PRACTICAL:1

Ex1: $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\cdot\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\cdot\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\cancel{4}^{2}\cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\cancel{\sqrt{a}}}$$

$$=\frac{2}{3\sqrt{3}}$$

2] $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\frac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})}=\frac{1}{2a}$

3] $\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\frac{\pi}{6}$

where $h\to 0$

$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{6}\right)-\sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi-6\left(h+\frac{\pi}{6}\right)}$

$\therefore \lim_{h\to 0}\frac{\cos h\cdot\cancel{\cos}\frac{\pi}{6}\cdot\sin h\cdot\sin\frac{\pi}{6}-\sqrt{3}\sin h\cos\frac{\pi}{6}+\cos h\cdot\sin\frac{\pi}{6}}{\pi-\cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$

$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\frac{1}{\sqrt{2}}-\sqrt{3}\left(\sin h\frac{\sqrt{3}}{2}+\cos h\frac{1}{\sqrt{2}}\right)}{\cancel{\pi}-6h+\cancel{\pi}}$

$$\lim_{h\to 0} \frac{\cos\frac{\sqrt{3}h}{2} \cdot \sin\frac{h}{2} - \sin\frac{3h}{2} \cdot \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin \cancel{4}h}{3.\cancel{12}h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1$$

$$= \frac{1}{3}$$

4) $$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By rationalizing Numerator and Denominator we get

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{\cancel{8}^{4}(\sqrt{x^2+3} + \sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-\frac{3}{x^2})}}$$

After applying limit we get,

$= 4$

5] (i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$ $\Big\}$ at $x = \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad \therefore f(\pi/2) = 0.$

f at $x = \pi/2$ define.

(ii) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}.$

By substituting method

$x - \dfrac{\pi}{2} = h$

$x = h + \pi/2.$

where $h \to 0$

$\lim_{h \to 0} \dfrac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$

$\lim_{h \to 0} \dfrac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$

$\therefore \lim_{h \to 0} \dfrac{\cos h \cdot 0 - \sin h}{-2h}$

$\lim_{h \to 0} \dfrac{\cancel{-}\sin h}{\cancel{-}2h}$

$\dfrac{1}{2} \lim_{h \to 0} \dfrac{\sin h}{h}$

$= \dfrac{1}{2}.$

1) $\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}\ \sin x}$

$\lim_{x\to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x\to \pi/2^-} \cos x.$

$\therefore$ LHS $\neq$ RHS.

$\therefore$ f is not continious at $x = \pi/2$

5] $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x = 3$ & $x = 6$.

at $x = 3$

$\therefore f(3) = \cancel{\frac{x^2-9}{x-3} = 0}$

f at $x = 3$ defined.

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6.$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$\therefore$ L.H.S = R.H.S

f is continious at $x=3$

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}^{3}}{\cancel{9}} = 3$$

6) i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x<0$
$\quad\quad = k$ $\quad x=0$ $\Big\}$ at $x=0$

Soln $\rightarrow$ f is continious at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k.$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k.$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k.$$

$$2(2)^2 = k$$

$$\therefore k = 8$$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\quad\quad = k$ $\quad x=0$ $\Big\}$ at $x=0$.

sol $\rightarrow$ $f(x) = (\sec^2 x)^{\cot^2 x}$

using

$$\tan^2 x - \sec^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$f \cot^2 x = \frac{1}{\tan^2 x}$$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we, know that

$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$= e$

$\therefore K = e$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$

$= k.$ $x = \pi/3$ } at $x = \pi/3$

$x - \dfrac{\pi}{3} = h$

$\therefore x = h + \dfrac{\pi}{3}$

where $h \to 0$

$f\left(\dfrac{\pi}{3}+h\right) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$\lim_{h\to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

using. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\lim_{h\to 0} \dfrac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3}\left(1-\tan\frac{\pi}{3}\cdot\tan h\right)-\left(\tan\frac{\pi}{3}+\tan h\right)}{1-\tan\frac{\pi}{3}\cdot\tan h}}{-3h}.$$

$$\lim_{h\to 0} \frac{\frac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\frac{(\sqrt{3}-3\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\sqrt{3}\tan h.}}{-3h.}$$

$$\lim_{h\to 0} \frac{\frac{(\cancel{\sqrt{3}}-3\tan h-\cancel{\sqrt{3}}-\tan h)}{1-\sqrt{3}\tan h}}{-3h.}$$

$$\lim_{h\to 0} \frac{\frac{(\sqrt{3}-3\tan h-\sqrt{3}-\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h.}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)}$$

$$=\frac{4}{3}\frac{1}{(1-\sqrt{3}(0))}$$

$$=\frac{4}{3}\left(\frac{1}{1}\right)=\frac{4}{3}$$

$$=\frac{4}{3}.$$

7) $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $\quad x \neq 0$

$= 9 \quad x = 0$ $\Big\}$ at $x=0$

$f(x) = \dfrac{1-\cos 3x}{x\tan x}$

$\lim_{x\to 0} = \dfrac{2\sin^2 3/2\, x}{x\tan x}$

$\lim_{x\to 0} = \dfrac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\dfrac{x\cdot\tan x}{x^2} \times x^2}$

