Q1) ~~Solve by Crouts method~~

$x+y-z=2;\ 2x+3y+5z=-3;\ 3x+2y-3z=6$

Sol^n:

$$\begin{bmatrix}1&1&-1\\2&3&5\\3&2&-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-3\\6\end{bmatrix}\qquad AX=B \quad (1)$$

$$A=LU \quad (2)$$

$$\begin{bmatrix}1&1&-1\\2&3&5\\3&2&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix}\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\u_{11}l_{21}&u_{12}l_{21}+u_{22}&u_{13}l_{21}+u_{23}\\u_{11}l_{31}&u_{12}l_{31}+u_{22}l_{32}&u_{13}l_{31}+u_{23}l_{32}+u_{33}\end{bmatrix}$$

$u_{11}=1$	$u_{11}l_{21}=2$	$u_{13}l_{21}+u_{23}=5$
$u_{12}=1$	$l_{21}=2$	$-2+u_{23}=5$
$u_{13}=-1$	$u_{12}l_{21}+u_{22}=3$	$u_{23}=7$
$l_{21}=2$	$2+u_{22}=3$	$u_{11}l_{31}=3$
$u_{22}=1$	$u_{22}=1$	$l_{31}=3$
$u_{23}=7$	$u_{12}l_{31}+u_{22}l_{32}=2$	$u_{13}l_{31}+u_{23}l_{32}+u_{33}=-3$
$l_{31}=3$	$3+l_{32}=2$	$-3-7+u_{33}=-3$
$l_{32}=-1$	$l_{32}=-1$	$-10+u_{33}=-3$
$u_{33}=7$		$u_{33}=7$

Now,

$$L=\begin{bmatrix}1&0&0\\2&1&0\\3&-1&1\end{bmatrix};\quad U=\begin{bmatrix}1&1&-1\\0&1&7\\0&0&7\end{bmatrix}$$

$LUX=B$
let, $UX=Y$ $\Big\}$ $LY=B$

$$\begin{bmatrix}1&0&0\\2&1&0\\3&-1&1\end{bmatrix}\begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix}=\begin{bmatrix}2\\-3\\6\end{bmatrix}$$

$\boxed{y_1=2}$

$2y_1+y_2=-3$

$\boxed{y_2=-7}$

$3y_1-y_2+y_3=6$

$6+7+y_3=6$

$\boxed{y_3=-7}$

from, $UX=Y$

$$\begin{bmatrix}1&1&-1\\0&1&7\\0&0&7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-7\\-7\end{bmatrix}$$

$x+y-z=2$

$y+7z=-7$

$7z=-7$

$\boxed{z=-1}$

$y-7=-7$

$\boxed{y=0}$

$x+y-z=2$

$x+0+1=2$

$\boxed{x=1}$

$x=1$
$y=0$
$z=-1$

2Q ~~Solve the given system by Gauss Seidel method.~~

$10x+2y+z=9$
$2x+20y-2z=-44$
$2x+3y+10z=22$

Soln:

$$x=\frac{1}{10}(9-2y-z) \quad -①$$
$$y=\frac{1}{20}(-44+2z-2x) \quad -②$$
$$z=\frac{1}{10}(22-2x-3y) \quad -③$$

First approximation.

Put $z=y=0$ into ①

$x=\frac{9}{10}=0.9$

Now, Put $x=0.9$ & $z=0$ into ②

$$y=\frac{1}{20}(-44-2\times0.9)$$
$$=\frac{1}{20}(-44-1.8)$$
$$=\frac{-45.8}{20}=-2.29$$

Now, Put $x=0.9, y=-2.29$ into ③

$$z=\frac{1}{10}(22-1.8+3\times2.29)$$
$$z=\frac{1}{10}(22-1.8+6.87)$$
$$=\frac{27.07}{10}=2.707$$

Second approximation.

Put $y=-2.29$ and $z=2.707$ into ①

$$x=\frac{1}{10}(9+2\times2.29-2.707)=1.0873$$

Put $x=1.0873$, $z=2.707$ into ②

$$y=\frac{1}{20}(-44+2\times2.707-2\times1.0873)$$
$$y=-2.03803$$

Put $x=1.0873, y=-2.04$ into ③

$$z=\frac{1}{10}(22-2\times1.0873+3\times2.04)$$
$$=2.59$$

Third approximation.

