Basic laws

1. Ohm's law 2. Kirchhoff's laws

→ Resisting flow of electric charge

physical property OR or ability to resist flow of current is resistance. (R).

$$\boxed{R = \rho \frac{l}{A}} \quad \Omega \text{ (ohm)}$$

→ **ohm's law** → It states that voltage across a resistor is directly proportional to current i flowing through Resistor.

$$\boxed{V \propto i}$$

$$\boxed{V = iR}$$

→ Kirchhoff's Laws → 1. Kirchoff's current law
2. Kirchhoff's voltage law

In a lumped electric circuit for any of its node and at time 't'.

1. **KCL** → It states that the algebric sum of currents entering a node is Zero.

Branch → N

$$\sum_{n=1}^{N} i_n = 0$$

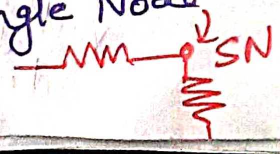
node

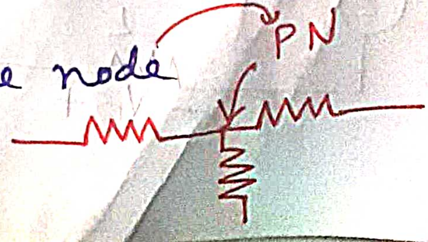
N → Number of branch connected to Node

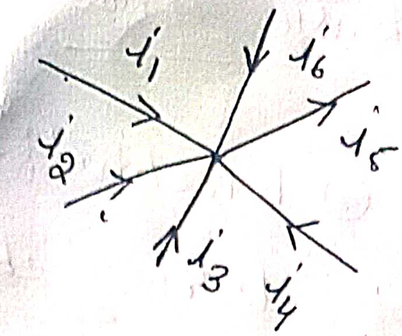
i_n → nth current entering or leaving Node.

* It always define at a node

Single Node   Principle node



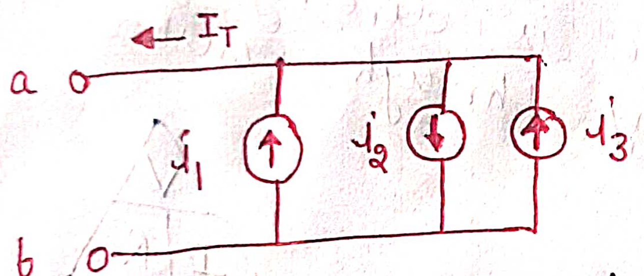




Entering Node:- i_1, i_2, i_6, i_3, i_4

Leaving Node:- i_5

$$\boxed{i_1 + i_2 + i_6 + i_3 + i_4 = i_5}$$



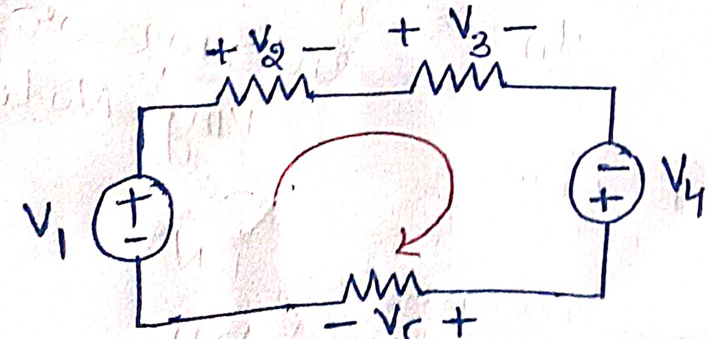
$$I_T + i_2 = i_1 + i_3 .$$

Kirchhoff's voltage law → It states that algebric sum of all voltages around a closed path (or loop) is Zero.

$$\boxed{\sum_{m=1}^{M} V_m = 0}$$

M = no of voltages in the loop or No of Branches in the loop.

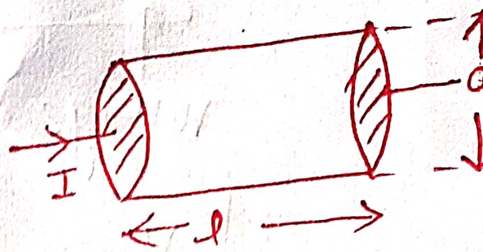
V_m = m-th voltage.



$$-V_1 + V_2 + V_3 - V_4 + V_5 = 0$$

$$\underbrace{V_2 + V_3 + V_5}_{\text{sum of voltage drop}} = \underbrace{V_1 + V_4}_{\text{sum of voltage rise}}$$

Ohm's law →



* At Constant Temprature current density is directly proportional to Electric field Intensity.

* At Constant temprature, potential difference across the element is directly proportional to current flowing into element.

current density $J = \frac{I}{A}$

Electric field Intensity (E) $= \frac{V}{l}$ $\frac{\text{volts}}{m}$.

$a = \frac{1}{\rho}$ mho/m.

$R = \rho l / A$

By ohm's law →

$J \propto E$

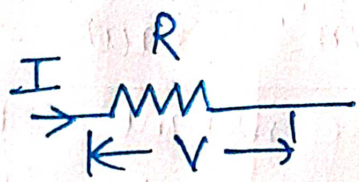
$$\boxed{J = aE}$$

$$\frac{I}{a} = \frac{1}{\rho} \cdot \frac{V}{l}$$

$$*** \quad \boxed{\frac{V}{I} = \frac{\rho l}{a} = R.}$$

$$\boxed{V \propto I}$$

voltage drop — current



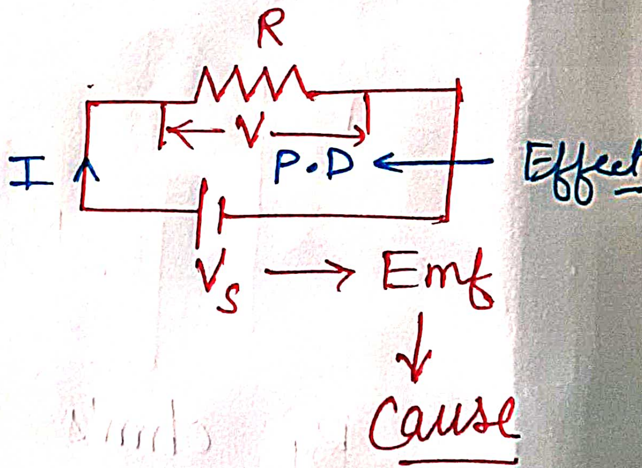
$$\boxed{V = RI}$$

$$\boxed{\frac{V}{I} = R = \text{Constant}} \leftarrow \text{Result}$$

Forms of ohm law :-

$$J = \sigma E$$
$$V = RI$$
$$V = R \cdot \frac{dq}{dt}$$

Note :- $\boxed{I \propto V_s}$



→ Emf is independent of current and Resistance Magnitude.

→ Potential difference depends upon current and Resistance Magnitude

$$\boxed{I \propto V_s} \leftarrow \text{EMF}$$

dependent (I) Independent (V_s)

$$\boxed{V \propto I}$$

P.D / drop (V)

Study plan

Kirchhoff's law → Simple node → 2 Branch
Kirchhoff's law → Principle node → at least 3 Branch

↓

Applied on lumped circuit

↓

It is Independent of nature of element.

Nodal & Mesh Analysis :

Nodal → KCL + ohm's law.

Mesh → KVL + ohm's law.

→ lumped circuit (KCL & KVL valid)

→ If Temprature Constant (ohm's law valid)

→ Nodal & Mesh Independent Method as KCL & KVL are Independent

→ ohm's law → R, L, C (lumped or distributed)

KCL & KVL → only for lumped elements (N/w) electric circuits

→ ohm's law → Not applicable for active elements.

Valid → Network + EMF

KCL → Network.

→ Not applicable for sources ∵ V, I nature is non linear.

→ only applicable linear passive elements R, L, C.

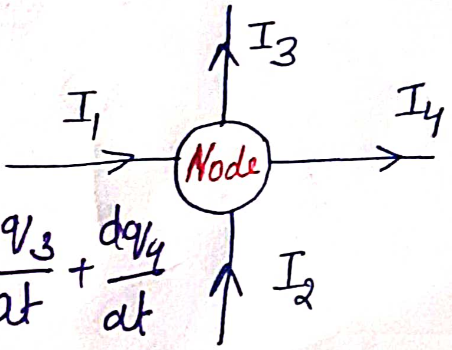
KCL

→ Based on conservation of charge
→ Current is ratio of change of charge.
→ Since charge is constant, current is zero.
→ According to KCL, Across any node sume of current is zero.

$$I = \text{rate of change of charge}$$
$$I = dq/dt$$
charge is constant
$$I = \frac{d}{dt}(\text{Constant})$$
$$\boxed{I = 0}$$

→ If Incoming current is taken as positive then outgoing current will be taken as as -ve. and vice-versa.

Incoming = +ve
outgoing = -ve.



$$I_1 + I_2 - I_3 - I_4 = 0$$
$$\underbrace{I_1 + I_2}_{\text{Incoming Current at Node}} = \underbrace{I_3 + I_4}_{\text{outgoing current.}}$$

$$* \frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{dq_3}{dt} + \frac{dq_4}{dt}$$

$$\boxed{q_1 + q_2 = q_3 + q_4}$$
↳ conservation of charge.

Nodal Analysis

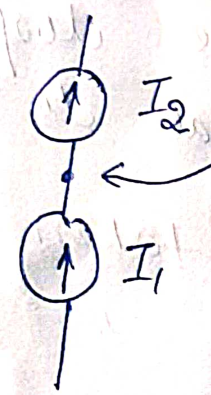
Nodal Analysis = KCL + ohm's law

KCL → applicable anywhere

ohm's law → Linear, Bilateral, Passive, lumped parameter. Tempreture = constant.

① Some Important Circuits :-

① Two current sources connected in series.



Apply KCL

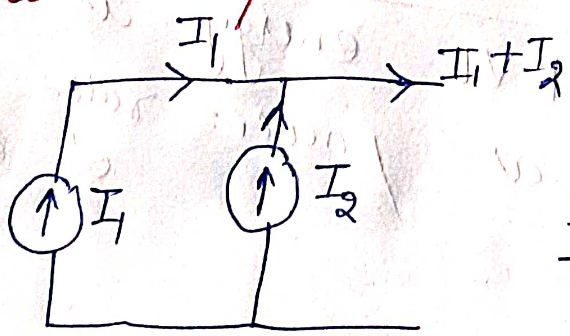
$\boxed{I_1 = I_2}$

5A IC = OC

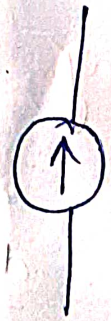
$10A \neq 5A$

10A (Wrong)

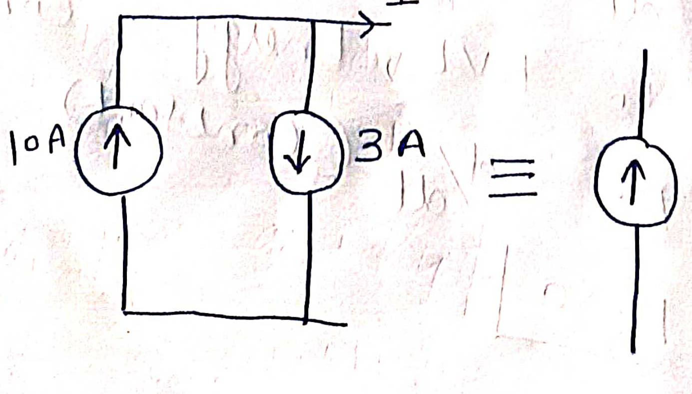
② When Two current sources are in parallel →



$\equiv$

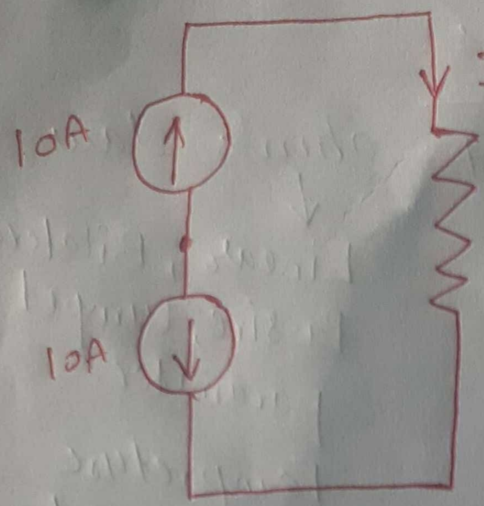


$I_1 + I_2$



$\equiv$ $= (10-3) = 7A.$

(Direction of large one)



→ Not satisfy KCL
→ circuit is wrong.

