1. (a) Consider the causal, complex coefficient transfer function shown below:

$$H_M(z) = \frac{\sum_{n=0}^{n=M-1} d^*[M-n]z^{-n} + z^{-M}}{1 + \sum_{n=1}^{n=M} d[n]z^{-n}}, M > 1 \quad (\mathbf{1})$$

(i) Show that the transfer function in **(1)** above is an allpass filter. **[5]**

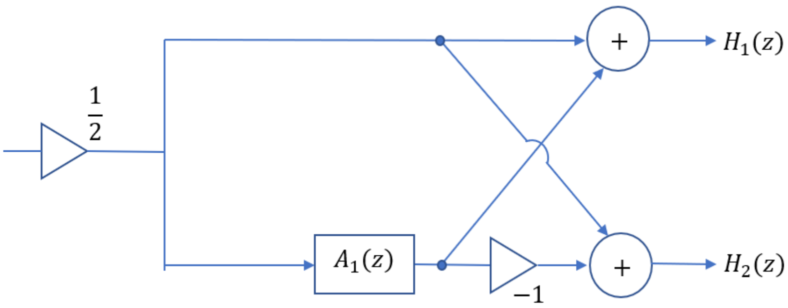
(ii) Comment on the locations of the zeros and the poles of a causal, real, stable, allpass filter's transfer function within the $z$ −plane, with respect to the unit circle. Justify your answer. **[5]**

(b) Given an allpass filter

$$H(z) = \frac{d + ez + fz^2}{1 + bz + cz^2}$$

with poles at $\frac{1}{2}$ and $\frac{1}{3}$, find $b, c, d, e, f$. **[5]**

(c) In the structure below, $A_1(z)$ is a first-order allpass transfer function. Find the type of filters $H_1(z)$ and $H_2(z)$ (lowpass etc.).




**[5]**

(d) Consider the cascade of two causal LTI systems with impulse responses $h_1[n] = a\delta[n] + b\delta[n-1]$ and $h_2[n] = c^n u[n]$, $a, b, c$ are real coefficients and $|c| < 1$. Determine the frequency response $H(e^{j\omega})$ of the overall system. Find all the possible sets of values of $a, b, c$ for which the overall transfer function has an amplitude equal to 1 for all frequencies **[5]**

2. (a) Design a length-5 FIR bandpass filter with an antisymmetric impulse response $h[n]$, satisfying the following magnitude response values: $\left|H(e^{j0.3\pi})\right| = 0.3$ and $\left|H(e^{j0.6\pi})\right| = 0.8$. **[4]**

(b) We have shown that a real-coefficient FIR transfer function $H(z)$ with a symmetric impulse response has a linear phase response. As a result, the all-pole IIR transfer function $G(z) = \frac{1}{H(z)}$ will also have a linear-phase response. What are the practical difficulties in implementing $G(z)$? Justify your answer. **[4]**

(c) A simple averaging filter is defined by the following input-output relationship:

$$y[n] = \frac{1}{N}(x[n] + x[n-1] + \cdots + x[n-(N-1)])$$

(i) Let $N = 4$. Determine the filter's transfer function and its zeros and poles. **[3]**

(ii) Determine a general form for zeros and poles for any $N$. **[3]**

(iii) A recursive implementation of a filter refers to writing the current output as a function of the previous output and the current and past inputs. By comparing $y[n]$ and $y[n-1]$ determine a recursive implementation of the filter. **[3]**

(d) An FIR digital filter has the transfer function

$$H(z) = (1 - z^{-1})^3(1 + z^{-1})^3$$

(i) Sketch the pole-zero diagram of this system. **[4]**

(ii) Sketch **roughly** the amplitude response of the above filter. Would you classify this as a low-pass, high-pass, band-pass, or band-stop filter? Please briefly explain. **[4]**

3. (a) The bilinear transformation from the $s$ −plane to the $z$ −plane is given by

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

(i) Explain using mathematical relations, where each point $s = \sigma + j\Omega$ on the $s$ −plane is mapped on the $z$ −plane through the bilinear transformation. **[5]**

(ii) Prove that the relationship between the continuous-time angular frequency and the discrete-time angular frequency is non-linear. **[5]**

