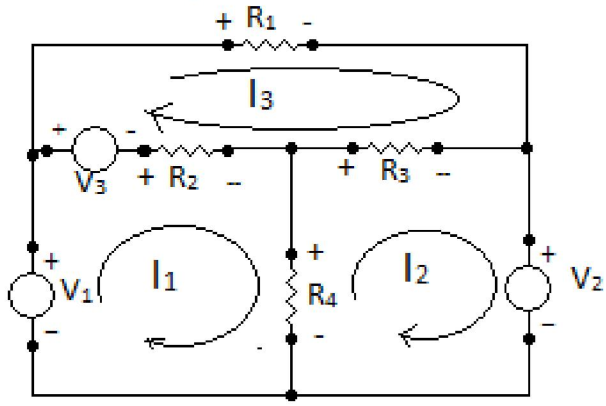
Let us consider a simple dc network as shown in figure below. To find the currents through different branches using Mesh (Loop) analysis method.

Step-I: Inspect the total no. of mesh in the circuit. (all independent closed paths)

Step-II: Label the mesh currents in any arbitrary direction in each mesh.

Step-III: Put the sign convention across each element. (current entering side across a resistor is marked as positive and current leaving side is marked as negative.

Step-IV: Apply KVL in each mesh. (Write KVL equation for each mesh) and solve the equation.



Applying KVL around mesh (loop)1:

$$-V_1 + V_3 + (I_1 - I_3)R_2 + (I_1 - I_2)R_4 = 0$$

Applying KVL around mesh (loop)2:

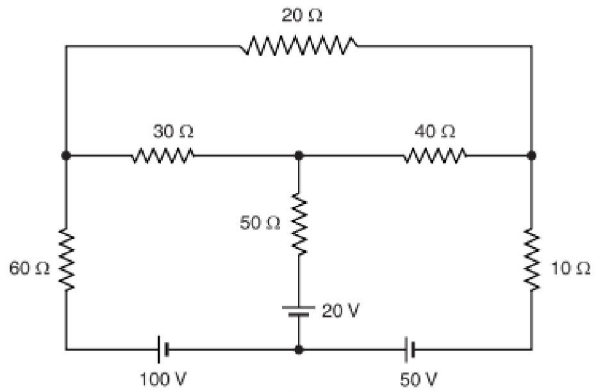
$$-(I_1 - I_2)R_4 + (I_2 - I_3)R_3 + V_2 = 0$$

Applying KVL around mesh (loop)3:

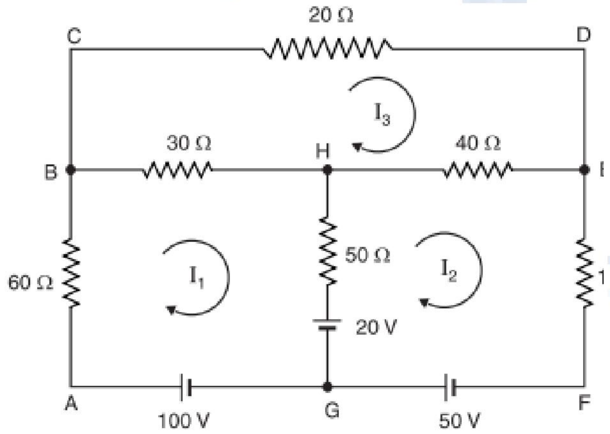
$$I_3R_1 - (I_2 - I_3)R_3 - (I_1 - I_3)R_2 - V_3 = 0$$

By solving these equations get the required answer.

Q:1. Calculate the current in each branch of the circuit shown in Fig. below.



Sol: Assign mesh currents I_1, I_2 and I_3 to mesh1 (ABHGA), mesh2 (HEFGH) and mesh3 (BCDEHB) respectively as shown in figure below.



On applying KVL in mesh1, we get,

$$-100 + 60I_1 + 30(I_1 - I_3) + 50(I_1 - I_2) + 20 = 0$$

$$140I_1 - 50I_2 - 30I_3 = 80 \ldots\ldots\ldots (i)$$

On applying KVL in mesh1, we get,

$$-20 - 50(I_1 - I_2) + 40(I_2 - I_3) + 10I_2 = 0$$

$$-50I_1 + 100I_2 - 40I_3 = 20 \ldots\ldots\ldots (ii)$$

On applying KVL in mesh1, we get,

$$-40(I_2 - I_3) - 30(I_1 - I_3) + 20I_3 = 0$$

$$-30I_1 - 40I_2 + 90I_3 = 90 \ldots\ldots\ldots (iii)$$

By solving equation (i), (ii) and (iii), we get,

$$\boldsymbol{I_1 = 1.65\ A}$$

$$\boldsymbol{I_2 = 2.12\ A}$$

$$\boldsymbol{I_3 = 1.5\ A}$$

Now, Current in 60 Ω resistor,

$$\boldsymbol{I_1 = 1.65\ A}$$

Current in 30 Ω resistor,

$$\boldsymbol{I_1 - I_3 = 1.65 - 1.5 = 0.15\ A}$$

Current in 50 Ω resistor,

$$\boldsymbol{I_1 - I_2 = 2.12 - 1.65 = 0.47\ A}$$

Current in 40 Ω resistor,

$$\boldsymbol{I_2 - I_3 = 2.12 - 1.5 = 0.62\ A}$$

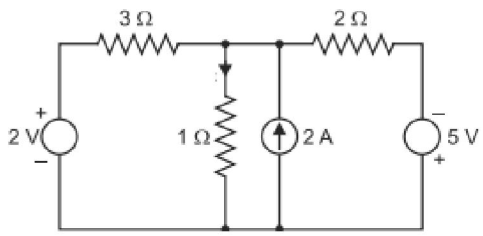
Current in 10 Ω resistor,

$$\boldsymbol{I_2 = 2.12\ A}$$

Current in 20 Ω resistor,

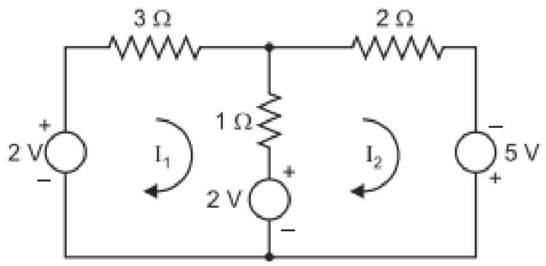
$$\boldsymbol{I_3 = 1.5\ A}$$

Q.2: Using mesh current method, determine current in 3 Ω in the circuit shown in figure below.



