1. $r^+ \cup r^*$

$\Rightarrow a^+ \cup a^*$

$= \{a^1, a^2 \ldots\} \cup \{\varepsilon, a^1\, a^2\, a^3 \ldots\}$

$= r^*$

2 $r^+ \cap r^*$

$= \{a^1\, a^2, a^3 \ldots\} \cap \{\varepsilon, a^1\, a^2\, a^3 \ldots\}$

$= r^+$

3. $r^* . r^+ = \{a^1\, a^2 a^3 \ldots\} . (\varepsilon . a^1\, a^2 \ldots\}$

$= \{a . aa, aaa \ldots\}$

$= r^+$

4. $(r^*)^* = \{a^0\, a^1\, a^2 \ldots\}^*$

$= \{\varepsilon\ \varepsilon\ a^1\ a^2 \ldots\}$

$= r^*$

5. $(r^*)^+ = \{\varepsilon, a, aa . aaa \ldots\}^+$

$= r^*$

6 $(r^+)^* = r^*$

7 $((r^*)^+)^* . r^+ = r^* . r^+ = r^+$

$\{r^+ + t^+)^* = r^*$

$(r^+)^{**+*} = r^*$

8. $(a+b)^* = (a^* + b^*)^*$

9 $(a+b)^* = (a+b^*)^* =$

10 $(a+b)^* = (a^* + b^*)^*$

ε a aa

$(r^+)^r$

01

$(1/a)$

11a1

⇒ But one RE can generate only one langu[age]
Same language we get diff. R.E

Two RE are equal when they have same languages

ϕ = It is a set that is null/Empty set. No element { }.

ε = It is a symbol which represent null string/Empty string. This is not a set it is string which length is 0.

$r = \varepsilon^*$ $\quad L(r) = \{\varepsilon^0, \varepsilon^1, \varepsilon^2, \varepsilon^3 \ldots\} \Rightarrow \{\varepsilon\}$.
$= \{\varepsilon . \varepsilon \ldots\}$
set have no duplicacy so.

$r = \varepsilon^+$ $\quad L(r) = \{\varepsilon^1 \varepsilon^2, \varepsilon^3 \ldots\}$
$= \{\varepsilon\}$

$r = \phi^*$ $\quad L(r) = \{\phi^0 \phi^1 \phi^2 \ldots\}$
$\{\varepsilon\} = \varepsilon$

$r = \phi^1$ $\quad L(r) = \{\phi^1 \phi^2 \ldots\} = \{\}$
$= \phi$

$(a+b)^* \neq (a \cdot b)^*$

$(a+b)^* \neq (a^* \, b)^*$

13. $(a+b)^* \neq (a \cdot b^*)^*$

14. $(a+b)^* = (a^* \cdot b^*)^*$

$(\epsilon, a, aa)(\epsilon, b, bb)$, bba, ba

(ϵ, a, b, ab) (ϵ, b, bba, a)

Q1. Start with ab

$ab(a+b)^*$

Q2. Start with bba

$bba(a+b)^*$

Q3. ends → abb $(a+b)^* abb$.

Q4. contain a substring aab

$(a+b)^* aab (a+b)^*$

Q5. start & ends a

$a + a(a+b)^* a$

Q6. " " same symbol

$a(a+b)^* a + b(a+b)^* b$

$+ a + b$

7. diff symbol .

$a(a+b)^* b \quad + \quad b(a+b)^* a$

8. $|w| = 3$. aaa, aba, bba ....

~~aaa~~

$(a+b)(a+b)(a+b) \rightarrow (a+b)^3.$

9. $|w| \geq 3$ $\quad (a+b)(a+b)(a+b).(a+b)^*$

$(a+b)^3 . (a+b)^*$

10. $|w| \leq 3$.

$\cancel{(a+b)^* (a+b)^* (a+b)^*}$

$(\varepsilon, (a+b)\ (a+b)^2\ (a+b)^3)$

$\{\varepsilon, a, b, ab, aa, ba, bb, aaa, aba, abb, \ldots\}$

$(\varepsilon+a+b)^{\cancel{*}\,3}$

$(\varepsilon+a+b)(\varepsilon+a+b)(\varepsilon+a+b)$

11. $|w|_a = 2$ No. of a excally two

$$b^* a b^* a b^*$$

12 $|w|_a \geqslant 2$

$$(a+b)^* a (a+b)^* a (a+b)^*$$

13. $|w|_a \leqslant 2$

$$b^* (a+\varepsilon) b^* (a+\varepsilon) b^*$$

OR $b^* + b^* a b^* + b^* a b^* a b^*$

14. 3^{rd} symbol from left end is b.

$$(a+b)^2 b (a+b)^*$$

15 28^{th} symbol from right end is a

$$(a+b)^* a (a+b)^{27}$$

16. $|w| = 0 \pmod 3$ string length divide by 3 then reminder is zero.

0, 3, 6, 9 --

$RE = ((a+b)^3)^*$

2, 5, 8, 11 - -

17 $|w| = 2 \pmod{3.}$

$$(a+b)^2 \cdot [(a+b)^3]^*$$

18 $|w|_b = 0 \pmod 2$

No. of b divisible by 2

$$a^* + (a^* b a^* b a^*)^*$$

19 $|w|_a = 1 \pmod 3$

$$b^* a b^* (b^* a b^* a b^* a b^*)^*$$

20 $|w|_b = 2 \pmod 3$

~~b~~

$$a^* b a^* b a^* (\cancel{b^* a b^* a b^* a b^*}) (a^* b a^* b a^* b)^*$$

Q.

$[(a+b)^3]^*$ $(a+b)^2$

$3 + 2$

Prove $(1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$

$= 0^*1(0+10^*1)^*$

$(1+00^*1)[\Lambda + (0+10^*1)^*(0+10^*1)]$

~~$(1+00^*1)[0+10^*1)$~~

$(1+00^*1)(0+10^*1)^*$

$1(\Lambda+00^*)(0+10^*1)^*$

$0^*1(0+10^*1)^*$