(b) A given real-coefficient, digital IIR lowpass filter has a rational transfer function $H_L(z)$ and a cutoff frequency $\omega_c$. The transfer function of $H_L(z)$ is transformed by replacing $z$ by $F(z) = \frac{az+b}{cz+d}$ to a real highpass rational transfer function $H_H(z) = H_L(F(z))$ of the same order as $H_L(z)$ but different cutoff frequency. By using the constraints $H_H(e^{j\pi}) = H_L(e^{j0}) = 1$ and $H_H(e^{j0}) = H_L(e^{j\pi}) = 0$, derive relationships among the parameters $a, b, c, d$ and explain what type of filter is $F(z)$. **[5]**

(c) Consider the Finite Impulse Response (FIR) filter transfer functions $G_1(z) = \frac{1}{4}(z + 2 + z^{-1})$ and $G_2(z) = \frac{1}{4}(-z + 2 - z^{-1})$. Explain the type of filters $G_1(z)$ and $G_2(z)$ (lowpass etc.), by using the following two approaches:

(i) Experimental approach. In this approach you will apply the filters to the signal $x[n] = u[n]$, with $u[n]$ the discrete unit step function and observe the effects they have on $x[n]$. **[5]**

(ii) Mathematical approach. In this approach you must find the frequency response of the two filters. **[5]**

4. (a) Consider the multirate structure of **Figure 1** below, where $H_0(z), H_1(z)$ and $H_2(z)$ are ideal, zero phase, real-coefficient lowpass, bandpass and highpass filters respectively, with frequency responses as follows:

$$H_0(e^{j\omega}) = u(\omega) - u(\omega - \frac{2\pi}{3})$$

$$H_1(e^{j\omega}) = u(\omega - \frac{\pi}{3}) - u(\omega - \frac{2\pi}{3})$$

$$H_2(e^{j\omega}) = u(\omega - \frac{\pi}{3}) - u(\omega - \pi)$$

The function $u(\omega)$ is the well-known unit step function, defined as

$$u(\omega) = \begin{cases} 1 & \omega \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

If the input is a real sequence with a Discrete Time Fourier Transform

$$X(e^{j\omega}) = (-\frac{3\omega^2}{\pi} + \frac{\pi}{3})\left(u(\omega) - u\left(\omega - \frac{\pi}{3}\right)\right)$$

sketch the Discrete Time Fourier Transform of the outputs $y_0[n], y_1[n]$ and $y_2[n]$. Notice that all frequency domain representations above are provided only within the range $[0, \pi]$. **[15]**

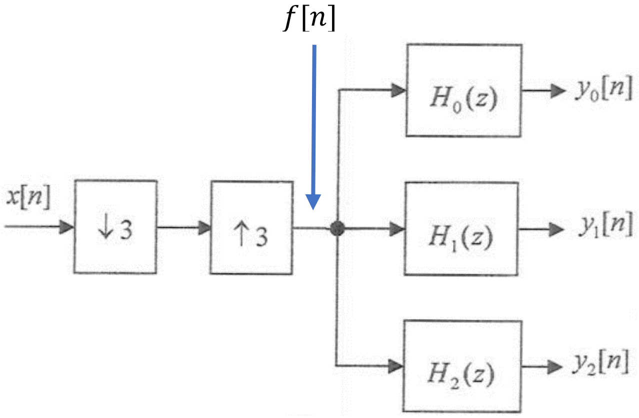



**Figure 1**

(b) Develop an expression for the output $y[n]$ as a function of the input $x[n]$ for the multirate structure of Figure 3 below. **[10]**

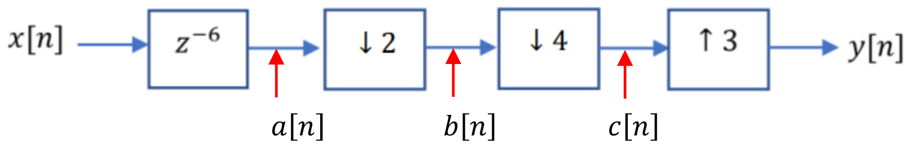



**Figure 2**