Similarly,

$$x=\frac{1}{10}(9+2\times2.04-2.59)=1.05$$
$$y=\frac{1}{20}(-44+2\times2.59-2\times1.05)=-2.05$$
$$z=\frac{1}{10}(22-2\times1.05+3\times2.05)=2.605$$

fourth approximation:-

Similarly,

$$x=\frac{1}{10}(9+2\times2.05-2.605)=1.05$$
$$y=\frac{1}{20}(-44+2\times2.605-2\times1.05)=-2.0445$$
$$z=\frac{1}{10}(22-2\times1.05+3\times2.044)$$
$$=2.603$$

$$\boxed{\begin{array}{l} x=1.05 \\ y=-2.044 \\ z=2.603 \end{array}}$$

Q3) Find LU decomposition & solve the system.

$3x+y+z=1$
$3x+5y+z=3$
$6x+y+11z=-3$

Soln:-

$$\begin{bmatrix} 3 & 1 & 1 \\ 3 & 5 & 1 \\ 6 & 1 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

$AX=B$ —①
$A=LU$ —②

$$\begin{bmatrix} 3 & 1 & 1 \\ 3 & 5 & 1 \\ 6 & 1 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{11}l_{21} & u_{12}l_{21}+u_{22} & u_{13}l_{21}+u_{23} \\ u_{11}l_{31} & u_{12}l_{31}+u_{22}l_{32} & u_{13}l_{31}+u_{23}l_{32}+u_{33} \end{bmatrix}$$

$u_{11}=3$
$u_{12}=1$
$u_{13}=1$
$l_{21}=1$
$u_{22}=4$
$u_{23}=0$
$l_{31}=2$
$l_{32}=-1/4$
$u_{33}=9$

$u_{11}l_{21}=3$
$l_{21}=1$
$u_{12}l_{21}+u_{22}=5$
$1+u_{22}=5$
$u_{13}l_{21}+u_{23}=1$
$1+u_{23}=1$

$u_{11}l_{31}=6$
$l_{31}=2$
$u_{12}l_{31}+u_{22}l_{32}=1$
$2+4l_{32}=1$
$l_{32}=-1/4$

$u_{13}l_{31}+u_{23}l_{32}+u_{33}=11$
$1\times2+0+u_{33}=11$
$u_{33}=11-2=9$

Now,

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -1/4 & 1 \end{bmatrix};\quad U=\begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$LUX=B$
let, $UX=Y$
$LY=B$.

$LY=B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -1/4 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

$y_1=1$
$y_1+y_2=3$
$y_2=2$
$2y_1-\frac{1}{4}y_2+y_3=-3$
$2-\frac{1}{2}+y_3=-3$
$4-1+2y_3=-6$
$3+2y_3=-6$
$2y_3=-9$
$y_3=\frac{-9}{2}=-4.5$

$$\begin{bmatrix} 3 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -9/2 \end{bmatrix}$$

$3x+y+z=1$
$4y=2$
$9z=-9/2$

$3x+\frac{1}{2}-\frac{1}{2}=1$

$z=-\frac{1}{2}$; $y=\frac{1}{2}$; $x=\frac{1}{3}$.

4. Q) Solve the System of linear Eq^n by Gauss Seidel method. (4)

$83x + 11y - 4z = 95$
$7x + 52y + 13z = 104$
$3x + 8y + 29z = 71$

$x = \frac{1}{83}(95 + 4z - 11y)$
$y = \frac{1}{52}(104 - 13z - 7x)$
$z = \frac{1}{29}(71 - 8y - 3x)$

First approximation;
Put $y = 0 = z$ into (1)
$x = \frac{95}{83} = 1.145$

Put $x = 1.145$ and $z = 0$ into (2)
$y = \frac{1}{52}(104 - 7 \times 1.145)$
$= \frac{95.985}{52} = 1.846$

Put $x = 1.145$ and $y = 1.846$ into (3)
$z = \frac{1}{29}(71 - 8 \times 1.846 - 3 \times 1.145)$
$= \frac{52.797}{29} = 1.821$

Second approximation;
Put $y = 1.846$ and $z = 1.821$ into (1)
$x = \frac{1}{83}(95 + 4 \times 1.821 - 11 \times 1.846)$
$= \frac{81.978}{83} = 0.988$

$y = \frac{1}{52}(104 - 13 \times \underset{(z)}{1.821} - 7 \times \underset{(x)}{0.988})$
$= 1.412$

$z = \frac{1}{29}(71 - 8 \times \underset{(y)}{1.412} - 3 \times \underset{(x)}{0.988})$
$= 1.956$

