1.1 Introduction of Newtonian Mechanics

The universe, in which we live, is full of dynamic objects. Nothing is static here Starting from giant stars to tiny electrons, everything is dynamic. This dynamicity of universal objects leads to variety of interactions, events and happenings. The curiosity of human being/scientist to know about these events and laws which governs them. Mechanics is the branch of Physics which is mainly concerned with the study of mobile bodies and their interactions.

A real breakthrough in this direction was made my Newton in 1664 by presenting the law of linear motion of bodies. Over two hundred years, these laws were considered to be perfect and capable of explaining everything of nature.

1.2 Frames of Reference

The dynamicity of universal objects leads to variety of interactions between these objects leading to various happenings. These happenings are termed as **events**. The relevant data about an event is recorded by some person or instrument, which is known as **"Observer"**.

The motion of material body can only described relative to some other object. As such, to locate the position of a particle or event, we need a coordinate system which is at rest with respect to the observer. Such a coordinate system is referred as frame of reference or observers frame of reference.

"A reference frame is a space or region in which we are making observation and measuring physical(dynamical) quantities such as velocity and acceleration of an object(event)." or

> "A frame of reference is a three dimensional coordinate system relative to which described the position and motion(velocity and acceleration) of a body(object)"

Fig.1.1 represents a frame of reference(S), an object be situated at point P have co-ordinate $x, y, z \; and \; t$ i.e. P(x, y, z, t), measured by an Observer(O). Where t be the time of measurement of the co-ordinate of an object. Observers in different frame of references may describe same event in different way.

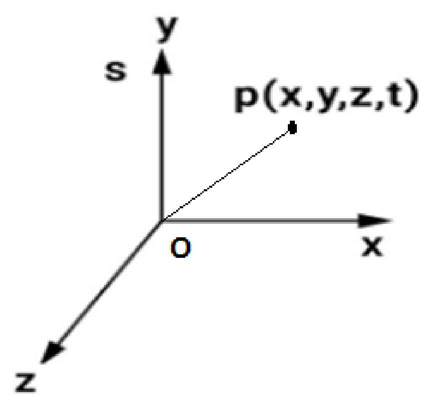


Fig 1.1: frame of reference

Example- takes a point on the rim of a moving wheel of cycle. for an observer sitting at the center of the wheel, the path of the point will be a circle. However, For an observer standing on the ground the path of the point will appear as cycloid.

There are two types of frames of reference.

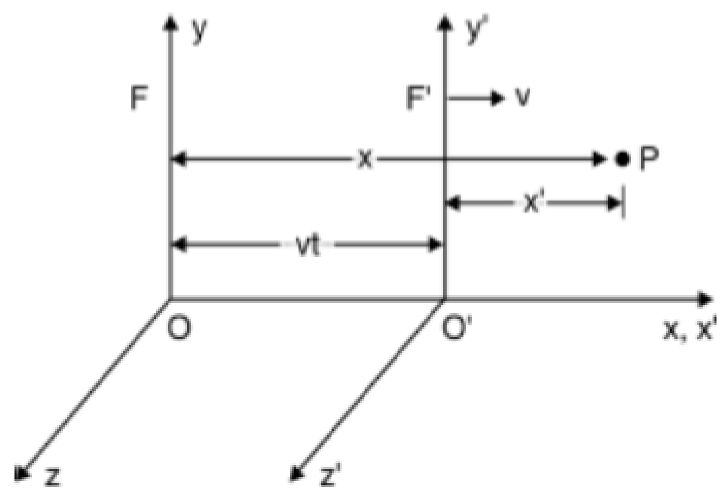
1. Inertial or non-accelerating frames of reference
2. Non-inertial or accelerating frames of reference

The frame of reference in which Newton's laws of motion hold good is treated as **inertial frame of reference**. However, the frame of reference in which Newton's laws of motion not hold good is treated as **non-inertial frame of reference**. Earth is non-inertial frame of reference, because it has acceleration due to spin motion about its axis and orbital motion around the sun.

1.3 Galilean Transformations

The Galilean transformations equations are used to transform the coordinates of position and time from one inertial frame to the other. The equations relating the coordinates of a particle in two inertial frames are called as Galilean transformations. Consider the two inertial frames of reference F and F'. Let the frame F' is moving with constant velocity v with reference to frame F. The frames F and F' are shown in Fig.1

Let some event occurs at the point P at any instant of time t. The coordinates of point P with respect to frame F are x, y, z, t and with respect to frame F' are x', y', z', t'. Let at t = t'= 0, the origin O of frame F and O' of frame F' coincides with one another. Also axes x and x' are parallel to v. Let y' and z' are parallel to y and z respectively.



From Fig.

$x=x'+ vt$……………………………...(1)

$x'= x - vt$……………………………...(2)

As there is no relative motion along y and z-axes, we can writ

$y'=y$….…(3),

$z'=z$….…(4), and $t'=t$….…(5)

These equations are called as Galilean transformation equations. The inverse Galilean transformation can be written as,

$x=x' + vt$, $y=y'$, $z'=z$ and $t=t'$

Hence transformation in position is variant only along the direction of motion of the frame and remaining dimensions(y and z) are unchanged under Galilean Transformation. At that era scientist were assumed time should be absolute.

b) Transformation in velocities components:

The conversion of velocity components measured in frame F into their equivalent components in the frame F' can be known by differential Equation (1) with respect to time we get,

$$u'_x = \frac{dx'}{dt} = \frac{d}{dt}(x - vt) = \frac{dx}{dt} - v$$

hence $u'_x = u_x - V$

Similarly, from Equation (3) and (4) we can write

$$u'_y = u_y \text{ and } u'_z = u_z$$

In vector form,

$$\mathbf{u'} = \mathbf{u} - \mathbf{v}$$

Hence transformation in velocity is variant only along the direction of motion of the frame and remaining dimensions(along y and z) are unchanged under Galilean Transformation.

c) Transformation in acceleration components:

The acceleration components can be derived by differentiating velocity equations with respect to time,

$$a'_x = \frac{du'_x}{dt} = \frac{d}{dt}(u_x - v)$$

$$a'_x = a_x$$

In vector form $\boxed{\mathbf{a' = a}}$

This shows that in all inertial reference frames a body will be observed to have the same acceleration. Hence acceleration are invariant under Galilean Transformation.

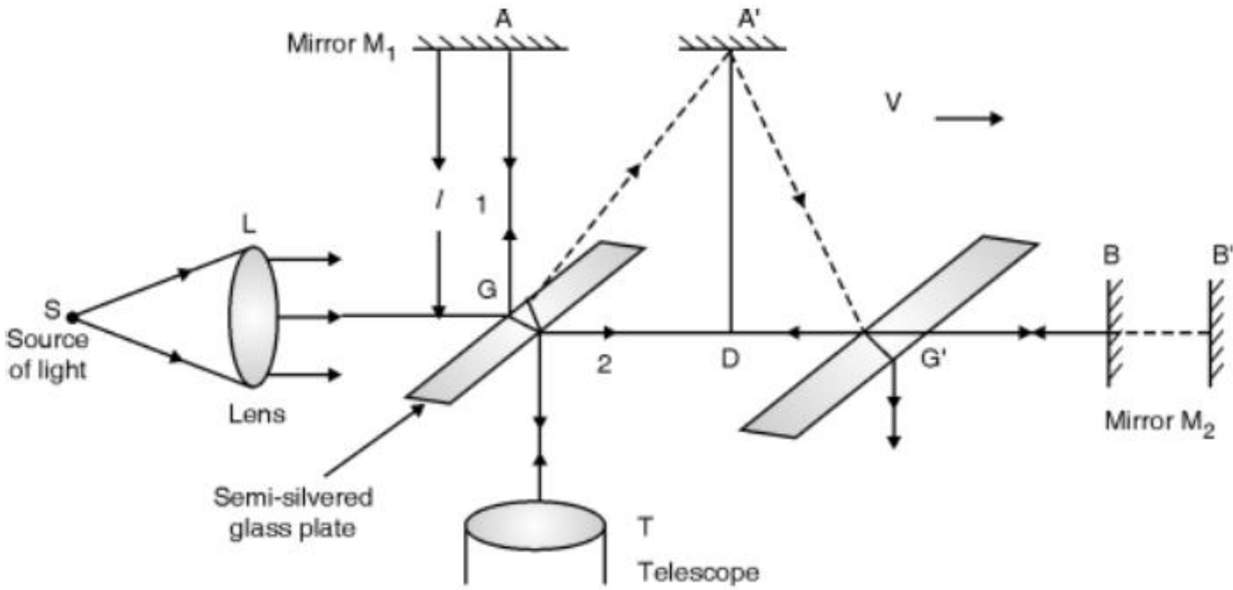
Failures of Galilean transformation:

- According to Galilean transformations the laws of mechanics are invariant. But under Galilean transformations, the fundamental equations of electricity and magnetism have very different forms.
- Also if we measure the speed of light c along x-direction in the frame F and then in the frame F' the value comes to be $\boxed{c' = c - v_x}$. But according to special theory of relativity the speed of light c is same in all inertial frames.

1.4 Michelson-Morley Experiment

"The objective of Michelson - Morley experiment was to detect the existence of stationary medium ether (stationary frame of reference i.e. ether frame.)", which was assumed to be required for the propagation of the light in the space.

In order to detect the change in velocity of light due to relative motion between earth and hypothetical medium ether, Michelson and Morley performed an experiment which is discussed below. The experimental arrangement is shown in Fig



Light from a monochromatic source S, falls on the semi-silvered glass plate G inclined at an angle 45° to the beam. It is divided into two parts by the semi silvered surface, one ray 1 which travels towards mirror M_1 and other is transmitted, ray 2 towards mirror M_2. These two rays fall normally on mirrors M_1 and M_2 respectively and are reflected back along their original paths and meet at point G and enter in telescope. In telescope interference pattern is obtained.

If the apparatus is at rest in ether, the two reflected rays would take equal time to return the glass plate G. But actually the whole apparatus is moving along with the earth with a velocity say v. Due to motion of earth the optical path traversed by both the rays are not the same. Thus the time taken by the two rays to travel to the mirrors and back to G will be different in this case.

Let the mirrors M_1 and M_2 are at equal distance l from the glass plate G. Further let c and v be the velocities at light and apparatus or earth respectively. It is clear from Fig. that the reflected ray 1 from glass plate G strikes the mirror M_1at A' and not at A due to the motion of the earth.

The total path of the ray from G to A' and back will be GA'G'.