KVL

→ It based on conservation of energy.
→ As per KVL, across close loop sum of voltage is equal to zero.

If | −ve to +ve travel taken = −ve
| +ve to −ve " " = +ve

OR

vice - versa.

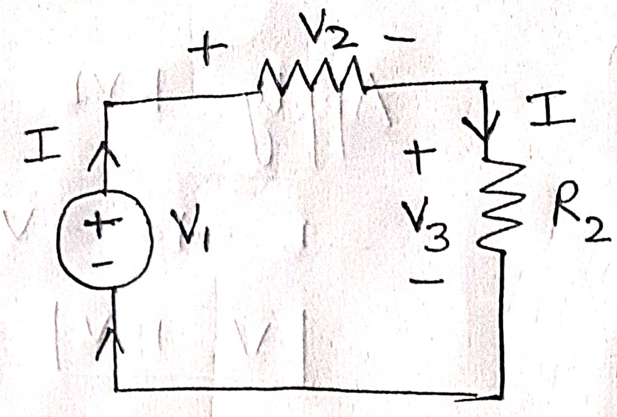
* According to KVL energy is constant
* power = 0 as power = rates of change of energy

$$P = \frac{dE}{dt}$$

According to KVL, Energy = Constant

$$P = d/dt\,(\text{Constant})$$

$$\boxed{P = 0}$$



$-ve$ to $+ve$ = Negative
$+ve$ to $-ve$ = positive

$$-V_1 + V_2 + V_3 = 0$$

$$\boxed{V_1 = V_2 + V_3}$$

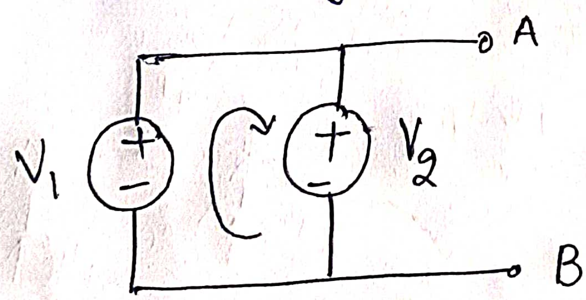
$$-\frac{dw_1}{dq} + \frac{dw_2}{dq} + \frac{dw_3}{dq} = 0$$

$$\underbrace{-w_1 + w_2 + w_3 = 0}_{\text{Conservation of energy}}$$

Mesh Analysis = KVL + ohm's law

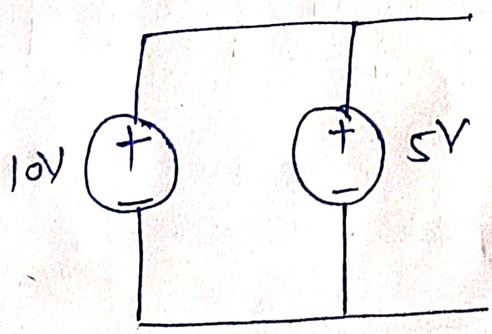
Mesh Analysis = ① Linear ② Bilateral ③ Passive ④ Temprature constant.

Case 1 → Two voltage source connected in parallel:-

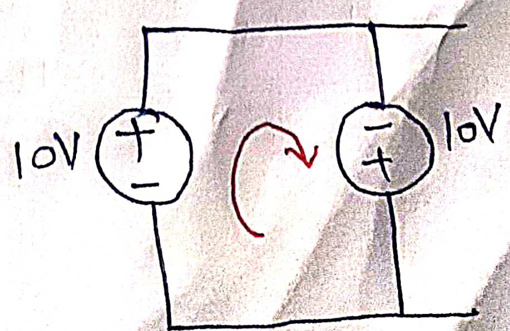


$$-V_1 + V_2 = 0$$

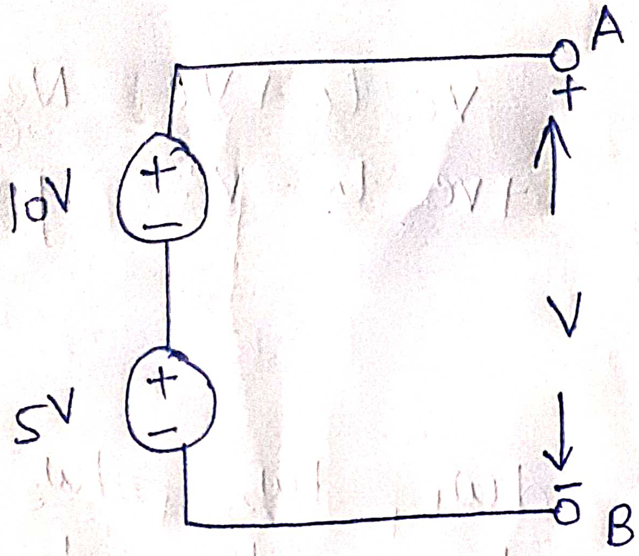
$$\boxed{V_1 = V_2}$$



← Wrong.



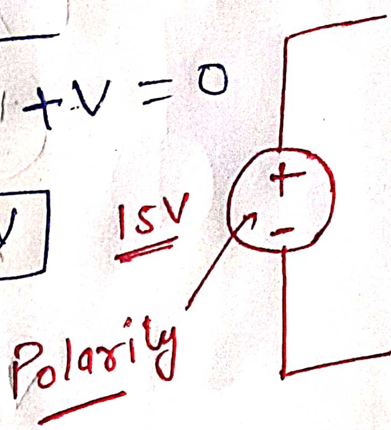
← Wrong

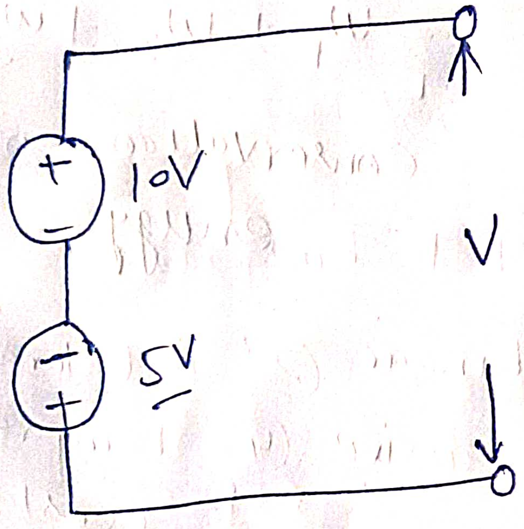


Apply KVL →

$$-5 - 10 + V = 0$$

$$\boxed{V = 15V}$$

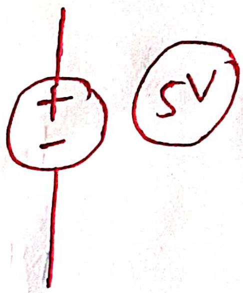




Apply KVL →

$$+5V - 10V + V = 0$$

$$\boxed{V = 5V}$$



MESH AND NODAL ANALYSIS

→ Basic Techniques used in finding solutions of a network.

→ Two Basic Circuit analysis techniques

- → Nodal Analysis → KCL + ohm' law
- → Mesh Analysis → KVL + ohm's law

→ **MESH Analysis** :- Based upon KVL.

Step 1 → Inspect total number of mesh in the circuit. (All Independent closed path)

Step 2 → Label the mesh current in any arbitrary direction in each mesh.

Step 3 → Put the sign convention across each element.

→ Current Entering side across a resistor is marked as positive.

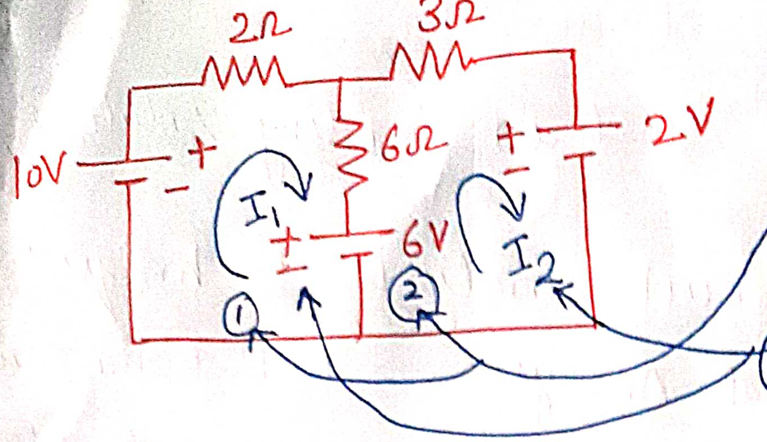
→ Current leaving side marked as negative.

Step 4 → Apply KVL at each Mesh.

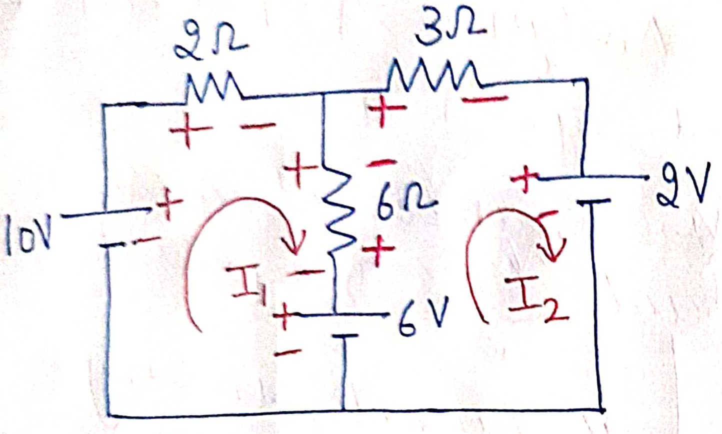
Note → ① The common branch carries the algebraic sum of mesh currents flowing through it.

② No of equations = No of Unknowns

Solve equations and determine mesh current.



1. Inspect Mesh
   i.e = 2.
   Total Mesh = 2
2. Label Mesh current either clockwise or Anticlockwise (prefer clockwise)
3. Put sign convention across each element.