$= 2\lim_{x\to 0} \dfrac{\left(\frac{3}{2}\right)^2}{1} = \cancel{2} \times \dfrac{9}{\cancel{4}\,2} = \dfrac{9}{2}$

$\lim_{x\to 0} f(x) = \dfrac{9}{2} \quad 9 = f(0)$

$\therefore f$ is not continious at $x=0$.

Redefine function.

$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x=0 \end{cases}$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

7) ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$

$= \pi/6 \quad x=0$ $\Big\}$ at $x=0$

$\lim_{x\to 0} \dfrac{e^{3x}-1}{x} \lim_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x\to 0} 3\cdot \dfrac{e^{3x}-1}{3x} \lim_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\log_e \frac{\pi}{\frac{180}{60}} = \frac{\pi}{60} = f(0)$

f is continious at $x=0$.

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$. $x=0$.

is continious at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x\to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

multiply with 2 Num & Denominator

$= 1 + \cancel{2} \times \frac{1}{\cancel{4}2} = \frac{3}{2} = f(0)$

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9) $f(x) = \dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \quad x \neq \pi/2$

$f(0)$ is continious at $x = \pi/2$

$\lim\limits_{x\to\pi/2} \dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$\lim\limits_{x\to\pi/2} \dfrac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim\limits_{x\to\pi/2} \dfrac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \lim\limits_{x\to\pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \dfrac{1}{2(2\sqrt{2})}$

$= \dfrac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$

AK
02/12/19

2. Derivative

Q1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

(i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(\cancel{a} + h - \cancel{a})\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\cancel{\tan a -} \left[\frac{\tan a + \tan h}{1 - \tan a \tan h}\right]}{h(\tan(a+h)(\tan a))]}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan^2 a \tan h - (\tan a + \tan h)]}{h(\tan(a+h) \times \tan a] \times \cancel{(1 - \tan a \tan h]}}$$

$$= \lim_{h\to 0} \frac{\tan^2 a - 0 - (1+0)}{h(\tan a \times \tan a) \times (1-0)}$$

$$= \lim_{h\to 0} (-1) \times \frac{\tan^2 a + 1}{h(\tan^2 a)}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= -\cos^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall\, a \in R$.

(2) cosec$x$

$f(x) = \text{cosec}\,x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\text{cosec}\,x - \text{cosec}\,a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{\sin a \sin (x-a)}$$

let $x - a = h$.

$x = a+h$

as $x \to a$, $h \to 0$

$$= \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{\sin a \sin(h)}$$

$$= \frac{\lim_{h\to 0} 2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\lim_{h\to 0} \sin a \sin(h)}$$

$$= \frac{\lim_{h\to 0} 2\cos\left(a+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\lim_{h\to 0} \sin a \sin(a+h)(h)}$$

$$= \lim_{h\to 0} \frac{2\cos\left(a+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)\times\left(-\frac{1}{2}\right)}{\sin a \cdot \sin(a+h)\left(-\frac{h}{2}\right)}$$

$$= \frac{-\cos(a)\cdot(1)}{\sin^2 a}$$

$$= \frac{-\cos a}{\sin^2 a}$$

$$= -\cot a \cdot \operatorname{cosec} a.$$

(3) $\sec x$

$f(x) = \sec x.$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(\cos x)(\cos a)(x-a)}$$

put $x = a+h$

As $x\to a$ $h\to 0.$

$$= \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{\cos(a+h)\cos a \times h}$$

$$= \lim_{h\to 0} \frac{2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a+h-a}{2}\right)}{\cos(a+h)\cos a \times h}$$

$$= \lim_{h\to 0} \frac{2\sin\left(a+\frac{h}{2}\right)\cdot\sin\left(\frac{h}{2}\right)\cdot\times\frac{1}{2}}{\cos(a+h)\cdot\cos\cdot\frac{h}{2}}$$

$= \dfrac{\sin(a+0)(1)}{\cos(a+0)\cos a}$

$= \dfrac{\sin a}{\cos^2 a}$

$=$ tanaceea

Q) If $f(x) = 4x+1, \quad x \leq 2$

$= x^2+5 \quad , x > 0$ at $x = 2$. then

find f is differentiable or not?