Sol: First convert 2A current source in parallel with 1Ω resistance into equivalent voltage source of voltage 2A×1Ω = 2V in series with 1Ω resistance.

Assign mesh currents I_1 and I_2 to mesh1 and mesh2 in Figure below.



On applying KVL in mesh1, we get,

$$-2 + 3I_1 + 1(I_1 - I_2) + 2 = 0$$

$$4I_1 - I_2 = 0 \ldots\ldots\ldots (i)$$

On applying KVL in mesh1, we get,

$$-2 - 1(I_1 - I_2) + 2I_2 - 5 = 0$$

$$-I_1 + 3I_2 = 7 \ldots\ldots\ldots (ii)$$

By solving equation (i) and (ii), we get,

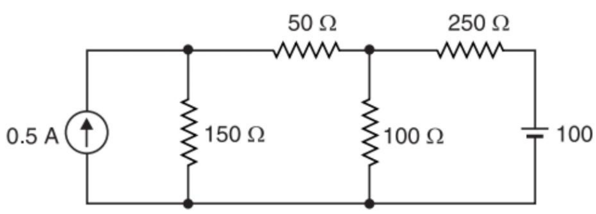
$$\boldsymbol{I_1 = \frac{7}{11}\ A}$$

$$\boldsymbol{I_2 = \frac{28}{11}\ A}$$

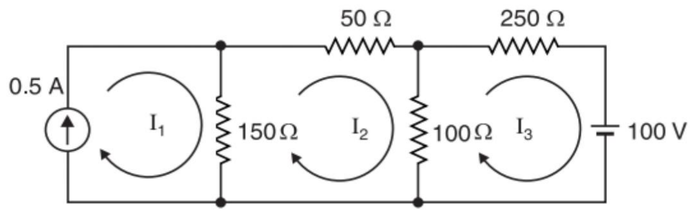
Hence, Current in 3 Ω resistor,

$$\boldsymbol{I_1 = \frac{7}{11}\ A}$$

Q.3: Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in figure.



Sol: Assign mesh currents I_1, I_2 and I_3 to mesh1, mesh2 and mesh3 respectively as shown in figure below.



As mesh1 contains a current source of 0.5 A, hence current in mesh1 is equal to 0.5 A. i.e.

$$\boldsymbol{I_1 = 0.5\ A}$$

On applying KVL in mesh2, we get,

$$-150(I_1 - I_2) + 50I_2 + 100(I_2 - I_3) = 0$$

$$-150I_1 + 300I_2 - 100I_3 = 0$$

$$-150\ (0.5) + 300I_2 - 100I_3 = 0$$

$$300I_2 - 100I_3 = 75 \ldots\ldots\ldots (i)$$

On applying KVL in mesh3, we get,

$$-100(I_2 - I_3) + 250I_3 + 100 = 0$$

$$-100I_2 + 350I_3 = -100 \ldots\ldots\ldots (ii)$$

By solving equation (i) and (ii), we get

$$\boldsymbol{I_2 = 0.171\ A \quad and \quad I_3 = -0.237\ A}$$

Current in 50 Ω resistor,

$$\boldsymbol{I_2 = 0.171\ A}$$

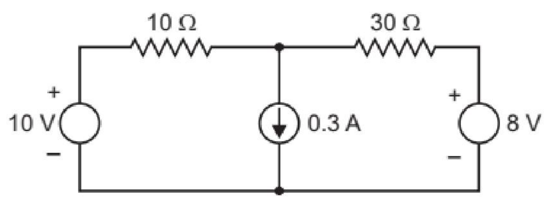
Current in 250 Ω resistor,

$$\boldsymbol{I_3 = -0.237\ A}$$

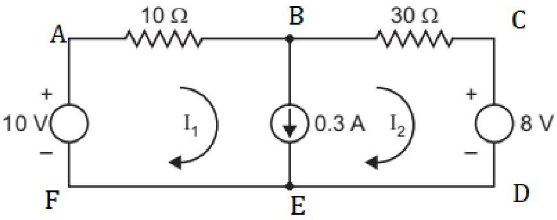
Current in 100 Ω resistor,

$$\boldsymbol{I_2 - I_3 = 0.171 + 0.237 = 0.408\ A}$$

Q.4: Use mesh current method to determine currents through each of the components in the circuit shown in figure.



Sol: Assign mesh currents I_1 and I_2 to mesh1 and mesh2 respectively as shown in figure below.



***Concept of super mesh**: if common branch of two mesh (ABEFA & BCDEB) contains a current source, then problem can be solved by applying KVL in super-mesh (ABCDEFA).*

On applying KVL in super-mesh, we get,

$$-10 + 10I_1 + 30I_2 + 8 = 0$$

$$10I_1 + 30I_2 = 2 \ldots\ldots\ldots (i)$$

From the common branch (containing current source,

$$\boldsymbol{I_1 - I_2 = 0.3} \ldots\ldots\ldots (ii)$$

By solving equation (i) and (ii), we get,

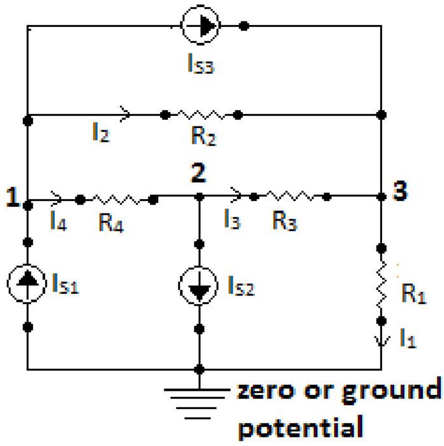
$$\boldsymbol{I_1 = 0.275\,A}$$

$$\boldsymbol{I_2 = -0.025\,A}$$

Nodal Analysis: Nodal analysis is based on Kirchhoff's current law.

Let us consider a simple dc network as shown in figure below to find the currents through different branches using **"Nodal analysis"** method.

Step-I: Inspect the total no. of node in the circuit and **select one node as the reference node** (assign as ground potential or zero potential) **and label the remaining nodes** as unknown node voltages with respect to the reference node.



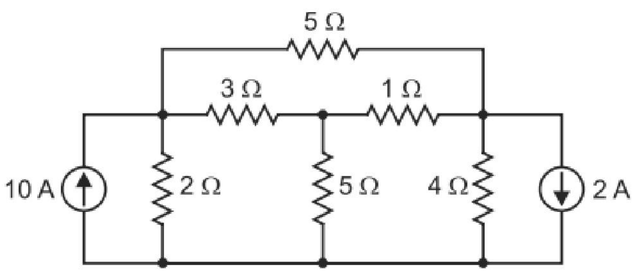
Step-II: Assign branch currents in each branch. (The choice of direction is arbitrary).

