Part 7

Pumping lemma
string

↳ Used to prove that language is not Regular
L

Collection of strings
symbol

↳ If substring of a string is repeated many times & if the resultant string is also available in Language 'L' then we can say it as Regular

Step 1.

↳ Consider language as a Regular.

↳ Assume a constant 'c' & select the string 'w' from L such that $|w| \geq c$

↳ Divide the w as xyz

↳ $|y| > 0$

↳ $|xy| \leq c$

↳ for $i \geq 1$ everystring xy^iz belongs to L.

Q. $\{a^n b^n \mid n \geq 1\}$ is not Regular.

$L = \{\varepsilon, ab, aabb, aaabbb, a^4b^4 \ldots\}$

$c = 6.$

$w = aaabbb.$ $|w| = 6 \geq c.$

$x = a$
$y = aab$
$z = bb$

$W = \underbrace{aa}_{x}\,\underbrace{ab}_{y}\,\underbrace{bb}_{z}$

$x = aa, \quad y = ab \quad , z = bb$

$|y| = 2 > 0$ ✓

$|xy| = |aa\,ab| = 4 \leq c$

$i=0 \quad xy^0z = xz = aabb$

$i=1 \quad xyz = aaabbb$

$i=2 \quad xy^2z = \underline{aa\,abab\,bb}$ This is not available in our lang. b'coz our language consist of the multiple of a's followed by multiple no. of b's & no. of a's & b's should be equal

Here no. of a's = no. of b's but a's not followed by b.

$W = \underbrace{a}_{x}\,\underbrace{aa}_{y}\,\underbrace{bbb}_{z}$

$x = a, \quad y = aa \quad z = bbb.$

$|y| = 2 > 0$

$|xy| = |aaa| = 3 \leq c$

$i=0 \quad xz = abbb$ → not available in L.

So $a^n b^n$ is not a Regular language.

$w = aaaa\,bbb$

$x = aaa$

$y = b$ $\quad |y| = 1 > 0$

$z = bb$ $\quad |xy| \neq |aaab| = 4 \leq c$

$L = \{a^p \mid p$ ...

Prime N...

$L = \{a^p \mid p \text{ is a prime number}\}$.

Prime Number $= 2, 3, 5, 7, 11 \ldots$

$L = \{aa, aaa, aaaaa \ldots\}$

$c = 5$

$w = aaaaa$

$|w| \geq c$ ✓

$w = \underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{a}_{z}$

$x = aa, \quad y = aa, \quad z = a$

$aa = |y| > 0$ ✓

$|xy| \leq c \qquad |aaaa| \leq c$ ✓

$i=0 \quad xy^0z = xz = aaa$

$i=1 \quad xy^1z = xyz = aaaaa$

$i=2 \quad xy^2z = aaaaaaa$

$i=3 \quad xy^3z = aaaaaaaaa$ ✗

mping Lemma.

Theorm is used to check whether the lang. is regular or not

Language can be either finite / non finite / in.

If Language is finite then FA exist.
Finite memory needed

It is always ~~finit~~ regular.

When L = infinite, it can be regular or not.

Pumping Lemma is one of the method to check whether the L is regular or not.

PL is a → -ve (negative) test.

Body → problem diagonis
+ve → prob. exists
-ve → " doesnot "

$\alpha \Longrightarrow$ PL Test $\xrightarrow{\text{Pass}}$ Regular ?
Undecidable.

↓ fail

Non Regular (Decidable).

PLemma Theorm → If L is an infinite language then there exists some +ve integer 'n' (Pumping length) such that any string

$w \in L$ has length greater than equal to n

ie. $|w| \geq n$ then w can be divided in three parts, $w = xyz$. satisfy following conditions:

1. for each $i \geq 0$, $xy^iz \in L$
2. $|y| > 0$ &
3. $|xy| \leq n$

② . $a=1$ $b=2$ $1+2=③$

$|w|$ $x|y|z$

$a+2$ = $a=5$ = $5/2$ 2.5

w

x y z

x yy z

x $yyyy$ z

$y > 0$.

