Closure Properties of Reg. Language. Part 8.

For eg. if we take an integer z, & two values in it like 1 + 2 = 3 → then the answer if it belongs to set integer then it is closed else not closed.

integer z → −∞ −5 ... 0, 1, 2

1+2 = 3 → closed

3/2 = 1.5 → does not belongs to integer.

So Division is not closed.

Similarly, if we have a set of Regular L.

If we take two language & perform any operation on it, & if the resultant also belongs to the set then it is closed or not —

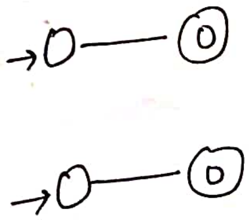
L1 op L2 = O/P

operation

1. Union ($L_1 \cup L_2$)
or L1 → FA . L2 → FA .

L1 → RE1
L2 → RE2] +
RE3 .

2.
L1
L2



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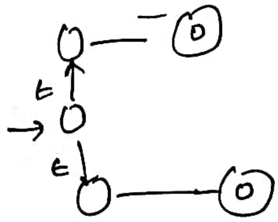
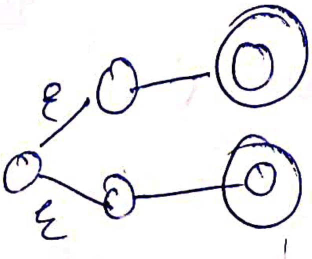




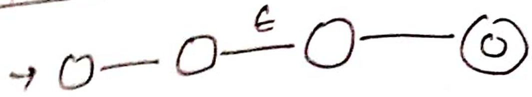
1+3 = 4

1/3 =

2. <u>Concatenation</u> $(L_1 \cdot L_2)$.

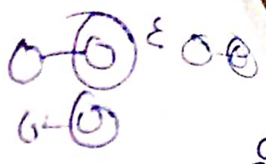
$L_1 \to$ 

<u>$L_1 \cdot L_2$</u>



$L_2 \to$ 

Either in terms of RE or FA.



3. <u>Closure</u> $(*)$ L^*

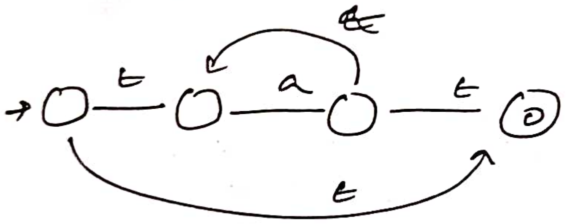
RL

$(L1^*) \to$ will also belongs to RL.

$L_1 = a$

$(a)^*$

If we are able to make finite automata of any language, then it is regular language



• <u>Complementation</u> $\bar{L} = \Sigma^* - L$.

$L \to$ Regular.

Complement (means final state $\xrightarrow{\text{converts}}$ non final
non final $\to$ final.)

$L \to$ DFA.

$\bar{L} \to$ will also result in DFA.

$U \to 1,2,3,4,5,6$ -3

$1,2,4,5,6$

$\bar{3}$

Intersection.

$L1 \cap L2 = \left(\overline{\overline{L1} \cup \overline{L2}}\right)$

$\downarrow$ closed $\qquad \downarrow$ closed.

$\overline{\overline{L_1} \cup \overline{L_2}} = \overline{R}$

Union = closed.

complement of $R = \overline{R}$ = regular

6. **Difference** $\quad L1 - L2 = L1 \cap (\overline{L_2})$

$L1 \downarrow R \qquad$ Regular

Intersection is also regular.

7. **Reversal.**

Regular language are closed under Reversal

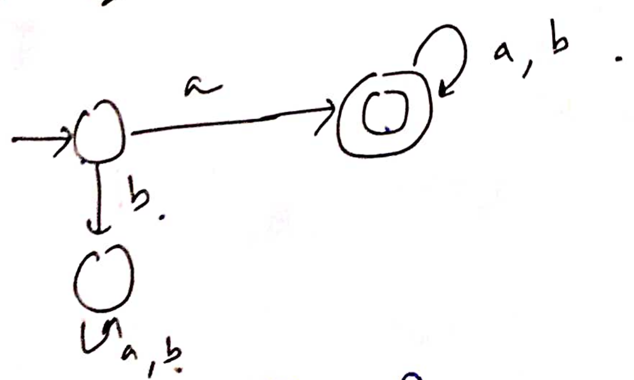
abc $\rightarrow$ cba (string)

L^R = means all strings in a language should be reversed.

Ex. L_1 = { set of all strings over (a, b) starting with a }. $\qquad L \in$ REG

RE $L_1 = a(a+b)^*$.

FA.



Ds.

A_1 = ABCD

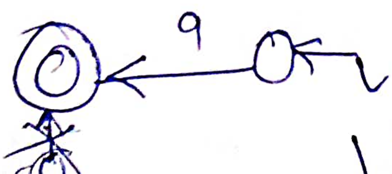
L_2 = D.