Step-III: Write the standard form of node equations (KCL) by inspecting the circuit.

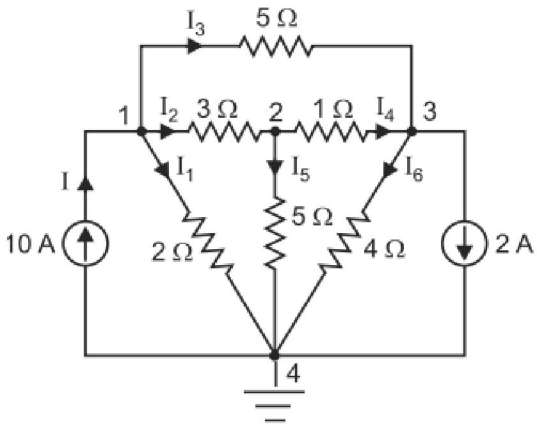
Step-IV: Express the branch currents in terms of node assigned voltages.

Step-V: Solve a set of simultaneous algebraic equation for node voltages and ultimately the Branch currents.

Q.5: Use nodal analysis to find the currents in various resistors of the circuit shown in figure.



Sol: The given circuit is redrawn in figure below with nodes marked 1, 2, 3 and 4.



Let us take node 4 as the reference node (at zero potential) and the voltage of node 1, 2 and 3 are V_1, V_2 and V_3 respectively. We shall apply *KCL* at nodes 1, 2 and 3 to obtain the solution.

- On applying KCL at node1, we get,

$$10 - I_1 - I_2 - I_3 = 0$$

Replacing current in terms of node voltages,

$$10 - \left(\frac{V_1}{2}\right) - \left(\frac{V_1 - V_2}{3}\right) - \left(\frac{V_1 - V_3}{5}\right) = 0$$

$$31V_1 - 10V_2 - 6V_3 = 300 \ldots \ldots \ldots (i)$$

- On applying KCL at node2, we get,

$$I_2 - I_4 - I_5 = 0$$

Replacing current in terms of node voltages,

$$\left(\frac{V_1 - V_2}{3}\right) - \left(\frac{V_2 - V_3}{1}\right) - \left(\frac{V_2}{5}\right) = 0$$

$$5V_1 - 23V_2 + 15V_3 = 0 \ldots \ldots \ldots (ii)$$

- On applying KCL at node3, we get,

$$I_3 + I_4 - I_6 - 2 = 0$$

Replacing current in terms of node voltages,

$$\left(\frac{V_1 - V_3}{5}\right) + \left(\frac{V_2 - V_3}{1}\right) - \left(\frac{V_3}{4}\right) - 2 = 0$$

$$4V_1 + 20V_2 - 29V_3 = 40 \ldots \ldots \ldots (i)$$

By solving equation (i), (ii) and (iii), we get,

$$\boldsymbol{V_1 = 12.06\,V}$$

$$\boldsymbol{V_2 = 5.1\,V}$$

$$\boldsymbol{V_3 = 3.8\,V}$$

Now, Current in each branch can be determined as,

$$\boldsymbol{I_1 = \frac{V_1}{2} = \frac{12.06}{2} = 6.03\,A}$$

$$\boldsymbol{I_2 = \frac{V_1 - V_2}{3} = \frac{12.06 - 5.1}{3} = 2.32\,A}$$

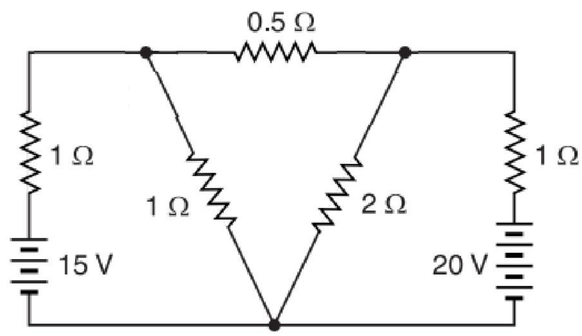
$$\boldsymbol{I_3 = \frac{V_1 - V_3}{5} = \frac{12.06 - 3.8}{5} = 1.65\,A}$$

$$\boldsymbol{I_4 = \frac{V_2 - V_3}{1} = \frac{5.1 - 3.8}{1} = 1.3\,A}$$

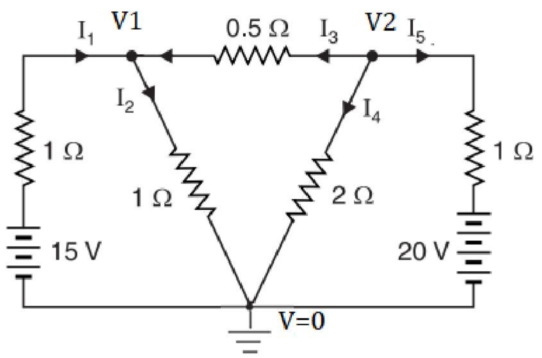
$$\boldsymbol{I_5 = \frac{V_2}{5} = \frac{5.1}{5} = 1.02\,A}$$

$$\boldsymbol{I_6 = \frac{V_3}{4} = \frac{3.8}{4} = 0.95\,A}$$

Q.6: Use nodal analysis to find the currents in various branches of the circuit shown in figure.



Sol: The given circuit is redrawn in figure below with nodes marked.



Let us take node 3 as the reference node (at zero potential) and the voltage of node 1 and 2 are V_1 and V_2 respectively. We shall apply *KCL* at nodes 1 and 2 to obtain the solution.

