1. (a) Consider the causal, real-coefficient transfer function:

$$H_M(z) = \frac{\sum_{n=0}^{n=M} b[n]z^{-n}}{\sum_{n=0}^{n=M} a[n]z^{-n}}, M > 0$$

(i) Give the condition that must hold for $H_M(z)$ to be an allpass filter. Justify you answer. **[3]**

(ii) Comment on the locations of the zeros of a causal, real, stable, allpass filter's transfer function within the z −plane, with respect to the unit circle and the poles. Justify your answer. **[3]**

(b) Consider a first-order, causal, real, stable, allpass transfer function $H(z)$ given as:

$$H(z) = \frac{p + z^{-1}}{1 + pz^{-1}}$$

(i) Determine the impulse response $h[n]$ for $n \geq 0$, associated with $H(z)$, if $h[n] = 0$ for $n < 0$. **[5]**

(ii) Find the phase response associated with $H(z)$. **[4]**

(iii) Determine the expression for $|H(z)|^2 - 1$. **[3]**

(iv) Find for which values of z the condition $|H(z)|^2 < 1$ holds. **[3]**

(c) What is a major drawback in designing an all-pole IIR linear-phase transfer function? Justify your answer. **[4]**

2. (a) Show that an antisymmetric linear-phase Finite Impulse Response (FIR) transfer function $H(z)$ of odd length N can be expressed as follows:

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}\left(\sum_{n=1}^{\frac{N-1}{2}} h\left[\frac{N-1}{2}-n\right](z^{n}-z^{-n})\right) \qquad (1)$$

[8]

(b) By using the relation

$$U_r\left(\frac{z+z^{-1}}{2}\right) = \frac{z^{r+1}-z^{-(r+1)}}{z-z^{-1}}$$

where $U_r(x)$ is the r −th order **Chebyshev Polynomial of the Second Kind** in x, express $H(z)$ of (a) above in the alternative form

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}(z-z^{-1})\sum_{n=0}^{M} a[n]\left(\frac{z+z^{-1}}{2}\right)^{n} \qquad (2)$$

in the case where $N = 9$. Determine the relation between $a[n]$ and $h[n]$ for $N = 9$. You can easily determine the value of the parameter M as a function of N, by ensuring that the orders of the polynomials of equations (1) and (2) are equal.

The **Chebyshev Polynomials of the Second Kind** satisfy the following recursive relationship:

$U_0(x) = 1$
$U_1(x) = 2x$
$U_r(x) = 2xU_{r-1}(x) - U_{r-2}(x), r \geq 2$

[8]

(c) Develop a realization of $H(z)$ based on equation (2) above in the form of **Figure 2.1** below, where $F_1(z^{-1})$ and $F_2(z^{-1})$ are causal filters. Determine the form of $F_1(z^{-1})$ and $F_2(z^{-1})$. The triangular sign indicates multiplication with the parameter shown next to it.
[**Hint:** To solve this part, it is convenient to write equation (2) as a function of M only instead of N only or both M and N]. **[9]**

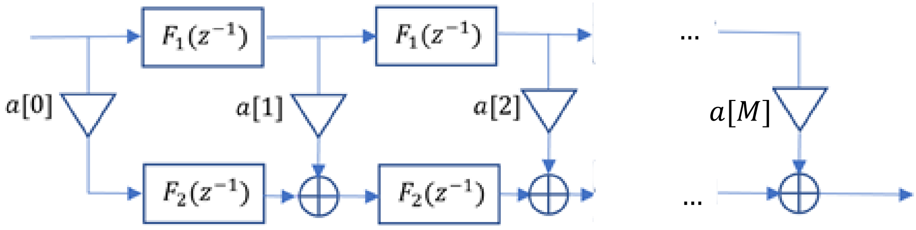

Figure 2.1

3. (a) The bilinear transformation from the s −plane to the z −plane is given by

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

(i) Illustrate, employing mathematical relationships, how the bilinear transformation maps every point $s = \sigma + j\Omega$ in the s −plane to the z −plane. **[5]**

(ii) Prove that the relationship between the continuous-time angular frequency and the discrete-time angular frequency is non-linear. **[5]**