Q. $L = a^n b^{2n}$. $n \geq 0$.

Lets say, we take a string out of it

aabbbb . $\in L$. ✓

Divide into 3 partss.

$\underset{x}{aa}$ $\underset{y}{bb}$ $\underset{z}{bb}$ (Assumption) anywhich way.

Now Pump aa $\underset{y}{bbbb}$ bb. $\longrightarrow$ $i=2$ $xy^iz \notin L$

Now this string belongs to Language?

Test fail → It is not regular.

Now aa bbbb . $\in L$?

$\underbrace{a}_{x}\ \underbrace{ab}_{y}\ \underbrace{bbb}_{z}$.

$xy^iz \in L$.

Pump y .

$y = 2$.

$\underbrace{a}_{x}\ \underbrace{abab}_{y}\ \underbrace{bbb}_{z}$ $\in L$??

False .

Condition fails , so this is not Regular Grammar !

Q. Using Pumping Lemma prove that the L $A = \{a^n b^n \mid n \geq 0\}$ is not regular .

Sol .

To prove L is not Regular follow the steps :-

1. Assume that A is regular .
2. It has to have a Pumping length (say P)
3. All strings longer than P can be pumped $|s| \geq P$.
4. Now find a string 's' in A such that $|s| \geq P$.
5. Divide s into xyz .
6. Show that $xy^iz \notin A$ for some i .
7. Then consider all ways that s can be divided into xyz .

Sol :- Assume that A is Regular.
Pumping Length = P. (Such that any string whose length $\geq P$ should be pumped)

$$S = a^P b^P.$$

S (Divide into 3 parts)

x y z.

P = 7 (Assume).

$$S = a^7 b^7$$
aaaaaaa bbbbbbb

First case : y is in 'a' part.

$$\underbrace{aa}_{x}\underbrace{aaaa}_{y}a \; \underbrace{bbbbbbb}_{z}$$

Case 2. → y is 'b' part.

$$\underbrace{aaaaaaa}_{a} \; \underbrace{bbbbbb}_{y}\underbrace{b}_{z}.$$

Case 3 y is in 'ab' part

$$\underbrace{aaa\,aaa}_{x}\,\underbrace{a\;bb}_{y}\,\underbrace{b\,bbbb}_{z}$$

Case 1. $xy^iz = xy^2z$
aa aaaa aaaa a bbb bbbb.

$11 \neq 7$.
(a's) (b's)

So this string doesn't lie in our L.

$\therefore \; xy^i z \Rightarrow xy^2 z$.

$\underbrace{aaaa\,aaa}_{x}$ $\quad \underbrace{b.bb.}_{y}bbbbb\underbrace{bbb}_{}$ $\quad \underline{b}$.

$\underline{a} = 7$ $\quad$ $b = 11$

$7 \neq 11$. $\quad$ Does not lie.

Case 3. $\quad xy^i z = xy^2 z$.

aaaaa $\quad$ aabbaabb $\quad$ bbbbb

$a = 9$ $\quad$ $b = 9$

But don't follow the pattern.

So this does not lie in our lang.

S can't be pumped. So our assumption is wrong.

L is regular or Not?

↳ finite ✓ R

↳ if comparision required ×. (memory required)

↳ if comparision required + Bounded (finite) ✓

Infinite (comp) → bounded R / not bounded NR

↳ A P series.

↳ G.P ×

1. $a^n \mid n \geq 1$ ✓ R.

2. $a^n b^m \quad n, m \geq 1$ ✓ ✓R

3. $a^n b^n \mid n \leq 10^{10^{10}}$ ✓R

4. $a^n b^n \mid n \geq 1$ × I → C → M R×

5. $ww^R \quad |w| = 2 \quad \in \{a, b\}$ ✓F R.

6. $ww^R \quad w \in (a, b)^*$ × I → C → R×.

7. $a^n b^m c^k \mid n, m, k \geq 1$ ~~I → C×~~ R ✓

8. $a^{2i} b^{2j} \mid i, j \geq 1$ I − C× R ✓

9. $a^n \mid n$ is even ✓ 0, 2, 4, 6, ε, a^2, a^4 .. a^6 I C× R

10. $a^n \mid n$ is odd ✓ I C× R ✓

11. $a^n \mid n$ is prime × AP series ×.