- On applying KCL at node1, we get,

$$I_1 - I_2 + I_3 = 0$$

Replacing current in terms of node voltages,

$$\left(\frac{-V_1 + 15}{1}\right) - \left(\frac{V_1}{1}\right) + \left(\frac{V_2 - V_1}{0.5}\right) = 0$$

$$4V_1 - 2V_2 = 15 \ldots\ldots\ldots (i)$$

- On applying KCL at node2, we get,

$$-I_3 - I_4 - I_5 = 0$$

Replacing current in terms of node voltages,

$$-\left(\frac{V_2 - V_1}{0.5}\right) - \left(\frac{V_2}{2}\right) - \left(\frac{V_2 - 20}{1}\right) = 0$$

$$4V_1 - 7V_2 = -40 \ldots\ldots\ldots (ii)$$

By solving equation (i) and (ii), we get,

$$\boldsymbol{V_1 = 9.25\ V}$$

$$\boldsymbol{V_2 = 11\ V}$$

Now, Current in each branch can be determine as,

$$I_1 = \frac{-V_1 + 15}{1} = \frac{-9.25 + 15}{1} = \mathbf{5.75\ A}$$

$$I_2 = \frac{V_1}{1} = \mathbf{9.25\ A}$$

$$I_3 = \frac{V_2 - V_1}{0.5} = \frac{11 - 9.25}{0.5} = \mathbf{3.5\ A}$$

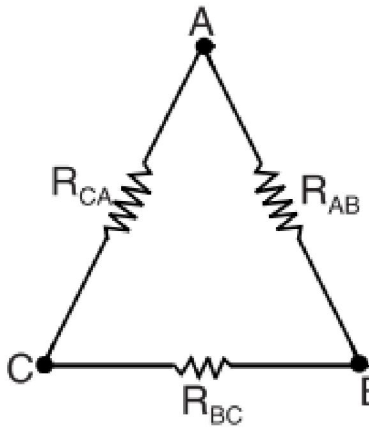
$$I_4 = \frac{V_2}{2} = \frac{11}{2} = \mathbf{5.5\ A}$$

$$I_5 = \frac{V_2 - 20}{1} = \frac{11 - 20}{1} = \mathbf{-9\ A}$$

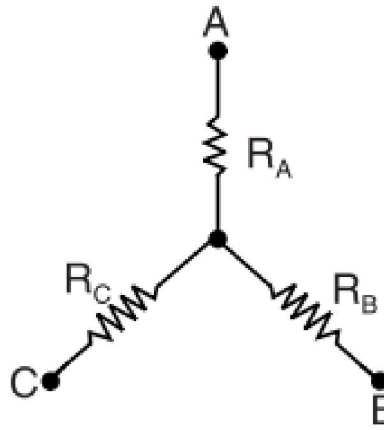
Star-Delta Transformation:

Complicated networks can be simplified by successively replacing delta mesh to star equivalent system and vice-versa.

In delta network, three resistors are connected in delta fashion (Δ).



in star network three resistors are connected in wye (Y) fashion.



Delta to Star Conversion: -

- In delta configuration, Resistance between A & B,

$$R_{A-B} = R_{AB} \| (R_{BC} + R_{CA}) = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})}$$

Similarly, Resistance between B & C,

$$R_{B-C} = \frac{R_{BC}(R_{AB} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})}$$

and, Resistance between C & A,

$$R_{C-A} = \frac{R_{CA}(R_{AB} + R_{BC})}{(R_{AB} + R_{BC} + R_{CA})}$$

- In star configuration, Resistance between A & B,

$$R_{A-B} = R_A + R_B$$

Similarly, Resistance between B & C,

$$R_{B-C} = R_B + R_C$$

and, Resistance between C & A,

$$R_{C-A} = R_C + R_A$$

- If star & delta are equivalent to each other, therefore resistance between each pair of terminals is same. Hence,

$$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \dots (i)$$

$$R_B + R_C = \frac{R_{BC}(R_{AB} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \dots (ii)$$

$$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{(R_{AB} + R_{BC} + R_{CA})} \dots (iii)$$

On addition of equation (i), (ii) and (iii), we get,

$$2(R_A + R_B + R_C) = \frac{2(R_{AB}.R_{BC} + R_{BC}.R_{CA} + R_{CA}.R_{AB})}{(R_{AB} + R_{BC} + R_{CA})}$$

$$(R_A + R_B + R_C) = \frac{(R_{AB}.R_{BC} + R_{BC}.R_{CA} + R_{CA}.R_{AB})}{(R_{AB} + R_{BC} + R_{CA})} \qquad \dots (iv)$$

Subtracting equation (ii) from equation(iv) gives,

$$\boldsymbol{R_A = \frac{R_{AB}.R_{CA}}{(R_{AB} + R_{BC} + R_{CA})}} \qquad \dots (v)$$

Subtracting equation (iii) from equation(iv) gives,

$$\boldsymbol{R_B = \frac{R_{AB}.R_{BC}}{(R_{AB}+R_{BC}+R_{CA})}} \qquad \dots(\boldsymbol{vi})$$

Subtracting equation (i) from equation(iv) gives,

$$\boldsymbol{R_C = \frac{R_{BC}.R_{CA}}{(R_{AB}+R_{BC}+R_{CA})}} \qquad \dots(\boldsymbol{vii})$$

Star to Delta Conversion: -

From equation (v), (vi) & (vii),

$$R_A.R_B + R_B.R_C + R_C.R_A = \frac{{R_{AB}}^2.R_{BC}.R_{CA} + R_{AB}.{R_{BC}}^2.R_{CA} + R_{AB}.R_{BC}.{R_{CA}}^2}{(R_{AB}+R_{BC}+R_{CA})^2}$$

$$R_A.R_B + R_B.R_C + R_C.R_A = \frac{R_{AB}.R_{BC}.R_{CA}(R_{AB}+R_{BC}+R_{CA})}{(R_{AB}+R_{BC}+R_{CA})^2}$$

$$R_A.R_B + R_B.R_C + R_C.R_A = \frac{R_{AB}.R_{BC}.R_{CA}}{(R_{AB}+R_{BC}+R_{CA})} \qquad \dots(viii)$$

Divide equation (viii) by equation (vii), we get,

$$\boldsymbol{R_{AB} = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_C}} \qquad \dots(\boldsymbol{ix})$$

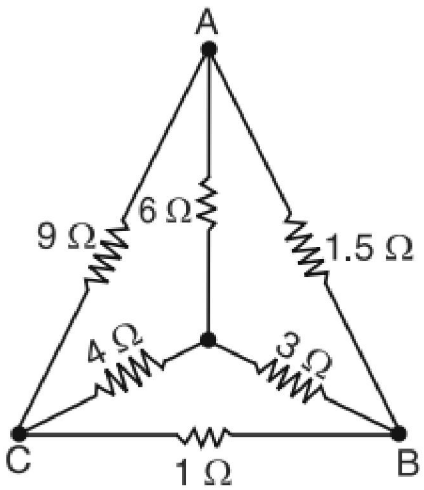
Divide equation (viii) by equation (v), we get,

$$\boldsymbol{R_{BC} = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_A}} \qquad \dots(\boldsymbol{x})$$

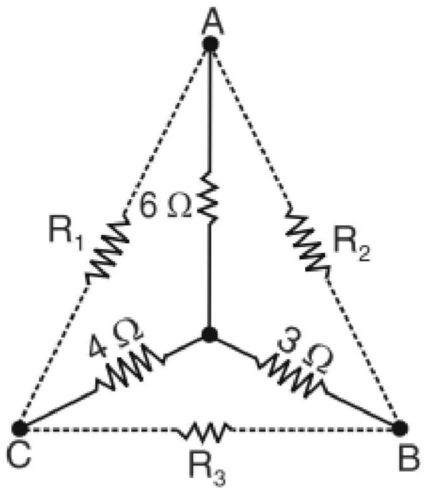
Divide equation (viii) by equation (vi), we get,

$$\boldsymbol{R_{CA} = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_B}} \qquad \dots(\boldsymbol{xi})$$

Q.7: A network of resistors is shown in Fig. 3.243 (i). Find the resistance (i) between terminals A and B (ii) B and C and (iii) C and A.



