PDA to CFG :-

if $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a PDA then CFG is defined as $G = (V, \Sigma, P, S)$

(1) Construction of set of non-terminal

$$V = \{S\} \cup \{[q, Z, q'] \mid q, q' \in Q, Z \in \Gamma\}$$

(2) (i) S-Production

$$S \to [q_0, Z_0, q], \quad q \in Q$$

(ii) For PoP operation

$$\delta(q, a, Z) \to (q', \wedge)$$

$$[q, Z, q'] \to a$$

(iii) For PUSH and No-operation.

$$\delta(q, a, Z) \to (q_1, Z_1 Z_2 \cdots Z_n)$$

$$[q, Z, q'] \to a\,[q_1, Z_1, q_2][q_2, Z_2, q_3] \cdots [q_n, Z_n, q']$$

where $q', q_2, q_3 \cdots q_n \in Q$

PDA to CFG:

Ex:1 Construct CFG from PDA

$A=(\{q_0,q_1\},\{a,b\},\{Z_0,Z\},\delta,q_0,Z_0,\phi)$

δ :-

$\delta(q_0,b,Z_0)=(q_0,ZZ_0)$ ✓

$\delta(q_0,b,Z)=(q_0,ZZ)$ ✓

$\delta(q_1,b,Z)=(q_1,\wedge)$ ✓

$\delta(q_0,\wedge,Z_0)=(q_0,\wedge)$ ✓

$\delta(q_0,a,Z)=(q_1,Z)$ ✓

$\delta(q_1,a,Z_0)=(q_0,Z_0)$ ✓

Z_0	Z
q_0q_0	q_0q_0
q_0q_1	q_0q_1
q_1q_0	q_1q_0
q_1q_1	q_1q_1

$\{2^2=4\}$

Soln :- Rule-1 :

$V=S,\ [q_0,Z_0,q_0]\ [q_0,Z_0,q_1]$

$[q_0,Z_0,q_0]\ [q_0,Z_0,q_1]$ ① A ② B

$[q_0,Z_0,q_0]\ [q_0,Z_0,q_1]$ ① E ② F

$[q_1,Z_0,q_0]\ [q_1,Z_0,q_1]$ ③ C ④ D

$[q_1,Z,q_0]\ [q_1,Z,q_1]$ ③ G ④ H

$S\to[q_0,Z_0,q_0]$

$S\to[q_0,Z_0,q_1]$

Non-Terminals
Total = S+8
= 9

Rule-2 :

$S\to[q_0,Z_0,q_0]$

$S\to[q_0,Z_0,q_1]$

* $\delta(q_0, b, Z_0) = (q_0, ZZ_0)$ {$2^2 = 4$}

its PUSH operation

$[q_0, Z_0, q_0] \to b\,[q_0, Z, q_0]\,[q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \to b\,[q_0, Z, q_1]\,[q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \to b\,[q_0, Z, q_0]\,[q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \to b\,[q_0, Z, q_1]\,[q_1, Z_0, q_1]$

Production ↑

* $\delta(q_0, \wedge, Z_0) = (q_0, \wedge)$

$[q_0, Z_0, q_0] \to \wedge$

* $\delta(q_0, b, Z) = (q_0, ZZ)$ {$2^2 = 4$}

its PUSH operation

$[q_0, Z, q_0] \Rightarrow b\,[q_0, Z, q_0]\,[q_0, Z, q_0]$

$[q_0, Z, q_0] \to b\,[q_0, Z, q_1]\,[q_1, Z, q_0]$

$[q_0, Z, q_1] \to b\,[q_0, Z, q_0]\,[q_0, Z, q_1]$

$[q_0, Z, q_1] \to b\,[q_0, Z, q_1]\,[q_1, Z, q_1]$

$[q_0\ Z_0\ q_0] = A$

$[q_0\ Z_0\ q_1] = B$

$[q_1\ Z_0\ q_0] = C$

$[q_1\ Z_0\ q_1] = D$

$[q_0\ Z\ q_0] = E$

$[q_0\ Z\ q_1] = F$

$[q_1\ Z\ q_0] = G$

$[q_1\ Z\ q_1] = H$

Date .....

