Slide 21

$$\Omega = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega}\,(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

we call this bandwidth

$$\log(\hat{\Omega}_o) = \frac{\log(\hat{\Omega}_{p1}) + \log(\hat{\Omega}_{p2})}{2}$$

average of passband edge frequencies in dB

$$\cancel{\Omega_p} = -\cancel{\Omega_p} \frac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega}\,(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

$$\hat{\Omega}\,(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) = \hat{\Omega}^2 - \hat{\Omega}_o^2$$

For the proof $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2}$ look at next slide

$$\hat{\Omega}\,(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) = \hat{\Omega}^2 - \hat{\Omega}_{p1}\hat{\Omega}_{p2}$$

$$\hat{\Omega}^2 \mp (\hat{\Omega}_{p2} - \hat{\Omega}_{p1})\hat{\Omega} - \hat{\Omega}_{p1}\hat{\Omega}_{p2} = 0$$

$ax^2 + bx + c = 0$

roots $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$$a = 1, \quad b = \mp(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}), \quad c = -\hat{\Omega}_{p1}\hat{\Omega}_{p2}$$

$$\hat{\Omega} = \frac{-(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) \pm \sqrt{(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})^2 + 4\hat{\Omega}_{p1}\hat{\Omega}_{p2}}}{2}$$

The green option refers to the case $\Omega = -\Omega_p$

$$\hat{\Omega} = \frac{-(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) \pm (\hat{\Omega}_{p2} + \hat{\Omega}_{p1})}{2} = \begin{cases} (+) & \hat{\Omega}_{p2} \\ (-) & -\hat{\Omega}_{p1} \end{cases}$$

~

$\Omega = -\Omega_p$

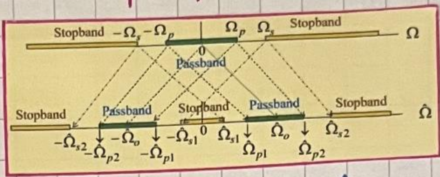
$$\hat{\Omega}\,(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) = \hat{\Omega}_o^2 - \hat{\Omega}^2$$

$$\hat{\Omega} = \frac{-(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) \pm (\hat{\Omega}_{p2} + \hat{\Omega}_{p1})}{2} = \begin{cases} (+) & \hat{\Omega}_{p1} \\ (-) & -\hat{\Omega}_{p2} \end{cases}$$

Slide 21 cont.

In this page I will prove that $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$

$$\Omega = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$



For $\Omega = \Omega_p$ we require that $\hat{\Omega} = \hat{\Omega}_{p2}$ or $\hat{\Omega} = -\hat{\Omega}_{p1}$

This makes sense since the original passband must be mapped to the new passband and its corresponding negative part. (Please look at Slide 22 of corresponding lecture, or look at figure above).
So lets replace Ω with Ω_p and $\hat{\Omega}$ with $\hat{\Omega}_{p2}$

$$\cancel{\Omega_p} = -\cancel{\Omega_p} \frac{\hat{\Omega}_o^2 - \hat{\Omega}_{p2}^2}{\hat{\Omega}_{p2}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

$$-(\hat{\Omega}_o^2 - \hat{\Omega}_{p2}^2) = \hat{\Omega}_{p2}^2 - \hat{\Omega}_{p2}\hat{\Omega}_{p1} \Rightarrow \hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2}$$

If you use the second mapping

$$\Omega_p = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}_{p1}^2}{-\hat{\Omega}_{p1}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

$$\hat{\Omega}_o^2 - \hat{\Omega}_{p1}^2 = \hat{\Omega}_{p1}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1}) \Rightarrow \hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2}$$

(same result)

If you use the mapping $\Omega_s \to \Omega_{s2}$ or $\Omega_s \to -\Omega_{s1}$ you will obtain:

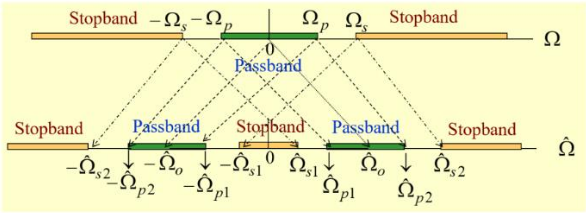
$$\Omega_s = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}_{s2}^2}{\hat{\Omega}_{s2}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})} \quad (1)$$

$$\Omega_s = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}_{s1}^2}{(-\hat{\Omega}_{s1})(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})} \quad (2)$$

Take the ratio (1)/(2): $\dfrac{(\hat{\Omega}_o^2 - \hat{\Omega}_{s2}^2)(-\hat{\Omega}_{s1})}{(\hat{\Omega}_o^2 - \hat{\Omega}_{s1}^2)\hat{\Omega}_{s2}} = 1 \Rightarrow$

$$-\hat{\Omega}_o^2\hat{\Omega}_{s1} + \hat{\Omega}_{s1}\hat{\Omega}_{s2}^2 = \hat{\Omega}_o^2\hat{\Omega}_{s2} - \hat{\Omega}_{s1}^2\hat{\Omega}_{s2} \Rightarrow$$

$$\hat{\Omega}_o^2(\hat{\Omega}_{s1} + \hat{\Omega}_{s2}) = \hat{\Omega}_{s1}\hat{\Omega}_{s2}(\hat{\Omega}_{s1} + \hat{\Omega}_{s2}) \Rightarrow \hat{\Omega}_o^2 = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$$


Stopband
$-\Omega_s$
$-\Omega_p$
Ω_p
Ω_s
Stopband
Ω
0
Passband
Stopband
Passband
Stopband
Passband
Stopband
$\hat{\Omega}$
$-\hat{\Omega}_{s2}$
$-\hat{\Omega}_{p2}$
$-\hat{\Omega}_o$
$-\hat{\Omega}_{p1}$
$-\hat{\Omega}_{s1}$
0
$\hat{\Omega}_{s1}$
$\hat{\Omega}_{p1}$
$\hat{\Omega}_o$
$\hat{\Omega}_{p2}$
$\hat{\Omega}_{s2}$