Sol: The star-connected resistances 6 Ω, 3 Ω and 4 Ω in figure can be converted into equivalent delta-connected resistances R_1, R_2 and R_3 as shown in fig. below.

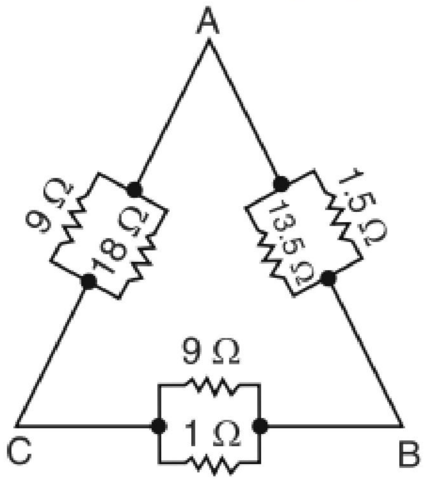


$$R_1 = \frac{6\,.\,4 + 4\,.\,3 + 3\,.\,6}{3} = \frac{54}{3} = 18\ \Omega$$

$$R_2 = \frac{6\,.\,4 + 4\,.3 + 3\,.6}{4} = \frac{54}{4} = 13.5\Omega$$

$$R_3 = \frac{6\,.\,4 + 4\,.\,3 + 3\,.\,6}{6} = \frac{54}{6} = 9\ \Omega$$

These delta-connected resistances R_1, R_2 and R_3 come in parallel with the original delta-connected resistances.

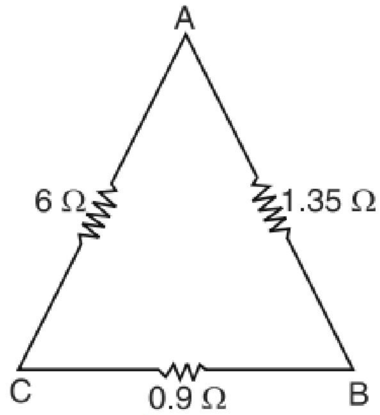


The parallel resistances in each leg of delta in can be replaced by a single resistor as shown in fig. below.

$$R_{AC} = 9\|18 = \frac{9 \times 18}{9 + 18} = 6\Omega$$

$$R_{BC} = 9\|1 = \frac{9 \times 1}{9 + 1} = 0.9\Omega$$

$$R_{AB} = 1.5\|13.5 = \frac{9 \times 18}{9 + 18} = 1.35\Omega$$



(*i*) Resistance between *A* and *B*

= 1·35 Ω || (6 + 0·9) Ω = 1·35 × 6·9/8·25 = **1·13** Ω

(*ii*) Resistance between *B* and *C*

= 0·9 Ω || (6 + 1·35) Ω = 0·9 × 7·35/8·25 = **0·8** Ω

(*iii*) Resistance between *A* and *C*

= 6 Ω || (1·35 + 0·9) Ω = 6 × 2·25/8·25 = **1.636** Ω

Network Theorems:

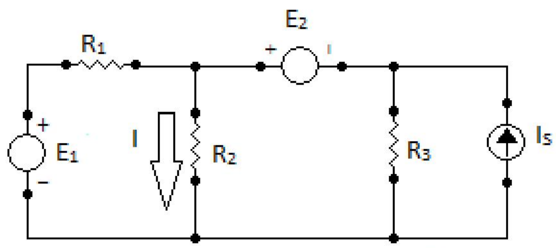
1. Superposition Theorem
2. Thevenin's Theorem
3. Norton's Theorem
4. Maximum Power Transfer Theorem

1. Superposition theorem:

According to superposition theorem,
"In any linear bilateral network containing two or more independent sources, the net response in any branch is the algebraic sum of responses caused by each independent source acting alone, with all other independent sources being replaced by their respective internal resistances."

To replace voltage source simply short circuit the terminal of voltage source and to replace current source simply open circuit the terminal of current source.

Consider a simple resistive network as shown in figure below. It has two independent practical voltage sources and one practical current source.



The current I will be the algebraic sum of three individual current produced by the three sources. i.e.,

$$I = I' + I'' + I'''$$

- ❖ Here I' is current due E_1 alone, with E_2 short-circuited and Is open-circuited.
- ❖ I'' is current due E_2 alone, with E_1 short-circuited and Is open-circuited.
- ❖ I''' is current due Is alone, with E_1 & E_2 short-circuited.

Remarks: Superposition theorem is most often used when it is necessary to determine the individual contribution of each source to a particular response.

Procedure:

Step (i) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

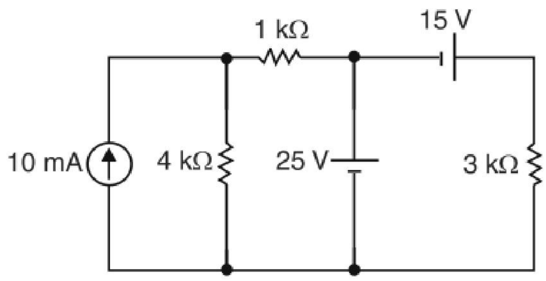
Step (ii) Determine the voltage across or current through the desired element/branch due to single source selected in step (*i*).

Step (iii) Repeat the above two steps for each of the remaining sources.

Step (iv) *Algebraically* add all the voltages across or currents through the element/branch under consideration.

The sum is the actual voltage across or current through that element/branch when all the sources are acting simultaneously.

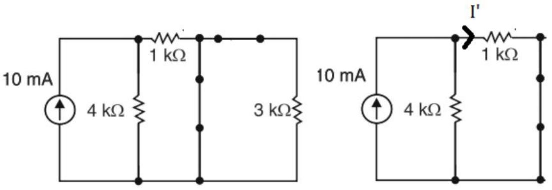
Q.8: Using the superposition principle, find the current through 1kΩ resistor in figure below.



