

Conversion FA to RE

Input \rightarrow FA

Output \rightarrow RE

Part 1.

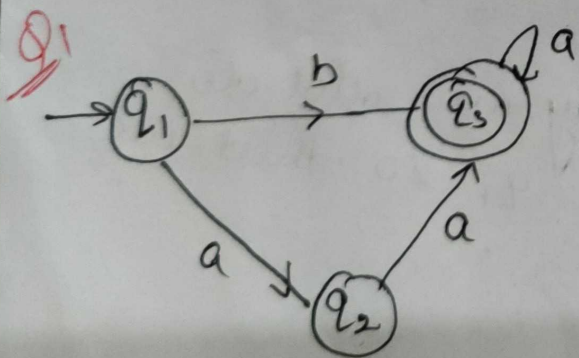
\rightarrow Arden's Method $R = \emptyset + RP$ then $R = \emptyset P^*$

\rightarrow state Eliminating method.

- \rightarrow 1. Write down equation for each state based on incoming edges
- \rightarrow 2. Add ϵ to the equation of initial state
- \rightarrow 3. Simplify the equation using Arden's theorem & find RE for final state.

Condition

- \rightarrow FA should not contain ϵ -Transition.
- \rightarrow FA should have only one initial state.



$$q_1 = \epsilon. \quad -1$$

$$q_2 = q_1 a \quad -2$$

$$q_3 = q_1 b + q_2 a + q_3 a. \quad -3$$

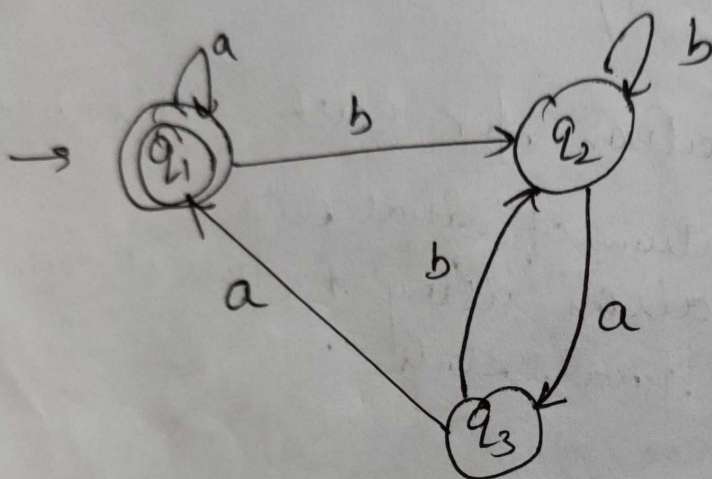
Apply ① & ② in 3.

$$\begin{aligned} q_3 &= \epsilon b + q_1 a a + q_3 a \\ &= \epsilon b + a a + q_3 a \\ &= b + a a + q_3 a \end{aligned}$$

$$q_3 = (b+aa) + q_3 a$$

$$R = \emptyset + RP \Rightarrow R = RP^*$$

$$q_3 = (b+aa)a^*$$



$$q_1 = q_1 a + q_3 a + \epsilon \quad \text{--- ①}$$

$$q_2 = q_2 b + q_3 b + q_3 b \quad \text{--- ②}$$

$$q_3 = q_2 a \quad \text{--- ③}$$

final state is q_1 so finally we did do substitute the equation in q_1 so that we need to get the RE.

~~eq~~ eq ③ in eq ①

~~$$q_1 = q_1 a + q_2 a$$~~

$$q_2 = q_1 b + q_2 b + q_2 ab.$$

$$q_2 = q_1 b + q_2 (b+ab)$$

by Arden's Theorem

$$\boxed{q_2 = q_1 b (b+ab)^*}$$

- 4

4^o in eq (3)

$$q_3 = q_1 b (b+ab)^* a \quad \text{--- (5)}$$

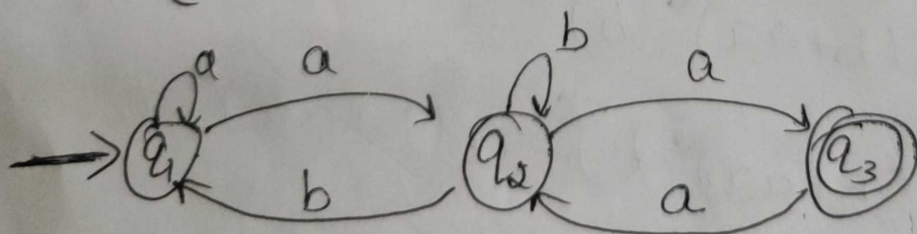
put eq (5) in eq (1)