$\therefore$ From Δ GA'D $\qquad (GA')^2=(AA')^2+(A'D)^2$...(1) $\qquad$ As (GD =AA')

If t be the time taken by the ray to move from G to A', then from Equation (1), we have

$$(c\,t)^2=(v\,t)^2+(l)^2$$

Hence $\boxed{t=\frac{l}{\sqrt{c^2-v^2}}}$

If t_1 be the time taken by the ray to travel the whole path GA'G', then

$$t_1 = 2t = \frac{2l}{\sqrt{c^2-v^2}} = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$t_1 = \frac{2l}{c}\left(\left(1+\frac{v^2}{2c^2}\right)\right) \text{ ....(2) Using Binomial Theorem}$$

Now, in case of transmitted ray 2 which is moving longitudinally towards mirror M_2. It has a velocity (c – v) relative to the apparatus when it is moving from G to B. During its return journey, its velocity relative to apparatus is (c + v). If t_2 be the total time taken by the longitudinal ray to reach G', then

$$T_{2=} \frac{l}{(c-v)} + \frac{l}{(c+v)} \text{ after solving}$$

$$t_2 = \frac{2l}{c}\left(\left(1+\frac{v^2}{c^2}\right)\right) \text{ ....(3)}$$

Thus, the difference in times of travel of longitudinal and transverse journeys is

$$\Delta t = t_2 - t_1 = = \frac{2l}{c}\left(\left(1+\frac{v^2}{c^2}\right)\right) - \frac{2l}{c}\left(\left(1+\frac{v^2}{2c^2}\right)\right)$$

$$\Delta t = \frac{lv^2}{c^3} \quad \text{..................(4)}$$

The optical path difference between two rays is given as,

Optical path difference (Δ) =Velocity × t = c x Δt

$$= c \text{ x } = \frac{lv^2}{c^3}$$

$$\Delta = \frac{lv^2}{c^2} \quad \text{..................(5)}$$

If λ is the wavelength of light used, then path difference in terms of wavelength is, $= \frac{lv^2}{\lambda c^2}$

Michelson-Morley perform the experiment in two steps. First by setting as shown in fig and secondly by turning the apparatus through 90^0. Now the path difference is in opposite direction i.e. path difference is $-\frac{lv^2}{\lambda c^2}$.

Hence total fringe shift $\boxed{\Delta N = \frac{\mathbf{2lv^2}}{\boldsymbol{\lambda c^2}}}$

Michelson and Morley using l=11 m, λ= 5800 x 10^{-10} m, v= 3 x 10^4 m/sec and c= 3 x 10^8 m/sec

$\therefore$ Change in fringe shift $\Delta N = \frac{2lv^2}{\lambda c^2}$ substitute all these values

=0.37 fringe

But the experimental were detecting no fringe shift. So there was some problem in theory calculation and is a negative result. The conclusion drawn from the Michelson-Morley experiment is that, there is no existence of stationary medium ether in space.

Negative results of Michelson - Morley experiment

1. **Ether drag hypothesis:** In Michelson - Morley experiment it is explained that there is no relative motion between the ether and earth. Whereas the moving earth drags ether alone with its motion so the relative velocity of ether and earth will be zero.
2. **Lorentz-Fitzgerald Hypothesis:** Lorentz told that the length of the arm (distance between the pale and the mirror M_2) towards the transmitted side should be $L(\sqrt{1 - v^2/c^2})$ but not L. If this is taken then theory and experimental will get matched. But this hypothesis is discarded as there was no proof for this.
3. **Constancy of Velocity of light:** In Michelson - Morley experiment the null shift in fringes was observed. According to Einstein the velocity of light is constant it is independent of frame of reference, source and observer.

Einstein special theory of Relativity (STR)

Einstein gave his special theory of relativity (STR) on the basis of M-M experiment

Einstein's First Postulate of theory of relativity:

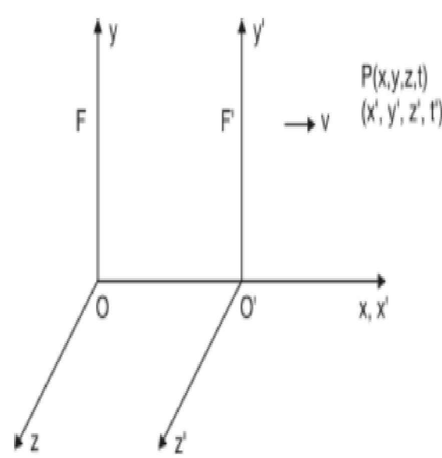
All the laws of physics are same (or have the same form) in all the inertial frames of reference moving with uniform velocity with respect to each other. (This postulate is also called the **law of equivalence**).

Einstein's second Postulate of theory of relativity:

The speed of light is constant in free space or in vacuum in all the inertial frames of reference moving with uniform velocity with respect to each other. (This postulate is also called the **law of constancy**).

1.5 Lorentz Transformation Equations

Consider the two observers O and O' at the origin of the inertial frame of reference F and F' respectively as shown in Fig. Let at time t = t' = 0, the two coordinate systems coincide initially. Let a pulse of light is flashed at time t = 0 from the origin which spreads out in the space and at the same time the frame F' starts moving with constant velocity v along positive X-direction relative to the frame F. This pulse of light reaches at point P, whose coordinates of position and time are (x, y, z, t) and (x',y', z', t') measured by the observer O and O' respectively. Therefore the transformation equations of x and x' can be given as,



$x' = k(x - vt)$…………………………....(1)

Where k is the proportionality constant and is independent of x and t.

The inverse relation can be given as,

x=k (x' + v t') ………………… (2)

As t and t' are not equal, substitute the value of x' from Equation (1) in Equation (2)

x=k [k (x – vt) + vt'] or $\frac{x}{k}=(kx-kvt+vt')$

$t'=\frac{x}{kv}-\frac{kx}{v}+kt$ or $t'=kt-\frac{kx}{v}\left(1-\frac{1}{k^2}\right)$ ………………………....(3)

According to second postulate of special theory of relativity the speed of light c remains constant. Therefore the velocity of pulse of light which spreads out from the common origin observed by observer O and O' should be same.

∴ x=c t and x' = ct'……………………………………...(4)

Substitute the values of x and x' from Equation (4) in Equation (1) and (2) we get

ct'=k (x – v t) = k (ct – v t) or ct'=kt (c – v) ....(5)

and similarly ct=k t' (c + v) ....(6)

Multiplying Equation (5) and (6) we get,

c^2 t t'=k^2t t' ($c^2 - v^2$) hence

after solving $\boxed{k=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$...............................(7)

Hence equation (7) substitute in equation (1), then Lorentz transformation in position will be

$$\boxed{x'=\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad y=y', \quad z'=z}$$

Calculation of Time: equation (7) substitute in equation (3),

$$t'=kt-\frac{kx}{v}\left(1-\frac{1}{k^2}\right)$$

From equation (7), $\frac{1}{k^2}=1-\frac{v^2}{c^2}$

or, $\frac{v^2}{c^2}=1-\frac{1}{k^2}$

then above equation becomes

$$t'=kt-\frac{kx}{v}\frac{v^2}{c^2}=kt-\frac{kxv}{c^2}$$

or, $$t'=k\left(t-\frac{xv}{c^2}\right)$$

Therefore $\boxed{t'=\frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$

Hence the Lorentz transformation equations becomes,

$$\boxed{x'=\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}},\quad y=y',\quad z'=z \text{ and } t'=\frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

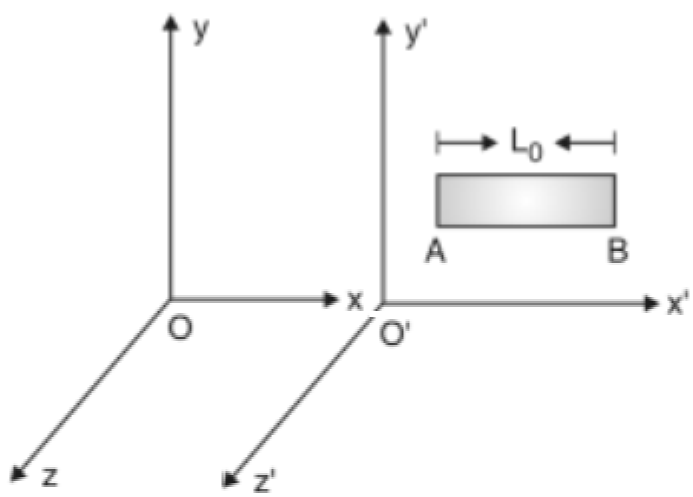
Under the condition **v << c** Lorentz transformation equation can be converted in to Galilean Transformation

$$\boxed{x'=x-vt,\quad y'=y,\quad z'=z \quad \text{and} \quad t'=t}$$

1.6 Applications of Lorentz Transformation

1.6.1 Length contraction

Consider a rod at rest in a moving frame of reference F' moving along x-direction with constant velocity v, relative to the fixed frame of reference F as shown in Fig.



The observer in the frame F' measures the length of rod AB at any instant of time t. This length L_o measured in the system in which the rod is at rest is called proper length, therefore L_o is given as,

$L_o = x_2' - x_1'$ ……………………………(1)

Where x_1' and x_2' are the coordinates of the two ends of the rod at any instant. At the same time, the length of the rod is measured by an observer O in his frame say L , then

$L = x_2 - x_1$ ……………………………..(2)

Where x_1 and x_2 are the coordinates of the rod AB respectively with respect to the frame F. According to Lorentz transformation equation

$$x_1'=\frac{x1-vt}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } x_2'=\frac{x2-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

Hence $$x_2'-x_1'=\frac{x_2-x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

By using Equations (1) and (2) we can write

$$L_o=\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

Hence $\boxed{L=L_o\sqrt{1-\frac{v^2}{c^2}}}$ ………………….(3)

From this equation $L << Lo$. Thus the length of the rod is contracted by a factor $\sqrt{1-\frac{v^2}{c^2}}$ as measured by observer in stationary frame F.

Special Case:

If v <<< c, then v^2/c^2 will be negligible in $L_o\sqrt{1-\frac{v^2}{c^2}}$ and it can be neglected

Then equation (3) becomes $L = L_O$.

Percentage of length contraction$=\frac{L_o-L}{L_o}\times 100$

1.6.2 Time dilation

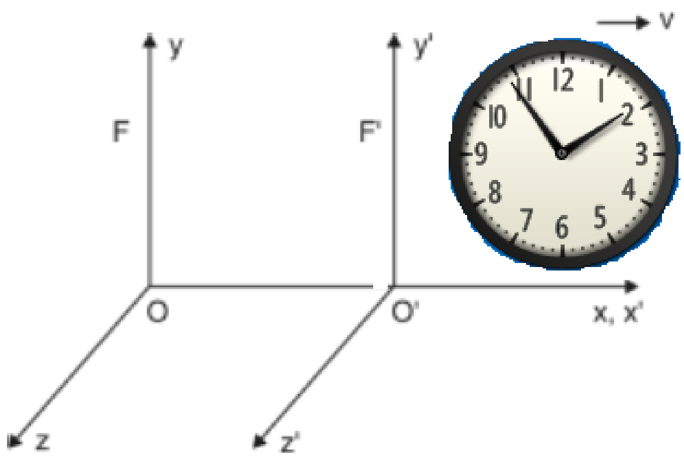
Let there are two inertial frames of references F and F'. F is the stationary frame of reference and F' is the moving frame of reference. At time t=t'=0 that is in the start, they are at the same position that is Observers O and O' coincides. After that F' frame starts moving with a uniform velocity v along x axis.

Let a clock is placed in the frame F'. The time coordinate of the initial time of the clock will be t_1 according to the observer in S and the time coordinate of the final tick (time) will be will be t_2 according to same observer.

The time coordinate of the initial time of the clock will be t'_1 according to the observer in F' and the time coordinate of the final tick (time) will be will be t'_2 according to same observer.