Sol: The given circuit contains one current source and two voltage source.

Case 1: When current source of 10 mA is connected alone.

The current through 1kΩ resistor due to current source acting alone is found by replacing 25-V and 15-V sources by short circuit as shown in fig. below



Here the resistor of 3Ω becomes shorted.

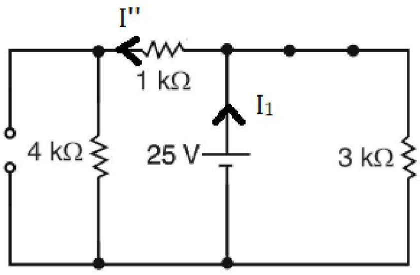
by current division rule, current through 1kΩ,

$$I' = (10 \times 10^{-3})\frac{4}{4+1} = 8 \times 10^{-3}A$$

I' is flowing in rightward direction.

Case 2: When voltage source of 25 V is connected alone.

The current through 1kΩ resistor due to 25V source acting alone is found by replacing the 10mA current source by an open circuit and 15 V source by a short circuit as shown in fig. below.



Current from the source,

$$I_1 = \frac{25}{3\|(1+4)K} = \frac{25}{(3\|5)K} = \frac{25}{(15/8)K}$$

$$I_1 = \frac{200}{15K} = \frac{40}{3} = 13.33mA$$

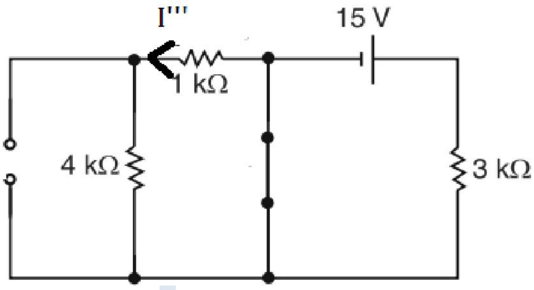
by current division rule, current through 1kΩ,

$$I'' = (13.33 \times 10^{-3})\frac{3}{5+3} = 5 \times 10^{-3}A$$

I'' is flowing in leftward direction.

Case 3: When voltage source of 15 V is connected alone.

The current through 1kΩ resistor due to 15V source acting alone is found by replacing the 10mA current source by an open circuit and 25 V source by a short circuit as shown in fig. below.



Here the resistor of 4Ω & 1Ω becomes shorted. The short circuit prevents any current from flowing in the 1 k Ω resistor.

Hence,

$$I''' = 0\ A$$

Now by using superposition theorem, net current in 1 kΩ due to all the source,

$$I = I' - I'' + I'''$$
$$I = (8 \times 10^{-3}) - (5 \times 10^{-3}) + 0$$
$$I = 3 \times 10^{-3}\ A$$
$$\boldsymbol{I = 3\ mA}$$

Limitations of superposition Theorem: -

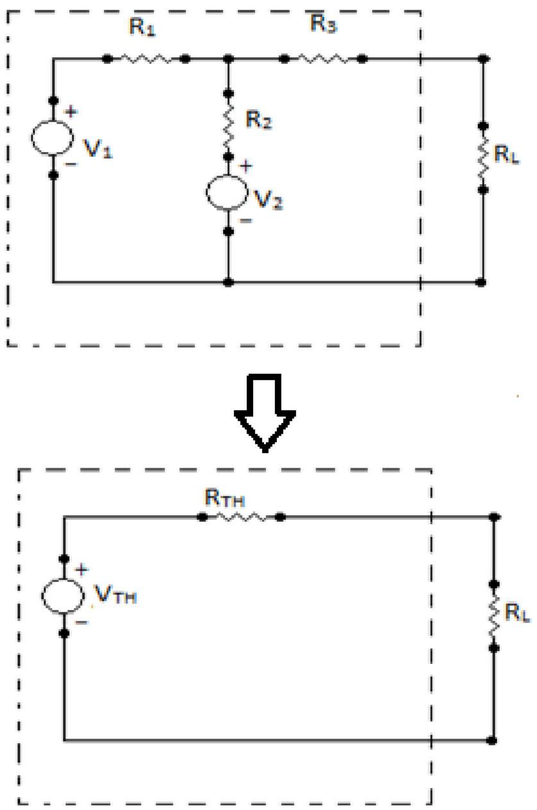
1: Superposition theorem doesn't work for power calculation. Because **power calculations** involve either the product of voltage and current, the square of current or the square of the voltage, they are not linear operations.

2: Superposition theorem cannot be applied for non-linear circuit

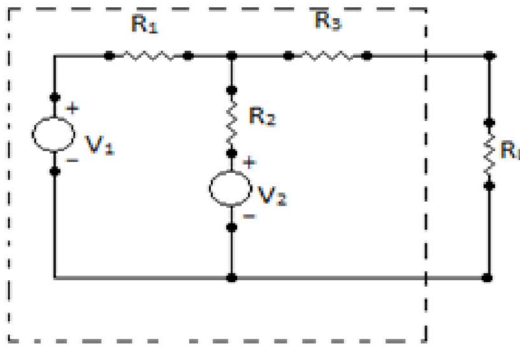
2. Thevenin's theorem:

According to Thevenin's theorem,
"Any two output terminals A & B, of an active linear network containing independent sources (includes voltage and current sources) can be replaced by a simple voltage source of magnitude V_{TH} in series with a single resistor R_{TH}."

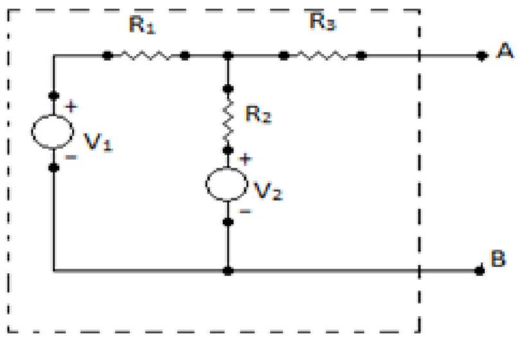
Where R_{TH} is the equivalent resistance of the network when looking from the output terminals with all sources (voltage and current) removed and replaced by their internal resistances and the magnitude of V_{TH} is equal to the open circuit voltage across the terminals.



Procedure for applying Thevenin's theorem: - To find a current I_L through the load resistance R_L using Thevenin's theorem, the following steps are followed:



Step-1: Disconnect the load resistance (*RL*) from the circuit and mark the terminal AB.

