Maths for Signals and Systems
Problem Sheet 2

Problem 1
Find the inverse, the eigenvalues and the determinant of A

$$A = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}$$

Problem 2

(i) Carry out the eigenvalue decomposition of the matrices $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$.

(ii) Carry out the eigenvalue decomposition of A^3, B^3 and A^{-1}.

Problem 3

(i) Assume that $A = S\Lambda S^{-1}$. What is the eigenvalue matrix of $A + 2I$?
(ii) What is the eigenvector matrix of $A + 2I$?
(iii) Carry out the eigenvector decomposition of $A + 2I$.

Problem 4
Let A be a real skew-symmetric matrix, i.e. $\mathrm{A} = -\mathrm{A}^{\mathrm{T}}$. Show that all eigenvalues of A are purely imaginary or zero.

Problem 5

Find the general formula for $\begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}^k$ by diagonalizing the matrix.

Problem 6
Let A and B be $n \times n$ real matrices. If the matrix $C = BA$ is invertible, prove that both A and B are invertible.

Problem 7
Let A be an invertible matrix with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_K$ and the corresponding eigenvectors $v_1, v_2, \ldots., v_K$. What can we say about the eigenvalues and eigenvectors of A^{-1}?

Problem 8
Let A be a real symmetric matrix. Assume that v_1 and v_2 are eigenvectors corresponding to distinct eigenvalues $\lambda_1 \neq \lambda_2$. Show that v_1 and v_2 are orthogonal.

Problem 9
Find the dimension and construct a basis for the four subspaces associated with:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 1 & 3 & 0 & 1 \\ 2 & 6 & -1 & 1 \end{bmatrix}$$

Problem 10

Consider the finite difference equations

$$x_{n+1} = -7x_n + 10y_n$$

and

$$y_{n+1} = -5x_n + 8y_n$$

Given that $x_0 = 1$ and $y_0 = 0.5$, find x_4 and y_4.