Similarly,
Third approximation:-
Put $z = 1.956$ & $y = 1.412$ in (1)
$x = \frac{1}{83}(95 + 4 \times 1.956 - 11 \times 1.412)$
$= 1.051$

Put $z = 1.956$ & $x = 1.051$ in (2)
$y = \frac{1}{52}(104 - 13 \times 1.956 - 7 \times 1.051)$
$= 1.344$

Put $y = 1.344$ & $x = 1.051$ in (3)
$z = \frac{1}{29}(71 - 8 \times 1.344 - 3 \times 1.051)$
$= 1.969$

Similarly, fourth approximation:-
Put $z = 1.969$, $y = 1.344$ in (1)
$x = \frac{1}{83}(95 + 4 \times 1.969 - 11 \times 1.344) = 1.051$

$y = \frac{1}{52}(104 - 13 \times 1.969 - 7 \times 1.051)$
$= 1.344$

$z = \frac{1}{29}(71 - 8 \times 1.344 - 3 \times 1.051)$
$= 1.969$

Here,
$$\boxed{\begin{aligned} x &= 1.051 \\ y &= 1.344 \\ z &= 1.969 \end{aligned}}$$

Q5.) Find the lowest degree Polynomial $f(x)$ that will fit the data.

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x	0	2	4	6	8
$f(x)$	5	9	61	209	501

Also find $y(5)$.

Solⁿ:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0=0$	$y_0=5$				
		$\Delta y_0=4$			
2	9		$\Delta^2 y_0=48$		
		$\Delta y_1=52$		$\Delta^3 y_0=48$	
4	61		$\Delta^2 y_1=96$		$\Delta^4 y_0=0$
		$\Delta y_2=148$		$\Delta^3 y_1=48$	
6	209		$\Delta^2 y_2=144$		
		$\Delta y_3=292$			
8	501				

Here, $x_0=0,\ y_0=5,\ h=4-2=2$

$$u=\frac{x-x_0}{h}=\frac{x-0}{2}=\frac{x}{2}$$

By Newton forward difference interpolation method

$$f(x)=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$f(x)=5+\frac{x}{2}.4+\frac{u(u-1)}{2}.48+\frac{u(u-1)(u-2)}{6}.48+\frac{u(u-1)(u-2)(u-3)}{24}.0$$

$$=5+2x+24u(u-1)+8u(u-1)(u-2)$$

$$=5+2x+24.\frac{x}{2}\left(\frac{x}{2}-1\right)+8.\frac{x}{2}\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)$$

$$=5+2x+12x\left(\frac{x}{2}-1\right)+4x\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)$$ (Evaluate this Eqⁿ for Polynomial degree (3))

$$f(5)=5+10+60\left(\frac{3}{2}\right)+20\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$=5+10+90+15$$

$$=20+10+90=120$$

Teacher's Signature :

Q6) Find Cubic Polynomial.

X	0	1	2	3
Y	1	2	1	10

Sol:

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
$x_0=0$	$y_0=1$			
1	2	$\Delta y_0=1$	$\Delta^2 y_0=-2$	$\Delta^3 y_0=12$
2	1	$\Delta y_1=-1$	$\Delta^2 y_1=10$	
3	10	$\Delta y_2=9$		

Here, $x_0=0,\ y_0=1,\ h=1,\ u=\frac{x-0}{1}=x$

Apply Newton forward Diff. interpolation.

$$f(x)=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$f(x)=1+x\cdot1+\frac{x(x-1)}{2}(-2)+\frac{x(x-1)(x-2)}{6}(12)$$

$$f(x)=2x^3-7x^2+6x+1$$

Q7) Find the Cubic Polynomial interpolation.
$f_0=5,\ f_1=1,\ f_2=9,\ f_3=25,\ f_4=55$, find f_5

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0=0$	$5=y_0$	$\Delta y_0=-4$	$\Delta^2 y_0=12$	$\Delta^3 y_0=-4$	$\Delta^4 y_0=10$
1	1	$\Delta y_1=8$	$\Delta^2 y_1=8$	$\Delta^3 y_1=6$	
2	9	$\Delta y_2=16$	$\Delta^2 y_2=14$		
3	25	$\Delta y_3=30$			
4	55				