Therefore the time of the object as seen by observer O' in F' at the position x' will be

$t_o = t'_2 - t'_1$ ……………………………….(1)



The time t' is called the proper time of the event. The apparent or dilated time of the same event from frame S at the same position x will be

$t = t_2 - t_1$ ……………………………….(2)

Now use Lorentz inverse transformation equations for, that is

$$t_1 = \frac{t'_1+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{……………………} \; (3)$$

$$t_2 = \frac{t'_2+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{……………………} \; (3)$$

By putting equations (3) and (4) in equation (2) and solving, we get

$t = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$

Substitute equation (1) in above equation,

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

This is the relation of the time dilation.

Special Case:

If v <<< c, then v^2/c^2 will be negligible in $\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and it can be neglected

Then $t = t_o$

Experimental evidence: The time dilation is real effect can be verified by the following experiment. In 1971 NASA conducted one experiment in which J.C. Hafele, as astronomer and R.F. Keating, a physicist circled the earth twice in a jet plane, once from east to west for two days and then from west to east for two days carrying two cesium-beam atomic clocks capable of measuring time to a nanosecond. After the trip the clocks were compared with identical clocks. The clocks on the plane lost 59 ± 10 ns during their eastward trip and gained 273 ± 7 ns during the westward trip.This results shows that time dilation is real effect.

1.6.3 Relativistic Addition of Velocities

One of the consequences of the Lorentz transformation equations is the counter-intuitive "velocity addition theorem". Consider an inertial frame S' moving with uniform velocity v relative to stationary observer S along the positive direction of X- axis. Suppose a particle is also moving along the positive direction of X-axis. If the particle moves through a distance dx in time interval dt in frame S, then velocity of the particle as measured by an observer in this frame is given by

$$u = \frac{dx}{dt} \qquad (1)$$

To an observer in S' frame, let the velocity be (by definition)

$$u' = \frac{dx'}{dt'} \qquad (2)$$

Now, we have the Lorentz transformation equations:

$$x' = \frac{x-vt}{\sqrt{(1-\frac{v^2}{c^2})}} \quad \text{and} \quad t' = \frac{t-\frac{vx}{c^2}}{\sqrt{(1-\frac{v^2}{c^2})}} \qquad (3)$$

Taking differentials of above equations, we get

$$dx' = \frac{dx - vdt}{dt - vdx/c^2} \qquad \text{and} \qquad dt' = \frac{dt - vdx/c^2}{\sqrt{(1 - v^2/c^2)}} \qquad (4)$$

Using eq (4) in eq.(2), we get

$$u' = \frac{dx - vdt}{dt - vdx/c^2} = \frac{\left(\frac{dx}{dt}\right) - v}{(1 - vdx/c^2 dt)} \qquad (5)$$

Or,

$$\boldsymbol{u'} = \frac{\boldsymbol{u - v}}{(\boldsymbol{1 - uv/c^2})} \qquad (6)$$

This is the the relativistic velocity addition formula. If the speeds u and v are small compared to the speed of light, above formula reduces to Newtonian velocity addition formula

$$u' = u - v$$

Inverse of the formula (6) enables us to find velocity of a particle in *S* frame if it is given in S' frame:

$$u = \frac{u' + v}{1 + u'v/c^2} \qquad (6)$$

1.7 Variation of Mass with Velocity

In Newtonian physics mass of an object used to be an absolute entity, same in all frames. One of the major unusual consequences of relativity was relativity of mass. In the framework of relativity, it can be shown that mass of an object increases with its velocity. We have the following equation expressing the variation of mass with velocity:

$$m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}} \qquad (1)$$

Where, m_0 is the mass of the object at rest, known as *rest mass* and v is its velocity relative to observer, As it is clear from above equation, if $v \ll c,\ m_0 \cong m$, in agreement with common experience.

Derivation:

We use relativistic law of momentum conservation to arrive at eq (1). Consider two inertial frames S and S', S' moving with respect to S with velocity v along positive X-axis. Let two masses m_1 and m_2 are moving with velocities u' and –u' with respect to moving frame S'.

Now, let us analyse the collision between two bodies with respect to frame S. If u_1 and u_2 are velocities of two masses with respect to frame S, then from velocity addition theorem,

$$u_1 = \frac{u' + v}{1 + u'v/c^2} \qquad \text{and} \qquad u_2 = \frac{-u' + v}{1 - u'v/c^2} \qquad (2)$$

At the time of collision two masses are momentarily at rest relative to frame S', but as seen from frame S they are still moving with velocity v. Since we assume momentum to be conserved even in relativity theory, as seen from S frame,

Momentum before collision = momentum after collision

$$m_1 \ u_1 + \ m_2 u_2 = (m_1 + \ m_2)v \qquad (3)$$

Substituting the values of u_1 and u_2 from equation (2), eq.(3) becomes

$$m_1\left(\frac{u^{'}+v}{1+u^{'}v/c^2}\right) \ + \ m_2\left(\frac{-u^{'}+v}{1-u^{'}v/c^2}\right) = (m_1 + \ m_2)v$$

Rearranging the terms,

$$m_1\left(\frac{u^{'}+v}{1+\frac{u^{'}v}{c^2}} - v\right) \ = \ m_2\left(v - \frac{-u^{'}+v}{1-u^{'}v/c^2}\right)$$

$$m_1\left(\frac{u^{'}+v-v-(u'v^2/c^2)}{1+\frac{u^{'}v}{c^2}}\right) \ = \ m_2\left(\frac{v-\frac{(-u^{'}v^2)}{c^2}+u^{'}-v}{1-u^{'}v/c^2}\right)$$

Or, $$m_1\left(\frac{u^{'}-(u'v^2/c^2)}{1+\frac{u^{'}v}{c^2}}\right) \ = \ m_2\left(\frac{u^{'}-\frac{(u^{'}v^2)}{c^2}}{1-u^{'}v/c^2}\right)$$

$$\frac{m_1}{m_2} = \frac{1+u^{'}v/c^2}{1-u^{'}v/c^2} \qquad (4)$$

Now , using set of equations (2), we can find RHS of equation to be equal to

$$\frac{1+u^{'}v/c^2}{1-u^{'}v/c^2} = \sqrt{\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}}} = \frac{m_1}{m_2}$$

If $u_2 = 0$, i.e. m_2(= m_0, say) is at rest with respect to S frame , above equation reduces to

$$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

We could have chosen two masses to be identical. In that case m_0 will also be the rest mass of m_1 .So we can apply above formula to a single with rest mass m_0 and moving mass m , related by

$$\boldsymbol{m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}}$$

This shows that mass of a body increases with its velocity.

1.8 Mass- Energy Equivalence

In Newtonian Physics, mass and energy are assumed to be quite different entities. There is no mechanism in Newtonian set up, how mass and energy can be converted into each other. Like many other, it is one of unusual consequences of special relativity that mass and energy are inter-convertible into each other and hence are equivalent.

Einstein's Mass- energy relation ($\mathbf{E = mc^2}$) (Derivation)

Consider a particle of mass m acted upon by a force ***F*** in the same direction as its velocity *v*. If F displaces the particle through distance ds, then work done dW is stored as kinetic energy of the particle dK, therefore

$$dW = dK = F.ds \qquad (1)$$

But from Newton's law,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} \qquad (2)$$

Using (2) in (1)

$$dK = m\frac{dv}{dt}ds + v\frac{dm}{dt}ds = m\frac{ds}{dt}dv + v\frac{ds}{dt}\,dm$$

Or,

$$dK = mvdv + v^2dm \qquad (3)$$

Now we have,

$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} = m_0(1-\frac{v^2}{c^2})^{-1/2} \qquad (4)$$

Taking the differential ,

$$dm = m_0(-1/2)(1-\frac{v^2}{c^2})^{-3/2}\left(\frac{-2vdv}{c^2}\right) = \frac{m_0vdv}{c^2(1-\frac{v^2}{c^2})^{3/2}}\ v$$

But $m_0 = m(1-v^2/c^2)^{1/2}$

$$\mathrm{dm} = \frac{\mathrm{m}(1-\mathrm{v}^2/\mathrm{c}^2)^{1/2}\mathrm{vdv}}{\mathrm{c}^2(1-\frac{\mathrm{v}^2}{\mathrm{c}^2})^{3/2}}$$

Or $dm = \frac{mvdv}{(c^2-v^2)}$

or,

$$mvdv = (c^2-v^2)dm \qquad (5)$$

Using eq (5) in eq (3)

$$dK = (c^2 - v^2)dm + v^2 dm = c^2 dm$$

Let the change in kinetic energy of the particle be K, as its mass changes from rest mass m_0 to to effective mass m, then

$$K = \int_0^K dK = \int_{m_0}^{m} c^2\, dm = c^2(m - m_0) = c^2\left(\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} - m_0\right)$$

This is the relativistic expression for kinetic energy of a particle. It says that kinetic energy of a particle is due to the increase in mass of the particle on account of its relative motion and is equal to the product of the gain in mass and square of the velocity of light. $m_0\, c^2$ can be regarded as the rest energy of the particle of rest mass m_0. The total energy E of a moving particle is the sum of kinetic energy and its rest mass energy.

$$E = m_0 c^2 + (m - m_0)c^2 = mc^2$$

Or,

$$\boldsymbol{E = mc^2}$$

This is the celebrated mass- energy equivalence relation. This equation is so famous that even common men identify Einstein with it.

Significance: This equation represents that energy can neither be created nor be destroyed, but it can change its form.

Evidence show its validity:

First verification of the $E = mc2$ was made by Cock-croft and Walton

^7_3Li	$+$	^1_1H	$\rightarrow$	$2\,^4_2He$
(7.0166 amu)		(1.0076 amu)		(4.0026 amu)

Mass defect = (7.0166 amu+1.0076 amu) -2x (4.0028 amu)

=0.0186 amu

= 0.0186 x 1.66 X 10^{-27} kg

$\boldsymbol{E = mc^2}$= 0.0186 x 1.66 X 10^{-27} x $(3 \times 10^8)^2$

$\boldsymbol{E}$ $=0.278 \times 10^{-11}\, joule$

$\boldsymbol{E}=\frac{(0.278\times10^{-11})}{(1.66\times10^{-19})}\, eV$

$\boldsymbol{E}$=17.36 MeV

(a huge amount of energy release)

1.9 Relation between Energy and Momentum

a) Relation between total Energy and Momentum

Sconce we know that

$$E = mc^2 = \frac{m0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring both side

$E^2\left(1-\frac{v^2}{c^2}\right)$=$m_0^2c^4$

$E^2c^2 - E^2v^2$=$\boldsymbol{m_0^2}c^6$

Science $p = mv$, v=$\frac{p}{m}$ substitute in above equation

$E^2c^2 - E^2\left(\frac{p}{m}\right)^2$=$\boldsymbol{m_0^2}c^6$, after solving

$$E^2 = p^2c^2 + \boldsymbol{m_0^2}c^4$$

This is the relation between relativistic total energy and momentum of the particle.

b) Relation between Kinetic Energy and Momentum

Science we know that $E^2 = p^2c^2 + \boldsymbol{m_0^2}c^4$ ------------------------(1)

Also we know that $E = E_0 + E_K$

$$E = m_0c^2 + E_K$$

Squaring both side

$E^2 = (mc^2 + E_K)^2$ -----------------------(2)

From equation (1) and (2), we get

$p^2c^2 + \boldsymbol{m_0^2}c^4$= $(m_0c^2 + E_K)^2$

$p^2c^2 + \boldsymbol{m_0^2}c^4$=$m_0{}^2c^4 + E_k^2 + 2m_0c^2E_K$

After solving

$p = \sqrt{\frac{E_k^2}{c^2} + 2m_0 E_K}$

Where E_K = kinetic energy

1.10 Mass-less Particle /Zero Rest Mass Particles:

A particle which has zero rest mass($m0$) is called massless particle

We have the energy-momentum relation,

$$E = \sqrt{m_0^2 c^4 + (p^2 c^2)} \qquad (1)$$

For a massless particle, $m_0 = 0$

Therefore, $E = pc$ or $p = E/c$ (2)

But $p = mv$, therefore

$$\frac{Ev}{c^2} = \frac{E}{c}$$

Which implies $v = c$

This says that a massless particles always move with the speed of light. Energy and momentum of a massless particle is given by equation (2). Massless particles can exist only as long as the move at the speed of light. Examples are photon, neutrinos and theoretically predicted gravitons.

