Digital Signal Processing

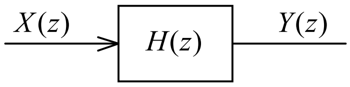
Tutorial Questions Booklet

Digital Signal Processing

Tutorial Sheet 1

Sampling and z-transforms

1*. Write down an expression in the z-domain for $Y(z)$ in terms of $X(z)$ and $H(z)$.



2*. Write down an expression for $X(z)$ in terms of $x(n)$.

3*. Write down an expression for $y(n)$ in terms of $x(n)$ and $h(n)$.

4**. Show that your expression for $y(n)$ in question 3 is related to your expression for $Y(z)$ in question 1 by the z-transform.

5***. Find the inverse z-transform of the system $X(z) = \dfrac{7z^2 - 5z}{z^2 - 2z - 3}$ for the following regions of convergence.

(a) $|z| < 1$

(b) $|z| > 3$

(c) $1 < |z| < 3$.

State in all cases whether the system is causal, anticausal or non-causal.

Digital Signal Processing

Tutorial Sheet 2

Discrete Fourier Transforms

1*. Find the Discrete Fourier Series coefficients for the sequence

$$x_p(n) = 10\sin(2\pi n/3)\ .$$

2**. Find the magnitude spectrum of the sequence [1, 1, 2, 3, 3]. Don't use software tools on a computer unless you write you own.

3**. A speech signal is sampled at 8 kHz. A DFT is computed of a block of 1024 samples of the data.

(a) What is the time duration of the block?

(b) What is the frequency resolution of the DFT?

(c) If only 512 data samples were taken but the block was zero padded to restore the size to 1024, what would be the new answers to (a) and (b)?

4***. A sequence $y(n)$ is constructed from a finite duration sequence $x(n)$ of length 8 samples in the following manner

$$y(n) = \begin{cases} x(n/2) & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases}$$

Determine $Y(k)$ in terms of $X(k)$ where $Y(k)$ and $X(k)$ are the DFTs of $y(n)$ and $x(n)$ respectively.

Digital Signal Processing

Tutorial Sheet 3

Convolution

1*. Given that $w(n) = x(n) * y(n)$, show that $W(z) = X(z).Y(z)$.

2**. Show that $x_{3p}(n) = \sum_{l=0}^{N-1} x_{1p}(l).x_{2p}(n-l)$ given that $X_p(k)$ is the DFS of $x_p(n)$ and that $X_{3p}(k) = X_{1p}(k).X_{2p}(k)$.

3***. Perform the linear convolution of the sequences $x(n)$ and $h(n)$ using (i) the overlap add procedure and (ii) the overlap save procedure.

$$x(n) = [3,2,1,4,1,2,3,2,1]$$
$$h(n) = [1,1]$$

The suggested block size is 3.

You may use MATLAB to compute the FFTs but otherwise work this problem on paper.

Digital Signal Processing

Tutorial Sheet 4

Digital Filters

1**. Design a lowpass causal linear-phase FIR filter with 8 taps and a cut-off frequency of $\pi/4$ using the frequency sampling method with a rectangular window.

2*. Find an expression for the d.c. gain of the first order lowpass filter

$$H(z) = k\frac{1+z^{-1}}{1-cz^{-1}}, \qquad |z| > |c|$$

3*. For the filter of Question 2, find the value of k for unity gain at d.c.

4**. For the filter of Question 2 with unity d.c. gain, find the value of c to give -3 dB gain at $\omega = \pi/2$.

5**. Sketch the magnitude response for the filter of Question 2 with $k = 1$ and

(i) $c = -0.9$

(ii) $c = -0.5$

(iii) $c = 0$

(iv) $c = 0.5$

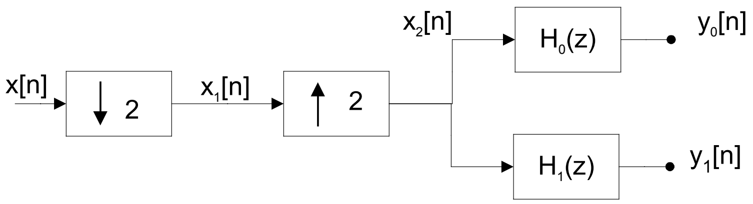
(v) $c = 0.9$.

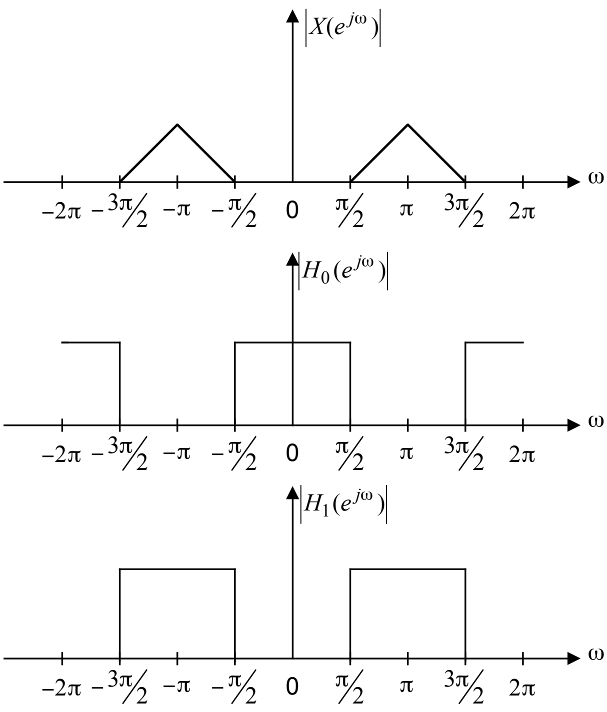
Digital Signal Processing

Tutorial Sheet 5

Multirate Signal Processing

1**. For the system below, sketch $X_1(z), X_2(z), Y_0(z)$ and $Y_1(z)$.





2*. Design a scheme for reducing the sampling rate of a signal to 0.75 of its orignal sampling frequency. Sketch the magnitude of the frequency response of any filters employed.

3**. Given a filter $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$, decompose this filter using type 1 polyphase decomposition into two filters $E_0(z)$ and $E_1(z)$. Sketch the signal flow graph for both the original filter and the polyphase filter.