→ LHD:-

$Df(2^-) = \lim\limits_{x \to 2^-} \dfrac{f(x) - f(2)}{x-2}$

$\lim\limits_{x \to 2^-} \dfrac{4x+1 - (8+1)}{x-2}$

$= \lim\limits_{x \to 2^-} \dfrac{4x+1-9}{x-2}$

$= \lim\limits_{x \to 2^-} \dfrac{4x-8}{x-2}$

$\lim\limits_{x \to 2^-} \dfrac{4\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 4$

RHD :-

$$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} x+2$$

$$= 4.$$

$\therefore$ RHD = LHD

Q3) If $f(x) = 4x+7, \quad x<3$

$\qquad = x^2+3x+1, \quad x \geq 3$ at $x=3$, then

find f is differentiable or not?

→ LHD:

$$Df(2^-) = \lim_{x\to 3^-} \frac{4x+7-(12+7)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4(x-3)}{x-3} = 4$$

$$Df(3^+) = \lim_{x\to 3^+} \frac{x^2+3x+1-(19)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{x-3}$$

$x+6$

$= 9.$

LHD ≠ RHD.

∴ f is not differentiable at $x=3$.

Q4 If $f(x) = 8x-5$, $x \le 2$.
$= 3x^2-4x+7$, $x > 2$ at $x=2$, then
find f is differentiable or not.

→ LHD:-

$$Df(2^-) = \lim_{x \to 2^-} \frac{8x-5-(11)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$$

→ LHD:-

$$Df(2^-) = \lim_{x \to 2^-} \frac{8x-5-(4)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{\cancel{20}\, x-2}$$

$$= \lim_{x \to 2^-} \frac{\cancel{8x-16}\ 8(x-2)}{x-2}$$

$= 8$

RHD:-

$$Df(2^+) = \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - (11)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3(2) + 2$$

$$= 8$$

09/12/19

PRATICAL - 3

Topic: Application of Derivatives

Q1) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$
(ii) $f(x) = x^2 - 4x$
(iii) $f(x) = 2x^3 + x^2 - 20x + 4$
(iv) $f(x) = x^3 - 27x + 5$
(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2) Find the intervals in which function is concave upwards.

(i) $y = 3x^2 - 2x^3$
(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
(iii) $y = x^3 - 27x + 5$
(iv) $y = 69 - 24x - 9x^2 + 2x^3$
(v) $y = 2x^3 + x^2 - 20x + 4$

Solutions:

Q1) (i) $f(x) = x^3 - 5x - 11$
$\therefore f'(x) = 3x^2 - 5$
$\therefore f$ is increasing iff $f'(x) > 0$.
$3x^2 - 5 > 0$
$3(x^2 - 5/3) > 0$
$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$-\sqrt{5}/3$ + $\sqrt{5}/3$

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$ and f is decreasing

iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore + (x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$

$-\sqrt{5}/3 \quad + \quad \sqrt{5}/3$

$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore (x-2) > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 (3x^2 + x - 10) > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore$ ~~f(x~~ $6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-2, 5/3)$

(2) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad -3 \quad - \quad 3 \quad +$

2] (a) $f(x) = 2x^3 - 9x^2 - 24x + 61$

$f'(x) = 6x^2 - 18x - 24$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$+ \quad -1 \quad 4 \quad +$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

3] $+ \quad - \quad + \quad -1 \quad 4$

$\therefore x \in (-1, 4)$

Q2 sol:-

i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is contave upword if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

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$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$.

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upword if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12 \cdot (x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad 1 \quad - \quad 2 \quad +$

$x \in (-\infty, 1) \cup (2, \infty)$

$y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upword iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

) $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12 - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0.$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

5] $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) > 0$

$\therefore$ There exist no interval

$x \in (-1/6, \infty)$

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PRATICAL 4:-

Application of Derivative & Newton's method

Q1) Find maximum & minimum value of following functi

(i) $f(x) = x^2 + \frac{16}{x^2}$ (ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q2) Find the value of following equation by Newton's method: correct upto the decimal.

(Take iterations only)

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$.

iii) $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$

Answers:

Q1)

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$ $\therefore \boxed{x = +/-2}$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$. $\therefore f(2) = 2^2 + 16/2^2$

$= 8$

$f''(-2) = 2 + 96/-2^4 = 8 > 0$.

$= 2 + 96/16$

$\therefore f$ has minimum value at $x = 2$

$\therefore$ function reaches minimum value at $x = 2$ & $x = -2$.