Numerical Problems

Question 1: Show that the distance between any two points in two inertial frame is invariant under Galilean transformation.

Answer: Consider two frame of reference S and S', frame S' moving with constant velocity v, relative to frame S in positive X-direction .

Point A and B placed in frame S and their co-ordinate relative O are (x_1, y_1, z_1) and (x_2, y_2, z_2).

After time t the coordinate of A and B relative to observer O' from frame S' will be (x'_1, y'_1, z'_1) and(x'_2, y'_2, z'_2)

Using Galilean Transformation

$x'_1 = x_1 - vt \;\; y'_1 = y_1$ and $z'_1 = z_1$

$x'_2 = x_2 - vt \;\; y'_2 = y_2$ and $z'_2 = z_2$

Now distance between two points from frame S' will be

$=\sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}$

Substitute all values then

$=\sqrt{(x_2 - vt - x_1 + vt)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

After simplification

$=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ i.e

$S'=S$

This show that, distance between two points is invariant under Galilean Transformation

Question 2: Show that the relativistic form of Newton's second law, when $\vec{F}$ is parallel to $\vec{v}$ is $\vec{F}=m_0 \frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$

Answer: Science we know that

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{dmv}{dt}$$

$\vec{F}=\frac{d}{dt}\left(\frac{m0\ v}{\sqrt{1-\frac{v^2}{c^2}}}\right)=\left[m_0\left(v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right)\right]$

$=m_0\left[\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} + \left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(0-\frac{2vdv}{c^2\,dt}\right)\right]$

$=m_0\frac{dv}{dt}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} + \frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\right]$

$\vec{F}=m_0\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[1+\frac{v^2}{c^2}-\frac{v^2}{c^2}\right]$

$\vec{F}=m_0\frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$, it is relativistic Newton's second law.

If $v \ll c$, then

$\vec{F}$=$m_0 \frac{dv}{dt}$, $hence\ \vec{F}$= $m_0\ \vec{a}$ (because at $\ll c$, $m \approx m_0$)

So it can also be written as $\vec{F}$= $m\ \vec{a}$

Question 3: **A particle of rest mass $\mathbf{m_0}$ moves with speed $\frac{\mathbf{c}}{\sqrt{2}}$. Calculate its mass, momentum, total energy and kinetic energy.**

Answer: Science we know that

$$\mathrm{m} = \frac{\mathrm{m_0}}{\sqrt{1-\frac{\mathrm{v^2}}{\mathrm{c^2}}}} = \frac{\mathrm{m_0}}{\sqrt{1-\frac{\mathrm{c^2}}{2\mathrm{c^2}}}} = \sqrt{2}\ \mathrm{m_0} = 1.41\mathrm{m_0}$$

The momentum of the particle is given by

$$\mathrm{p} = \mathrm{mv} = 1.41\mathrm{m_0} \times \frac{\mathrm{c}}{\sqrt{2}} = \mathrm{m_0\ c}$$

Total energy of the particle

$$\mathrm{E} = \mathrm{mc^2} = 1.41\mathrm{m_0} \times \mathrm{c^2} = 1.41\mathrm{m_0c^2}$$

Kinetic energy

$$\mathrm{EK} = \mathrm{E} - \mathrm{m0c2} = 1.41\mathrm{m_0c^2} - \mathrm{m_0c^2} = 0.41\mathrm{m_0c^2}$$

Question 4: **A man weight 50 kg on the earth, when he is in a rocket ship in flight his mass is 50.5 kg ass measured by an observer on earth. What is the speed of rocket.**

Answer: Given that

m_0=50 kg , $m = 50.5\ kg\ and\ c = 3 \times 108 m/s$

$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ substitute all values and after solving

$v = 4.23 \times 10^7$m/s

Question 5 : **Compute the mass m and speed v of an electron having kinetic energy $\mathbf{1.5}\ \boldsymbol{MeV}$. Given rest mass of electron $\boldsymbol{m_0} = \mathbf{9.1 \times 10^{-31}}\boldsymbol{kg},\ \boldsymbol{c} = \mathbf{3 \times 108}\ \boldsymbol{m/s}$.**

Answer: Given that

$$\mathrm{Ek} = 1.5\ \mathrm{MeV} = 1.5 \times 10^6 \times 1.6 \times 10^{-19}\mathrm{joule}\ , \mathrm{m_0} = 9.1 \times 10^{-31}\mathrm{kg}$$

$$\text{and } \mathrm{c} = 3 \times 108\ \mathrm{m/s}$$

Kinetic energy$(E_k) = (m{-}m_0)c^2$

Kinetic energy$(E_k) = (m{-}m_0)c^2$

$m - m_0 = \frac{E_k}{c^2}$ *hence* $m = \frac{E_k}{c^2} + m_0$,substitute all values and after solving

m= $3.58 \times 10^{-30}\mathrm{kg}$

Using the relation $m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ where m= $3.58 \times 10^{-30}\mathrm{kg}$ *and* $\mathrm{m_0} = 9.1 \times 10^{-31}\mathrm{kg}$

Then after solving $v = \mathrm{c} = 2.9 \times 108\ \mathrm{m/s}$

Question 6: . Calculate the amount of work to be done to increase the speed of an electron from 0.6c to 0.8c. Given that rest mass energy of electron (E_0) $= 0.50\ MeV$

Answer:

Kinetic energy$(E_k) = (m{-}m_0)c^2$=$\left(\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} - 1\right)m_0c^2$

Where, $E_0 = m_0c^2 = 0.50MeV$

For speed 0.6c,

E_{k1}=$\left(\frac{1}{\sqrt{\left(1-\frac{(0.6c)^2}{c^2}\right)}} - 1\right) \times 0.50\ Mev$

=1.25 MeV

For speed 0.8c,

E_{k2}=$\left(\frac{1}{\sqrt{\left(1-\frac{(0.8c)^2}{c^2}\right)}} - 1\right) \times 0.50\ Mev$

=3.35 MeV

Hence, The amount of work to be done(electron increase the speed 0.6c to 0.8c)=$E_{k2} - E_{k1}$

= 2.1Mev

5.1 Introduction to fiber optics

An optical fiber is a hair thin cylindrical fiber made of glass or any transparent dielectric material. It is used for optical communication as a waveguide. It transmits signals in form of light. The optical fiber optical communication system is shown in Fig 5.1.

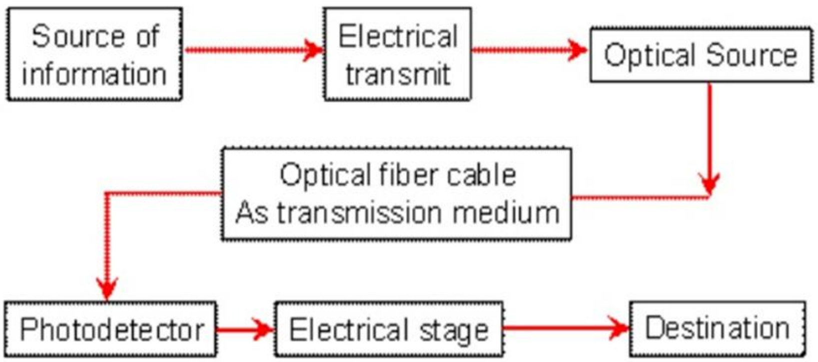


Fig.5.1 optical fiber optical communication system

Optical fiber is backbone of communication system it carry a signal with a speed up to 1Tbit/sec or 100 million conversation simultaneously.

5.1.1 Structure of optical fiber

Core- Central tube of very thin size made up of optically transparent dielectric medium and carries the light form transmitter to receiver.

Cladding- Outer optical material surrounding the core having refractive index lower than core. It helps to keep the light within the core throughout the phenomena of total internal reflection.

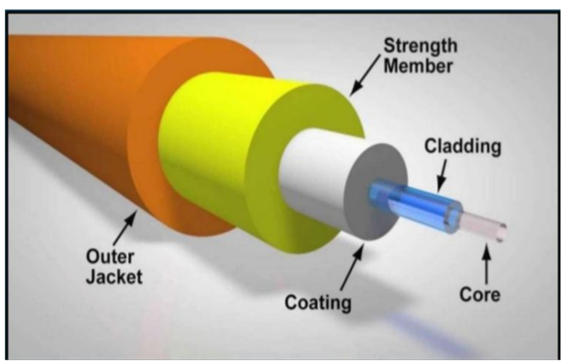


Fig. 5.1 Structure of optical fiber

Jacket- Plastic coating that protects the fiber from external interference is made of silicon rubber .

5.1.2 Propagation Mechanism and communication in optical fiber:

Optical fiber is a wave guide used for optical communication. It is made of transparent dielectric materials whose function is to guide the light wave. An optical fiber consists of an inner cylindrical portion of glass, called core. The function of core is to carry the light from one end to another end by the principle of total **internal reflection**. When an ray of light travels from a denser to a rarer medium such that the angle of incidence is greater than the critical angle, the ray reflects back into the same medium this phenomena is called TIR. The core is surrounded by another cylindrical covering called cladding. The refractive index of core is greater than the refractive index of cladding. Cladding helps to keep the light within the core. In the optical fiber the rays undergo repeated total number of reflections until it emerges out of the other end of the fiber, even if fiber is bending. The propagation of light inside the optical fiber is shown in Fig.5.2

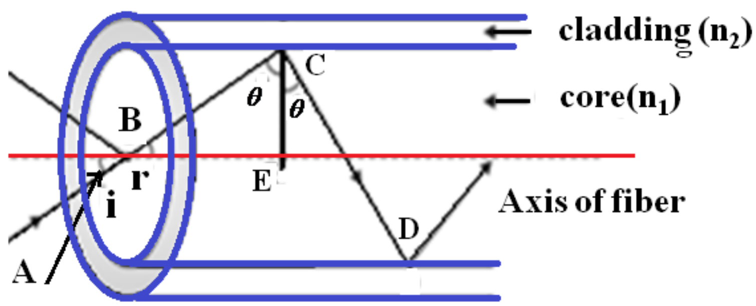


Fig. 5.2 Propagation Mechanism in optical fiber

Let i be the angle of incidence of the light ray with the axis and r the angle of refraction. If θ be the angle at which the ray is incident on the fiber boundary, then θ = (90 – r).