$f(5)=114.99.$

$x_0=0,\ y_0=5,\ h=1,\ u=\frac{x-0}{1}=x$

$$f(x)=y_0+u\Delta y_0+\frac{u(u-1)}{2}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$f(x)=5+x(-4)+\frac{x(x-1)}{2}(12)+\frac{x(x-1)(x-2)}{6}(-4)+\frac{x(x-1)(x-2)(x-3)}{24}(10)$$

$$=5-4x+6(x^2-x)-\frac{2}{3}(x^3-3x^2+2x)+\frac{5}{12}(x^4-6x^3+11x^2-6x)$$

$$=5-4x+6x^2-6x-\frac{2x^3}{3}+2x^2-\frac{4}{3}x+\frac{5}{12}x^4-\frac{5}{2}x^3+\frac{55}{12}x^2-\frac{5}{2}x$$

$$=\frac{5}{12}x^4+\left(-\frac{2}{3}-\frac{5}{2}\right)x^3+\left(6+2+\frac{55}{12}\right)x^2+\left(-10-\frac{4}{3}-\frac{5}{2}\right)x+5$$

$$\boxed{f(x)=\frac{5}{12}x^4-\frac{19}{6}x^3+\frac{151}{12}x^2-\frac{83}{6}x+5}$$

Q8.) Apply Runge kutta method to find an approximate value of y where $x=0.2$ given that

$\frac{dy}{dx} = x+y$; $y=1$ and $x=0$

Solⁿ. $f(x,y) = x+y$

$x_0 = 0$; $y_0 = 1$

find $y(0.2) = ?$

$h = 0.2$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where, $k_1 = hf(x_0, y_0)$

$k_1 = 0.2(0+1) = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_2 = hf(0+0.1, 1+0.1)$

$= hf(0.1, 1.1)$

$= 0.2(0.1+1.1)$

$= 0.2(1.2)$

$= 0.24$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= hf(0+0.1, 1+0.12)$

$= hf(0.1, 1.12)$

$= 0.2(0.1+1.12)$

$= 0.244$

$k_4 = hf\left(x_0 + \frac{h}{2}, y_0 + k_3\right)$

$= hf(0+0.1, 1+0.244)$

$= hf(0.1, 1.244)$

$= 0.2(0.1+1.244)$

$= 0.2 \times 1.344$

$= 0.2688$

$y_1 = y(0.2) = 1 + \frac{1}{6}(0.2 + 2\times0.24 + 2\times0.244 + 0.2688)$

$= 1 + \frac{1}{6}(1.4368) = 1.2395$

Q9.) Apply Runge kutta method; find y when $x=0.2$, given that $\frac{dy}{dx} = x+y^2$, $y=1$ when $x=0$.

Solⁿ: $f(x,y) = x+y^2$; $x_0 = 0$, $y_0 = 1$

find $y(0.2) = ?$

$h = 0.2$

$y_1 = y(0.2) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Teacher's Signature :

$K_1 = hf(x_0, y_0) = hf(0,1)$
$= 0.2(0+1^2) = 0.2\times 1$
$= 0.2$

$K_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$
$= 0.2f(0+0.1, 1+0.1)$
$= 0.2f(0.1, 1.1)$
$= 0.2\left(0.1+(1.1)^2\right)$
$= 0.262$

$K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$
$= 0.2f\left(0.1, 1+\frac{0.262}{2}\right) = 0.2f(0.1, 1.131)$
$= 0.2\left(0.1+(1.131)^2\right) = 0.2\times 1.3792$
$k_3 = 0.27584$

$k_4 = hf(0+h, y_0+k_3) = 0.2f(0+0.2, 1+0.2758)$
$= 0.2f(0.2, 1.2758) = 0.2\left(0.2+(1.2758)^2\right)$
$= 0.2(0.2+1.6277) = 0.3655$

$y_1 = y(0.2) = 1+\frac{1}{6}(0.2+2\times 0.262+2\times 0.27584+0.3655)$
$= 1+0.27353 = 1.27353$

Q10) Calculate an approximate value of $\int_0^{\pi/2} \sin x\,dx$; using (1) Trapezoidal rule
(2) Simpsons rule using 11 ordinates.