$$q_1 = q_1 a + q_1 b (b+ab)^* a + \epsilon$$

$$q_1 = q_1 (a + b (b+ab)^* a) + \epsilon \quad \text{By Arden's theorem.}$$

$$= \epsilon (a + b (b+ab)^* a)^*$$

Q Prove strings recognized are $(a + a(b+aa)^* b)^* a (b+aa)^* a$



$$q_1 = q_1 a + q_2 b + \Lambda$$

$$q_2 = q_1 a + q_2 b + q_3 a$$

$$q_3 = q_2 a$$

Put ③ into ②

$$Q_2 = Q_1 a + Q_2 b + Q_2 a a$$

$$Q_2 = Q_1 a + Q_2 (b + a a)$$

ARDEN'S Theorem

$$R = Q + RP \quad \boxed{R = QP^*}$$

$$Q_2 = Q_1 a (b + a a)^* \quad \text{--- ④}$$

Put ④ into ①

$$Q_1 = Q_1 a + Q_1 a (b + a a)^* b + \Lambda$$

$$Q_1 = Q_1 (a + a (b + a a)^* b) + \Lambda$$

Arden's Theorem $R = Q + RP = QP^*$

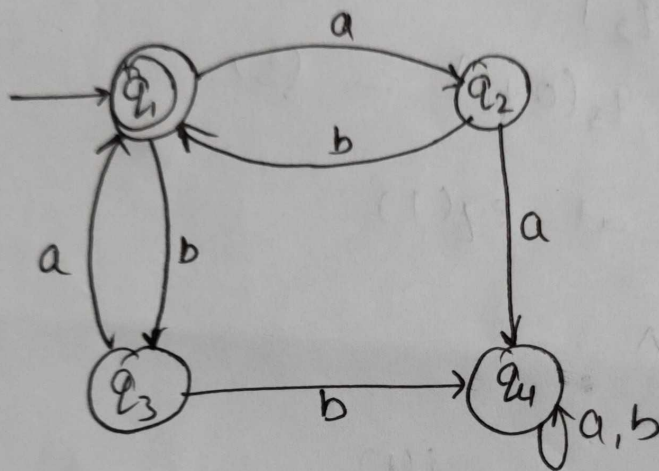
$$Q_1 = \Lambda (a + a (b + a a)^* b)^*$$

$$Q_1 = (a + a (b + a a)^* b)^*$$

$$Q_2 = (a + a (b + a a)^* b)^* a (b + a a)^*$$

$$Q_3 = (a + a (b + a a)^* b)^* a (b + a a)^* a$$

Prove that the finite automata whose transition diagram as shown in fig 5.14 accepts the set of all strings over an Alphabet $\{a, b\}$ with an equal number of a's & b's such that each prefix has at most one more a than the b's & at most one more b than the a's



$$q_1 = q_2b + q_3a + \epsilon \quad \text{---(1)}$$

$$q_2 = q_1a \quad \text{---(2)}$$

$$q_3 = q_1b \quad \text{---(3)}$$

$$q_4 = q_4a + q_4b + q_3a + q_2a \quad \text{---(4)}$$

② & ③ put in ①

$$q_1 = q_1ab + q_1ba + \epsilon$$

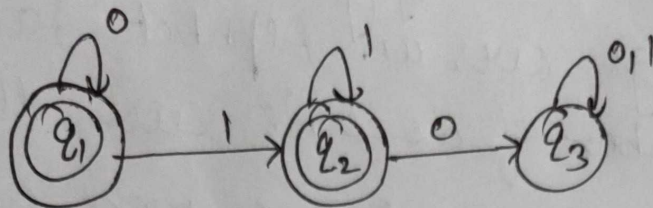
$$q_1 = q_1(ab + ba) + \epsilon$$

Arden's theorem

$$q_1 = \epsilon(ab + ba)^*$$

$$R = Q + RP \Rightarrow R = QP^*$$

Q Describe in English the set accepted by the finite Automata whose transition dig.