Loop ① →

$$-10 + 2I_1 + 6(I_1 - I_2) + 6 = 0$$

$$2I_1 + 6I_1 - 6I_2 = 4$$

$$\boxed{8I_1 - 6I_2 = 4} \rightarrow ①$$

Loop ② →

$$-6 + 6(I_2 - I_1) + 3I_2 + 2 = 0$$

$$\boxed{9I_2 - 6I_1 = 4} \rightarrow ②$$

4. Apply KVL at each Mesh
5. Solve these equations

$$8I_1 - 6I_2 = 4 \quad \times 6 \qquad = \cancel{48I_1} - 36I_2 = 24$$

$$9I_2 - 6I_1 = 4 \quad \times 8 \qquad = -\cancel{48I_1} + 72I_2 = 32$$

$$+36I_2 = 56$$

$$I_2 = \frac{56}{36} = \frac{14}{9}$$

$$\cancel{8I_1 + \frac{4}{3} = 4}$$

$$\cancel{8I_1 = \frac{8}{3}}$$

$$\cancel{I_1 = \frac{1}{3} = 0.33A}$$

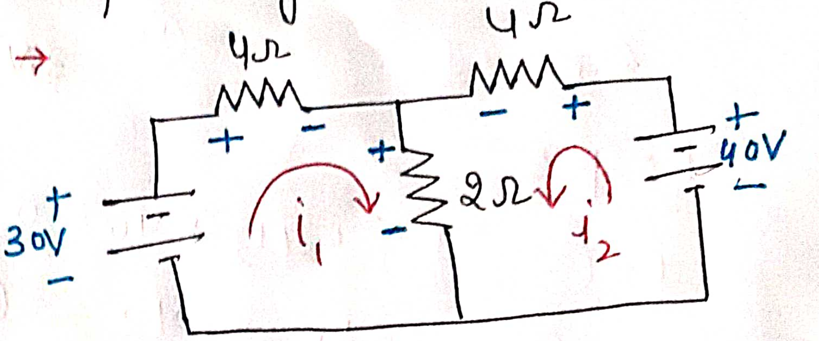
$$8I_1 - 6 \times \frac{14}{9} = 4$$

$$8I_1 = 4 + \frac{28}{3}$$

$$I_1 = \frac{40}{3} \times \frac{1}{8} = \frac{5}{3} = 1.667A$$

Numericals

Q:- Determine the current in all the branches of Network using loop Analysis.



Loop Analysis :- Loop 1 →

$$-30 + 4I_1 + 2(I_1 + I_2) = 0$$

$$6I_1 + 2I_2 = 30 \rightarrow (1)$$

Loop 2 →

$$-40 + 4I_2 + 2(I_1 + I_2) = 0$$

$$6I_2 + 2I_1 = 40 \rightarrow (2)$$

Equation (1) & Equation (2)

$$\cancel{6I_1} + 2I_2 = 30$$

$$2I_1 + 6I_2 = 40 \times 3$$

$$\cancel{6I_1} + 18I_2 = 120$$

$$- \quad - \quad -$$

$$+16I_2 = +90$$

$$*** \boxed{I_2 = \frac{90}{16} = \frac{45}{8}\text{ A}}$$

$$\cancel{6}^{3} \times \frac{45}{\cancel{8}_{4}} + 2I_1 = 40$$

$$135 + 8I_1 = 160$$

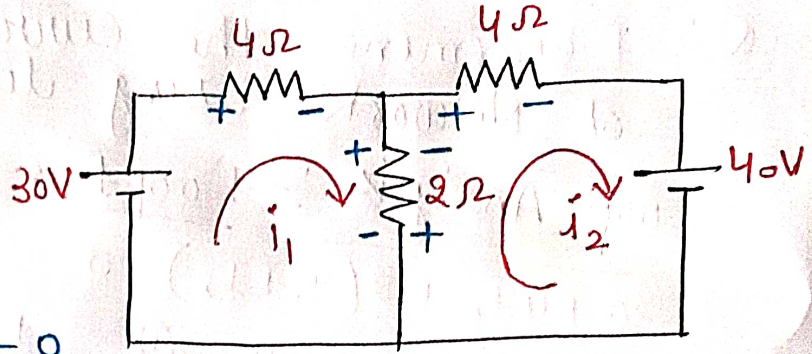
$$8I_1 = 25$$

$$*** \boxed{I_1 = \frac{25}{8}\text{ A}}$$

Loop 1 →

$-30+4I_1+2(I_1-I_2)=0$

$6I_1-2I_2=30 \rightarrow (1)$



Loop 2 →

$+2(I_2-I_1)+4I_2+40=0$

$6I_2-2I_1=-40 \rightarrow (2)$

Solve equation form by ~~equation~~ Loop1 & Loop2 →

$6\cancel{I_1}-2I_2=30$

$-2I_1+6I_2=-40 \times 3$

$-6\cancel{I_1}+18I_2=-120$

$I_2 = \dfrac{-120+30}{18-2} = \dfrac{-90}{16} \Rightarrow -\dfrac{45}{8}$ A

*** $\boxed{I_2 = \dfrac{-45}{8}\ \text{A}}$

Solve for I_1 equation 1 →

$6I_1-2I_2=30$

put value of I_2 in equation (1)

$6I_1-2\times\dfrac{-45}{8}=30$

$6I_1 = 30 + \dfrac{90}{8}$

$6I_1 = \dfrac{240+90}{8} = \dfrac{\cancel{330}\ 150}{8}$

$I_1 = \dfrac{\cancel{330}\ 150}{8}\times\dfrac{1}{6} = \dfrac{\cancel{55}}{\cancel{8}} = \dfrac{25}{8}$ A

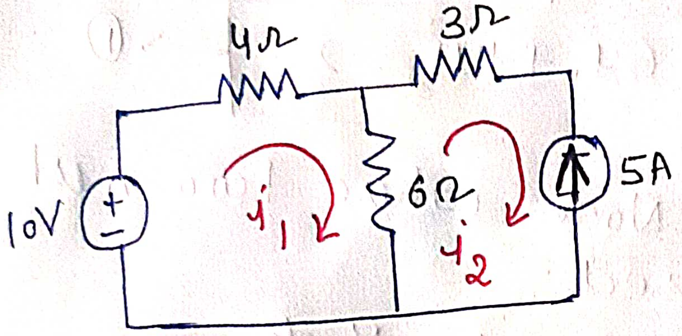
* Mesh Analysis with current source :-

→ presence of current source reduces no of equations

→ KVL + KCL + ohm's law.

Special Cases

Case 1 → When current source exist only in one Mesh.



$$-10+4i_1+6(i_1-i_2)=0$$

But $i_2 = -5A$

put in equation loop 1

$$-10+4i_1+6(i_1+5)=0$$

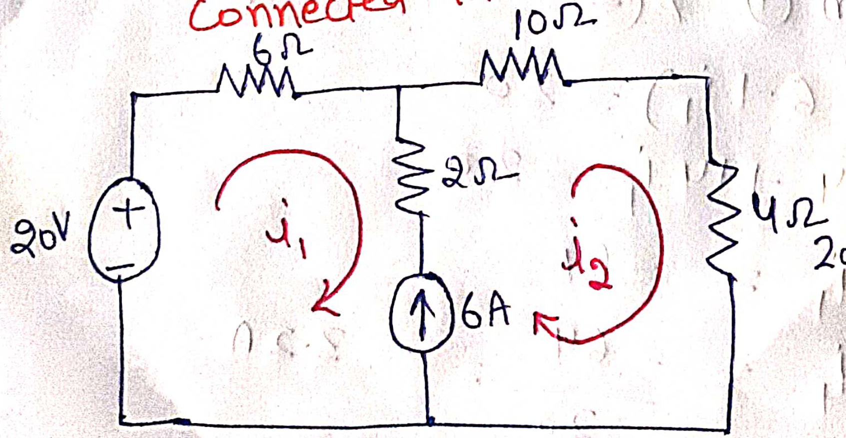
$$-10+4i_1+6i_1+30=0$$

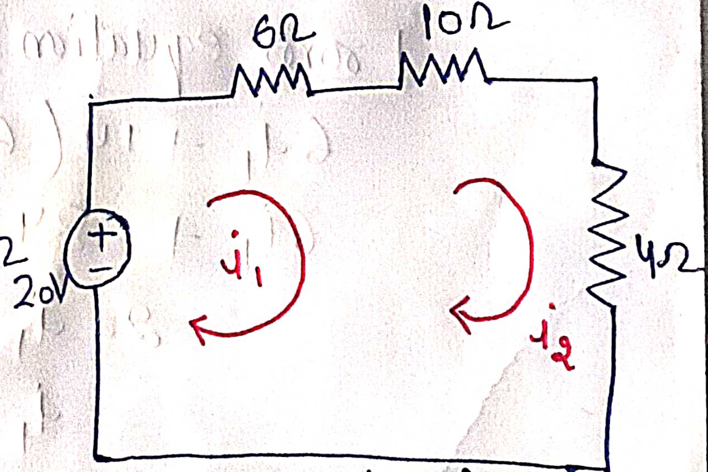
$$10i_1=-20$$

$$\boxed{i_1=-2A} \quad **$$

Case 2 → When current source exists between two meshes. We create a supermesh

→ By excluding the current source and element connected in series.



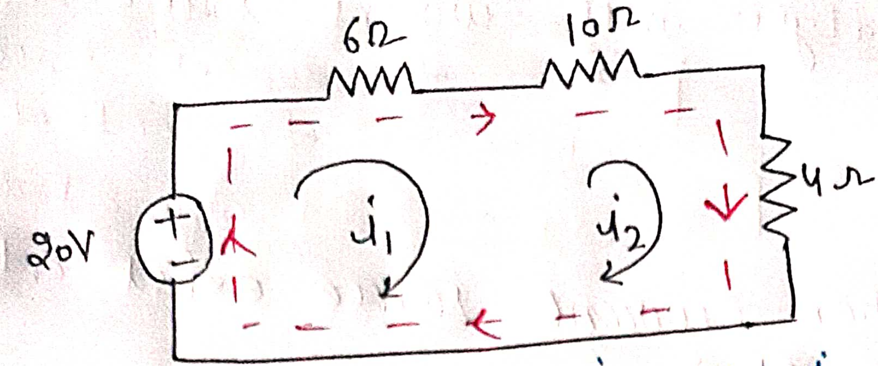


Note :- Super Mesh create when two mesh have a common current source.

properties of super Mesh → ① It Require Application of KVL & KCL.

② A super mesh has no current of its own

③ The current source in super mesh provides constraint equation necessary to solve for Mesh current.

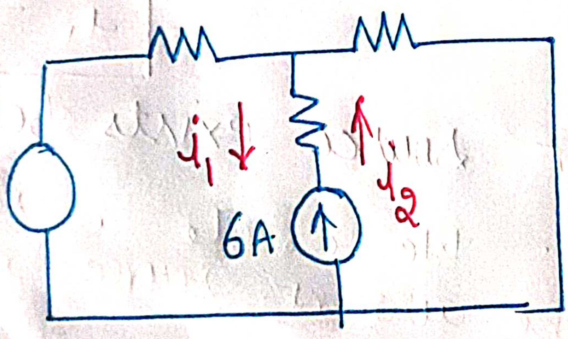


Apply KVL → $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

$$\boxed{6i_1 + 14i_2 = 20} \rightarrow (1)$$

Apply KCL → to a Node where two mesh intersect.