Let n_1 and n_2 be the refractive index of the core and clad of fiber. If $\theta \geq \theta_c$ critical angle then the ray is totally internally reflected where $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

5.2 Acceptance angle, acceptance cone and numerical aperture.

5.2.1 Acceptance angle

The acceptance angle is the maximum angle from the fiber axis at which light may enter the fiber so that it will propagate in the core by total internal reflection. If a ray is rotated around the fibre axis keeping i acceptance angle same, then it describes a conical surface called cone as shown in Fig.2. Now only those rays which are coming into the fiber within this cone having a full angle 2 i will only be totally internally reflected and this confined within the fiber for propagations. Therefore this cone is called as **acceptance cone.** Consider a cylindrical optical fiber wire which consists of inner core of refractive index n_1 and an outer cladding of refractive index n_2 where $n_1 > n_2$. The typical propagations of light in optical fiber are shown in figure 5.3.

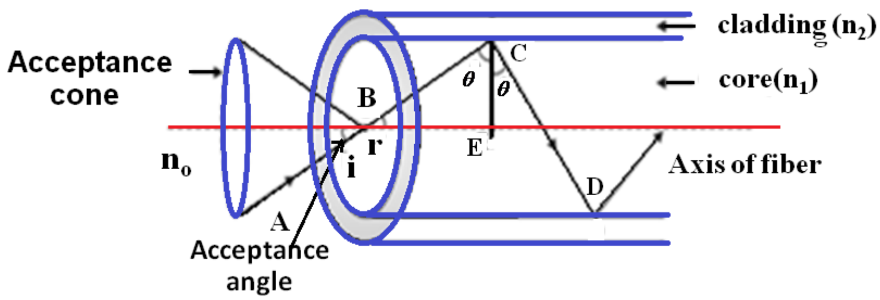


Fig. 5.3 Propagation Mechanism in optical fiber

Now we will calculate the angle of incidence i for which $\theta > \theta_c$ (critical angle) so that the light rebounds within the fiber. Applying Snell's law of refraction at entry point of the ray AB.

$$n_0 \sin i = n_1 \sin r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where n_0 is the refractive index of medium from which the light enters in the fiber. From triangle BCE,

$$r = (90 - \theta)$$

$$\therefore \quad \sin r = \sin(90 - \theta)$$

$$\sin r = \cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting the value of sin r from Equation (2) in Equation (1), We get,

$$n_0 \sin i = n_1 \cos\theta$$

$$\sin i = \frac{n_1}{n_0}\cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If i is increased beyond a limit, θ will drop below the critical value θ_c and the ray will escape from the side walls of the fiber. The largest value of i which is i_{max} occurs when $\theta = \theta_c$. Applying this condition in Equation (3),

$$\sin i_{max} = \frac{n_1}{n_0}\cos\theta_c \quad \ldots\ldots (4)$$

where $\cos\theta_c = \sin^{-1}\frac{n_2}{n_1}$

$$\cos\theta_c = \sqrt{1-\sin^2\theta_c} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

∴ Equation (4), we have

$$\sin i_{max} = \frac{n_1}{n_0}\frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin i_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \ldots\ldots (5)$$

Almost all the time the ray is launched from air medium, then $n_0 = 1$ and $i_{max} = i$

$\sin i = \sqrt{(n_1^2 - n_2^2)}$

Where i is called acceptance angle of the fiber.

∴ $i = \sin(\sqrt{n_1^2 - n_2^2})$ .Hence the acceptance angle is defined as the maximum angle from the fiber axis at which light may enter the fiber so that it will propagate in the core by total internal reflection.

Now the light contained within the cone having a full angle 2i are accepted and transmitted through fiber. The cone associated with the angle 2i is called acceptance cone as shown in Fig.5.2

5.2.2 Numerical aperture: Numerical aperture (NA) is a measure of the amount of light rays that can be accepted by the fiber and is more generally used term in optical fiber. Numerical aperture 'NA' determines the light gathering ability of the fiber. So it is a measure of the amount of light that can be accepted by the fiber. This is also defined as,

NA=sin i

$$\therefore\ NA = \sqrt{n_1^2 - n_2^2} \quad \ldots\ldots (6)$$

The NA may also be derived in terms of relative refractive index difference Δ as,

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta = 1 - \frac{n_2}{n_1}$$

Hence $\frac{n_2}{n_1} = 1 - \Delta$ Now From Equation (6)

$$\therefore \; NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

Now substitute the value of $\frac{n_2}{n_1}$ then after solving above equation will be

$$\therefore \; NA = n_1\sqrt{2\Delta}$$

Hence the numerical aperture is depends upon the relative refractive index

5.2.3 Parameter/V-Number:

The number of modes of multimode fiber cable depends on the wavelength of light, core diameter and material composition .This can be determined by the normalized frequency parameter (V) express as

$$V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{\pi d}{\lambda}(NA)$$

Where d is diameter of fiber core diameter and λ is wavelength of light and NA is numerical aperture. For a single mode fiber V less than 2.04 and for multimode it is greater than 2.04.Mathematically the numbers of modes are related to V-number

(a)For steps index optical fiber.

$$N_{si} = \frac{V^2}{2}$$

(b) Graded index optical fiber is

$$N_{di} = \frac{V^2}{4}$$

5.3 Classification of optical fiber:

Optical fiber is classified into two categories: based on:-

1) The number of modes-

- Single mode fiber(SMF) and
- Multi-mode fiber(MMF)

2) The refractive index-

- Step index optical fiber
- Graded- index optical fiber

5.3.1 Types of optical fiber on basis of number of modes

i) Single mode fiber-

In single mode fiber only one mode can propagate through the fiber shown in fig.5.4. It has small core diameter (5um) and cladding diameter is up to 70um. The difference in refractive index between the core and clad are very small. Because of this, the number of light reflections created as the light passes through the core decreases, lowering attenuation and creating the ability for the signal to travel further. Due to negligible dispersion it is typically used in long distance, higher bandwidth applications.

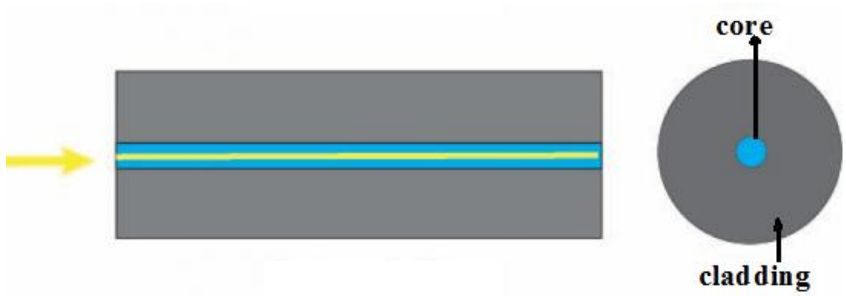


Fig. 5.4 Single mode optical fiber

(ii) Multi-mode fiber

It allows a large number of modes for light ray travelling through it. The **core** diameter is 40um and that of cladding is 70um. The relative refractive index difference is also large than single mode fiber shown in fig.5.5.

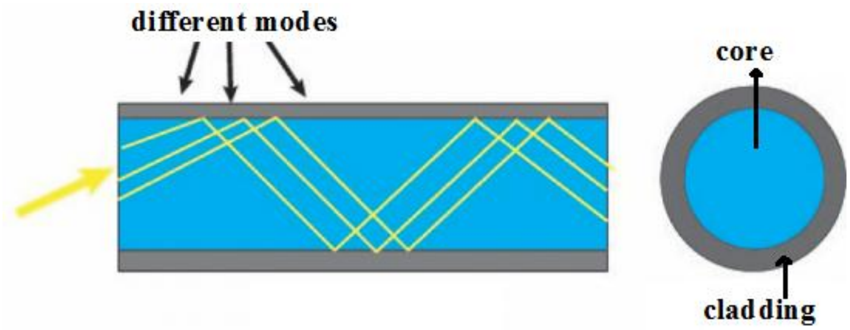


Fig. 5.5 Multi mode fiber

5.3.2 Types of optical fiber on the basis of refractive index

- There are two type of optical fiber:-

1) Step index optical fiber
   - Single Mode
   - Multi Mode
2) Graded- index optical fiber

i) Step Index fiber:- Step-index optical fiber-the refractive index of core and cladding are constant. These types of fibers have sharp boundaries between core and cladding with defined indices of refraction shown in fig.5.6 a. The entire core uses single index of refraction. The light ray propagates through it in the form of Zig Zag path .which cross the fiber axis during every reflection at the core cladding boundary.

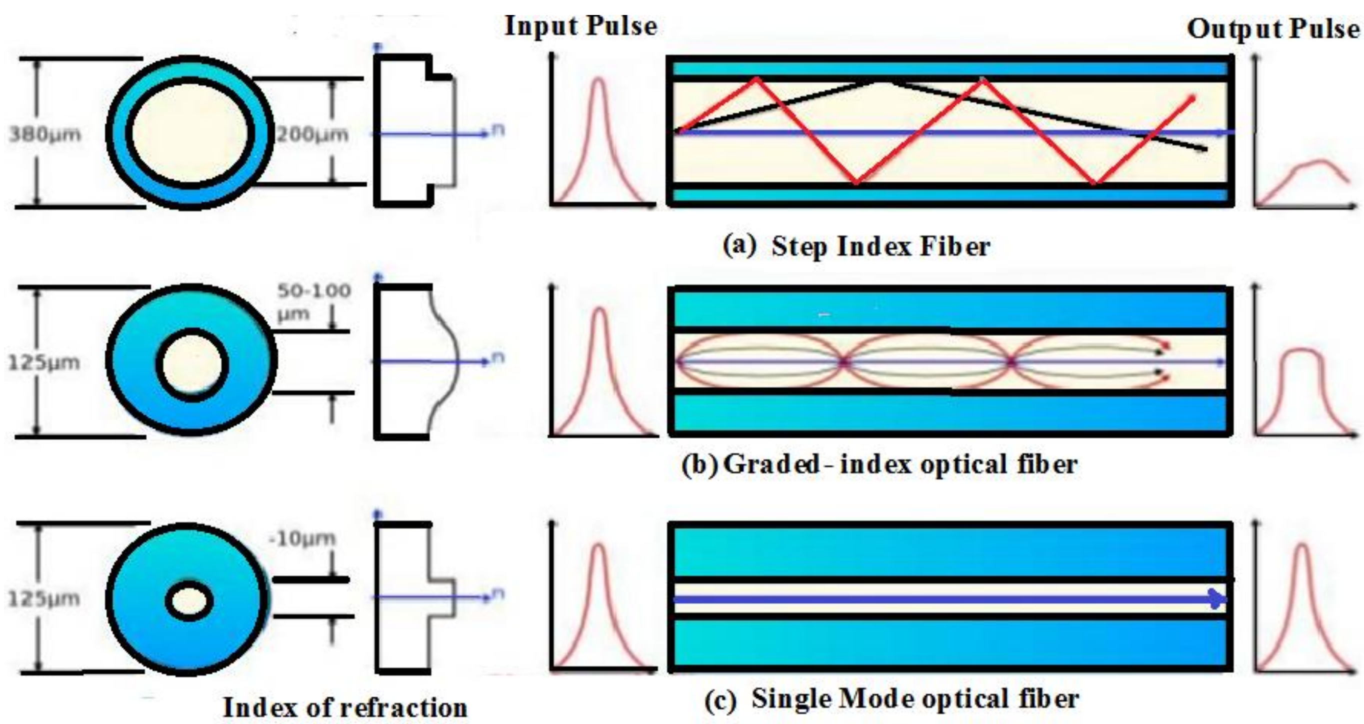


Fig. 5.6 (a)Multi mode step index fiber (b)graded index multimode optical fiber (c)single mode step index optical fiber.

ii) Multimode graded Index Fiber:

In this type of fiber core has a non uniform refractive index that gradually decrease from the center towards the core cladding interface. The cladding has a uniform refractive index. The light rays propagate through it in the form of helical rays. They never cross the fiber axis. In Graded-index fiber's refractive index decreases gradually away from its center, finally dropping to the same value as the cladding at the edge of the core the propagation mechanism are shown in Fig 5.6 b.