$\overline{L_2}$ = ABC

$L_1 \cap \overline{L_2}$

(ABCD $\cap$ ABC)

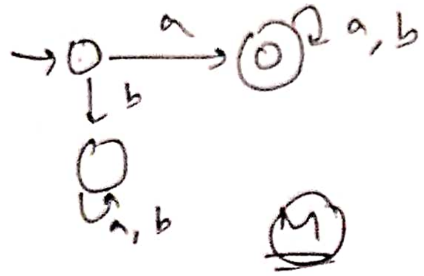
= ABC

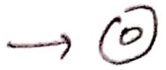


Now to reverse, means reverse the DFA

Steps.

1. Make initial state of M as final state of M'



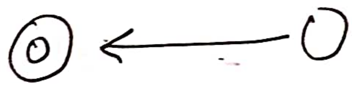


M'

2. Final state of M become initial state of M'

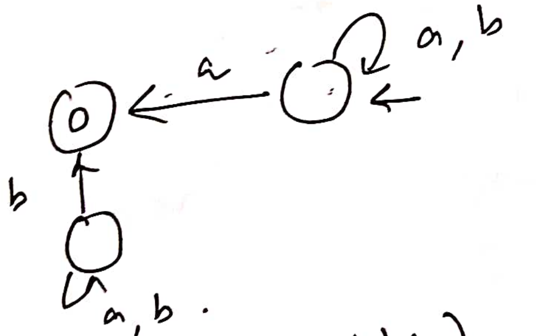
M'

3. Reverse the direction of edges of M to make M'



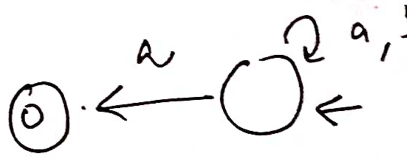
M'

4. No change in loop & remove unnecessary states.



(unreachable state)

⇒



$(a+b)^* a$

Reverse

Quotient Operation < Left Quotient (Cut Prefix) / Right Quotient (Cut Suffix)

L1, L2 → languages on same I/P symbol.

$$\frac{L1}{L2} = \{ x \mid xy \in L_1 \text{ for some } y \in L_2 \}$$

Right Quotient (cutting in Right).

means, $\frac{xy}{y}$

$$\frac{L1}{L2} = \{ y \mid xy \in L_1 \text{ for some } x \in L_2 \}$$

Left Quotient (cutting in left)

←

means, $\frac{xy}{x}$

ex. $L1 = \{10, 100, 1010, 101110\}$ } same $\Sigma = (0,1)$

$L2 = \{10\}$.

$\frac{L_1}{L_2}$

$\frac{L1}{L2}$ (left) $= \frac{10}{10} = \epsilon$

$\frac{100}{10} = 0$. (left side se match karo)

$\frac{1010}{10} = 10$

$\frac{101110}{10} = 1110$

$\frac{\cancel{10}}{\cancel{10}} = \epsilon$

$\frac{100}{10} = \epsilon$

$\frac{L_1}{L_2}$ (Right) = $\frac{10}{10}$, $\frac{100}{10}$, $\frac{1010}{10}$,

no match so

(Right side se match) = ϵ, ϵ, 10, 1011

Q. $L_1 = a^* b$ (When * given open the story)

$L_2 = ab^*$

open $L_1 = \{ b, ab, aab, aaab, \ldots \}$

$L_2 = \{ \cancel{ab} \}$ a, ab, abb, abbb ... }

$\frac{L1}{L2}$ (Right) = ϵ (for all)

= a.

$\frac{L1}{L2}$ (Left) = ϵ, b, ab, aab ...

= a

Now L_2 = ab & so on ..

$L1$ = Regular $L2$ = Regular.

$\frac{L1}{L2}$ = when quotient calculating we are cutting something not adding then the answer will also be regular.

Init operation (Initial / Prefix)

Set of all prefix of $w \in L$.

Let $L = \{ab, ba\}$.

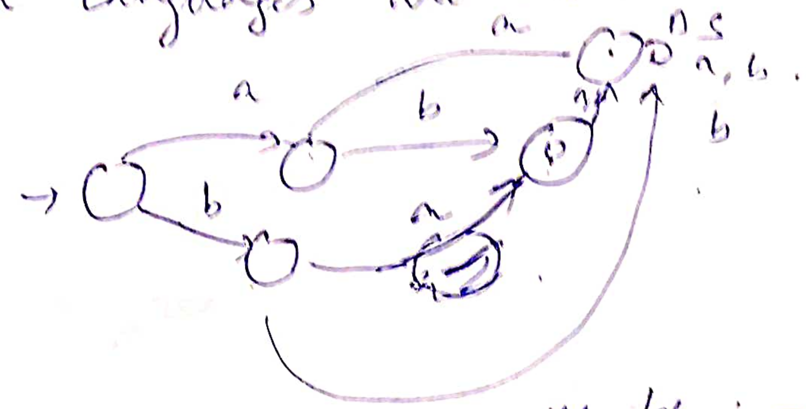
$ab \in L$
$ba \in L$

$ab = \epsilon, a, ab$.
$ba = \epsilon, b, ba$.
$abc \to \epsilon, a, ab, abc$.