Apply KCL to Node



As 6A has upword direction

i.e $i_2 > i_1$

$i_2 - i_1 = 6$

$$\boxed{i_2 = 6 + i_1} \rightarrow (2)$$

From equation (1) & (2)

$$6i_1 + 14(6 + i_1) = 20$$

$$6i_1 + 84 + 14i_1 = 20$$

$$20i_1 = -64$$

$$i_1 = \frac{-64}{20} = -3.2\,A$$

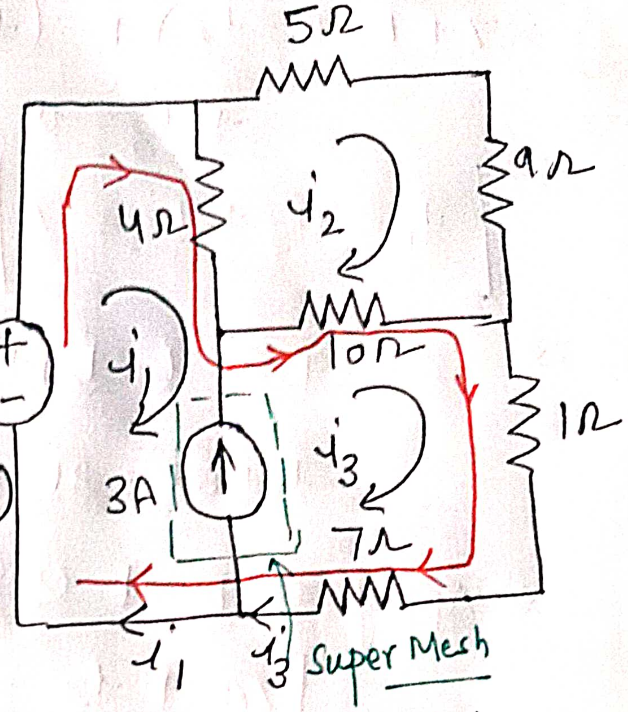
$$\boxed{i_2 = 2.8\ A}$$ **

* loop 1 & 3 Super Mesh

$-10+4(i_1-i_2)+10(i_3-i_2)$
$+i_3+7i_3=0$

$4i_1-4i_2+10i_3-10i_2+i_3+7i_3=10$

$4i_1-4i_2+18i_3-10i_2=10 \rightarrow (1)$



IN MESH (2)

$5i_2+9i_2+10(i_2-i_3)+4(i_2-i_1) \rightarrow (2)$

$14i_2+10i_2-10i_3+4i_2-4i_1=0$

$28i_2-10i_3-4i_1=0 \rightarrow (2)$

Find current i_1 in circuit given above

$\boxed{i_3 = i_1+3A}$ By applying KCL.

$28i_2-10(i_1+3)-4i_1=0$

$28i_2-10i_1-30-4i_1=0$

$28i_2-14i_1=30 \rightarrow (3)$

$4i_1-14i_2+18(i_1+3)=10$

$4i_1-14i_2+18i_1+54=10$

$2\times 22i_1-14i_2=-44 = 44i_1-\cancel{28}i_2=-88$

$-14i_1+2\cancel{8}i_2=30$

$30i_1=-58$

$i_1=-\frac{58}{30} \Rightarrow -1.93A$

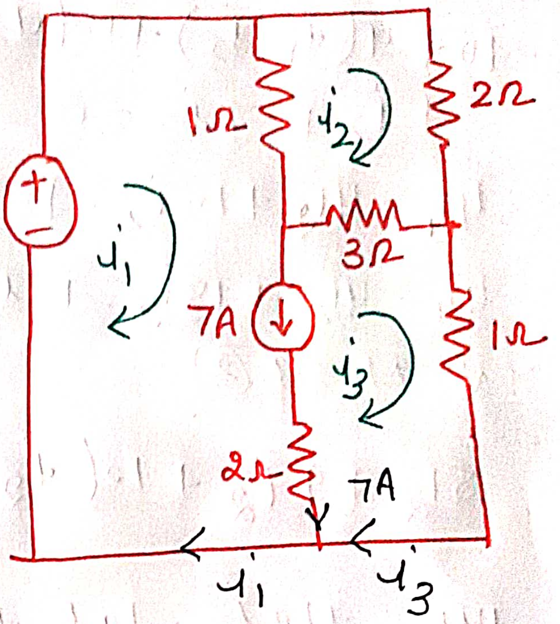
* EXAMPLE OF SUPERMESH *

* Mesh ① & ③ has super Mesh.

$-7+1(i_1-i_2)+3(i_3-i_2)$

$+i_3=0$

$\boxed{i_1-4i_2+4i_3=7} \rightarrow ①$



* $2i_2+3(i_2+i_3)+1(i_2-i_1)$ [Mesh-2]

$-i_1+6i_2-3i_3=0 \rightarrow ②$

* Apply KCL at common Node

$\boxed{7A+i_3=i_1}$

Put in ①

$7+i_3-4i_2+4i_3=7$

$5i_3-4i_2=0 \rightarrow ③$

$-7-i_3+6i_2+3i_3=0$

$6i_2-4i_3=7 \times 4$

$-4i_2+5i_3=0 \times 6$

$\cancel{24i_2}-16i_3=28$

$-\cancel{24i_2}+30i_3=0$

$14i_3=28$

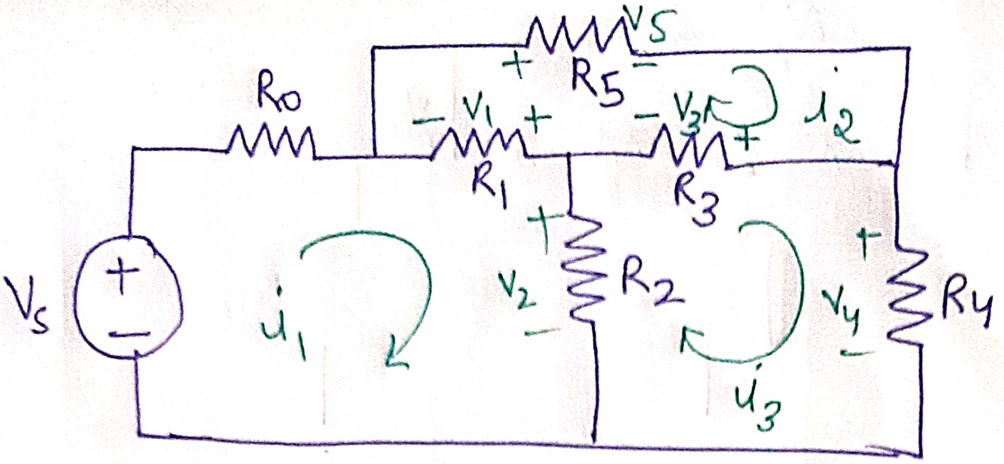
$\boxed{i_3=2A}$

$i_1=2A+7A=9A$

$9-4i_2+8=7$

$+4i_2=+10$

$i_2=\frac{10}{4}=2.5A$



For Mesh 1 →

$$-V_s + R_0 i_1 + R_1(i_1 - i_2) + R_2(i_1 - i_3) = 0 \quad (1)$$

For Mesh 2 →

$$+ i_2 R_5 + R_3(i_2 - i_3) + R_1(i_2 - i_1) = 0$$

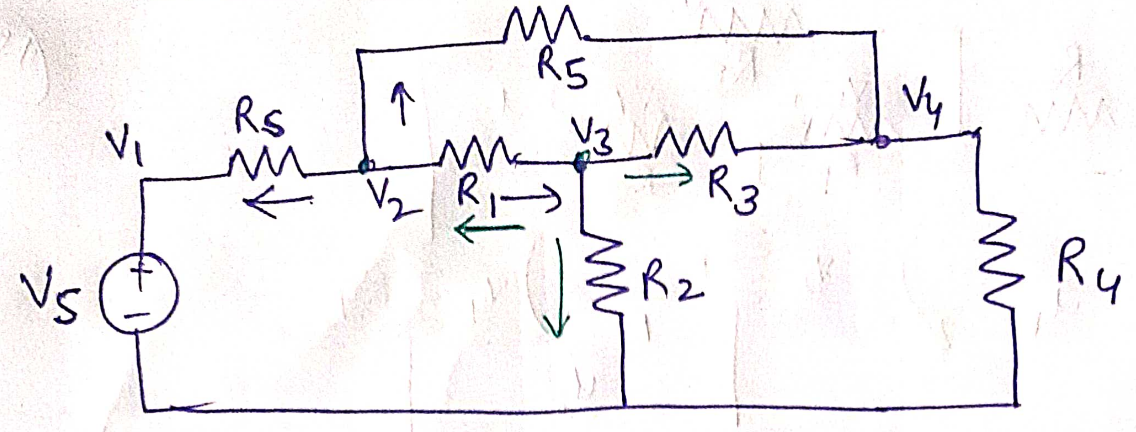
For Mesh 3 →

$$+ R_4 i_3 + R_2(i_3 - i_1) + R_3(i_3 - i_2) = 0$$

$$V_s = i_1(R_0 + R_1 + R_2) - R_1 i_2 - R_2 i_3 = 0$$

$$0 = i_2(R_5 + R_3 + R_1) - R_3 i_3 - R_1 i_1 = 0$$

$$0 = i_3(R_4 + R_2 + R_3) - i_1 R_2 - i_2 R_3 = 0$$



$$I_{R_1} = \frac{V_3 - V_2}{R_1}$$

$$I_{R_2} = \frac{V_3}{R_2}$$

$$I_{R_3} = \frac{V_3 - V_4}{R_3}$$

$$I_{R_1} + I_{R_2} + I_{R_3} = 0$$

$$\frac{V_3 - V_2}{R_1} + \frac{V_3}{R_2} + \frac{V_3 - V_4}{R_3} = 0$$

$$\frac{V_2 - V_s}{R_s} + \frac{V_2 - V_3}{R_1} + \frac{V_2 - V_4}{R_5} = 0$$

$$\frac{V_4 - V_3}{R_3} + \frac{V_4 - V_2}{R_5} + \frac{V_4}{R_4} = 0$$

$$\left(\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_5}\right)V_2 - \frac{V_3}{R_1} - \frac{V_4}{R_5} = \frac{V_s}{R_s} \rightarrow (1)$$

$$-\frac{V_2}{R_1} + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_3 - \frac{V_4}{R_3} = 0$$

$$-\frac{V_2}{R_5} - \frac{V_3}{R_3} + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_4 = 0$$

Nodal Analysis → KCL + ohm's law

Step 1 → Inspect the total number of node in the circuit.
→ Select one Node as reference Node.
→ Label the Remaining Node. as Unknown node voltages w.r.t the reference node [datum or ground].

Step-II → Assign Branch currents in each element (Branch). [The choice of direction is arbitrary.]

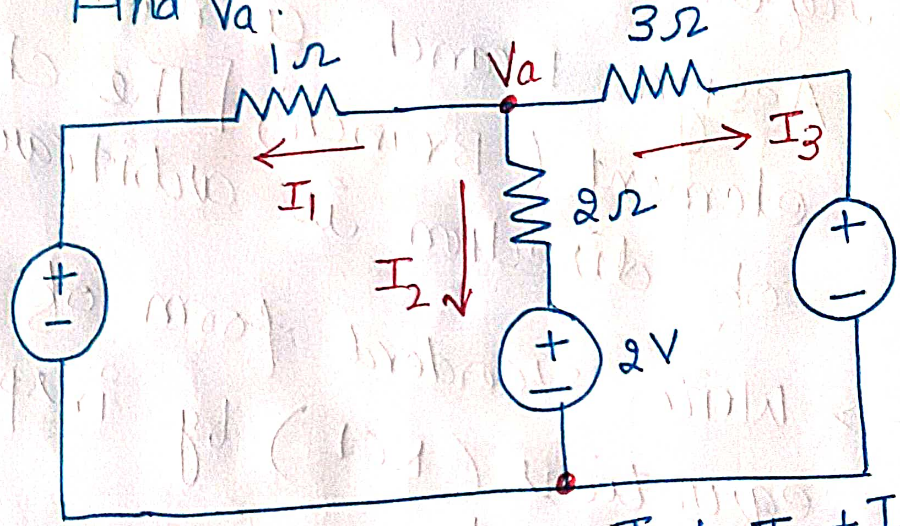
Step-III → Write standerd form of Node equations (KCL) by inspecting the circuit.

Step-IV → Express the branch currents in terms of Node assigned voltages.

Step V → Solve a set of simultaneous algebric equation for Node voltages and ultimately the Branch current.

$$\boxed{i = \frac{V_{higher} - V_{lower}}{R}}$$ → current flow from higher potential to lower potential in a resistor.