12. a^{n^2} × I No Pattern R×

13. a^{2^n} ×

14. $a^i b^{j^2}$ ×

abc cba.

ab ba.

w = ab

w^R = ba

Q $L = \{a^{n^2} \mid n \geq 0\}$ is regular or not.

S $L = \{\varepsilon, a, aaaa, aaaaaaaaa - - - \}$

$c = 4$

$|w| = |aaaa| \geq c$ ✓

$4 \geq c$

$w = \underbrace{a}_{x}\underbrace{aa}\underbrace{a}_{z}$ $\quad x = a, \; y = aa \quad z = a$

$|y| > 0$

$|xy| \leq c$

$|y| = |aa| \geq 2$ ✓

$|xy| = 3 \leq 4$ ✓

$i = 0 \quad xy^iz = xz = aa$ ✗

which is not in L so L is not regular language.

a

$a^{2^n} = \underbrace{a, a^2, a^4}, a^8, a^{16}, a^{32}$

GP (2)

a^{2}, a^{4}, a^{9}, a^{9} ...

a^{9}, a^{4}, a^{1}

a^{2}, a^{1}

2, ~~4, 16~~

Q

Pigeonhole Principle.

It states that if n pigeons fly into m pigeonhole & n > m then atleast one hole must contain two or more pigeons.

o o o 3 pigeons n

□ □ 2 holes m.

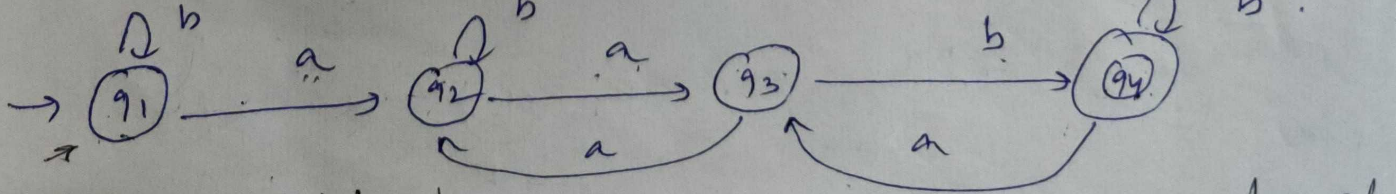
In automata theory, pigeons are the strings of a's, the pigeonhols are the states & the correspondence associated each string with the state to which A goes when the string is I/P.

This principle is used to prove that certain infinite language are not regular.

3 < 4.

~~3 → 4~~

eg. DFA with 4 states. bba ab.



b'abbab baab

If we take a string.

a, aa, aab → no state is repeated.

If we take (aabb, bbaab, abbabb, abbbabb abb ...

$S \to S + S$

$S \to id$

then states is repeated

This means if string w has length

$|w| \geq 4$.

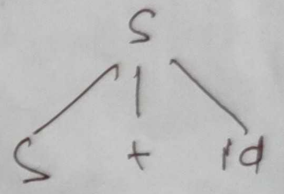
then the transition of string w are more than the states of DFA.

w ≥ no. of states

then q → must be repeated in the walk of w id.

$S \to S + id$

$S \to id$



CFG :- $L(G) \in NT \to (NT \cup T)^*$

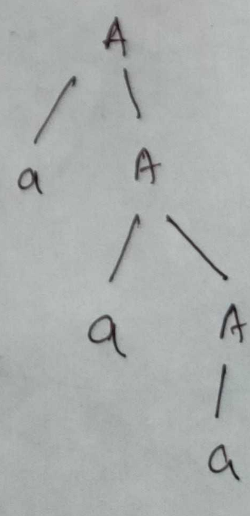
$NT \to T$

$NT \to \Lambda$

$NT \to aT$

$NT \to NT$

$A \to aA \mid a$



$L_1 \to AAS$

aab

a

≤2

≤2

LDT

RDT

⇒ NT → L

→ R.

Ambiguity.