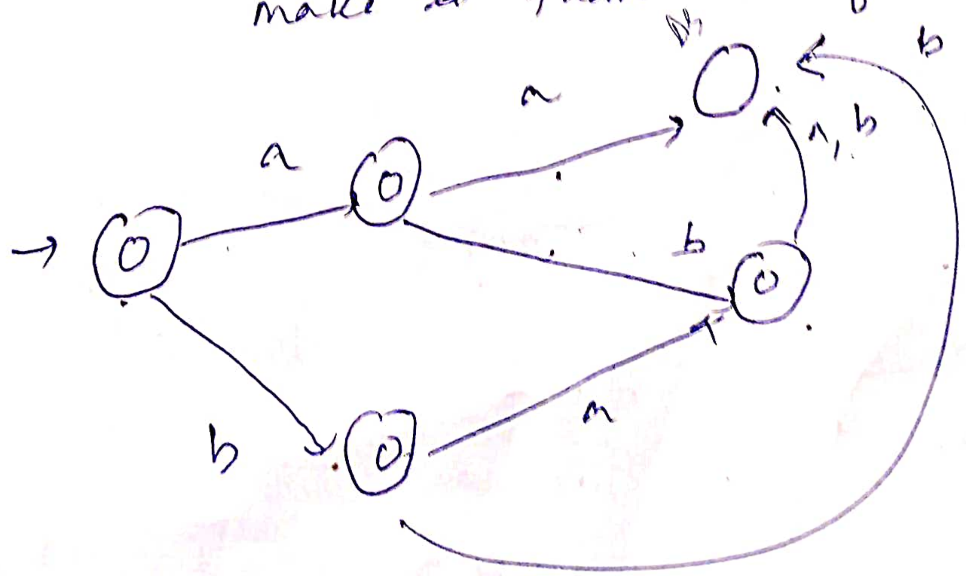
Init operation $\to \{\epsilon, a, ab, b, ba\}$.

Regular Languages are closed under Init.

DFA -



For init operation, DFA will be :-
Non final states except dead state (unreachable)
make them as final.



Now ϵ, a, b, ab, ba will be accepted

10. **Infinite Union** :- Regular languages are not closed under infinite union.

eg. $L_1 = \{a^1 b^1\}$ $\cup$ $L_1 \cup L_2 = \{a^1 b^1, a^2 b^2\}$

$L_2 = \{a^2 b^2\}$

$\cup$ $\{$ ~~$a^3 b^3$~~

$L_3 = \{a^3 b^3\}$

$\cup$

$L_4 = \{a^4 b^4\}$. $a^n b^n$

⋮

Infinite time Union

$\{a^n b^n \quad n \geq 1\}$.

But this is not regular language. coz there is a comparison, that no. of a should be equal to no. of b.

eg. $a^1 \cup a^2 \cup a^3 \ldots \cup a^n$

$= a^n \quad n \geq 1$.

It is regular language.

But one ex is closed, & the other is not closed, then we can say Regular Languages are not closed under Infinite Union.

Homomorphism → means same.

Substitute function

Ek set mein doosre set ki value substitute karni hai.

$h(L) = \{ \underbrace{h(w)}_{\text{find out homomophic image}} \mid \underbrace{w}_{\text{strings}} \in L \}$.

homomorphism of a lang.

where $h : \Sigma$ to Γ^* is called homomorphism
tow

Q. where $\Sigma(0,1)$ input, $\Gamma = (a, b)$ substitute to Γ.

~~$h(0) = aa$~~, ~~$h(1) = bb$~~ ←

whenever see 0 → substitute by a
& 1 → " b

$h(0) = aa$ $h(1) = bb$

if $L = \{00, 101\}$.

find $h(L) = \{ \underbrace{aa}_{0}\underbrace{aa}_{0}, \underbrace{bb}_{1}\underbrace{aa}_{0}\underbrace{bb}_{1} \}$.

RL → RE exists.

RE → $(0+1)^* 1^*$.

$(aa+bb)^*(bb)^*$ — { Homorphic image }

If R/E is given then it is specified like it ⇒ it is also regular exp.

12 Inverse Homomorphism

① Let $h(0) = a$ $h(1) = b$ $h(2) = ab$.

$\Sigma = \{0, 1\}$ $\Gamma = \{a, b\}$

Let $L = \{abab\}$

$h^{-1}(L) = \{0101, 22, 012, 201 \ldots\}$

$h(L) = \{abab, abab, abab, abab\}$
$= \{abab\}$

② $h(0) = aa$ $h(1) = bb$.

$\Sigma = \{0, 1\}$ $\Gamma = \{a, b\}$

$L = \{aa, aabb, baab, ababa\}$

X

$h^{-1}(L) = \{0, 01\}$ not given

X

$h(h^{-1}L) = \{aa, aabb\}$

not equal.

$h^{-1}(L) \in L$.