Q① → Consider Network shown in below figure
Find Va:



At Va apply KCL → $I_1 + I_2 + I_3 = 0$

$$\frac{V_a - 10}{1} + \frac{V_a - 2}{2} + \frac{V_a - (-1)}{3} = 0$$

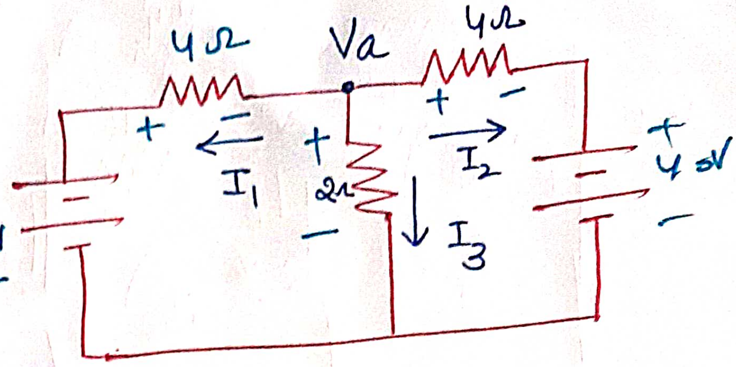
$$\frac{6V_a - 60 + 3V_a - 6 + 2V_a + 2}{6} = 0$$

$$11V_a = 64$$

$$\boxed{V_a = \frac{64}{11}} = 5.82V$$

By Nodal Analysis →

Apply KCL at Node Va →



$I_1 + I_2 + I_3 = 0$

$$\frac{Va - 30}{4} + \frac{Va}{2} + \frac{Va - 40}{4} = 0$$

$Va - 30 + 2Va + Va - 40 = 0$

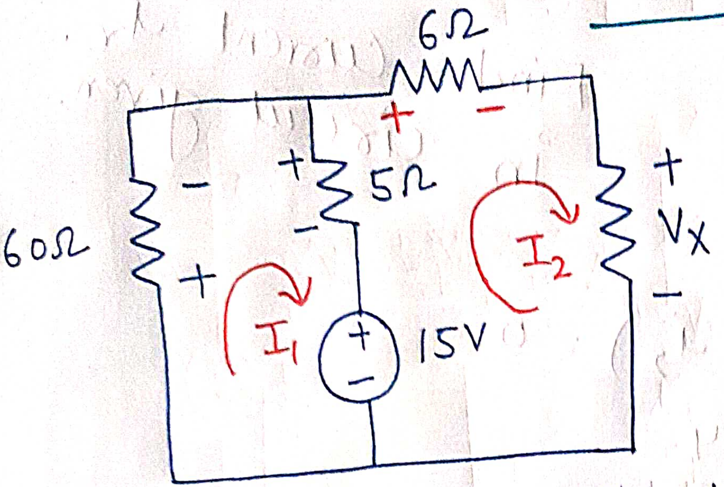
$4Va \cancel{*} 70 = 0$

$$Va = \frac{+70}{4} = \underline{17.50\ V}$$

$$I_1 = \frac{17.50V - 30V}{4} = \frac{-12.50}{4} = -\frac{25}{8}\ A$$

$$I_2 = \frac{17.50 - 40V}{4} = \frac{22.50}{4} = -\frac{45}{8}\ A.$$

* Mesh Analysis *



Find voltage V_x in given figure.

For loop 1 → $60I_1 + 5(I_1 - I_2) + 15V = 0$

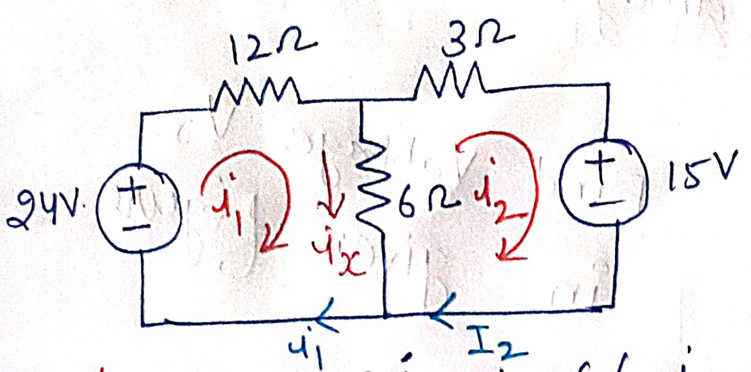
$$\boxed{65I_1 - 5I_2 = -15V} \rightarrow ①$$

For loop 2 → $-15 + 17I_2 - 5I_1 = 0$

$$\boxed{17I_2 - 5I_1 = 15} \rightarrow ②$$

$$\cancel{65I_1} - 5I_2 = -15$$
$$-5I_1 + 17I_2 = 15 \times 13$$
$$\cancel{-65I_1} + 17 \times 13 I_2 = 195$$
$$(221 - 5) I_2 = 195 - 15$$
$$I_2 = \frac{180}{216} = \frac{90}{108} = \frac{45}{54} = \frac{5}{6}$$

*** $\boxed{V_x = I_2 R = I_2 \times 6\Omega = \frac{5}{6} \times 6\Omega = 5V}$



Find current i_x in circuit given.

Loop 1 → $-24 + 12i_1 + 6(i_1 - i_2) = 0$

$$18i_1 - 6i_2 = 24$$

$$\boxed{3i_1 - \cancel{2}\, i_2 = 4} \rightarrow (1)$$

Loop 2 → $6(I_2 - I_1) + 3I_2 + 15 = 0$

$$\boxed{9I_2 - 6I_1 = -15} \rightarrow (2)$$

$$-I_2 + 3i_1 = 4 \quad \times 2 = -2I_2 + 6\cancel{I_1} = 8$$

$$9I_2 - 6\cancel{i_1} = -15$$

$$7I_2 = -7$$

$$\boxed{I_2 = -1}$$

Put Value of I_2 in equation (1)

$$3i_1 + 1 = 4$$

$$3i_1 = 3$$

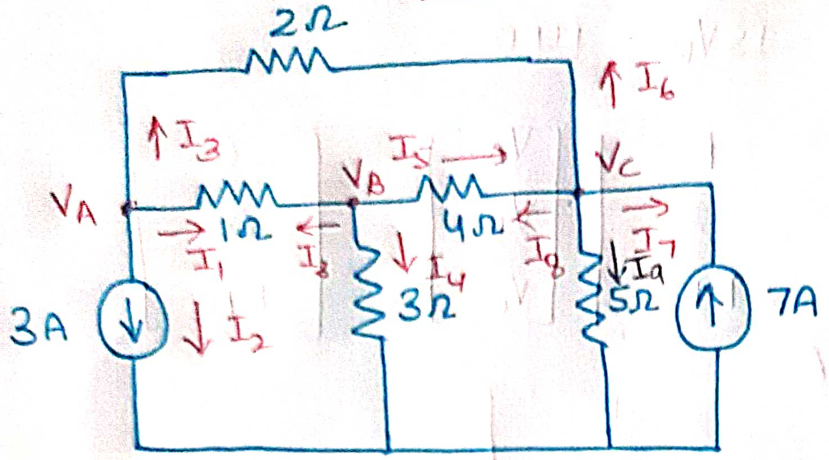
$$\boxed{i_1 = 1}$$

According to loop. $I_x = I_1 - I_2$

$$I_x = 1 - (-1) = 2.$$

$$\boxed{I_x = 2A}$$

TUTORIAL + ASSIGNMENT



At Node V_A →

Apply KCL → $I_1 + I_2 + I_3 = 0$

$$\frac{V_A - V_B}{1} + 3 + \frac{V_A - V_C}{2} = 0$$

$$2V_A - 2V_B + 6 + V_A - V_C = 0$$

$$\boxed{3V_A - 2V_B - V_C = -6} \rightarrow (1)$$

At Node V_B →

$$I_3 + I_4 + I_5 = 0$$

$$\frac{V_B - V_A}{1} + \frac{V_B}{3} + \frac{V_B - V_C}{4} = 0$$

$$12V_B - 12V_A + 4V_B + 3V_B - 3V_C = 0$$

$$\boxed{19V_B - 12V_A - 3V_C = 0} \rightarrow (2)$$

At Node V_C → $\cancel{I_9} + I_7 + I_8 + I_6 = 0$

$$\frac{V_C}{5} + -7 + \frac{V_C - V_B}{4} + \frac{V_C - V_A}{2} = 0$$

$$\frac{4V_C - 140 + 5V_C - 5V_B + 10V_C - 10V_A}{20} = 0$$

$$\boxed{19V_C - 10V_A \cancel{+} 15V_B = 140} \rightarrow (3)$$

$$3V_A - 2V_B - V_C = -6$$
$$19V_B - 12V_A - 3V_C = 0$$
$$19V_C - 10V_A - 15V_B = 140$$

$$\underbrace{\begin{bmatrix} 3 & -2 & 1 \\ -12 & 19 & -3 \\ -10 & -15 & 19 \end{bmatrix}}_{\Delta} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 140 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} -6 & -2 & 1 \\ 0 & 19 & -3 \\ 140 & -15 & 19 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 3 & -6 & 1 \\ -12 & 0 & -3 \\ -10 & 140 & 19 \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} 3 & -2 & -6 \\ -12 & 19 & 0 \\ -10 & -15 & 140 \end{bmatrix}$$

$$V_A = \frac{\Delta_1}{\Delta} \quad V_B = \frac{\Delta_2}{\Delta} \quad V_C = \frac{\Delta_3}{\Delta}.$$

$$\Delta = 136$$

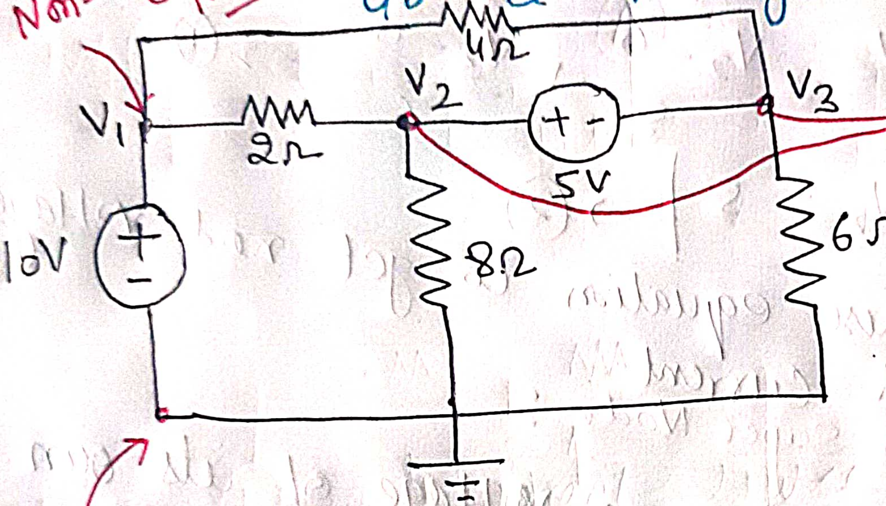
$$V_A = \frac{712}{136} = 5.235A$$

Nodal Analysis with voltage Sources :-

$\rightarrow$ KCL + KVL + ohm's law

Case 1 $\rightarrow$ If voltage source is connected Between the reference node and a Non - reference node, we simply set voltage at non-reference node to a voltage of voltage source.

Non-reference Node



$\rightarrow$ Non-reference node (super Node)

KCL + KVL Both Apply.

$$V_2 - V_3 = 5V$$

$$\boxed{V_2 = V_3 + 5V}$$

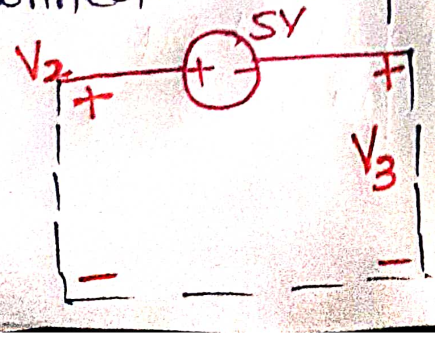
Reference Node

$$\therefore \boxed{V_1 = 10V}$$

Case 2 $\rightarrow$ If voltage source is connected Between two non reference nodes.