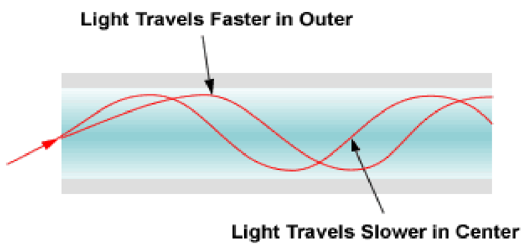


Fig. 5.6 Propagation in graded index multimode optical fiber

The rays in graded index light signals travels helical path instead of zigzag. The ray near to the core axis travels small distance with less velocity as compared to those who are far away from the core axis. The higher refractive index of core reduced the velocity of signal as compare to the other region away from the core. Therefore arrival timing at output end of different signals are almost equal. Hence the model dispersion minimizes.

5.4 Losses in optical fiber

The information carrying capacity of optical fiber are dependents upon the losses in the optical fiber A beam of light carrying signals travels through the core of fiber optics and the strength of the light will become lower leads to signal strength becomes weaker. This loss of light power is generally called fiber optic loss. It is basically comes in the form of

1) Attenuation

2) Dispersion

5.4.1 Attenuation

The reduction in amplitude of signal as it is guided through the fiber known as attenuation. It is expressed in decibel (dB)

$$dB = 10\log_{10}\frac{P_{out}}{P_{in}}$$

$$\alpha = -\frac{dB}{L} = -\frac{1}{L}10\log_{10}\frac{P_{out}}{P_{in}}$$

Where L=in Km

α=attenuation coefficients of fiber in dB/Km Since attenuation is loss,there for it is expressed as

$$P_{out} = P_{in}10\log^{\frac{-\alpha L}{10}}$$

Attenuation in optical fiber communication is due to following way

i) Absorption:

Absorption is the most prominent factor causing the attenuation in optical fiber. The absorption of light may be because of interaction of light with one or more major components of glass or caused by impurities within the glass. Following are the three main sources of absorption of light.

a)Impurities in fiber material:

from transition metal ions & particularly from OH ions with absorption peaks at wavelengths 2700 nm, 400 nm, 950 nm & 725nm.

b)Intrinsic absorption:

Electronic absorption band (UV region) & atomic bond vibration band (IR region) in basic SiO2.

Radiation defects

ii) Scattering Losses

The variation in material density, chemical composition, and structural inhomogeneity scatter light in other directions and absorb energy from guided optical wave. The random variation of molecular position of glass create the random inhomogenties of refractive index that act as a tiny center of scattering .The amplitude of scattering field is proportional to the square of frequency. The essential mechanism is the Rayleigh scattering.

iii) Bending Loss (Macro bending & Micro bending)

Bending losses occur due to imperfection and deformation present in the fiber structure. These are of two types

a)Micro bending: If microscopic bends of the fiber axis that can arise when the fibers are incorporated into cables. The power is dissipated through the micro bended fiber. There is repetitive coupling of energy between guided modes & the leaky or radiation modes in the fiber.

b)Macrobending Loss: These types of loss are due to excessive bending and crushing of fiber. The curvature of the bend is much larger than fiber diameter. Light wave suffers sever loss due to radiation of the evanescent field in the cladding region. As the radius of the curvature decreases, the loss increases exponentially until it reaches at a certain critical radius. For any radius a bit smaller than this point, the losses suddenly become extremely large. The higher order modes radiate away faster than lower order modes.

5.5 Dispersion

When an optical signal or pulse is sent into the fiber the pulse spreads /broadens as it propagates through the fiber called dispersion. It is also defined as the distortion of light wave or pulse as it travels from one end of the fiber to the other end of fiber. The data or information to be transmitted through fiber is first coded in the forms of pulse after these pulses are transmitted through the optical fiber. Finally, these pulses are received at the receiver and decoded. The light pulses, entering at different angles at input of fiber take different times to reach at the output end. Consequently the pulses are broadening at the output end. The pulses at input end, output ends are shown in Fig 5.7. “The deformation in the pulse is called pulse dispersion.”

Fig. 5.7 Dispersion in optical fiber

In communication, dispersion is used to describe any process by which any electromagnetic signal propagating in a physical medium is degraded because the various wave characteristics (i.e., frequencies) of the signal have different propagation velocities within the physical medium. The pulse dispersion is of following types.

(1)Intermodal dispersion or modal dispersion

(2)Interamodal dispersion or chromatic dispersion

5.1.1 Intermodal dispersion or modal dispersion:

Modal dispersion exists in multimode fibers. When a ray of light is launched into the fiber, the pulse is dispersed in all possible paths through the core, so called different modes. Each mode will be different wavelength and has different velocity as shown in the figure. Hence, they reach the end of the fiber at different time. This results in the elongation or stretching of data in the pulse. The higher order modes travel a long distance and arrive at the receiver end later than the lower order modes. In this way one mode travel more slowly than other mode shown in Fig 5.8 . Thus causes the distorted pulse. This is called intermodal dispersion. The mechanism of modal dispersion is, when light incident the fiber, it propagates in different mode.

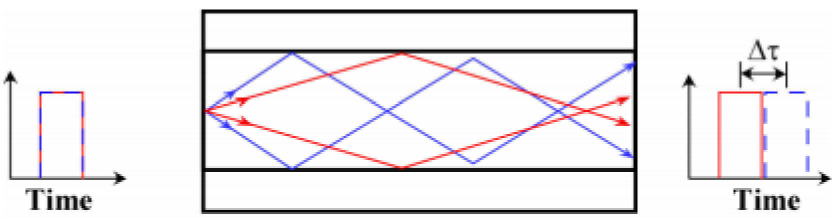


Fig. 5.8 Intermodal dispersion or modal dispersion in optical fiber

Hence in multi mode fiber many signals travel at same time. These signals enter the fiber with different angles to the fiber axis, these signals travels different distance with same velocity. A ray of light OA be incident at angle i on the entrance aperture of the fiber. The rays is refracted in to core along AB and makes an angle r with the axis of core and make an angle θ with the axis of core. Now the ray is strike at the upper core cladding at B .After the ray is totally internally reflected back inside the core and strikes at C and D.Let t be the time taken by the light ray to cover the distance B to C and than C to D with velocity v

$$t = \frac{BC + CD}{v}$$

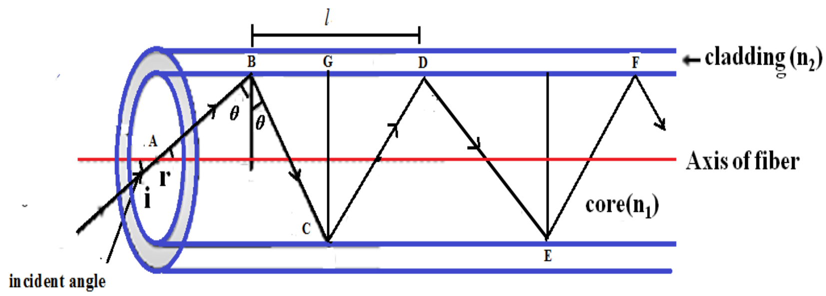


Fig. 5.9 signal propagation in step index multimode optical fiber.

If n_1 be the core and c is the speed of light in vacuum

$$BC = \frac{BG}{\cos r}$$

$$CD = \frac{GD}{\cos r}$$

$$BC + CD = \frac{BG + GD}{\cos r} = \frac{l}{\cos r}$$

$$t = \frac{l}{\cos r} \times \frac{n_1}{c}$$

When the angle of incident in core clad interface is equal to the critical angle than the maximum time taken by the ray to reach is equal to the

$$r = \theta_C$$

$$\sin\theta_C = n_2 / n_1$$

$$t_{\max} = \frac{l}{\cos\theta_C} \times \frac{n_1}{c} = t_{\max} = \frac{l}{n_2} \times \frac{n_1{}^2}{c}$$

The minimum time to reach the signal to come outside when the incident angle at the core air interface is

$$r = 0$$

$$t_{\min} = \frac{n_1 l}{c}$$

The total time differences between two rays are $\Delta t = t_{\max} - t_{\min}$

$$\Delta t = \frac{l}{n_2} \times \frac{n_1^{\ 2}}{c} - \frac{n_1 l}{c}$$

$$\Delta t = \frac{n_1 l}{c}[\frac{n_1}{n_2} - 1] = \frac{n_1 l}{c}\Delta$$

The relative refractive index between core and clad are related to numerical aperture are

$$NA = n_1\sqrt{2\Delta}$$

$$\Delta t = \frac{l(NA)^2}{2n_1 c}$$

From Fig. The arrival timing are different for different signals. As a result output pulse broadens. The broadening of output pulse by travelling in multimode fiber is called model dispersion. It is not suitable for long distance communication due to large dispersion

(2)Material dispersion or spectral dispersion: This is wavelength based effect. Also we know the refractive index of core depends upon wavelength or frequency of light when a input pulse with different components travels with different velocities inside the fiber, the pulse broadens. This is known as material dispersion shown in Fig.5.9

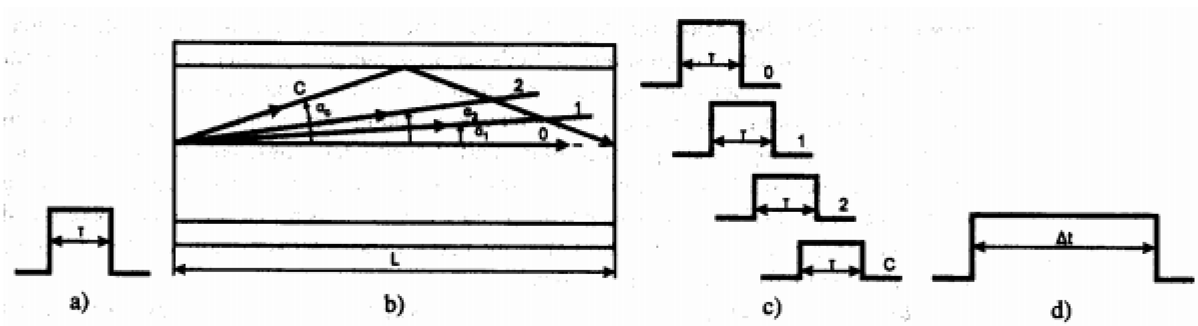


Fig. 5.9 Material dispersion in step index multimode optical fiber

(3)**Wave guide dispersion:** Due to wave guide structure the light rays in the fiber follow different paths. Therefore they take different time interval to travel these paths. This dispersion is called as waveguide dispersion.