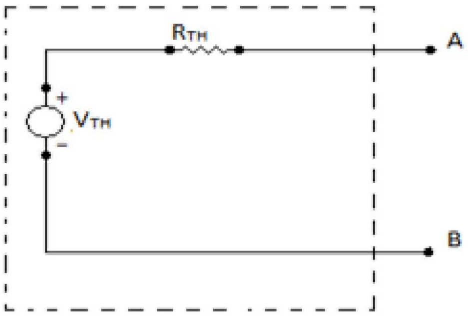


Step-2: Calculate the open-circuit voltage V_{TH} at the load terminals. In general, select a path from A to B, and calculate voltage across that path. we can apply any of the techniques (mesh-current, node-voltage and superposition method) to find V_{TH}.

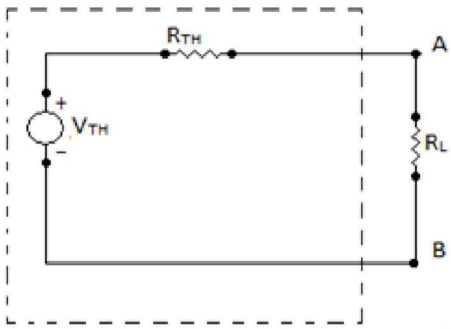
Step-3: Redraw the circuit with each practical source replaced by its internal resistance (Note: voltage sources should be short-circuited (just remove them and replace with plain wire) and current sources should be open-circuited (just removed).

Step-4: Calculate the resistance R_{TH} that would exist between the load terminals.

Step-5: Place V_{TH} in series with R_{TH} to form the Thevenin's equivalent circuit as shown below.



Step-6: Reconnect the original load to the Thevenin voltage circuit as shown below the load's voltage, current and power may be calculated by a simple arithmetic operation only.



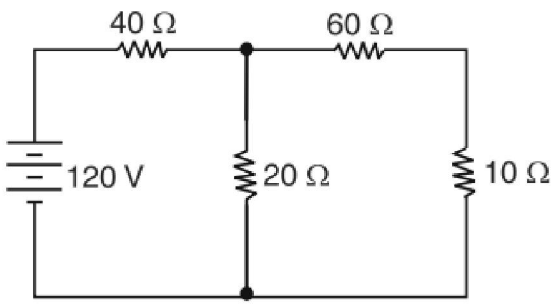
Load current,

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

Voltage across the load,

$$V_L = I_L R_L = \frac{V_{TH}}{R_{TH} + R_L} . R_L$$

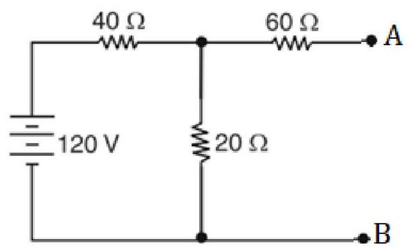
Q.9: Using Thevenin's theorem, find the current in 10Ω resistor in the circuit shown in Fig.



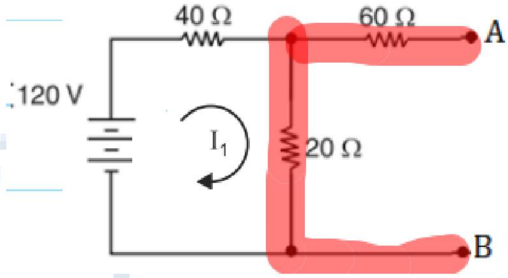
Sol:

➢ **Calculation of V_{Th}:**

1. Remove load resistor of 10Ω and mark the terminals as A & B.



2. Select any path from A to B and determine voltage across that path, it will be equal to V_{Th}.



Current through 60 Ω = 0 A

Current through 20 Ω,

$$I_1 = \frac{120}{40 + 20} = 2\,A$$

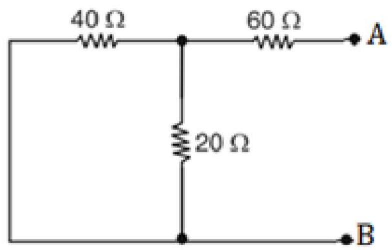
Now,

$$V_{Th} = (60 \times 0) + (20 \times I_1)$$
$$V_{Th} = (60 \times 0) + (20 \times 2)$$

$$\boldsymbol{V_{Th} = 40\,V}$$

➢ **Calculation of R_{Th}:**

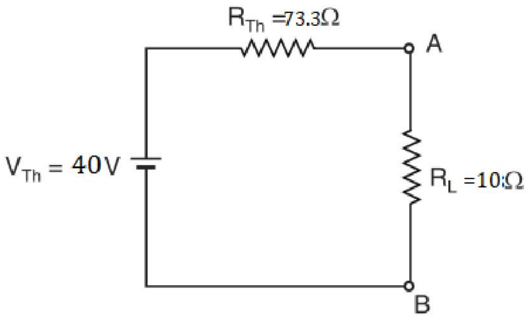
Short circuit the voltage source and determine the equivalent resistance across AB.



$$R_{Th} = (40 \| 20) + 60$$

$$\boldsymbol{R_{Th} = \frac{800}{60} + 60 = 73.33\,\Omega}$$

- **Calculation of Load current:**
  Draw the Thevenin's equivalent network, replace load resistor and calculate load current.



$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$I_L = \frac{40}{73.3 + 10}$$

$$\boldsymbol{I_L = 0.48\ A}$$

For more Problems / Questions ON Thevenin'S THEOREM refer class notes.

Norton's theorem: - Norton's theorem states that any two output terminals A & B,of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a simple current source of magnitude I_N in parallel with a single resistor R_N.

Where the magnitude of current source is the current measured in the short circuit placed across the terminal pair *A* & *B*. The parallel resistance R_N is the equivalent resistance looking into the terminal pair *A* & *B* with all independent sources has been replaced by their internal resistances.

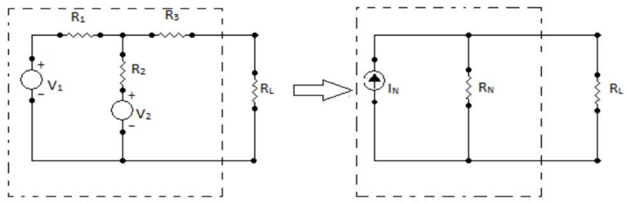


Figure: 23

Procedure for applying Norton's theorem:- To find a current IL through the load resistance RL using Norton's theorem, the following steps are followed:

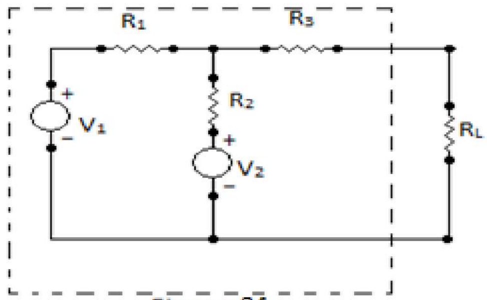


Figure: 24

Step-1: Disconnect the load resistance (R_L) from the circuit and mark the terminal AB.