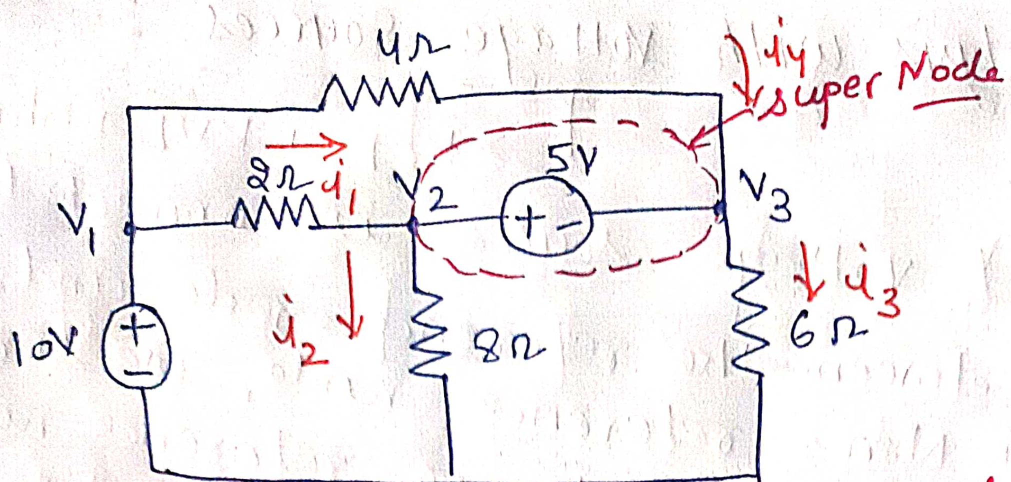
$\rightarrow$ super Node

Super Node :- It is formed by enclosing a voltage source connected between two non-reference nodes and any elements connected in parallel with it.



$$-V_2 + 5V + V_3 = 0$$

$$\boxed{V_2 = V_3 + 5V}$$



$$\boxed{i_1 + i_4 = i_2 + i_3}$$

[2 & 3 Node → a super Node
KCL must satisfy at supernode

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2 - 0}{8} + \frac{V_3 - 0}{6} \rightarrow ①$$

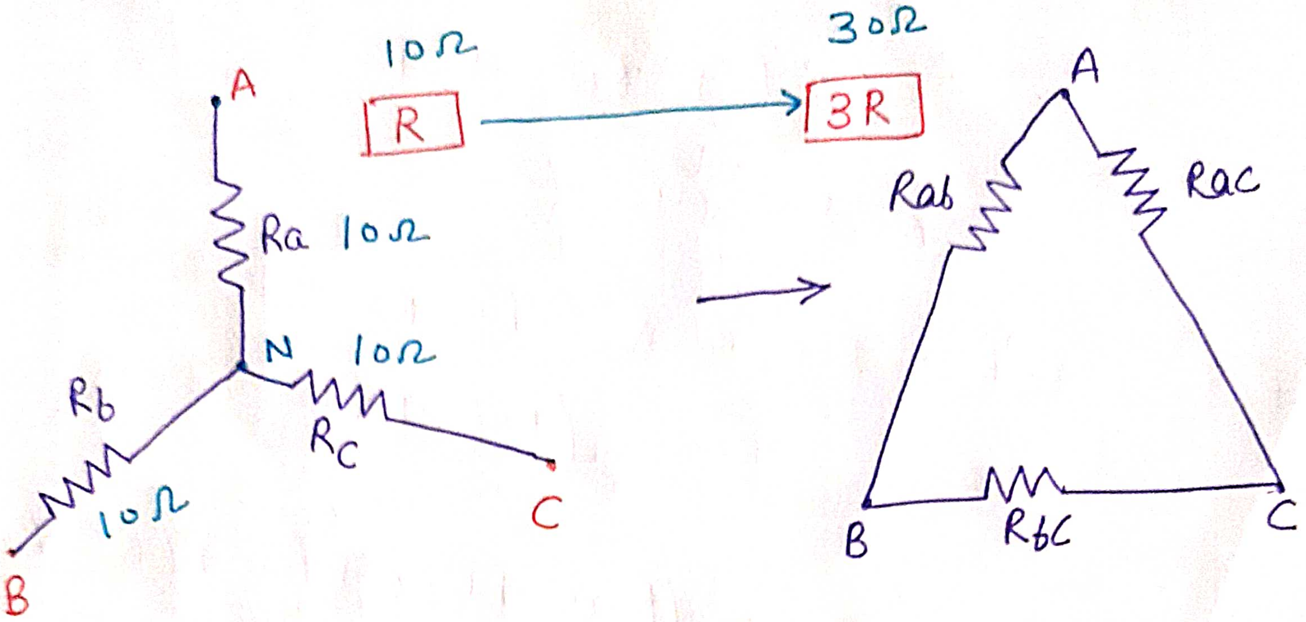
$$\boxed{V_2 - V_3 = 5} \rightarrow ②$$

Solve these equation to get node voltages and branch current.

properties of a Super Node :-

① A super node has no voltage of its own.
② A super node require the application of KCL & KVL.
③ Voltage source inside the super node provides equation needed to solve for Node voltages.

Star to delta Conversion



$$R_{ab} = \frac{(10 \times 10) + 10 \times 10 + 10 \times 10}{10} = 30\Omega$$

$$R_{bc} = R_{ac} = 30\Omega$$

$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{ac} + R_{bc}} \rightarrow (1) \qquad R_b = \frac{R_{bc} \times R_{ab}}{R_{bc} + R_{ab} + R_{ac}} \rightarrow (2)$$

$$R_c = \frac{R_{ab} + R_{bc}}{R_{ab} + R_{ac} + R_{bc}} \rightarrow (3)$$

Multiply equation (1) & (2)

$$R_a \times R_b = \frac{R_{ab}^2 \times R_{ac} \times R_{ac}}{(R_{ab} + R_{bc} + R_{ca})^2} \rightarrow (4)$$

$$R_a \times R_c = \frac{R_{ab} \times R_{bc} \times R_{ac}^2}{(R_{ab} + R_{bc} + R_{ac})^2} \rightarrow (5)$$

$$R_b \times R_c = \frac{R_{ab} \times R_{bc}^2 \times R_{ac}}{(R_{ab} + R_{bc} + R_{ac})^2} \rightarrow ⑥$$

Add ④ - ⑤ - ⑥

$$R_a R_b + R_b R_c + R_c R_a = \frac{R_{ab}^2 R_{bc} R_{ac} + R_{ab} R_{bc}^2 R_{ac} + R_{ab} R_{bc} R_{ca}^2}{(R_{ab} + R_{bc} + R_{ac})^2}$$

$$= \frac{(R_{ab} + R_{bc} + R_{ac})(R_{ab} R_{bc} R_{ca})}{(R_{ab} + R_{bc} + R_{ac})^2}$$

$$= \frac{R_{ab} \times R_{bc} \times R_{ca}}{R_{ab} + R_{bc} + R_{ca}} \rightarrow ⑦$$

R_a

R_b

R_c

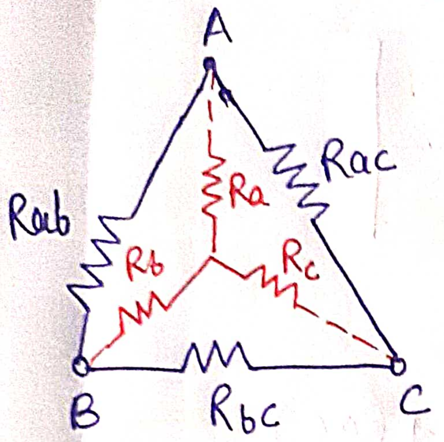
$$R_a R_b + R_b R_c + R_c R_a = R_b \times R_{ca}$$

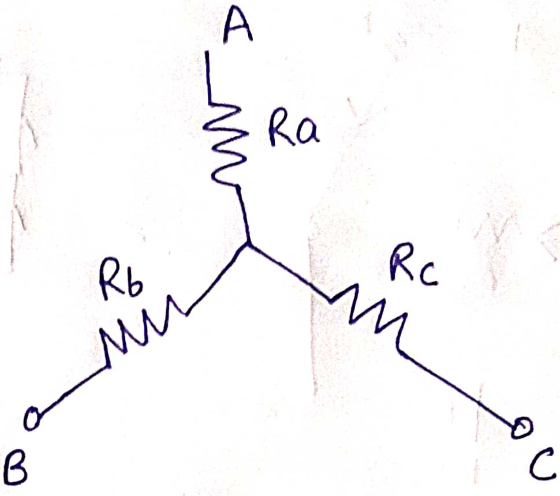
$$\boxed{\frac{R_a R_b + R_b R_c + R_c R_a}{R_b} = R_{ca}} \quad **$$

$$\boxed{\frac{R_a R_b + R_b R_c + R_c R_a}{R_c} = R_{ab}} \quad **$$

$$\boxed{\frac{R_a R_b + R_b R_c + R_c R_a}{R_a} = R_{bc}} \quad **$$

Detla – Star Network



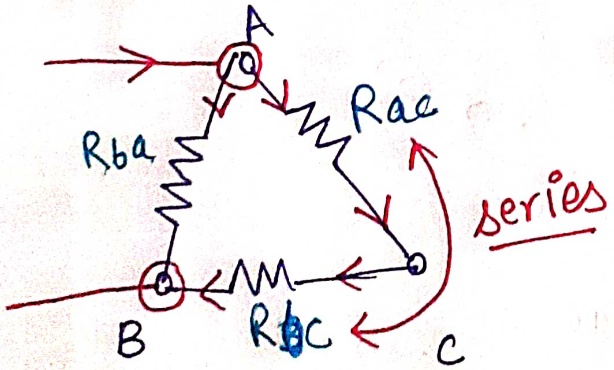


(triange or Δ)

$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{bc} + R_{ac}} \rightarrow \frac{\text{Multiplication of adjacent Resistance}}{\text{Total Resistance } \Delta}$$

$$R_b = \frac{R_{ab} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_c = \frac{R_{ac} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$



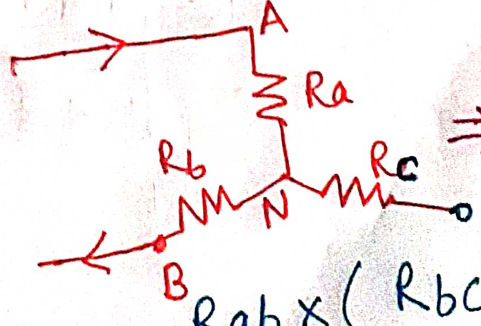
$$R_{AB} = R_{ab} \parallel (R_{ac} + R_{bc})$$

$$R_{AB} = \frac{R_{ab} \times (R_{bc} + R_{ac})}{R_{ab} + R_{ac} + R_{bc}}.$$

$\longrightarrow$ in Δ.

