MATH 235
Theorems

Sachin Kumar[*]
University of Waterloo

Winter 2023[†]

[*] *skmuthuk@uwaterloo.ca*
[†] Last updated: February 26, 2023

Contents

1 Abstract Vector Spaces

2 Linear Transformations

2.1 Rank and Nullity

Theorem 2.1.1
Let V and W be vector space over $\mathbb{F}$, and let $L : V \to W$ be a linear map. Then

1. $L(\vec{0}) = \vec{0}$
2. $\text{range}(L)$ is a subspace of W
3. $\ker(L)$ is a subspace of V

Theorem 2.1.2 (Rank-Nullity Theorem)
Let V and W be vector spaces over $\mathbb{F}$ with $\dim(V) = n$. Let $L : V \to W$ be a linear map.

$$\text{rank}(L) + \text{nullity}(L) = n$$

2.2 Linear Maps as Matrices

Theorem 2.2.1
Let V be an n-dimensional vector space with ordered basis $\mathcal{B}$. Let W be an m-dimensional vector space with ordered basis $\mathcal{C}$. Then, for every linear map $L : V \to W$, there exists $A \in M_{m\times n}(\mathbb{F})$ such that $[L(\vec{v})]_{\mathcal{C}} = A[\vec{v}]_{\mathcal{B}}$ for all $\vec{v} \in V$, which we denote ${}_{\mathcal{C}}[L]_{\mathcal{B}}$ is given by

$${}_{\mathcal{C}}[L]_{\mathcal{B}} = \left[[L(\vec{b}_1)]_{\mathcal{C}} \dots [L(\vec{b}_n)]_{\mathcal{C}}\right]$$