Sol^n

$h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} = \frac{\pi}{10}$ (Let $n=5$)

$x_n = x_0+nh$	$y_n = \sin x_n$
$x_0 = 0$	$y_0 = 0$
$x_1 = x_0+h = \frac{\pi}{2n} = \frac{\pi}{10}$	$y_1 = 0.309$
$x_2 = x_0+h = \frac{\pi}{5}$	$y_2 = 0.5877$
$x_3 = \frac{3\pi}{10}$	$y_3 = 0.809$
$x_4 = \frac{2\pi}{5}$	$y_4 = 0.951$
$x_5 = \frac{\pi}{2}$	$y_5 = 1$

$\int_0^{\pi/2} \sin x\,dx = \frac{h}{2}\left[y_0+2(y_1+y_2+\dots+y_{n-1})+y_n\right]$
$= \frac{\pi}{20}\left[y_0+y_5+2(y_1+y_2+y_3+y_4)\right]$
$= \frac{\pi}{20}\left[0+1+2(0.309+0.5877+0.809+0.951)\right]$
$= \frac{\pi}{20}\left[1+2(2.6567)\right] = \frac{\pi}{20}[6.3134]$
$= 0.15707\times 6.3134$
$= \cancel{0.9917}\ 0.991706$

Q12) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$rd rule and Simpson's $\frac{3}{8}$th rule.

Sol^n: Let $n=6$

$h = \frac{6-0}{6} = 1$

$x_n = x_0 + nh$	$y_n = \frac{1}{1+x_n^2}$
$x_0 = 0$	$y_0 = 1$
$x_1 = 1$	$y_1 = 0.5$
$x_2 = 2$	$y_2 = 0.2$
$x_3 = 3$	$y_3 = 0.1$
$x_4 = 4$	$y_4 = 0.0588$
$x_5 = 5$	$y_5 = 0.0385$
$x_6 = 6$	$y_6 = 0.027$

By using Simpson's $\frac{1}{3}$rd Rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.0385)+2(0.2+0.0588)\right]$$

$$= \underline{1.3662}$$

By Simpson's $\frac{3}{8}$th rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{8}\left[(1+0.027)+3(0.5+0.2+0.0588+0.0385)+2\times 0.1\right]$$

$$= \underline{1.3571}$$

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Q1. Solve NLPP.

Optimize $Z = x_1^2 + x_2^2 + x_3^2$

s.t. $x_1 + x_2 + 3x_3 = 2$

$5x_1 + 2x_2 + x_3 = 5$

$x_1, x_2, x_3 \geq 0$

Soln: $L(x_1, x_2, x_3, \lambda_1, \lambda_2) = Z(x_1, x_2, x_3) - \lambda_1 h_1(x_1, x_2, x_3) - \lambda_2 h_2(x_1, x_2, x_3)$

$= x_1^2 + x_2^2 + x_3^2 - \lambda_1(x_1 + x_2 + 3x_3 - 2) - \lambda_2(5x_1 + 2x_2 + x_3 - 5)$

$\frac{\partial L}{\partial x_1} = 2x_1 - \lambda_1 - 5\lambda_2 = 0$ | $x_1 = \frac{5\lambda_2 + \lambda_1}{2}$

$\frac{\partial L}{\partial x_2} = 2x_2 - \lambda_1 - 2\lambda_2 = 0$ | $x_2 = \frac{2\lambda_2 + \lambda_1}{2}$

$\frac{\partial L}{\partial x_3} = 2x_3 - 3\lambda_1 - \lambda_2 = 0$ | $x_3 = \frac{3\lambda_1 + \lambda_2}{2}$

$\frac{\partial L}{\partial \lambda_1} = x_1 + x_2 + 3x_3 = 2$

$\frac{\partial L}{\partial \lambda_2} = 5x_1 + 2x_2 + x_3 = 5$

$5\lambda_2 + \lambda_1 + 2\lambda_2 + \lambda_1 + 3(3\lambda_1 + \lambda_2) = 4$

$10\lambda_2 + 11\lambda_1 = 4$ —— (1)

$5(5\lambda_2 + \lambda_1) + 2(2\lambda_2 + \lambda_1) + (3\lambda_1 + \lambda_2) = 10$

$25\lambda_2 + 5\lambda_1 + 4\lambda_2 + 2\lambda_1 + 3\lambda_1 + \lambda_2 = 10$

$30\lambda_2 + 10\lambda_1 = 10$ —— (2)