5.6 Applications of optical fiber.

The important applications of optical fiber are,

(1)Optical fibre communication has large bandwidth; it is capable of handling a number of channels. Hence it has wide applications in communication.

(2)The optical fibre system is used in defence services because high security is maintained.

(3)Optical fibre systems are particularly suitable for transmission of digital data generated by computers.

(4)It is used for signaling purpose.

(5)Optical fibres are used in medical endoscopy.

5.7 LASER

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. Laser is **"light amplification by stimulated emission of radiation"**.

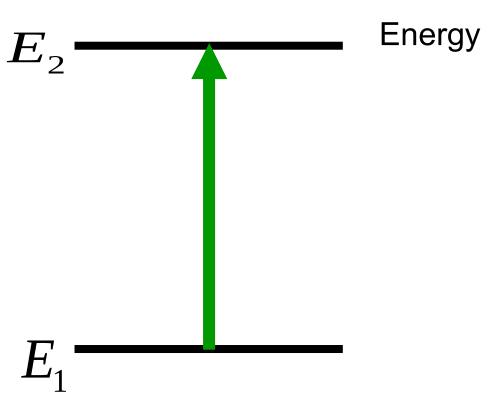
5.7.1 Laser Characteristics

- The light emitted from a laser has a very high degree of coherence. whereas the light emitted from conventional light source is incoherent because the radiation emitted from different atoms do not bear any definite phase relationship with each other.
- The light emitted from a laser is highly monochromatic.
- Degree of non-monochromaticity is $\xi = \frac{\Delta \nu}{\nu_0}$
- Lasers emit light that is highly directional, that is, laser light is emitted as a relatively narrow beam in a specific direction. Ordinary light, such as from a light bulb, is emitted in many directions away from the source.
- The intensity of Laser light is tremendously high as the energy is concentrated in a very narrow region and stays nearly constant with distance. The intensity of light from conventional source decreases rapidly with distance.

5.7.2 Einstein's A,B Coefficients.

In 1916, Einstein considered the various transition rates between molecular states (say, 1 & 2)

a) Absorption: When an atom encounters a photon of light, it can absorb the photon's energy and jump to an excited state. An atom in a lower level absorbs a photon of frequency hν and moves to an upper level.



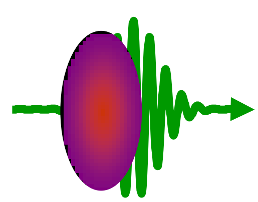


Fig. 5.10 Stimulated Absorption

Number of Absorption per unit time per unit volume is $B_{12} N_1 u(\nu)$ where N_i is the number of atoms (per unit volume) in the i^{th} state, U(ν) dν radiation energy per unit volume within frequency range ν and ν+dν B_{12} the coefficient of proportionality and is a characteristic of the energy levels.

b) Spontaneous emission: Spontaneous emission is the process in which a quantum mechanical system (such as a molecule, an atom or a subatomic particle) transits from an excited energy state to a lower energy state (e.g., its ground state) and emits a quantized amount of energy in the form of a photon. An atom in an upper level can decay spontaneously to the lower level and emit a photon of frequency hν. if the transition between E_2 and E_1 is radiative. This photon has a random direction and phase. The rate of Spontaneous emission (per unit volume) = $A_{21} N_2$ where A_{21} is the proportionality constant

Fig. 5.11. Spontaneous emission

Molecules typically remain excited for no longer than a few nanoseconds.

c)Stimulated Emission

When a photon encounters an atom in an excited state, the photon can induce the atom to emit its energy as another photon of light, resulting in two photons. An incident photon causes an upper level atom to decay, emitting a "stimulated" photon whose properties are identical to those of the incident photon. The term "stimulated" underlines the fact that this kind of radiation only occurs if an incident photon is present. The schematic diagram of process is shown in fig.5.12.The amplification arises due to the similarities between the incident and emitted photons.

Fig. 5.12. Stimulated emission

The rate of stimulated emission are $B_{21}\, N_2\; u(v)$

5.7.3 Relation between Einstein's A, B Coefficient: Einstein first proposed stimulated emission in 1916. In thermal equilibrium, the rate of upward transitions equals the rate of downward transitions:

$$B_{12}\, N_1\, u(v) = A_{21}\, N_2 + B_{21}\, N_2\, u(v)$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\frac{N_1}{N_2} - 1} \ldots\ldots\ldots(1)$$

In equilibrium, the ratio of the populations of two states is given by the Maxwell- Boltzman distribution . $N_2 / N_1 = \exp(-\Delta E/kT)$, where $\Delta E = E_2 - E_1 = hv$, k is the Boltzmann Constant .As a result, higher-energy states are always less populated than the ground state. Acc. to **Planck's blackbody radiation formula**

$$u(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{KT}} - 1} \ldots\ldots\ldots\ldots\ldots(2)$$

Comparing equation 1 and (2)

$$B_{12} = B_{21} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{8\pi h\nu^3}{c^3} B_{21} = A_{21} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The number of spontaneous emission to the number of stimulated emission can be written as.

$$\frac{A_{21}}{B_{21}u(\nu)} = e^{\frac{h\upsilon}{KT}} - 1$$

Case 1 : If $h\nu << KT$

The number of spontaneous emission to the number of stimulated emission $\approx \frac{h\upsilon}{KT}$

Case 2: If $h\nu >> KT$. The number of spontaneous emission to the number of stimulated emission $\approx e^{\frac{h\upsilon}{KT}} - 1$

For normal source at $T \sim 10^3 K, \lambda = 6000Å, \frac{h\nu}{kT} \approx 23$

The number of spontaneous emission to the number of stimulated emission ~ 10^{10} .The spontaneous emission are dominant.

5.7.4 Components of Lasers

1) Active Medium:

The active medium may be solid crystals such as ruby or Nd:YAG, liquid dyes, gases like CO_2 or Helium/Neon, or semiconductors such as GaAs. Active mediums contain atoms whose electrons may be excited to a metastable energy level by an energy source shown in Fig. 5.13

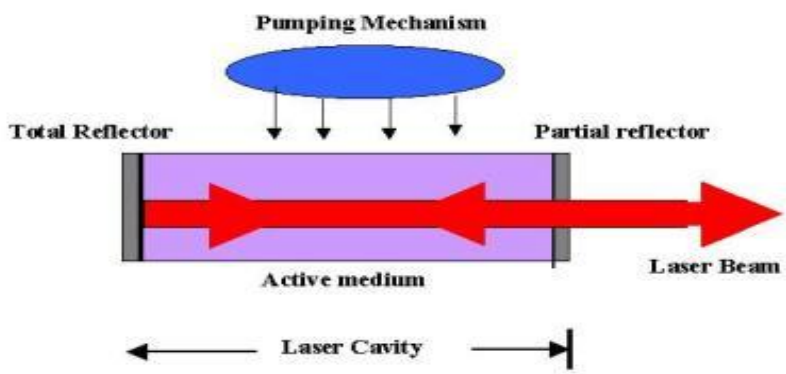


Fig. 5.13. Component of Laser

2 Excitation/Pumping Mechanism

Excitation of the lasing atoms or molecules by an external source of light (such as a lamp) or another laser.Excitation mechanisms pump energy into the active medium by one or more of three basic methods; optical, electrical or chemical.

3 Optical resonator/ cavity

a)High Reflectance Mirror

A mirror which reflects essentially 100% of the laser light.

b) Partially Transmissive Mirror

A mirror which reflects less than 100% of the laser light and transmits the remainder.

Gas lasers consist of a gas filled tube placed in the laser cavity. A voltage (the external pump source) is applied to the tube to excite the atoms in the gas to a **population inversion**. The number of atoms in any level at a given time is called the population of that level. Normally, when the material is not excited externally, the population of the lower level or ground state is greater than that of the upper level. When the population of the upper level exceeds that of the lower level, which is a reversal of the normal occupancy, the process is called **population inversion**. The light emitted from this type of laser is normally continuous wave (CW).

Lasing Action

1. Energy is applied to a medium raising electrons to an unstable energy level.
2. These atoms spontaneously decay to a relatively long-lived, lower energy, metastable state.
3. A population inversion is achieved when the majority of atoms have reached this metastable state.
4. Lasing action occurs when an electron spontaneously returns to its ground state and produces a photon.

5.8 Population Inversion

In physics the redistribution of atomic energy levels that takes place in a system so that Laser action can occur. Normally, a system of atoms is in temperature equilibrium and there are always more atoms in low energy states(E_1) than in higher ones(E_2). Although absorption and emission of energy is a continuous process, the statistical distribution (population) of atoms in the various energy states is constant. When this distribution is disturbed by pumping energy into the system, a population inversion will take place in which more atoms will exist in the higher energy states than in the lower. Normally, when the material is not excited externally, the population of the lower level or ground state (N1) is greater than that of the upper level (N2).When the population of the upper level exceeds that of the lower level, which is a reversal of the normal occupancy, the process is called Population inversion shown in Fig 5.13.

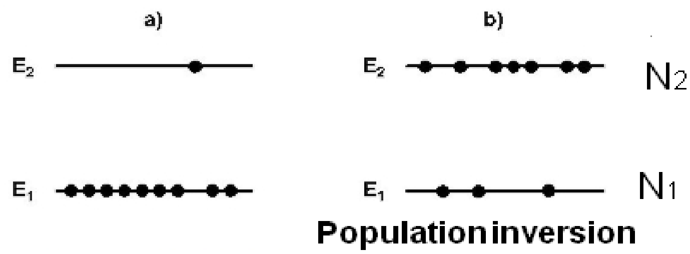


Fig. 5.13. Population Inversion

To understand the concept of a population inversion, it is necessary to understand some thermodynamics and the way that light interacts with matter. To do so, it is useful to consider a very simple assembly of atoms forming a laser medium. Assume there are a group of N atoms, each of which is capable of being in one of two energy state: either

1. The ground state, with energy E_1; or
2. The excited state, with energy E_2, with $E_2 > E_1$.
3. The number of these atoms which are in the ground state is given by N1, and the number in the excited N2. Since there are N atoms in total i.e $N=N_1+N_2$
4. The energy difference between the two states, given by $\Delta E_{12} = E_2 - E_1$
5. which determines the characteristic frequency ν_{12} of light which will interact with the atoms; This is given by the relation $\Delta E_{12} = E_2 - E_1 = h\nu_{12}$

where h being Plank's constant If the group of atoms is in thermal equlibrium, the ratio of the number of atoms in each state is given by Maxwell Boltzman

$$\frac{N_2}{N_1} = \exp((-E2 - E1)/KT)$$

where T is the thermodynamic temperature of the group of atoms, and k is Boltzmann's constant.

5.9 Three-level Laser

In a three-level system, the **laser transition** ends on the ground state. The unpumped gain medium exhibits strong absorption on the laser transition. A **population inversion** and consequently net laser **gain** result only when more than half of the ions (or atoms) are **pumped** into the upper laser level. the threshold pump power is thus fairly high. The schematic diagram of three levels laser are shown as Fig. 5.14.