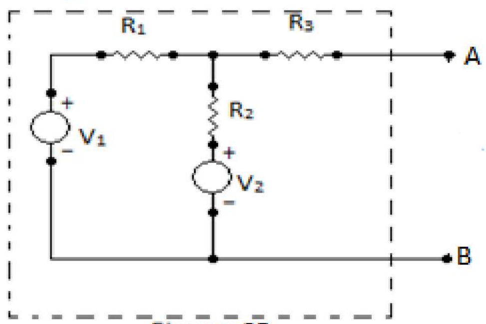


Figure: 25

Step-2: Short the output terminal after disconnecting the load resistance (R_L) from the terminals *A* & *B* and then calculate the short circuit current I_N. In general, we can apply any of the techniques (mesh-current, node-voltage and superposition method)

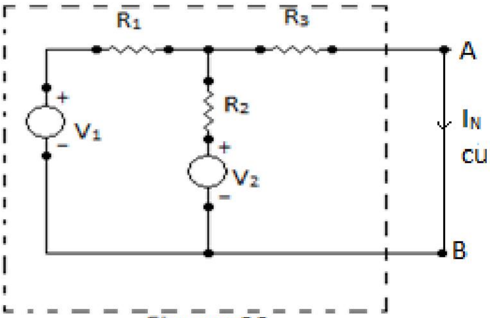


Figure: 26

Step-3: Redraw the circuit with each practical source replaced by its internal resiststance (Note: voltage sources should be short-circuited (just remove them and replace with plain wire) and current sources should be open-circuited (just removed).

Step-4: Calculate the resistance R_N that would exist between the load terminals.

Step-5: Place R_N in parallel with current source I_N to form the Norton's equivalent circuit.

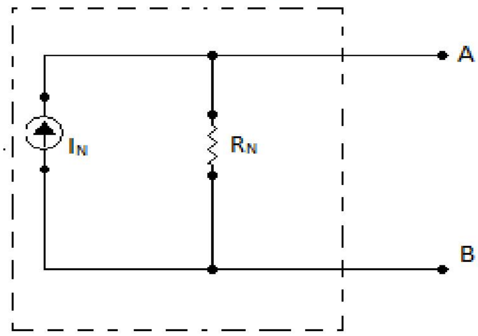


Figure: 27

Step-6: Reconnect the original load to the Norton current circuit as shown below; the load's voltage, current and power may be calculated by a simple arithmetic operation only.

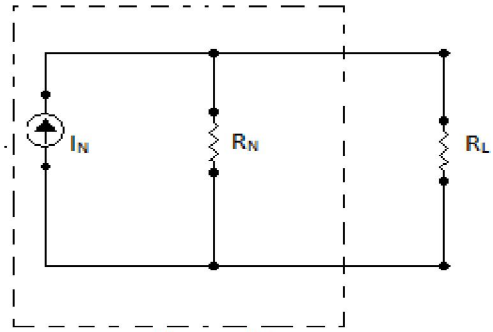


Figure: 28

Load current,

$$I_L = \frac{R_N}{R_N+R_L} \cdot I_N$$

Voltage across the load, $V_L = I_L \cdot R_L$

Power absorbed by the load,

$$P_L = I_L^2 \cdot R_L$$

For Problems / Questions ON NORTON"S THEOREM refer class notes.

<u>**Maximum power transfer theorem:**</u> <u>Maximum power transfer theorem states that the power transfer by a network to a load resistance R_L will be maximum, if the value of load resistance is equal to the thevenin's resistance (internal resistance) R_{TH} of the network.</u>

Let us consider an electric network as shown in fig. below, the problem is to find the choice of the resistance R_L so that the network delivers maximum power to the load.

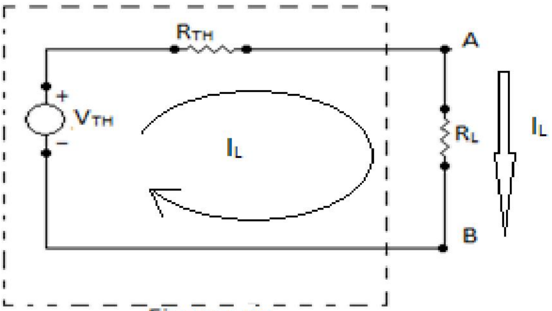


Figure: 29

In fig. 29 a variable load resistance R_L is connected to an equivalent thevenin circuit of original circuit.

The current for any value of load resistance R_L Is,

$$I_L = \frac{V_{TH}}{R_{TH}+R_L}$$

Then the power delivered to load is given by,

$P_L = I_L^2 \,.\, R_L$

$P_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 . R_L$

$P_L = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} . R_L$

The load power depends on both R_L and R_{TH}; however, R_{TH} is constant for the equivalent Thevenin network. So power delivered by the equivalent Thevenin network to the load resistor is entirely depends on the value of R_L .

To find the value of R_L that absorbs a maximum power from the Thevenin circuit, we differentiate PL with respect to R_L.

$$\frac{dP_L}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH}+R_L)^2 - 2R_L\,(R_{TH}+R_L)}{(R_{TH}+R_L)^4}\right] = 0$$

$$(R_{TH} + R_L)^2 = 2R_L\,(R_{TH} + R_L)$$

$$R_{TH}^2 + R_L^2 + 2\,R_{TH}\,.R_L = 2R_L\,R_{TH} + 2\,R_L^2$$

$$\boxed{R_L = R_{TH}}$$

The expression for maximum power dissipated to load resistance is given by,

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} . R_L$$

$$\boxed{P_{MAX} = \frac{V_{TH}^2}{4R_L}}$$

The total power delivered by the source is,

$P_T = I_L^2\,(R_{TH} + R_L\,)$

$P_T = 2\,I_L^2\,R_L$

Efficiency under maximum power condition is,

$$\text{Efficiency} = \frac{I_L^2 R_L}{2I_L^2 R_L} \times 100\ \%$$

Efficiency = 50%

Remarks: The Thevenin equivalent circuit is useful in finding the maximum power that a linear circuit can deliver to a load.

For Problems / Questions ON Maximum power transfer theorem refer class notes.