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = +1, -1$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60 \therefore 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5 = 6 - 5 = 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60 \quad \therefore -30 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-5)^5 = 5$

$\therefore f$ has the maximum value 5 at $x = -1$

f has minimum value 1 at $x = 1$.

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0 \quad \therefore 3x^2 - 6x = 0 \quad \therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0 \quad \therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6 = -6 < 0$

$\therefore f$ has maximum value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$.

$\therefore f''(2) = 6(2) - 6 \quad \therefore 6 > 0$

$\therefore f$ has maximum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1 = -3$

$\therefore f$ has maximum value, at $x = 0$ &

f has minimum value -3 at $x = 2$

(x) $f(x) = 2x^3 - 3x^2 - 12x + 1$
$f'(x) = 6x^2 - 6x - 12$
consider, $f'(x) = 0$ $\therefore 6x^2 - 6x - 12 = 0$
$6(x^2 - x - 2) = 0$ $\therefore x^2 + x - 2x - 2 = 0$
$\therefore x(x+1) - 2(x+1) = 0$ $\therefore (x-2)(x+1) = 0$
$\therefore x = 2$ or $x = 1$ $\therefore f''(x) = 12\cancel{x}-6$ $12x - 6$.
$f''(2) = 12(2) - 6$ $\therefore 2x - 6$ $\therefore 18 > 0$
f has minimum value at $x = 2$
$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$= 16 - 12 - 24 + 1$
$= -19$
$f''(-1) = 12(-1) - 6$
$= -12 - 6$ $= -18 < 0$.
f has maximum value at $x = -1$.

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1 = -8$
$\therefore f$ has maximum value 8 at $x = -1$ &
f has minimum value -19 at $x = 2$.

$x_0 = 0$ (given)

Q2) $f(x) = x^3 - 3x^2 - 55x + 95$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$\therefore x_1 = 0 + 95/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 95$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1) = 0.1727 - 0.0829/55.9467$

$= 0.1212$

(ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9 = -9$

$f(3) = 3^3 - 4(3) - 9 = 27 - 12 - 9 = 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 \ldots$$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9 = 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$
$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1) = 2.7392 - \frac{0.596}{18.5096} = 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 17.9851$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$
$= 2.7071 - \frac{0.0102}{17.9861} = 2.7071 - 0.0061 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 =$
$= 17.9851$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$
$= 2.7071 - \frac{0.0102}{17.9851} = 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 17.8943$
$= 17.8943$

$f' x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$= 2.7015 + 0.0050 = 2.7065$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $\quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation.

By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2.2}{5.2} = 2 - 0.4230$$

$= 1.577$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10 = -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.577 + \frac{0.3755}{8.2164}$

$= 1.577 + 0.0822 = 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.659$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.0204}{0.7143} = 1.66$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) +$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

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$f''(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 18.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.6618 + \frac{0.0004}{7.6977} = 1.6618.$

$\therefore$ root of equation is 1.6618.

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PRATICAL - 5

Topic: Integration

Q1) Solve the following integration

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2 - 3\sin x + 3\sqrt{x})\,dx$

iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2+4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx.$

(x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

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$$\int \frac{1}{x^2+2x-3}\,dx$$

$$\int \frac{1}{\sqrt{x^2+2x-3}}\,dx$$

$$\int \frac{1}{\sqrt{x^2+2x+1-4}}$$

$$a^2+2ab+b^2 = (a+b)^2$$

$$\int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$$

substitute

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $\therefore t = x+1$

$$\int \frac{1}{\sqrt{t^2-4}}\,dt$$

using

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c$

$$\int (4e^{3x}+1)\,dx$$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{ax}\,dx = \frac{1}{a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c.$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$ $\qquad$ * $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3 + 3x + 4\, dx}{\sqrt{x}}$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx$

Split the denominator.

$\int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\, dx.$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \cdot \sin(2t^4)\, dt.$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \frac{1}{2\times 4t^3}\, du$

$= \int t^{\cancel{4}} \sin(2t^4) \times \frac{1}{8}\, du$

$= \frac{t^4 \sin(2t^4)}{8}\, du$

substitute t^4 with $u/2$

$= \int \frac{4/2 \times \sin(u) du}{8}$

$= \int u \times \frac{\sin 4}{16} \; du$

$= \frac{1}{16} \int u \times \sin(u)\, du$

$\int u\, du = uv - \int v\, du.$

$dv = u = u$

$dv = \sin(u) \cdot du$

$du = 1. \; dv = v = -\cos(u)$

$\frac{1}{16} (u \times -\cos(u)) = \int -\cos(u) du$

$= \frac{1}{16} (u \times (-\cos u)) + \int \cos(u) du.$

$\# \int \cos x\, dx = \sin x$

$= \frac{1}{16} (u(-\cos u)) + \sin(u))$

resubstituting the value

$= 1/16 \times (2t^4 (-\cos(t^4) + \sin(2t^4)))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C.$