$(10\lambda_2 + 11\lambda_1 = 4) \times 3$

$30\lambda_2 + 10\lambda_1 = 10$

$\cancel{30\lambda_2} + 33\lambda_1 = 12$

$\cancel{30\lambda_2} + 10\lambda_1 = 10$

$23\lambda_1 = 2 \Rightarrow \boxed{\lambda_1 = \frac{2}{23}}$

$10\lambda_2 = 4 - \frac{22}{23}$

$\cancel{10}\lambda_2 = \frac{7\cancel{0}}{23}$

$\boxed{\lambda_2 = \frac{7}{23}}$

$x_1 = \frac{\frac{35}{23} + \frac{2}{23}}{2} = \frac{37}{46} = 0.804$

$x_2 = \frac{\frac{14}{23} + \frac{2}{23}}{2} = \frac{16}{46} = 0.3478$

$x_3 = \frac{\frac{6}{23} + \frac{7}{23}}{2} = \frac{13}{46} = 0.283$

$H_1 = 2 > 0$

$H_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$

$H_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 8 > 0$

Teacher's Signature : The Point is minima

Q7: $f(x) = 5x_1 + 2x_2^2 + x_3^2 - 3x_3x_4 + 4x_4^2 + 2x_5^4 + 3x_5x_6 + 6x_6^2 + 3x_6x_7 + x_7^2$ 2

$f'(x) = \frac{\partial f}{\partial x_1} = 5$

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Q4) NLPP Problem.

Expt. No. Optimize $Z = 6x_1x_2 - 10x_3$

S.t. $2x_1 + x_2 + 3x_3 = 10$

$x_1, x_2, x_3 \geq 0$

Sol^n^:

$L(x_1, x_2, x_3, \lambda) = Z(x_1, x_2, x_3) - \lambda h_1(2x_1 + x_2 + 3x_3 - 10)$

$6x_1x_2 - 10x_3 - \lambda(2x_1 + x_2 + 3x_3 - 10)$

$\frac{\partial L}{\partial x_1} = 6x_2 - 2\lambda = 0$

$\frac{\partial L}{\partial x_2} = 6x_1 - \lambda = 0$

$\frac{\partial L}{\partial x_3} = -10 - 3\lambda = 0$

$\frac{\partial L}{\partial \lambda} = 2x_1 + x_2 + 3x_3 = 10$

$10 + 3\lambda = 0$

$\lambda = \frac{-10}{3}$

$x_2 = \frac{-\cancel{20}^{10}}{\cancel{18}_9} = \frac{-10}{9}$

$x_1 = \frac{-10}{18}$

$\frac{-\cancel{20}^{10}}{\cancel{18}_9} - \frac{10}{9} + 3x_3 = 10$

$\Rightarrow \frac{-20}{9} + 3x_3 = 10$

$\Rightarrow -20 + 27x_3 = 90$

$27x_3 = 110$

$x_3 = \frac{110}{27}$

$H_1 = 0$; $H_2 = \begin{vmatrix} 0 & 6 \\ 6 & 0 \end{vmatrix} = -36$; $H_3 = \begin{vmatrix} 0 & 6 & 0 \\ 6 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

Can't say about the Point.

Q5) NLPP.

Optimize $Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$

S.t. $x_1 + x_2 + x_3 = 7$.

$x_1, x_2, x_3 \geq 0$

Sol^n^:

$L(x_1, x_2, x_3, \lambda) = Z(x_1, x_2, x_3) - \lambda h(x_1 + x_2 + x_3 - 7)$

$x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3 - \lambda(x_1 + x_2 + x_3 - 7)$

$\frac{\partial L}{\partial x_1} = 2x_1 - 10 - \lambda = 0$

$\frac{\partial L}{\partial x_2} = 2x_2 - 6 - \lambda = 0$

$\frac{\partial L}{\partial x_3} = 2x_3 - 4 - \lambda = 0$

$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 = 7$

$x_1 = \frac{10+\lambda}{2}$, $x_2 = \frac{6+\lambda}{2}$, $x_3 = \frac{4+\lambda}{2}$

$\frac{10+\lambda}{2} + \frac{6+\lambda}{2} + \frac{4+\lambda}{2} = 7$

$20 + 3\lambda = 14$

$3\lambda = -6$

$\boxed{\lambda = -2}$

$x_1 = \frac{10-2}{2} = 4$

$x_2 = \frac{4}{2} = 2$

$x_3 = \frac{\cancel{2}}{\cancel{2}} = 1$

Teacher's Signature : ______

$H_1 = |2|$ $H_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = |4|$ $H_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 8$

$H_1 > 0$; $H_2 > 0$; $H_3 > 0$ Point is minima. $(4, 2, 1)$

Minimum value of $Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$.

