Maths for Signals and Systems
Problem Sheet 1

Problem 1

Find two 3-D vectors that are perpendicular to the vector $(1,0,0)$ and to each other.

Problem 2

(a) Can a matrix of the form below have an inverse? We assume that the elements shown with letters of the alphabet are not zero. Justify your answer.

$$\begin{bmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{bmatrix}$$

(b) What about a matrix of the form

$$\begin{bmatrix} a & 0 & b \\ 0 & 0 & c \\ d & e & 0 \end{bmatrix}$$

Problem 3

Find the determinant of the following matrix and investigate whether the matrix is singular for certain values of the parameter a.

$$A = \begin{bmatrix} 1-a & 1 & 1 \\ 1 & 1-a & 1 \\ 1 & 1 & 1-a \end{bmatrix}$$

Problem 4

Which of the following sets are vector spaces? (if not, explain why not)

a. The set of points $(\mathrm{x},\mathrm{y}) \in \mathrm{R}^2$ with $\mathrm{y} = 2\mathrm{x} + \mathrm{x}^2$
b. The set of points $(\mathrm{x},\mathrm{y}) \in \mathrm{R}^2$ with $\mathrm{y} = 1 - \mathrm{x}$
c. The set of matrices $\begin{pmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \\ \mathrm{d} & \mathrm{e} & \mathrm{f} \end{pmatrix}$ with a+2e=0

Problem 5

In R^n, if ***x*** and ***y*** are orthogonal, show that the Pythagorean Theorem holds:

$$||\boldsymbol{x} + \boldsymbol{y}||^2 = ||\boldsymbol{x}||^2 + ||\boldsymbol{y}||^2$$

Problem 6

Consider the following sub-space for R^4:

$$S = \text{span}\left\{ \begin{bmatrix} 0 \\ -1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ -5 \\ -5 \end{bmatrix} \right\}.$$

Find a basis for S and determine the dimension of S
Use Gram-Schmidt method to orthogonalize your basis

Problem 7
Compute the following determinants**:**

a. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$

b. $\begin{vmatrix} 2 & 3 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

c. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 8 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & 5 & 8 \end{vmatrix}$

Problem 8
Find a basis for the null space of

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 & -5/3 \\ 1/2 & 1 & 1/2 & -1/6 \\ -1 & -1 & 0 & 1 \end{bmatrix}$$

Problem 9
Consider the matrix

$$\boldsymbol{A} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

Compute its inverse.