6) $\int \sqrt{x} (x^2 - 1) dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx.$

$= \int x^{1/2} \cdot x^2 - x^{1/2} dx$

$= \int x^{3/2} dx = \int x^{1/2} dx.$

$= \frac{2x^3 - \sqrt{x}}{7} + \frac{2 - \sqrt{x}}{3} + C.$

7) $\int \frac{1}{x^3} \cdot \sin\left(\frac{1}{x^2}\right) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\Rightarrow -\frac{2}{x^3} \, dx = dt$

$I = -\frac{1}{2} \int -\frac{2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2} \int \sin t$

$= -\frac{1}{2} (-\cos t) + c$

$= \frac{1}{2} \cos t + c$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

8) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \int \frac{\cos x}{\sqrt[3]{\sin x}}$

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let $\sin x = t$

$\cos x \, dx = dt$

$I = \int \frac{dt}{dt^2}$

$= \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} \, dt$

$= 3t^{1/3} + c$

$= 3(\sin x)^{1/3} + c$

$= 3 \cdot \sqrt[3]{\sin x} + c$

9. $\int e^{\cos^2 x} \cdot \sin^2 x \, dx$

$I = \int e^{\cos^2 x} \cdot \sin^2 x \, dx$

let $\cos^2 x = t$

$-2 \sin x \cdot \sin x \cdot dx = dt$

$-2 \sin 2x \, dx = dt$

$I = \int - \sin 2x \times e^{\cos^2 x} dx$

$= -\int e^t dt$

$= e^t + c$

resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

10) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

let $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x) \, dx = dt$

$(x^2 - 2x) \, dx = dt/3$

$I = \frac{1}{t} \, dt$

$= \frac{1}{3} \int dt/t$

$\frac{1}{3}\log t + c$

$= \frac{1}{3}\log(x^3 - 3x^2 + 1) + c$

06/01/2020

PRATICAL - 6

Topic :- Application of integration and Numerical integration

Find the length of the following ~~curve~~ curve

~~$x = t+\sin t$, $y = 1-\cos t$~~

1) $x = t-\sin t$, $y = 1-\cos t$ $\quad t \in [0, 2\pi]$

2) $y = \sqrt{4-x^2}$ $\quad x \in [-2, 2]$

3) $y = x^{3/2}$ in $[0, 4]$

4) $x = 3\sin t$, $y = 3\cos t$ $\quad t \in [0, 2\pi]$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

using simpson's Rule solve the following

$\int_0^2 e^{x^2} dx$ with $n = 4$

$\int_0^4 x^2 dx$ with $n = 4$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

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Ans i) $x = t - \sin t$

$y = 1 - \cos t$

$t \in [0, 2\pi]$

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$

$$\frac{dx}{dt} = 1 - \cos t$$

$y = 1 - \cos t$

$$\frac{dy}{dt} = 0 - (-\sin t)$$

$$\frac{dy}{dt} = \sin t$$

$$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt$$

$$= \int_0^{2\pi} 2 \sin \frac{t}{2}\, dt$$

$$= \left[-4 \cos\left(\frac{t}{2}\right)\right]_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4$$

$$= 8$$

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$y = \sqrt{4 - x^2} \quad x \in [-2, 2]$

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dy}{dx} \quad = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

$y = x^{3/2}$ in $[0, 4]$

$f'(x) = \frac{3}{2} \cancel{x^{1/2}} \cdot x^{1/2}$

$[f'(x)]^2 = 9/4x$

$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^x \sqrt{1 + \frac{9}{4}x}\, dx$

put $u = 1 + \frac{9}{4}x, \quad du = \frac{9}{4}dx$

$L = \int_1^{1+9/4x} \frac{4}{9}\sqrt{u}\, du$

$= \left[\frac{4}{9} \cdot \frac{2}{3}\left(u^{3/2}\right)\right]_1^{1+9/4x}$

$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$

4] $x = 3\sin t \quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\int n\, dt$

$= 3\,[n]_0^{2\pi}$

$= 3\,[2\pi - 0]$

$L = 6\pi$ units.