$= 4^2 - 10 \times 4 + 4 - 12 + 1 - 4$

$= 16 - 40 + 4 - 12 + 1 - 4$

$= 21 - 56 = -35$

Q6). Optimize $Z = 4x_1 + 9x_2 - x_1^2 - x_2^2$

s.t. $4x_1 + 3x_2 = 15$

$3x_1 + 5x_2 = 14$

$x_1, x_2 \geq 0$

Sol^n:

$L(x_1, x_2, \lambda_1, \lambda_2) = Z(x_1, x_2) - \lambda_1 h_1(4x_1 + 3x_2 - 15) - \lambda_2 h_2(3x_1 + 5x_2 - 14)$

$= 4x_1 + 9x_2 - x_1^2 - x_2^2 - \lambda_1(4x_1 + 3x_2 - 15) - \lambda_2(3x_1 + 5x_2 - 14)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - 4\lambda_1 - 3\lambda_2 = 0$

$\frac{\partial L}{\partial x_2} = 9 - 2x_2 - 3\lambda_1 - 5\lambda_2 = 0$

$\frac{\partial L}{\partial \lambda_1} = 4x_1 + 3x_2 = 15$

$\frac{\partial L}{\partial \lambda_2} = 3x_1 + 5x_2 = 14$

$x_1 = \frac{4 - 4\lambda_1 - 3\lambda_2}{2}$, $x_2 = \frac{9 - 3\lambda_1 - 5\lambda_2}{2}$

$4\left(\frac{4 - 4\lambda_1 - 3\lambda_2}{2}\right) + 3\left(\frac{9 - 3\lambda_1 - 5\lambda_2}{2}\right) = 15$

$16 - 16\lambda_1 - 12\lambda_2 + 27 - 9\lambda_1 - 15\lambda_2 = 30$

$16 - 25\lambda_1 - 27\lambda_2 + 27 = 30$

$25\lambda_1 + 27\lambda_2 = 13$

$= 3\left(\frac{4 - 4\lambda_1 - 3\lambda_2}{2}\right) + 5\left(\frac{9 - 3\lambda_1 - 5\lambda_2}{2}\right) = 14$

$= 12 - 12\lambda_1 - 9\lambda_2 + 45 - 15\lambda_1 - 25\lambda_2 = 28$

$= -27\lambda_1 - 34\lambda_2 = -27 \Rightarrow 27\lambda_1 + 34\lambda_2 = 27$

$\lambda_1 = -2.37$

$\lambda_2 = 2.67$

$x_1 = 3.33$

$x_2 = 1.38$

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Q 7). NLPP.

Optimize $Z = 4x_1 - 4x_1x_2 + x_3^2 + 2x_2^2$

s.t. $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0$

Sol:

$L(x_1, x_2, x_3, \lambda_1, \lambda_2) = Z(x_1, x_2, x_3) - \lambda_1 h_1 (x_1 + x_2 + x_3 - 15) - \lambda_2 h_2 (2x_1 - x_2 + 2x_3 - 20)$

$= 4x_1 - 4x_1x_2 + x_3^2 + 2x_2^2 - \lambda_1(x_1 + x_2 + x_3 - 15) - \lambda_2(2x_1 - x_2 + 2x_3 - 20)$

$\frac{\partial L}{\partial x_1} = 4 - 4x_2 - \lambda_1 - 2\lambda_2 = 0$ — (1)

$\frac{\partial L}{\partial x_2} = -4x_1 + 4x_2 - \lambda_1 + \lambda_2 = 0$ — (2)

$\frac{\partial L}{\partial x_3} = 2x_3 - \lambda_1 - 2\lambda_2 = 0$ — (3)

$\frac{\partial L}{\partial \lambda_1} = x_1 + x_2 + x_3 = 15$ — (4)

$\frac{\partial L}{\partial \lambda_2} = 2x_1 - x_2 + 2x_3 = 20$ — (5)

from (1) & (2)

(1) + (2)

$4 - 4x_1 - 2\lambda_1 - \lambda_2 = 0$

$x_1 = \frac{4 - 2\lambda_1 - \lambda_2}{4}$

$x_2 = \frac{4 - \lambda_1 - 2\lambda_2}{4}$

$x_3 = \frac{\lambda_1 + 2\lambda_2}{2}$

$\lambda_1 = -10.3$

$\lambda_2 = 10.78$

$x_1 = 6.91$

$x_2 = -1.81$

$x_3 = 0.24$

$\frac{4 - 2\lambda_1 - \lambda_2}{4} + \frac{4 - 4\lambda_1 - 2\lambda_2}{4} + \frac{2\lambda_1 + 4\lambda_2}{4} = 15$

$4 - \cancel{2\lambda_1} - \lambda_2 + 4 - 4\lambda_1 - 2\lambda_2 + \cancel{2\lambda_1} + 4\lambda_2 = 60$