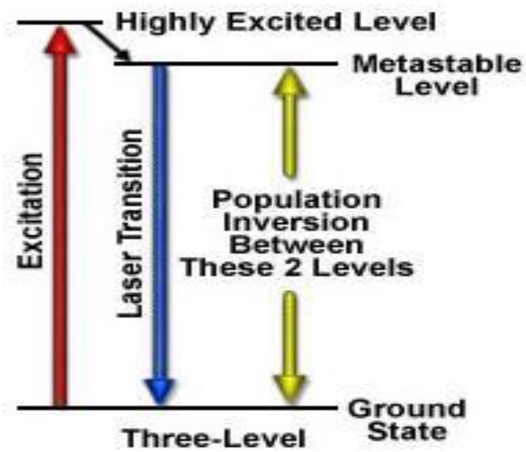


Fig. 5.14. Diagram of Three Level Laser

* The population inversion can be achieved only by pumping into a higher-lying level, followed by a rapid radiative or non-radiative transfer into the upper laser level,
* Avoids stimulated emission caused by the pump wave. (For transitions between only two levels, simultaneous pump absorption and signal amplification cannot occur.) hence we can say in three level laser
* Initially excited to a short-lived high-energy state.
* Then quickly decay to the intermediate metastable level.

Population inversion is created between lower ground state and a higher-energy metastable state. An example of a three-level laser medium is **ruby** (Cr^{3+}:Al_2O_3), as used by Maiman for the first laser

5.10 Four-level laser

A lower threshold pump power can be achieved with a four-level laser medium, where the lower laser level is well above the ground state and is quickly depopulated e.g. by multi-phonon transitions. Ideally, no appreciable population density in the lower laser level can occur even during laser operation. Reabsorption of the laser radiation is avoided (provided that there is no absorption on other transitions). This means that there is no absorption of the gain medium in the unpumped state, and the gain usually rises linearly with the absorbed pump power.

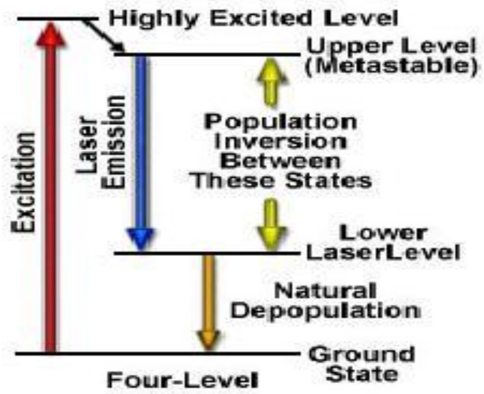


Fig. 5.15. Diagram of Four Level Laser

The most popular four-level solid-state gain medium is Nd:YAG. All lasers based on neodymium-doped gain media, except those operated on the ground-state transition around 0.9–0.95 μm, are four-level lasers.Neodymium ions can also be directly pumped into the upper laser level, e.g. with pump light around 880 nm for Nd:YAG. Even though effectively only three levels are involved, the term three-level system would not be used here

5.11 Types of Laser

The Laser are classified

- According to the state of laser medium: solid, liquid or gas.
- According to the wavelength: Infra-red, visible, ultra-violet (UV) or x-ray lasers.
- According to the nature of output: Pulsed or Continuous (direct) wave lasers.
- According to the energy level system: Two level lasers(e.g., semi-conductor lasers), three level lasers(e.g., Ruby laser), four level lasers(Nd: YAG, He-Ne, CO2 lasers).
- According to the manner of pumping: e.g., electric discharge , optical pumping, chemical reaction, direct conversion

5.11.1 Ruby laser

- ➢ First laser to be operated successfully
- ➢ Lasing medium: Matrix of Aluminum oxide doped with chromium ions
- ➢ Energy levels of the chromium ions take part in lasing action
- ➢ A three level laser system

The construction of ruby Laser is shown in Fig. 5.16.

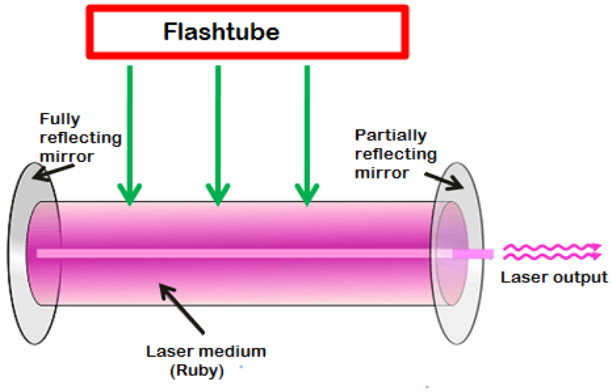


Fig. 5.16. Construction of Ruby Laser

Working: Ruby is pumped optically by an intense flash lamp. This causes Chromium ions to be excited by absorption of radiation around 0.55 µm and 0.40µm.

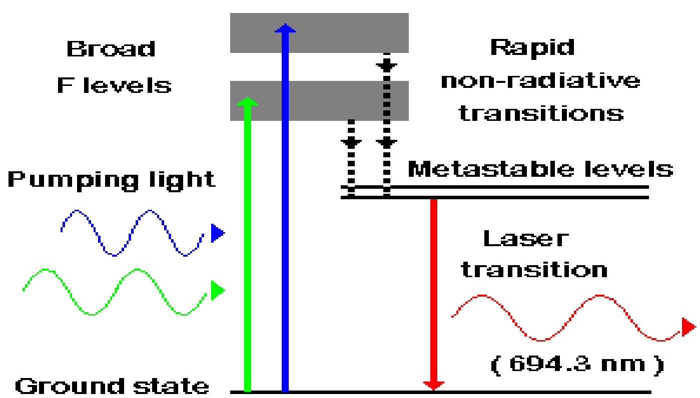


Fig. 5.17. Energy level Diagram of chromium ions in Ruby Laser

Chromium ions are excited from ground levels to energy levels E_1 and E_2 shown in Fig.5.17.Than the excited ions decay non-radiatively to the level M – upper lasing level M- metastable level with a lifetime of ~ 3ms.The Laser emission occurs between level M and ground state G at an output wavelength of 694.3 nm.Ruby laser is one of the important practical lasers which has long lifetime and narrow linewidth (Linewidth – width of the optical spectrum or width of the power Spectral density) The output lies in the visible region, where photographic emulsions and photodetectors are much more sensitive than they are in infrared region. It has applications in holography and laser ranging. Due to flash lamp operation leads to a pulsed output of the laser. In between flashes, the lasing action stops then the output is highly irregular function of time leads to Laser spiking. The Intensity of Ruby Laser has random amplitude fluctuations of varying duration.

5.11.2 He-Ne laser

A helium–neon laser or HeNe laser, is a type of gas laser whose gain medium consists of a mixture of helium and neon(10:1) inside of a small bore capillary tube, usually excited by a DC electrical discharge. It is four level laser give the continuous laser output. The construction of He-Ne Laser is shown in Fig.5.18. The best-known and most widely used He-Ne laser operates at a wavelength of 632.8 nm in the red part of the visible spectrum. The first He-Ne lasers emitted light at 1.15 μm, in the infrared spectrum, and were the first gas lasers.

Schematic diagram of a Helium–Neon laser

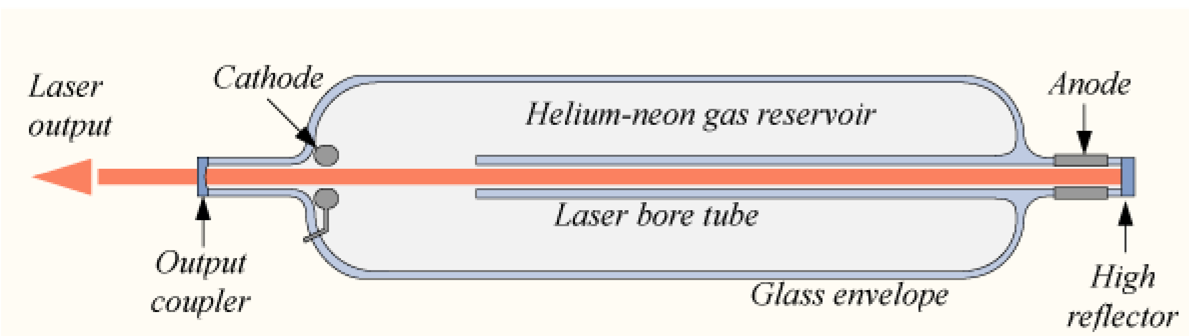


Fig. 5.18. Construction of He-Ne Laser

The gain medium of the laser, is a mixture of helium and neon gases, in approximately a 10:1 ratio, contained at low pressure in a glass envelope. The gas mixture is mostly helium, so that helium atoms can be excited. The excited helium atoms collide with neon atoms, exciting some of them to the state that radiates 632.8 nm. Without helium, the neon atoms would be excited mostly to lower excited states responsible for non-laser lines. A neon laser with no helium can be constructed but it is much more difficult without this means of energy coupling. Therefore, a HeNe laser that has lost enough of its helium (e.g., due to diffusion through the seals or glass) will lose its laser functionality since the pumping efficiency will be too low. The energy or pump source of the laser is provided by a high voltage electrical discharge passed through the gas between electrodes (anode and cathode) within the tube. A DC current of 3 to 20 mA is typically required for CW operation. The optical cavity of the laser usually consists of two concave mirrors or one plane and one concave mirror, one having very high (typically 99.9%) reflectance and the output coupler mirror allowing approximately 1% transmission.

Commercial HeNe lasers are relatively small devices, among gas lasers, having cavity lengths usually ranging from 15 cm to 50 cm (but sometimes up to about 1 metre to achieve the highest powers), and optical output power levels ranging from 0.5 to 50 mW.

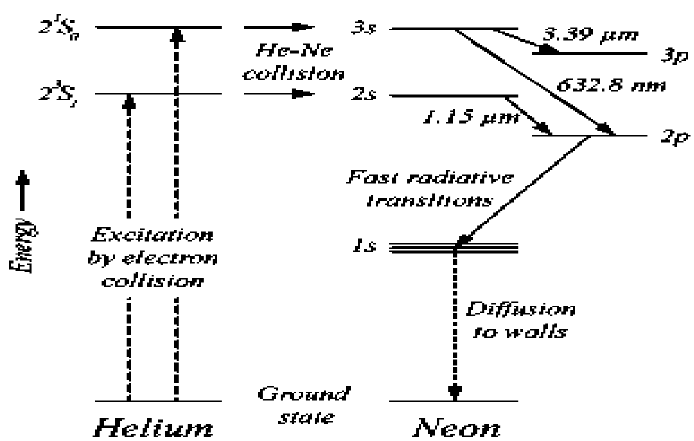


Fig. 5.19. Energy level Diagram of He-Ne Laser

The mechanism producing population inversion and light amplification in a He-Ne laser plasma originates with inelastic collision of energetic electrons with ground state helium atoms in the gas mixture. As shown in the accompanying energy level diagram in Fig.5.19, these collisions excite helium atoms from the ground state to higher energy excited states, among them the 2^3S_1 and 2^1S_0 long-lived metastable states. Because of a fortuitous near coincidence between the energy levels of the two He metastable states, and the $3s^2$ and $2s^2$ (Paschen notation) levels of neon, collisions between these helium metastable atoms and ground state neon atoms results in a selective and efficient transfer of excitation energy from the helium to neon.

Applications of laser

1. Scientific

a.Spectroscopy

b.Lunar laser ranging

c.Photochemistry

d. Laser cooling

e. Nuclear fusion

2. Military

a. Death ray

b. Defensive applications

c. Strategic defense initiative

d. Laser sight

e. Illuminator

f. Target designator