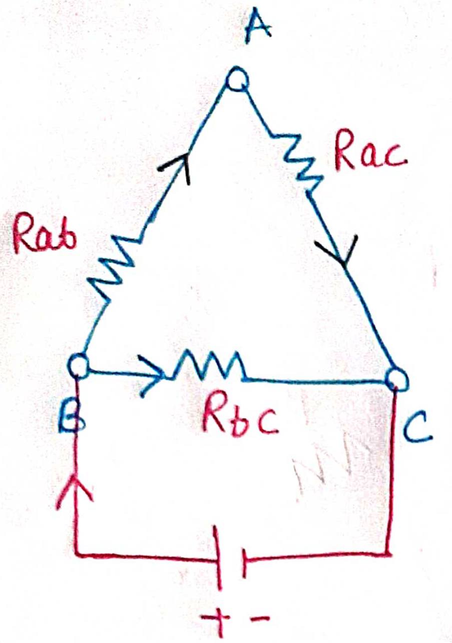
R_{bc} & R_{ac} → series

R_{ab} in parallel with R_{bc} & R_{ac}.

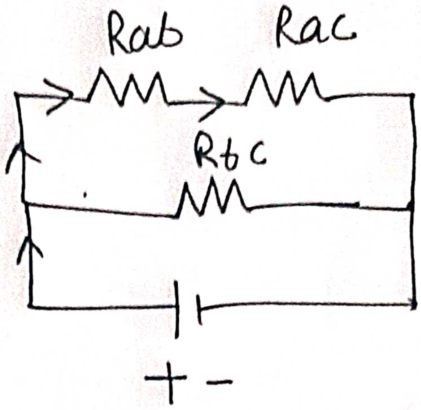


$\Rightarrow R_a + R_b$

Equation ① $\rightarrow$ $\therefore$ $R_a + R_b = \dfrac{R_{ab} \times (R_{bc} + R_{ac})}{R_{ab} + R_{ac} + R_{bc}}$.



$\Rightarrow$



Rab + Rac (series)

$\underline{R_{bc} \parallel (R_{ab} + R_{ac})}$

$$\frac{R_{bc} \times (R_{ab} + R_{ac})}{R_{ab} + R_{bc} + R_{ac}} = R_b + R_c \longrightarrow (2)$$

<u>Simillarly</u> in Between Ra & Rc.

$$\frac{R_{ac} \times (R_{ab} + R_{bc})}{R_{ab} + R_{bc} + R_{ac}} = R_a + R_c \longrightarrow (3)$$

$$R_a + R_b = \frac{(R_{ac} + R_{bc})R_{ab}}{R_{ab} + R_{bc} + R_{ac}} \longrightarrow (1)$$

$$R_b + R_c = \frac{(R_{ac} + R_{ab}) \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}} \longrightarrow (2)$$

$$R_a + R_c = \frac{(R_{ab} + R_{bc}) \times R_{ac}}{R_{ab} + R_{bc} + R_{ca}.} \longrightarrow (3)$$

③ (19)

Adding ③ equations

$$2(R_a + R_b + R_c) = \frac{2(R_{ab} * R_{bc} + R_{bc} \times R_{ac} + R_{ac} \times R_{ab})}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_a + R_b + R_c = \frac{R_{ab} \times R_{bc} + R_{bc} \times R_{ac} + R_{ac} \times R_{ab}}{R_{ab} + R_{bc} + R_{ca}} \longrightarrow (4)$$

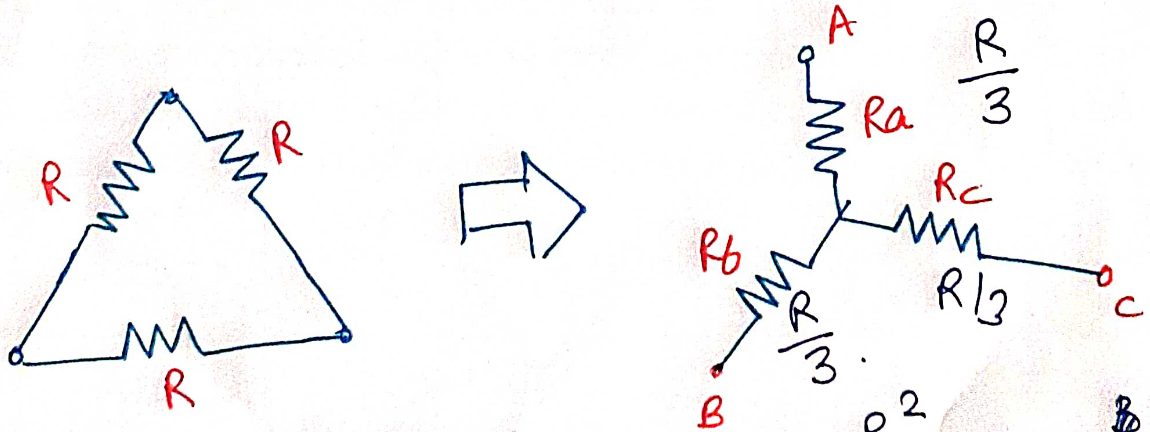
For R_c :- $R_a + R_b + R_c$ — $R_a + R_b$

i.e eq (4) — (1)

$$R_c = \frac{R_{ac} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_b = \frac{R_{bc} + R_{ab}}{R_{ab} + R_{bc} + R_{ca}}.$$



$$R_a = \frac{R \times R}{R + R + R} \Rightarrow \frac{R^2}{3R} = \frac{R}{3}.$$

(IP)

NETWORK THEOREM

- Superposition Theoram
- Thevenin's Theoram
- Norton's Theoram
- Maximum Power Transfer Theoram

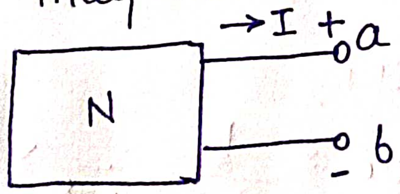
① Thevenin's Theorem →

Statement :- A linear active RLC network which contains one or more Independent or dependent voltage and current sources can be replaced by a single voltage source Voc in series with equivalent Impedance Zeq.

OR

It states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with R_{th}. Where V_{th} is the open circuit voltage at the terminals and R_{th} is the Input or equivalent resistance at the terminals when the independent sources are turned off.

Linear Active Bilateral two port N/W →

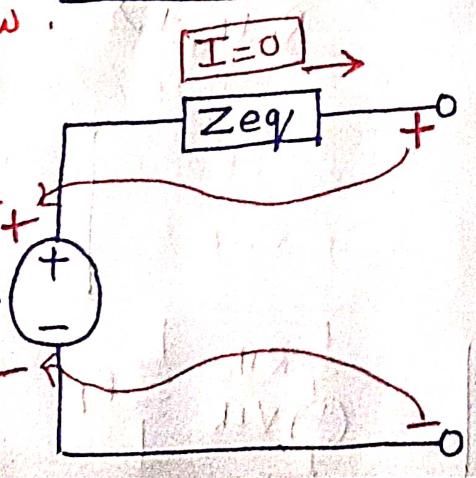


Voc → open circuit voltage B/w a &b.
Zeq → Equivalent Impedance B/W a&b.

When :-

① All Independent Voltage source are Short Circuited.
② All independent current source are open Circuited.
③ All dependent sources remain as they are.

Thevenin Equivalent Model → Voc



Thevenin's equivalent Model

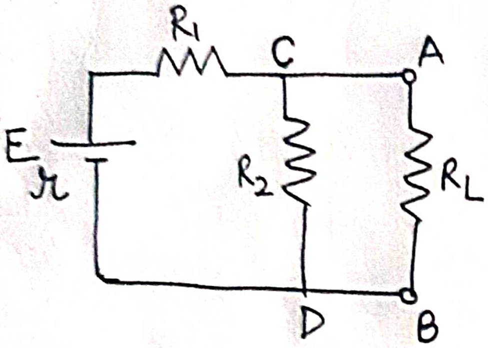
This theorem is not applicable to :-

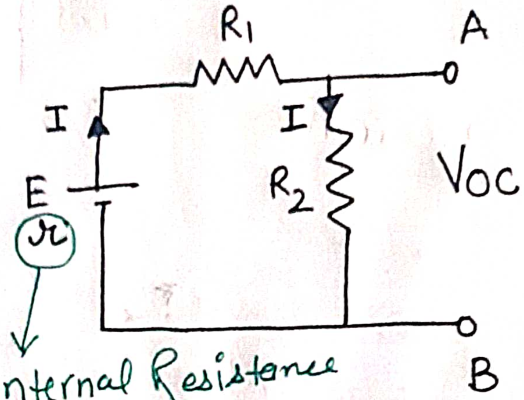
For the Network contains

① Non - linear elements

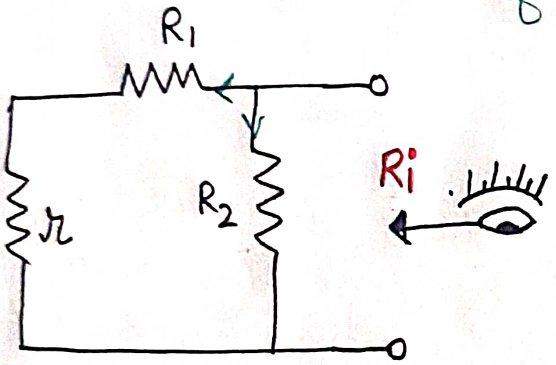
② Unilateral elements

diode circuits.





internal Resistance of battery.



Step ① Suppose we have to find current through R_L.

Remove R_L open circuit terminals A & B. Draw the Circuit again.

Step② → Calculate open circuit voltage Voc which appears across terminals A & B when they are open circuited (i.e R_L is remove)

Voc → drop across R_2 $= IR_2$

$$I = \frac{E}{R_1 + R_2 + r} \qquad \therefore \; V_{oc} = \frac{E}{R_1 + R_2 + r} \times R_2$$

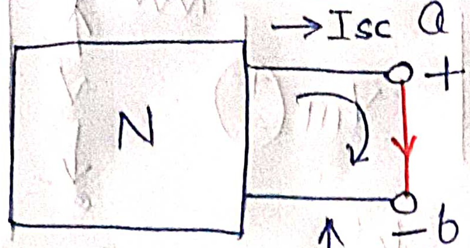
Step③ → Equivalent Resistance $R = R_2 \parallel (R_1 + r)$

$$*** \; \boxed{R = \frac{R_2 (R_1 + r)}{R_2 + R_1 + r}} = R_{th}.$$

Step④ → R_L Connected Back →

Imp → $\boxed{I = \frac{V_{th}}{R_{th} + R_L}}$

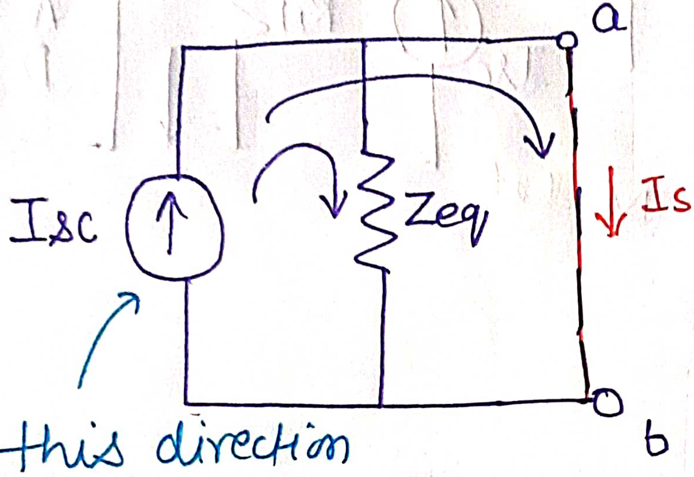
Norton's theorem →



↑ current correct direction

I_{sc} → Short Circuit current between a & b when $\boxed{V = 0}$

Z_{eq} → same as that for thevenin's Model.

