Discrete Structure
First Sessional
Solution

Section-A

Q1. Attempt all parts.

a. State pigeonhole principle with suitable example.

Ans: **Pigeon hole principle state that** If nk+1 objects are placed in n boxes, then one of the boxes must contain at least k+1 objects.

Examples

- In a class of 13 students, at least two must be born in the same month. *Here, the 13 students are "pigeons" and the 12 months are "pigeonholes".*

b. Let A = {1,2,3}. Find the power set of A

Ans: Total number of subset are = 2^3

= 8

Hence, P(A) = {∅,{1},{2},{3},{1,2},{2,3},{1,3},{1,2,3}}

c. Write dual of (U∩A)∪(B∩A)=A.

Ans: (φ∪A)∩(B∪A)=A

(d. If f(x) = x^2 and g(x) = 2x+1. Find fog(x).

Ans: fog(x) = (2x+1)^2

Section-B

Q2. Attempt all parts.

a. Let A = {a,b,c,d} and B = {a,b,l,m}. Find A ⊕ B

Ans: **Symmetric Difference of Sets:** The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by A ⊕ B i.e.

A⊕B = (A∪B)-(A∩B)

Example: Let A = {a, b, c, d} B = {a, b, l, m} A ⊕ B = {c, d, l, m}

b. Find the Domain, Co-Domain, and Range of function.

Let x={1,2,3,4}

y={a,b,c,d,e}

f={(1,b),(2,a),(3,a),(4,c)

Ans: Domain of function: {1, 2, 3, 4}

Range of function: {a, b, c, d}

Co-Domain of function: {a, b, c, d, e}

Section-C

Q3. Answer any two of the following.

a. If a set A = {1, 2}. Determine all relations from A to A. Is relation AXA an equivalence relation?

Ans: There are $2^2 = 4$ elements i.e., {(1, 1), (1, 2), (2, 1), (2, 2)} in A x A. So, there are $2^4 = 16$ relations from A to A. i.e.

∅,

{(1, 1)}, {(1, 2)}, {(2, 1)},{(2, 2)} ,

{(1, 1), (1, 2)}, {(1, 1), (2, 1)}, {(1, 1), (2, 2)},

{(1, 2), (2, 1)}, {(1, 2), (2, 2)}, {(2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1)}, {(1, 1), (1, 2), (2, 2)},

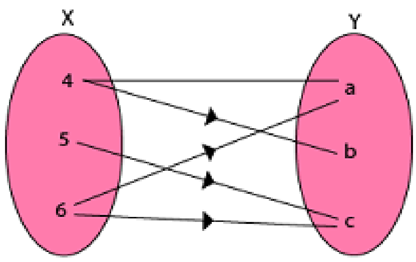
{(1, 1), (2, 1), (2, 2)}, {(1, 2), (2, 1), (2, 2)},

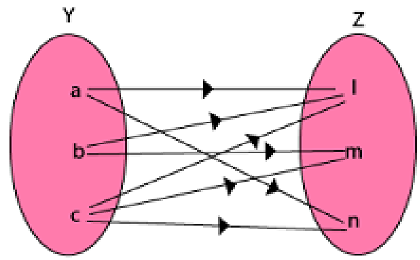
{(1, 1), (1, 2), (2, 1), (2, 2)}.

b. Let X = {4, 5, 6}, Y = {a, b, c} and Z = {l, m, n}. Consider the relation R_1 from X to Y and R_2 from Y to Z. Find the composition of relation (i) R_1 o R_2 (ii) R_1o R_1^{-1}

Ans: So let R_1 = {(4, a), (4, b), (5, c), (6, a), (6, c)}

R_2 = {(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)}





(i) The composition relation R_1 o R_2 as shown in fig:

R_1 o R_2 ={(4, l), (4, n), (4, m), (5, l), (5, m), (5, n), (6, l), (6, m), (6, n)}

(ii) The composition relation R_1o R_1^{-1}as shown in fig:

R_1o R_1^{-1} = {(4, 4), (5, 5), (5, 6), (6, 4), (6, 5), (4, 6), (6, 6)}

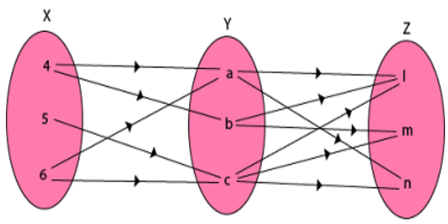


Fig : R_1 o R_2

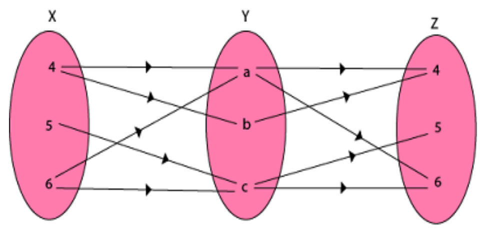


Fig : R_1 o R_1^{-1}

C.. Ans: i) Let $P = (A \cup B)'$ and $Q = A' \cap B'$Let x be an arbitrary element of P then $x \in P \Rightarrow x \in (A \cup B)' \Rightarrow x \notin (A \cup B) \Rightarrow x \notin A$ and $x \notin B \Rightarrow x \in A'$ and $x \in B' \Rightarrow x \in A' \cap B' \Rightarrow x \in Q$
Therefore, $P \subset Q$ …………….. (i)
Again, let y be an arbitrary element of Q then $y \in Q \Rightarrow y \in A' \cap B' \Rightarrow y \in A'$ and $y \in B' \Rightarrow y \notin A$ and $y \notin B \Rightarrow y \notin (A \cup B) \Rightarrow y \in (A \cup B)' \Rightarrow y \in P$
Therefore, $Q \subset P$ …………….. (ii)
Now combine (i) and (ii) we get; $P = Q$ i.e. $(A \cup B)' = A' \cap B'$

ii). Proof of De Morgan's law: $(A \cap B)' = A' \cup B'$

Let $M = (A \cap B)'$ and $N = A' \cup B'$

Let x be an arbitrary element of M then $x \in M \Rightarrow x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

$\Rightarrow x \in N$

Therefore, $M \subset N$ …………….. (i)

Again, let y be an arbitrary element of N then $y \in N \Rightarrow y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow y \in M$

Therefore, $N \subset M$ …………….. (ii)

Now combine (i) and (ii) we get; $M = N$ i.e. $(A \cap B)' = A' \cup B'$

Q4

A ans: Let us assume that $\sqrt{2}$ is a rational number.So it can be expressed in the form p/q where p, q are co-prime integers and $q \neq 0$

$\sqrt{2} = p/q$

Here p and q are coprime numbers and $q \neq 0$

Solving

$\sqrt{2} = p/q$

On squaring both the sides we get,

$\Rightarrow 2 = (p/q)^2$

$\Rightarrow 2q^2 = p^2$……………………………..(1)

$p^2/2 = q^2$

So 2 divides p and p is a multiple of 2.

$\Rightarrow p = 2m$

$\Rightarrow p^2 = 4m^2$ ………………………………..(2)

From equations (1) and (2), we get,

$2q^2 = 4m^2$

$\Rightarrow q^2 = 2m^2$

$\Rightarrow q^2$ is a multiple of 2

$\Rightarrow$ q is a multiple of 2

Hence, p, q have a common factor 2. This contradicts our assumption that they are co-primes. Therefore, p/q is not a rational number

$\sqrt{2}$ is an irrational number.

4b: (I) R ∪ Δ is the smallest relation having reflexive property, Hence R = R ∪ Δ = {(k, k), (k, l), (l, l), (l, m), (m, m), (m, k)}

(ii). For symmetric closure we know that R∪R**-1

So here R = {(k,k),(k,l),(l,m),(m,k)} and

R**-1 = {(k,k),(l,k),(m,l),(k,m)}

Therefore R∪R**-1 ={(k,k),(k,l),(l,m),(m,k),((l,k),(m,l),(k,m)}

(iii) for transitive closure R*= R∪R**2∪R**3

R= { (k,k),(k,l),(l,m),(m,k)}

R**2 = {(k,k),(l,k),(m,k),(k,l),(m,l),(k,m)}

R***3 = {(k,k),(l,k),(m,k),(k,l),(l,l),(m,l),(k,m),(m,m)}

So R* =R ∪ R**2∪R**3

={(K,k),(k,l),(k,m),(l,k),(l,l),(l,m),(m,k),(m,l),(m,m)}

Q5.

Solution

A)). : The characteristics equation is s^2-4s+4=0 or $(s-2)^2$=0 s = 2, 2

Therefore, the homogeneous solution of the equation is given by $a_{r(n)}=(C_1+C_2r).2^r$………….. equation (i)

Putting r = 0 and r = 1 in equation (i), we get $a_0=(C_1+0).2^0 = 1 \therefore C_1=1$

$a_1=(C_1+C_2).2=6 \therefore C_1+C_2=3 \Rightarrow C_2=2$

Hence, the particular solution is $a_{r(P)}=(1+2r).2^r$.

B. Solution : Find its characteristic equation

$$r^3 + 3r^2 + 3r + 1 = 0$$

$$(r + 1)^3 = 0 \quad \text{then } r_1 = -1$$

Then an = $(A + B\,n + Cn^2)(-1)^n$ is a solution.

Now we should find constants using initial conditions. And find the value of A, B and C

A = 1 B = 3 C = -2

Then put the value of A, B and C in above equation

$a_n = (1 + 3n - 2n^2)\,(-1)^n$ is a solution.