(b) A given real-coefficient, digital IIR lowpass filter has a rational transfer function $H_L(z)$ and a cutoff frequency ω_c. The transfer function of $H_L(z)$ is transformed by replacing z by $F(z) = \frac{az+b}{cz+d}$ to a real highpass rational transfer function $H_H(z) = H_L(F(z))$ of the same order as $H_L(z)$ but different cutoff frequency. By using the constraints $H_H(e^{j\pi}) = H_L(e^{j0}) = 1$ and $H_H(e^{j0}) = H_L(e^{j\pi}) = 0$, derive relationships among the parameters a, b, c, d and explain what type of filter is $F(z)$. **[5]**

(c) Consider the two LTI causal digital filters with impulse responses given by:

$$h_A[n] = 0.5\delta[n] - \delta[n-1] + 0.5\delta[n-2]$$
$$h_B[n] = 0.25\delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2]$$

Explain the type of filters $h_A[n]$ and $h_B[n]$ (lowpass etc.), by using the following two approaches:

(i) Practical approach. In this approach you will apply the filters to the input signal $u[n]$, with $u[n]$ the discrete unit step function and observe the effects they have on $u[n]$. **[5]**

(ii) Mathematical approach. In this approach you must find and sketch the amplitude response of the two filters. **[5]**

4. (a) Examine the multirate configuration illustrated in **Figure 4.1** below. In this setup, $H_0(z), H_1(z)$ and $H_2(z)$ are lowpass, bandpass, and highpass filters, each possessing ideal, zero-phase characteristics and real coefficients. Their corresponding frequency responses are outlined as follows:

$$H_0(e^{j\omega}) = u(\omega) - u(\omega - \frac{\pi}{3})$$

$$H_1(e^{j\omega}) = u(\omega - \frac{\pi}{3}) - u(\omega - \frac{2\pi}{3})$$

$$H_2(e^{j\omega}) = u(\omega - \frac{2\pi}{3}) - u(\omega - \pi)$$

The function $u(\omega)$ is the unit step function, defined as

$$u(\omega) = \begin{cases} 1 & \omega \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

If the input is a real sequence with a Discrete Time Fourier Transform

$$X(e^{j\omega}) = (-\frac{3\omega^2}{\pi} + \frac{\pi}{3})\left(u(\omega) - u\left(\omega - \frac{\pi}{3}\right)\right)$$

sketch the Discrete Time Fourier Transform of the outputs $y_0[n], y_1[n]$ and $y_2[n]$.
It's important to note that all frequency domain representations mentioned are limited to the range $[0, \pi]$. **[15]**

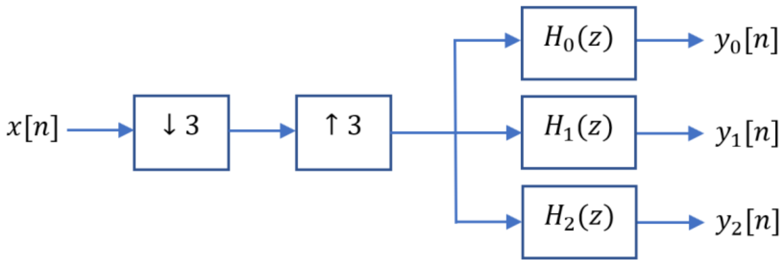

Figure 4.1

(b) Analyse the structure of **Figure 4.2** below and determine its input-output relations. **[10]**

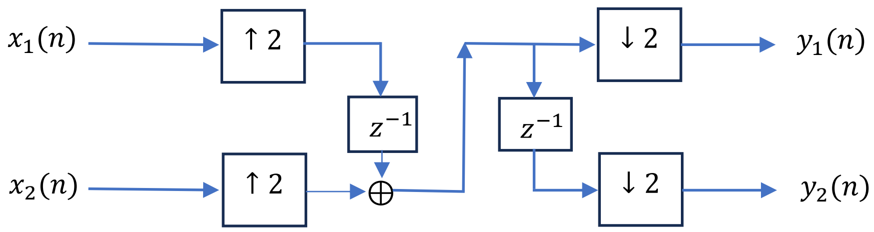

Figure 4.2