$8 - 4\lambda_1 + \lambda_2 = 60$

$\lambda_2 - 4\lambda_1 = 52$

$2\left(\frac{4 - 2\lambda_1 - \lambda_2}{4}\right) - \left(\frac{4 - \lambda_1 - 2\lambda_2}{4}\right) + 2\left(\frac{\lambda_1 + 2\lambda_2}{2}\right) = 20$

$8 - \cancel{4\lambda_1} - \cancel{2\lambda_2} - 4 + \lambda_1 + \cancel{2\lambda_2} + \cancel{4\lambda_1} + 8\lambda_2 = 80$

$4 + \lambda_1 + 8\lambda_2 = 80$

$\lambda_1 + 8\lambda_2 = 76$

$H_1 = |0|$; $H_2 = \begin{vmatrix} 0 & -4 \\ -4 & 4 \end{vmatrix} = -16$; $H_3 = \begin{vmatrix} 0 & -4 & 0 \\ -4 & 4 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 4(-8) = -32$

Teacher's Signature :

Q. NLPP.

Q) Optimize $Z = kx^{-1}y^{-2}$ (6)

S.t. $x^2+y^2=3$

$x, y, \lambda, 0$

$L(x, y, \lambda_1) = Z(x,y) - \lambda h(x^2+y^2-3)$

$= kx^{-1}y^{-2} - \lambda(x^2+y^2-3)$

$\frac{\partial L}{\partial x} = -ky^{-2}x^{-2} - 2x\lambda = 0$

$\frac{\partial L}{\partial y} = -kx^{-1}y^{-3} - 2y\lambda = 0 \Big) \frac{y}{x}$

$\frac{\partial L}{\partial \lambda} = x^2+y^2=3$

$\cancel{-ky^{-2}x^{-2}} - 2x\lambda = 0$

$\cancel{-ky^{-2}x^{-2}} - 2\frac{y^2}{x}\lambda = 0$

$+ \quad +$

$\frac{2y^2}{x}\lambda - 2x\lambda = 0$

$2y^2\lambda - 2x^2\lambda = 0$

$\cancel{2}y^2\lambda - \cancel{2}x^2\lambda = 0$

$\lambda(y^2 - x^2) = 0$

$\lambda = 0, \quad y^2 = x^2, \quad 2x^2 = 3$

$x = \sqrt{\frac{3}{2}} = y$

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Q9) NLPP.

Expt. No. Optimize $Z = x_1^2 + x_2^2 + x_3^2$ Page No. ________

St: $4x_1 + x_2^2 + 2x_3 = 14$

$x_1, x_2, x_3 \geq 0$

Solⁿ: $L(x_1, x_2, x_3, \lambda) = Z(x_1, x_2, x_3) - \lambda . h(4x_1 + x_2^2 + 2x_3 - 14)$

$x_1^2 + x_2^2 + x_3^2 - \lambda(4x_1 + x_2^2 + 2x_3 - 14)$

$\frac{\partial L}{\partial x_1} = 2x_1 - 4\lambda = 0$ | $\frac{\partial L}{\partial \lambda} = 4x_1 + x_2^2 + 2x_3 = 14$

$\frac{\partial L}{\partial x_2} = 2x_2 - 2\lambda x_2 = 0$ | $x_1 = \frac{\cancel{4}^2\lambda}{\cancel{2}} = 2\lambda$; | $x_2(2 - 2\lambda) = 0$, $x_2 = 0$

$\frac{\partial L}{\partial x_3} = 2x_3 - 2\lambda = 0$ | $x_3 = \lambda$

$4(2\lambda) + (0)^2 + 2\lambda = 14$

$8\lambda + 2\lambda = 14$

$10\lambda = 14$

$\lambda = \frac{\cancel{14}7}{\cancel{10}5} = \frac{7}{5}$

$x_1 = \frac{14}{5}$, $x_2 = 0$, $x_3 = \frac{7}{5}$

$H_1 = |2|$

$H_2 = \begin{vmatrix} 2 & 0 \\ 0 & 2-2\lambda \end{vmatrix} = 2(2 - 2 \times \frac{7}{5}) = 2(2 - \frac{14}{5}) = -\frac{8}{5}$

$H_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & -\frac{8}{5} & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2\left(\frac{-16}{5}\right) = -\frac{32}{5}$

$H_1 > 0$, $H_2 < 0$, $H_3 < 0$

Teacher's Signature : ________