$$q_1 = q_1 0 + \Lambda \quad \text{--- (1)}$$

$$q_2 = q_1 1 + q_2 1 \quad \text{--- (2)}$$

$$q_3 = q_2 0 + q_3 (0+1) \quad \text{--- (3)}$$

Apply Arden's theorem at eq(1)

$$q_1 = q_1 0 + \Lambda$$

$$q_1 = 0^* \quad \text{--- (4)}$$

$$q_2 = q_1 1 + q_2 1$$

~~Apply~~ eq (4) in (2)

$$q_2 = 0^* 1 + q_2 1$$

Arden's theorem

$$q_2 = (0^* 1)^*$$

As final state q_1 & q_2 so.

$$q_3 = q_2 0 + q_3 (0+1)$$

$$q_3 = (0^* 1)^* 0 + q_3 (0+1)$$

Apply Arden's theorem

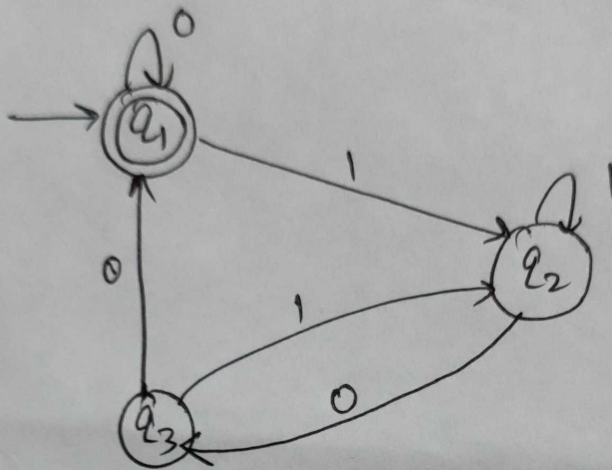
$$q_3 = (0^* 1)^* 0 (0+1)^*$$

$$\begin{aligned} q_1 + q_2 &= 0^* + (0^* 1)^* \\ &= 0^* (\Lambda + 11^*) \\ &= 0^* 1^* \end{aligned}$$

$$\boxed{\Lambda + RR^* = R^*}$$

The string accepted by FA is The string of any number of 0's followed by a string of any number of 1's.

Q.



$$q_1 = q_1 0 + q_3 0 + \Lambda \quad \text{--- (1)}$$

$$q_2 = q_1 1 + q_2 1 + q_3 1 \quad \text{--- (2)}$$

$$q_3 = q_2 0 \quad \text{--- (3)}$$

So put eq (3) in (1)

$$q_2 = q_1 1 + q_2 1 + q_2 0 1$$

$$= q_1 1 + q_2 (1 + 01)$$

$$q_2 = q_1 1 (1 + 01)^*$$

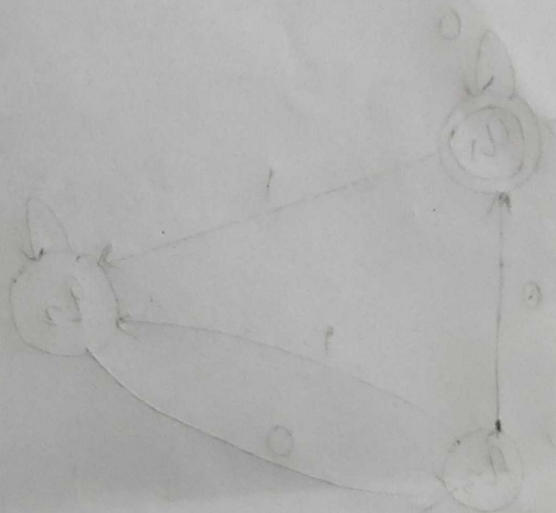
Arden's theorem.

$$q_1 = q_1 0 + q_2 0 0 + \Lambda$$

$$= q_1 0 + q_1 1 (1+01)^* 00 + \Lambda$$

$$q_1 = q_1 (0 + 1(1+01)^* 00) + \Lambda$$

$$q_1 = \Lambda (0 + 1(1+01)^* 00)^*$$



$$(1) \quad \Lambda + 0q_0 + 0q_1 = q_0$$

$$(2) \quad 1q_0 + 1q_1 + 1q_2 = q_1$$

$$(3) \quad 0q_0 = q_2$$

$$(4) \quad q_0 = 0q_0 + 1q_1 + 1q_2$$

$$(5) \quad (10+1)q_0 + 1q_1 = q_1$$

$$(6) \quad (10+1)q_0 + 1q_1 = q_1$$

$$(7) \quad (10+1)q_0 = q_1$$

$$\Lambda + 00q_0 + 0q_1 = q_2$$