ELEC60004/70068 Machine Reasoning
Tutorial 4 – KE Reasoning

1.

A. Prove, using the proof procedure KE, the following formulas:

i) $((p \wedge q) \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$
ii) $p \rightarrow ((p \wedge q) \vee (p \wedge \neg q))$

B. Using the KE calculus for propositional logic, show whether the following formulas are either theorems or non-theorems. Annotate your proof with the rules used at each step. For a non-theorem, show a counter-model which makes the formula false

(i) smoke -> smoke

(ii) (smoke -> fire) -> (-smoke -> -fire)

(iii) (smoke -> fire) -> ((smoke /\ alarm) -> fire)

(iv) ((smoke /\ alarm) -> fire) -> ((smoke -> (alarm -> fire))

(iv) (smoke -> fire) -> ((smoke -> alarm) /\ (alarm -> fire))

C. Using the KE calculus for propositional logic, show whether the following formulas are either theorems or non-theorems. Annotate your proof with the rules used at each step. For a non-theorem, show a counter-model which makes the formula false.

(i) (p -> (q \/ r)) <-> ((p -> q) \/ (p > r)))
(ii) (p -> (q \/ r)) <-> ((p -> q) /\ (p -> r)))

2.

In a particular football tournament, the first round involves groups of four teams, each team plays the others once, top two qualify for the quarter finals. In one group, after two out of three group matches involving AC Milan, Barcelona, Chelsea, and Hammersmith Academicals, suppose the following situation has arisen concerning qualification from the group stage for the quarter finals:

- *If Chelsea, AC Milan or Barcelona lose, then they do not qualify.*
- *If Hammersmith win or draw, then they qualify. Hammersmith qualify if they lose and AC Milan and Barcelona draw.*
- *AC Milan qualify if they win, or if they draw and Chelsea fail to win.*
- *Chelsea qualify if they win, or if they draw and Barcelona win.*
- *Barcelona qualify* ***only*** *if both they and Hammersmith win.*

(a) Formalise these statements in propositional logic, using the propositional symbols *a*, *b*, *c* and *h* for AC Milan, Barcelona, Chelsea and Hammersmith Academicals respectively.

(b) In the final matches, Chelsea drew with Hammersmith and AC Milan lost to Barcelona. Knowing that "if a team wins a football match, then it does not lose or draw that match", specify all the relevant facts.

(c) Using the KE calculus, prove that Chelsea and Hammersmith qualified, and that AC Milan and Barcelona did not. Clearly annotate the proof to show the derivation, including use of the beta-simplification rule.

3

Consider the following sentences.

> *If Ken passes the AI exam, then Jem passes the HCI exam.*
> *If Jem passes the HCI exam, then Ken passes the SE exam.*
> *Ken passes either the AI exam, or the SE exam, but not both.*

Formalise these statements in propositional logic.

Show, using the calculus KE, that *Ken passes the SE exam*.

Using the KE calculus, or otherwise, show whether (or not) *Jem passes the HCI exam*.

4.

As well as using KE for proof by refutation, KE can be used for model building, i.e. apply the KE rules and look for (ideally) one open branch.

Now consider the island of knights and knaves. Everyone is either a knight, or a knave, but not both. Knights always tell the truth, and knaves always lie.

You meet two islanders, *Tony* and *Gordon*. They say to you:

Tony: I am a knave if (and only if) Gordon is a knave.
Gordon: We are of different kinds.

Who is which?

5.

Pascal's Wager (in its modern form): Prove using KE

If I am paranoid, then if they are out to get me, then I win.

If I am paranoid, then if they are not out to get me, then I don't lose.

If I am not paranoid, then if they are not out to get me, then I don't win.

If I am not paranoid, then if they are out to get me, then I lose.

Therefore if I am paranoid then I win or I don't lose, and If I am not paranoid, then I don't win or I lose.

6.

In the **Obligatio Game**, a finite number n of rounds is chosen, depending on the severity of the exam.

The examiner then gives the candidate successive assertions, $\phi_1, \phi_2, \ldots \phi_n$, that she has to either *accept* or *reject* as each one is put forward. In the former case, ϕ_i is added to the candidate's stock of commitments; in the latter case, the negation $-\phi_i$ is added.

The candidate passes the exam if she maintains the consistency of her stock of commitments throughout all n assertions.

Suppose the candidate is exposed (successively) to the following three assertions ($n=3$):

1 p -> -(q \/ r)
2 p /\ q
3 q <-> r

Using the KE calculus, show what sequences of answers would ensure that the student passes the exam.

Requirement: use KE for model building, for all combinations of *accept* or *reject* of the assertions.

7. Use predicate KE to prove the following syllogism

No retile has fur.
All snakes are reptiles.
Therefore, no snake has fur.