5) $x = \frac{1}{6}y^3 + \frac{1}{2y} \quad y \in [1, 2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y_2}.$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy.$

$= \int_1^2 \frac{y^{4+1}}{2y^2}\, dy.$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

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$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$ units

Q2) 1) $\int_0^2 e^{x^2}\,dx = 16.4526$

In each case ~~with~~ the width of the sub-interval be $dx = \frac{2-0}{4} = \frac{1}{2}$

and so the sub interval will be $[0, 0.5], [0.5, 1] [1, 1.5]$ $[1.5, 2]$.

By simpson's Rule

$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$

$\approx \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right]$

≈ 17.3536.

2) $\int_1^4 x^2\,dx \; n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$

$= \frac{1}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$

$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$

$= \frac{64}{3}$;

5

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6.$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/8$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	6.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{\Delta x}{3} (y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4$

$= \frac{\pi/18}{3} (0 + 4(0.4167 + 0.707 + 0.875)$
$+ 6.930)$

$\approx 2(0.584 + 0.801) + 0.930$

≈ 0.681

PRATICAL-7

Q1) solve the following differential equations

(1) $x \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$p(x) = \frac{1}{x}$ $\quad$ $Q(x) = \frac{e^x}{x}$

If $e \int_x^{1/x} dx$

$y(IF) = Q(x)(IF)\,dx + c$

$= \int \frac{e^x}{x} x \cdot dx + c$

$= \int e^x dx + c.$

$xy = e^x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

soln :- $\frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$p(x) = 2 \; Q(x) = e^{-x}$

$\int P(x) = dx$

$If = e \int 2dx.$

$= e^{2x}$

$y(If) = \int Q(x)\,(If)\,dx + c$

$= \int e^x\,dx + c$

$= y \cdot e^{2x}$

$= e^x + c$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

Soln:-

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$$P(x) = 2(x). \qquad Q(x) = \frac{\cos x}{x^2}.$$

$$If = e^{\int P(x)dx}$$

$$= e^{\cancel{\int 2dt}}$$

$$= e^{\int 2x dx}$$

$$y.(If) = \int Q(x)(If)dx + c$$

$$= \int \frac{\cos x}{x^2} \cdot x^2 dx + c$$

$$= \int \cos x + c$$

$$\therefore x^2 y = \sin x + c$$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Soln:-

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$

$$P(x) = 3/x \qquad Q(x) = \sin x/x^3$$

$$P(x) = \int 3/x \, dx.$$

$$= x^3$$

$$If = e^{\int P(x)dx}$$

$$= x^3$$

$$y(If) = \int Q(x)(If)dx + c$$

$$= \int \frac{\sin x}{x^3} \cdot x^3 dx + c$$

$$= \int \sin x + c$$

$$x^3 y = -\cos x + c.$$

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5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

Soln :

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2}x}$

$P(x) = 2$

$Q(x) = \frac{2x}{e} = 2x e^{-2x}$

$(IF) = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int 2x e^{-2x} e^{2x} + c$

$y e^{2x} = \int 2x + c = x^2 + c$

6) $\sec^2 x . \tan y\,dx + \sec^2 y \tan x\,dy = 0$

Soln :-

$= \sec^2 x . \tan y\,dx = -\sec^2 y \tan x\,dy$

$= \frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 x}{\tan y}\,dy$

$= \cancel{\int \frac{\sec^2 x}{\tan x}\,dx} = -\int \frac{\sec^2 x}{\tan y}\,dy$

$= \int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 x}{\tan y}\,dy$

$= \log|\tan x| = -\log|\tan y| + c$

$= \log|\tan x + \tan y| = c$

$\tan x + \tan y = e^c$

7) $\dfrac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$x-y+1 = v$

$1 - \dfrac{dy}{dx} = \dfrac{dv}{dx}$

$1 - \dfrac{dv}{dx} = \dfrac{dy}{dx}$

$1 - \dfrac{dv}{dx} = \sin^2 v$

$\dfrac{dv}{dx} = 1 - \sin^2 v$

$\dfrac{dv}{dx} = \cos^2 x$

$\dfrac{dv}{\cos^2 x} = dx$

$\int \sec^2 x \, dv = \int dx$

$\tan v = x + c$

$\tan(x-y+1) = x + c$

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$= \frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{(v+2)}$$

$$= \int \frac{v+2}{v+1}\,dv = 3\,dx$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = \int 3\,dx$$

$$v \log|v| = 3x+c$$

$$2x+3y+\log|2x+3y+1| = 3x+c$$

$$3y = x - \log|2x+3y+1| + c.$$

PRATICAL - 8

TOPIC: Euler's method

$\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2) = ?$

$f(x) = y + e^x - 2 \quad x_0 = 0 \quad y(0) = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205		
4	2	9.8215		