Page No. .....

$\delta(q_0, a, Z) = (q_1, Z)$

$[q_0, Z, q_0] \Rightarrow a[q_1, Z, q_0]$

$[q_0, Z, q_1] \rightarrow a[q_1, Z, q_1]$

$\delta(q_1, b, Z) = (q_1, \wedge)$

$[q_1, Z, q_1] \rightarrow b$

$\delta(q_\phi, a, Z_0) = (q_0, Z_0)$

$[q_\phi, Z_0, q_0] = a[q_0, Z_0, q_0]$

$[q_\phi, Z_0, q_1] = a[q_0, Z_0, q_1]$

Production : CFG

① $S \rightarrow A \mid B$

② $A \rightarrow bEA \mid bFC$

③ $B \rightarrow bEB \mid bFD$

④ $C \rightarrow aA$

⑤ $E \rightarrow bEE \mid bFG \mid aG$

⑥ $F \rightarrow bEF \mid bFH \mid aH$

⑦ $D \rightarrow aB$

⑧ $H \rightarrow b$ ⑨ $A \rightarrow \wedge$

Ans

Ex2:. PDA to CFG

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, a)$
$\delta(q_1, b, a) = (q_1, a)$
$\delta(q_1, a, a) = (q_1, \wedge)$
$\delta(q_1, \wedge, Z_0) = (q_1, \wedge)$

Soln:.

CFG = (V, T, P, S)

① $S \rightarrow (q_0, Z_0, q_0)$
② $S \rightarrow (q_0, Z_0, q_1)$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$ $(2^2 = 4)$

$[q_0, Z_0, q_0] = a\,[q_0, a, q_0]\,[q_0, Z_0, q_0]$
$[q_0, Z_0, q_0] = a\,[q_0, a, q_1]\,[q_1, Z_0, q_0]$
$[q_0, Z_0, q_1] = a\,[q_0, a, q_0]\,[q_0, Z_0, q_1]$
$[q_0, Z_0, q_1] = a\,[q_0, a, q_1]\,[q_1, Z_0, q_1]$

$\delta(q_0, b, a) = (q_1, a)$ $\{z' = z\}$

$[q_0, a, q_0] = b[q_1, a, q_0]$

$[q_0, a, q_1] = b[q_1, a, q_1]$

$\delta(q_1, a, a) = (q_1, \wedge)$ $\{z^{\leftarrow} = 0\}$

$[q_1, a, q_0) = a$

$\delta(q_1, \wedge, z_0) = (q_1, \wedge)$

$[q_1, z_0, q_0] = \wedge$

$\delta(q_1, b, a) = (q_1, a)$ $\{z' = z\}$

$[q_1, a, q_0] = b[q_1, a, q_0]$

$[q_0, a, q_1] = b[q_1, a, q_1]$

$\delta(q_0, a, a) = (q_0, aa)$ $\{2^2 = 4\}$

$[q_0, a, q_0] = a[q_0, a, q_0]\ [q_0, a, q_0]$

$[q_0, a, q_0] = a[q_0, a, q_1]\ [q_1, a, q_0]$

$[q_0, a, q_1] = a[q_0, a, q_0]\ [q_0, a, q_1]$

$[q_0, a, q_1] = a[q_0, a, q_1]\ [q_1, a, q_1]$

$[q_0, z_0, q_0] = A$

$[q_0, z_0, q_1] = B$

$[q_1, z_0, q_0] = C$

$[q_\phi, z_0, q_1] = D$

$[q_0, a, q_0] = E$

$[q_0, a, q_\phi] = F$

$[q_1, a, q_0] = G$

$[q_1, a, q_1] = H$