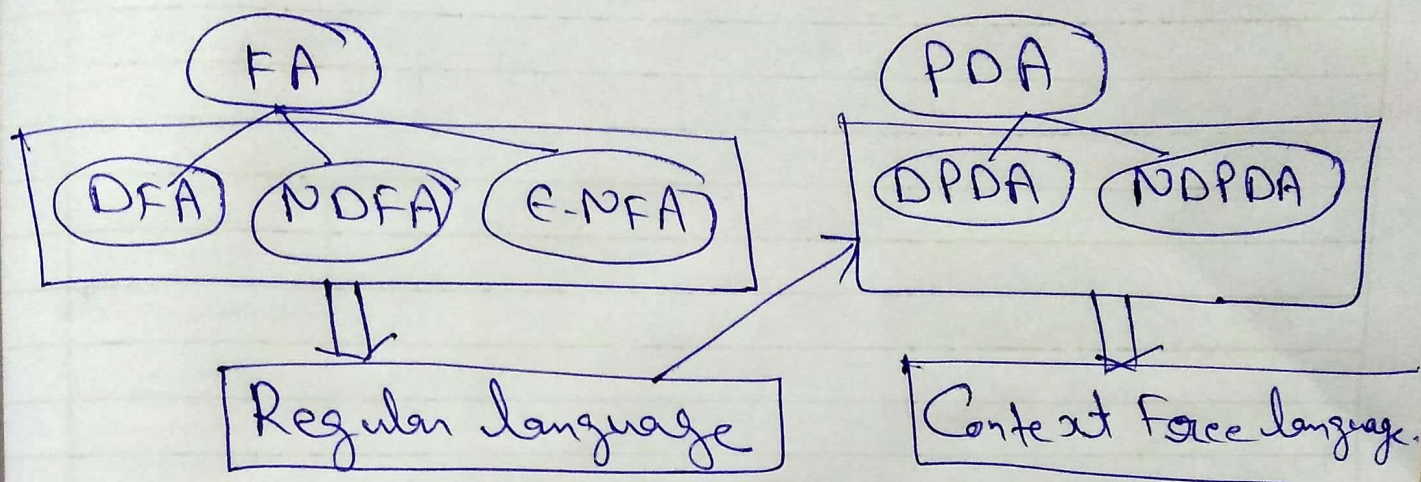


# (Push Down Automata) (PDA)

PDA :- A PDA is a F.A with extra memory called stack which helps PDA to recognize CFL.



- \* PDA is more powerful than F.A
- \* R.L also accept PDA. PDA is a

$$\boxed{\text{PDA} = \text{FA} + \text{Stack}}$$

A PDA has three Components

- 1) an Input tape
- 2) a Control Unit
- 3) a Stack with infinite size.

A Stack has two operation.

- 1) PUSH :- a new symbol is added at the Top
- 2) POP :- The top symbol is read and removed.



\* A PDA may or may not be read input symbol, but it has to read the top of the stack in every transition. ②

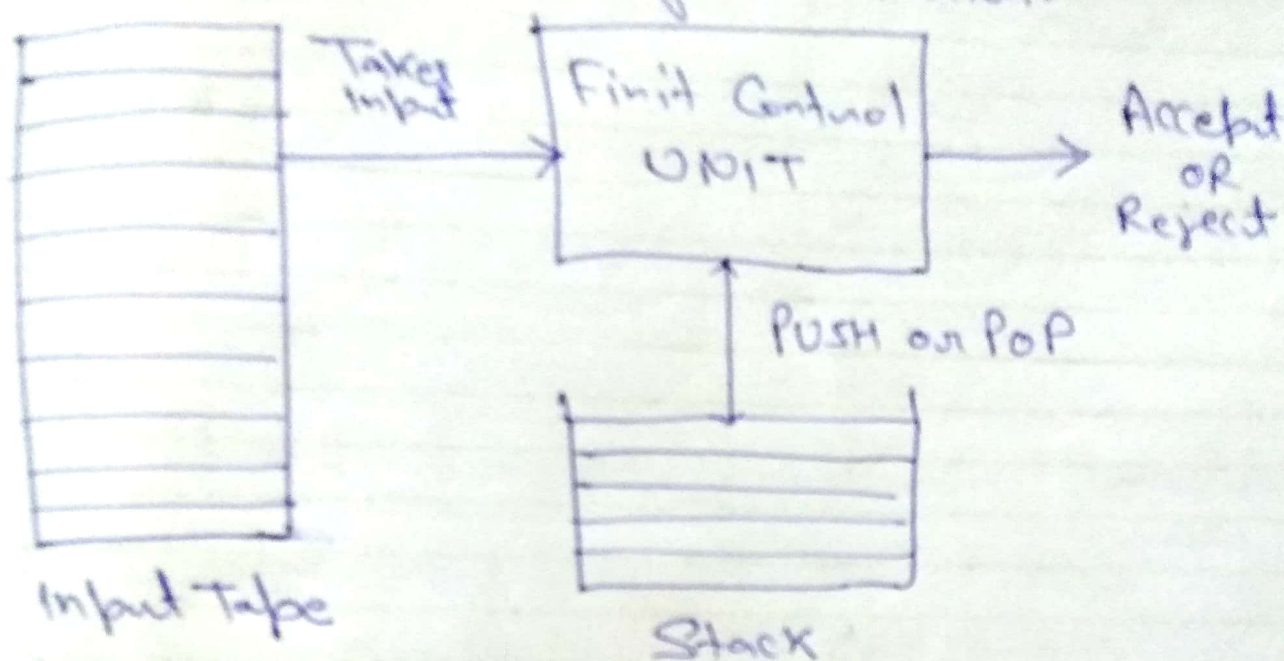


Fig. Structure of PDA

A PDA can be described as 7-tuple

$$PDA = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$Q$  = finite no. of states