$\therefore y(2) = 9.8215$

$\frac{dy}{dx} = 1 + y_2, \quad y(0) = 1 \quad h = 0.2$ find $y(1) = ?$

$y_0 = 0, \quad y_0 = 0 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

Q 3) $\frac{dy}{dx} = \frac{\sqrt{x}}{y}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$\therefore y(1) = 1.5051$

Q 4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$ - $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

$y_0 = 2 \quad x_0 = 1 \quad h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y(1) = 1 \quad h = 0.2$

$x_0 = 1 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

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PRATICAL - 9

Topic: Limits of Partial Order derivatives

Q1. Evaluate the following limits:-

(i) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By Applying limit

$= \dfrac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \dfrac{-64+3+1-1}{4+5}$

$= \dfrac{-61}{9}$

ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, Denominator $\neq 0$.

$\therefore$ By Applying limits,

$= \dfrac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \dfrac{1(4+0-8)}{2}$

$= \dfrac{4}{2}$

$= -2$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$.

$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2.z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On Applying Limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q2.

(i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f(x) = \frac{d}{dx}(f(x,y))$

$= \frac{d}{dx}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore fx = 2xye^{x^2+y^2}$

$fy = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore fy = 2yxe^{x^2+y^2}.$

ii) $f(x, y) = e^{x} \cos y$

$f_x = \frac{d}{dx}(f(x, y))$

$= \frac{d}{dx}(e^{x} \cos y)$

$\therefore f_x = e^{x} \cos y$

$f_y = \frac{d}{dy}(f(x, y))$

$= \frac{d}{dy}(e^{x} \cos y)$

$\therefore f_y = -e^{x} \sin y$

(iii) $f(x, y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{d}{dx}(f(x, y))$

$= \frac{d}{dx}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6xy$

$f_y = \frac{d}{dy}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q3) (i) $f(x, y) = \frac{2x}{1+y^2}$

$f_x = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{2 + 2y^2 - 0}{(1+y^2)^2}$

$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y^2)}$$

$$= \frac{2}{(1+y^2)}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{d}{dy}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

Q4) (i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{d}{dx}\left(\left(-x^2y - 2x(y^2 - xy)\right)\right)$$

$$= \frac{x^4\left(\frac{d}{dx}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{d}{dx}}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y}{x^6}$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad \text{——— (2)}$$

$$f_{xy} = \frac{d}{dy}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{——— (3)}$$

$$f_{yx} = \frac{d}{dx}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(x^2)}{(x^2)^2} \quad \text{——— (4)}$$

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$$= \frac{-x^2 - 4xy + 2x}{x^4}$$

$\therefore$ from (3) & (4)

$f_{xy} = f_{yx}$

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{d}{dx}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{d}{dy}(x^2 + 3x^2y^2 - \log(x^2+1))$$

$$= 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\frac{d}{dx}(2x) - 2x\frac{d(x^2)}{dx}}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \text{—— (1)}$$

$$f_{yy} = \frac{d}{dy}(6x^2y)$$

$$= 6y^2.$$

$$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right) \quad \text{—— (2)}$$

$$= 0 + 12xy - 0$$

$$= 12xy$$

$$f_{yx} = \frac{d}{dx}(6x^2y)$$

$$= 12xy$$

from (3) & (4)

$\therefore f_{xy} = f_{yx}.$

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(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}$

$= x\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$ —— (1)

$= -y^2\sin(xy) + e^{x+y}$

$f_{yy} = \frac{d}{dy}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ —— (2)

$f_{xy} = \frac{d}{dy}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ —— (3)

$f_{yx} = \frac{d}{dx}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ —— (4)

$\therefore$ from (3) & (4)

$\cancel{f_{xy}} \neq f_{yx}$

Q5) (i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad$ $\therefore f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$

$= \frac{x+y}{\sqrt{2}}$.

ii)

(ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$ $\qquad f_y = 0 - 0 + \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0$ $\qquad f_y$ at $(\frac{\pi}{2}, 0) = \sin(\frac{\pi}{2})$

$= -1$

$L(x,y) = f(a,b) + f_x(a,b)(x, a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \cancel{\pi/2} - x + \cancel{\pi/2} + y$

$= 1 - x + y$ "

(iii)

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\qquad f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + \cancel{f_x} \; f_x(a,b)(x-a) + f_y(a,b)(y$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$ "

PRATICAL-10

Topic: Directional derivative, Gradient vector & maxima, minima, Tangent and Normal vectors.

Q) Find the directional derivative of the following function at given points & in the direction of given vector.