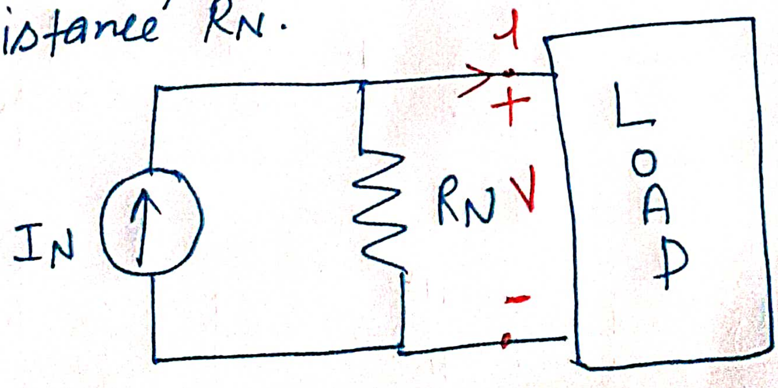


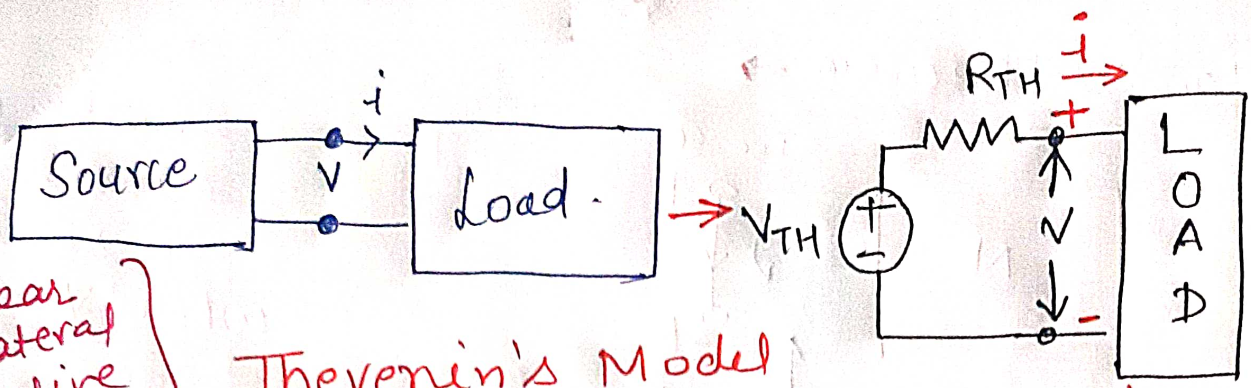
← Norton's Model

this direction of current indicates current through ab.

$$\boxed{Z_{eq} = \frac{V_{oc}}{I_{sc}}}$$

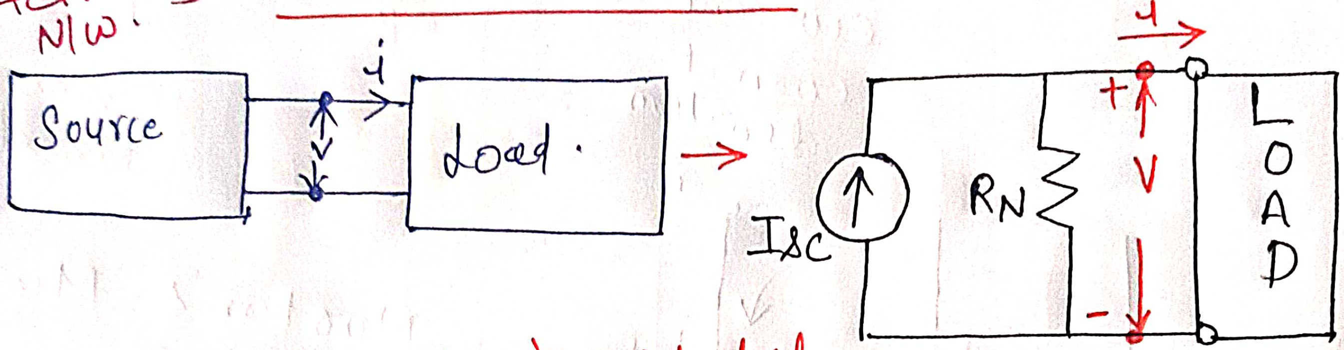
Statement :- When viewed from load, any network composed of ideal voltage and current sources and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source I_N in parallel with an equivalent resistance R_N.





Linear
Bilateral
Active
N/W.

Thevenin's Model



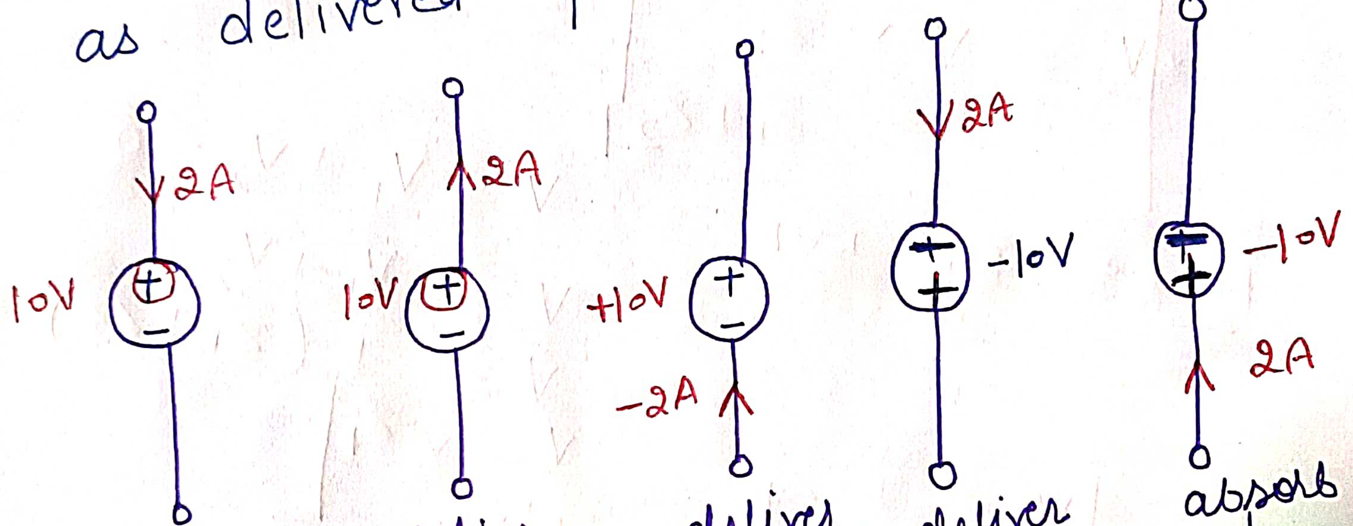
Norton's Model

Concept of power absorbed and Power delivered

→ if current enter into positive terminal of voltage source then it is reffered as absorbed Power

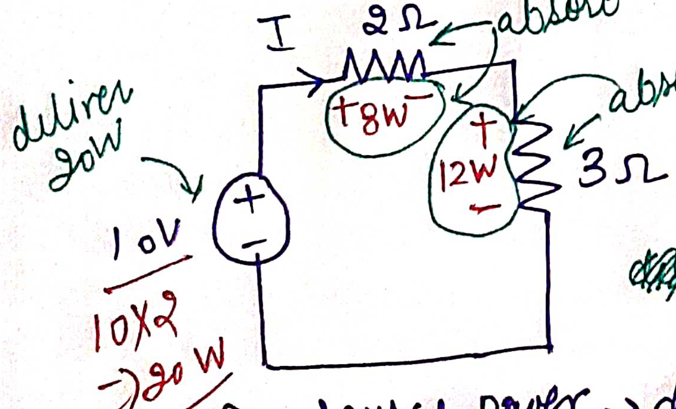
→ if current leaves positive terminal of voltage source then it is reffered as delivered power.

$\boxed{R, L, C}$ ← always absorb power



Absorbed ↓ 20W | deliver ↓ 20W | deliver ↓ −20W | deliver ↓ −20W | absorb ↓ −20W

Note:- Pabs can be written as − Pdeliver
" " − Pabs.
Pdeliver " "



$10\times2 \to 20\,W$

↖ source power → deliver

$I = 10V/5 = 2A$

$V_{2\Omega} = 2\times2 = 4V$

$V_{3\Omega} = 3\times2 = 6V.$

→ $P_{2\Omega} = 4V\times2 = 8W$

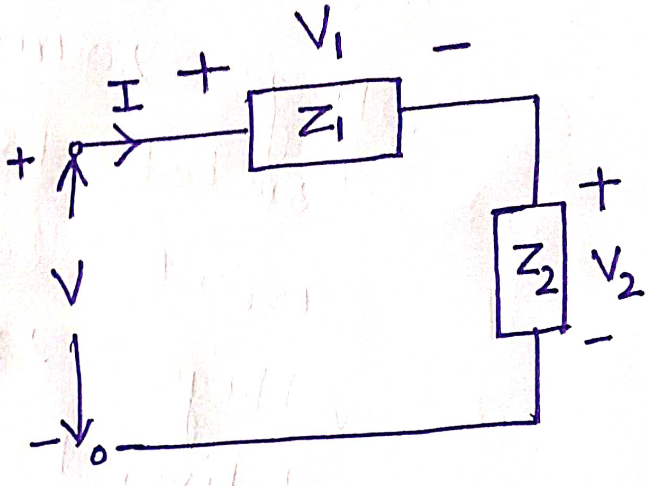
$P_{3\Omega} = 6V\times2 = 12W$

$\boxed{\text{Total Power deliver} = \text{Total power absorb.}}$

DIVISION RULE →

① **Voltage division Rule :-**

↳ use in series equivalent circuit for distribution of voltage.



$$* \; V_1 = V \times \frac{Z_1}{Z_1 + Z_2}$$

$$* \; V_2 = V \times \frac{Z_2}{Z_1 + Z_2}$$

Proof :- By KVL ;

$$-V + V_1 + V_2 = 0$$

$$V = V_1 + V_2$$

$$V = I(Z_1 + Z_2)$$

$$V_1 = I Z_1 = \frac{V}{Z_1 + Z_2} \times Z_1$$

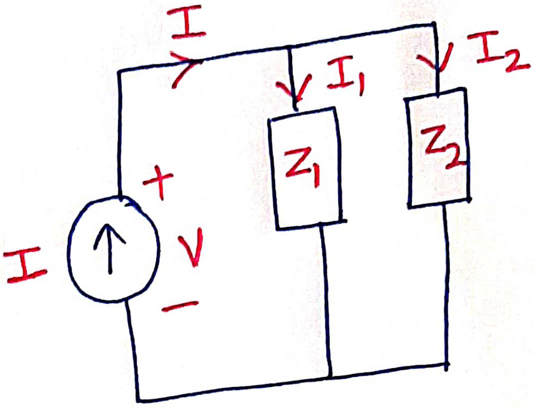
$$V_2 = I Z_2 = \frac{V}{Z_1 + Z_2} \times Z_2 .$$

Current Division Rule

→ It is used in parallel circuit for distribution of current.

** $\boxed{I_1 = \frac{I \times Z_2}{Z_1 + Z_2}}$

** $\boxed{I_2 = \frac{I \times Z_1}{Z_1 + Z_2}}$



Proof :-

$$I = I_1 + I_2$$

$$I = \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$I = V\left[\frac{Z_1 + Z_2}{Z_1 Z_2}\right]$$

*** $\boxed{V = I \times \frac{Z_1 Z_2}{Z_1 + Z_2}}$

** $I_1 = \frac{V}{Z_1} = \frac{I \times \cancel{Z_1} Z_2}{Z_1 + Z_2} \times \frac{1}{\cancel{Z_1}} = \frac{I \times Z_2}{Z_1 + Z_2}$

** $\boxed{I_2 = \frac{V}{Z_2} = \frac{I \times Z_1}{Z_1 + Z_2}}$.