$\Sigma$  = Input symbols

$\Gamma$  = Stack symbols

$\delta$  = Transition function  $Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow Q \times \Gamma^*$

$q_0$  = Initial state ( $q_0 \in Q$ )

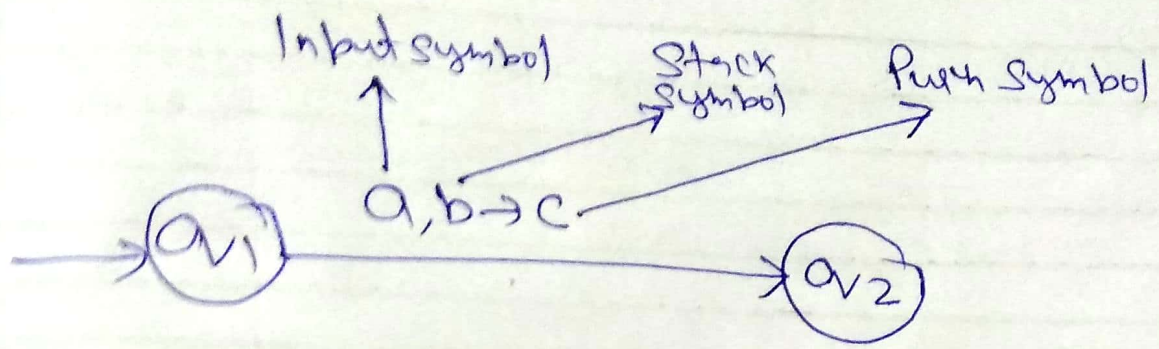
$Z_0$  = initial stack symbol

$F$  = accepting states ( $F \subseteq Q$ )



(3)

The following diagram shows a transition in a PDA from a state  $q_1$  to  $q_2$  labeled as  $a, b \rightarrow c$ :



This means at state  $q_1$ , if we encounter an input 'a' and top symbol of the stack is b, then we pop 'b', push 'c' on top of the stack and move to state  $q_2$ .

Transition Function:

$$\delta(q_0, a, z_0) \rightarrow q_1, az_0$$

\* Turnstile Notation :

\* It is denoted by  $(\vdash)$

\* It is represent one or ~~more~~ many moves of PDA. It is called process of transition.



Ex 1: Let PDA  $(Q, \Sigma, S, \delta, q_0, F)$  (PDA) (4)  
 $Q = (q_0, q_1, q_f)$ ,  $\Sigma = \{a, b\}$ ,  $S = \{q, z_0\}$   
 $F = \{q_f\}$

$\delta(q_0, a, z_0) = \{(q_0, az_0)\}$   
 $\delta(q_0, a, a) = \{(q_0, aa)\}$   
 $\delta(q_0, b, a) = \{(q_1, \wedge)\}$   
 $\delta(q_0, b, \wedge) = \{(q_1, \wedge)\}$   
 $\delta(q_f, \wedge, z_0) = \{(q_0, \wedge)\}$

Input String aabb

Sol<sup>n</sup>:

$$\begin{aligned}
 \delta(q_0, \underline{a}abb, z_0) &\vdash (q_0, abb, az_0) \\
 &\vdash (q_0, bb, aaz_0) \\
 &\vdash (q_1, b, aaz_0) \\
 &\vdash (q_1, \wedge, z_0) \\
 &\vdash (q_0, \wedge, \wedge)
 \end{aligned}$$

String is accepting because Push down Store (PDS) is empty.

\* PDA is accepting by empty stack



Ex 2:

(PDA)

⑤

$$\delta(q_0, a, z_0) = (q_0, a z_0)$$

$$\delta(q_0, a, a) = (q_0, a a)$$

$$\delta(q_0, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, b, a) = (q_1, \Lambda)$$

$$\delta(q_1, \Lambda, z_0) = (q_f, z_0)$$

Input String a a a b b b

Sol:

$$(q_0, \underline{a} a a b b b, z_0) \vdash (q_0, a \underline{a} b b b, a z_0)$$

$$\vdash (q_0, a \underline{a} b b b, a a z_0)$$

$$\vdash (q_0, a b \underline{b} b, a a a z_0)$$

$$\vdash (q_1, a b \underline{b}, a a z_0)$$

$$\vdash (q_1, a \underline{b}, a z_0)$$

$$\vdash (q_1, a, z_0)$$

$$\vdash (q_f, \Lambda, z_0)$$

String accepting

\* PDA is accepting by final state



# Instantaneous Description :-

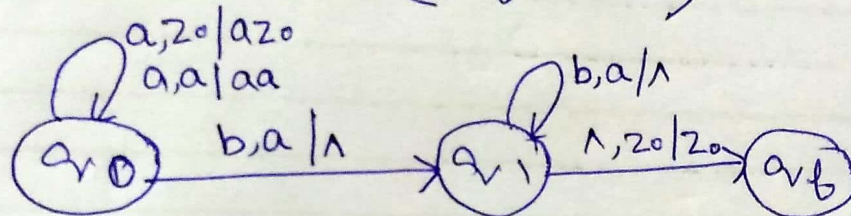
(6)

- 1) Graphical Representation
- 2) Tuple Representation.

Ex

$$\begin{aligned} s(q_0, a, z_0) &= (q_0, az_0) \\ s(q_0, a, a) &= (q_0, aa) \\ s(q_0, b, a) &= (q_1, \wedge) \\ s(q_1, b, a) &= (q_1, \wedge) \\ s(q_1, \wedge, z_0) &= (q_b, z_0) \end{aligned}$$

Soln :



Graphical Representation

Tuple Representation.

$$s(q_0, a, z_0)$$

$$s(q_1, \wedge, z_0)$$