(i) $f(x,y) = x+2y-3 \qquad a = (1,-1) \qquad u = 3i-j$

Here, $u = 3i-j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{20}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1)+2(-1)-3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right)$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-4} + h/\sqrt{10} \cancel{+4}}{\cancel{h}}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right).$$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}} \right)$$

$$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}} \right)^2 - 4\left(3 + \frac{h}{\sqrt{26}} \right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$\frac{25}{26}h^2 + \frac{40h - 4h}{\sqrt{26}} + 5.$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5.$

$D_u f'(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$

$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$,,

(iii) $2x + 3y$ $a = (1,2)$, $u = (3i + 4j)$.

Here $u = 3i + 4j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$.

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$.

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + \cancel{8} - \cancel{8}}{h}$

$= \frac{18}{5}$

Q2) Find gradient vector for the following function at given point $a = (1, 1)$

(i) $f(x, y) = x^y + y^x$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x, y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, \; x^y \log x + x y^{x-1})$

$f(1, 1) = (1+, 1+0)$

$= (1, 1)$

ii) $f(x, y) = (\tan^{-1} x) \cdot y^2$ $a = (1, -1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x, y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, \; 2y \tan^{-1} x\right)$

$f(1, -1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4_2}(-\cancel{2})\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

i) $f(x, y, z) = xyz - e^{x+y+z}$, $a = (1, -1, 0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x, y, z) = f_x, f_y, f_z$

$f(1, -1, 0) = yz - e^{x+y+z}, \; xz - e^{x+y+z}, \; xy - e^{x+y+z}$

$= ((-1)(0) - e^{1+(-1)+0}, \; (1)(0) - e^{1+(-1)+0}, \; (1)(-1)$

$= (0 - e^0, \; 0 - e^0, \; -1 - e^1)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent & normal to each of the following using curves at given points

$\therefore x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x.$

$(x_0, y_0) = (1, 0)$ $\therefore x_0 = 1, \; y_0 = 0.$

eq^n of tangent.

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \cdot 2(1) + e^0 \cdot 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x - 1) + 1(y - 0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the required eq^n of tangent.

eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1)+2(y)+d=0$ at $(1,0)$

$\therefore 1+2y+d=0$

$= 1+2(0)+d=0$

$d+1=0$

$\therefore d=-1$

(ii) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0.$

$= 2x-2.$

$f_y = 0+2y-0+3+0.$

$= 2y+3$

$\cancel{(x_0, y_0) = 2(2)-2=2.}$

$f(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2)-2=2$

$f_y(x_0, y_0) = 2(-2)+3=-1$

eq^n of tangent.

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(1(y+2))=0.$

$2x-2-y-2=0$

$2x-y-4=0 \longrightarrow$ It is required eq^n of tangent

eq^n of Normal

$ax+by+c=0.$

$bx+ay+d=0.$

$=-1(x)+2(y)+d=0$

$= -x+2y+d=0$ at $(2,-2)$

$\therefore 2-4+d=0$

$= -6+d=0$

$\therefore d=6$

Q4) Find the eq^n of tangent & normal line to each of the following surface.

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$.

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore$ $x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$\therefore 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \longrightarrow$ This is required eq^n of tangent

Eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+4}{0}$$

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at $(1,-1,2)$

at $(1,-1,2)$

ii) $3xyz - x - y + z = -4$

$3xyz - x - y + z + 4 = 0$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3xz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$ $\therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$(x_0, y_0, z_0) = (1, -1, 2)$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ $\longrightarrow$ This is required eq^n of tangent

Eq^n of normal at $(-7, 5, -2)$

$= \dfrac{x - x_0}{f_x} = \dfrac{y - y_0}{f_y} = \dfrac{z - z_0}{f_z}$

$= \dfrac{x-1}{-7} = \dfrac{y+1}{5} = \dfrac{z-2}{-2}$ //

e) Find the local maxima & minima for the following function.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad \text{—— (1)}$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad \text{—— (2)}$

multiply eqn 1 ~~with 2~~.

$\therefore \cancel{4x - 2y = -4}$

$\cancel{2y - 3x = 4}$

$x = 0$

Substitute value of x in eqn (1)

$2(0) - y = -2 \quad \therefore y = 2$,

critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

here $r > 0$

$rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

(ii) $f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — (1)

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

multiply eqn (1) with 3

(2) with 4

$\cancel{12x^2} + 9y = 0$

$\cancel{12x^2} - 8y = 0$

$\cancel{17y} = 0$

$\therefore y = 0$

substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$= 2(0)^4 + 3(0)^2(0)(9$

$= 0 + 0 - 0$

$= 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$\cancel{x = -\frac{2}{2}} \quad \therefore x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} = 4$

$\therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

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