Question 1

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.
Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.)

Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

Answer (1.a.)

We are provided with a fixed sample of 10 drugs. For this sample set, we are evaluating the success for question: "Is the drug not able to do a satisfactory job?" For this question, we will have a yes or no response. Also, the probability of the result "yes or no" is same for each trial. Such scenarios can be accurately portrayed by Binomial Probability Distribution.

Listing down the three conditions for evaluating:
"The probability that at most 3 drugs are not able to do a satisfactory job. "

1. Fixed number of trials: In our scenario, the fixed number of trials is 10. Each trial will be performed in a similar way, however; result may vary for the trials.
2. Each of the trials is grouped into two classifications (success or failure): For Binomial Probability Distribution; success is defined in terms of what we have to determine. In our scenario, Success will be treated when the answer is yes for "Is the drug not able to do a satisfactory job?"
3. Probabilities of success should remain same in all trails: The probability of success trial should remain same throughout the observation process, which is denoted by P. For example, in our problem statement; probability of drugs not able to do satisfactory job (which is defined as success is our scenario) is 1/5.

The formula for finding Binomial Probability:

$$\mathbf{P(x=r) = {}^{n}_{r}C(P)^{r}(1-P)^{(n-r)}}$$

Question 1.b.)

Calculate the required probability

Answer (1.b.)

The required probability is calculated using the formula:

$$P(x=r) = {}^{n}_{r}C(P)^{r}(1-P)^{(n-r)}$$

Now, analyzing our problem statement:
We are provided that, Probability of Satisfactory to Not Satisfactory is 4:1;

i.e.

Probability of drugs producing Satisfactory result = $\frac{4}{5}$ = 0.8

Probability of drugs not producing Satisfactory result = $\frac{1}{5}$ = 0.2

To Calculate: P(x $\leq$ 3)

Equation 1:

P(x $\leq$ 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)

Calculating values for equation 1,

P(x=0)

P(x=0) = ${}^{10}_{0}C(0.2)^{0}(0.8)^{(10-0)}$

P(x=0) = 1 x 0.1073741824

P(x=0) = 0.11

P(x=1)

P(x=1) = ${}^{10}_{1}C(0.2)^{1}(0.8)^{(10-1)}$

P(x=1) = 10 x 0.2 x 0.134217728

P(x=1) = 0.26843544

P(x=1) = 0.27

P(x=2)

P(x=2) = ${}^{10}_{2}C(0.2)^{2}(0.8)^{(10-2)}$

P(x=2) = 45 x 0.04 x 0.167772160

P(x=2) = 0.30198989

P(x=2) = 0.30

P(x=3)

P(x=3) = ${}^{10}_{3}C(0.2)^{3}(0.8)^{(10-3)}$

P(x=3) = 120 x 0.008 x 0.2097152

P(x=3) = 0.20132659

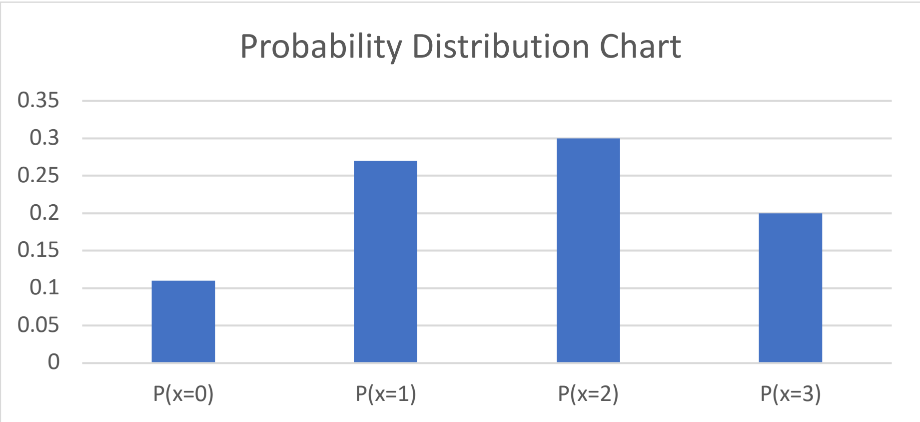
P(x=3) = 0.20

Substituting the values in Equation 1:

P(x $\leq$ 3) = 0.11 + 0.27 + 0.30 + 0.20

= 0.88

So, **P(x $\leq$ 3) = 0.88**



Question 2

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.)

Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

Answer (2.a.)

Population Mean range can be calculated as below:

Population Mean = Sample Mean $\pm$ Margin of Error

Central Limit Theorem establishes that for "n>30", the sampling distribution becomes normally distributed and show some interesting properties mentioned below. This holds true in our scenario since n is 100 here which is greater than 30.

Properties of Central Limit Theorem:

1. Sampling Distribution's Mean ($\mu\bar{X}$) = Population Mean (μ)
2. Sampling Distribution's Standard Deviation (Standard Error) = $\sigma / \sqrt{n}$, where σ is the population's standard deviation and n is the sample size
3. For n > 30, the sampling distribution becomes a normal distribution

Using these assumptions, we can calculate margin of error, which we can use to calculate population mean range and this population mean range is known as confidence interval.

So, confidence interval = ($\bar{x} + \frac{z^*s}{\sqrt{N}}$, $\bar{x} - \frac{z^*s}{\sqrt{N}}$).

Question 2.b.)

Find the required range

Answer (2.b.)

Available Information

- Sample Size = 100
- Mean time = 207 sec
- Standard deviation = 67 sec
- Confidence Level = 95%

Here, range for population mean would be calculated as: $\bar{x} \pm margin\ of\ error$

i.e. $\bar{x} \pm \frac{z^*s}{\sqrt{N}}$

i.e. ($\bar{x} + \frac{z^*s}{\sqrt{N}}$, $\bar{x} - \frac{z^*s}{\sqrt{N}}$)

where z* = 1.96 for 95% confidence level.

Now, let's calculate margin of error:

$$\frac{z^{*}s}{\sqrt{N}} = \frac{1.96 \times 65}{\sqrt{100}}$$
$$= \frac{127.4}{10}$$
$$= 12.74$$

After substituting the values, population mean range will be:

(207 – 12.74 , 207 + 12.74)

Confidence Interval = (194.26 , 219.74)

Question 3

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

Answer (3.a.)

Available Information

- μ = 200 sec
- n = 100
- $\bar{x}$ = 207 sec
- Standard Deviation = 65 sec
- Significance Level = 5% implies $\alpha = 0.05$

Hypothesis

H_0 (Null hypothesis): time $\leq$ 200 sec

As, Null hypothesis is the complement to the original claim

H_1 (Alternate hypothesis): time > 200 sec

As, Alternate Hypothesis is always opposite of H_0

Calculation of Test Statistics

Critical Value Method

Step 1: Calculate value of z_c from the given value of α.

Cumulative probability of the critical point (total area till that point) = 1 - 0.05 = 0.950

Now, 0.950 is not there in z-table, so look for numbers nearest to 0.950.

We observe that z-score for 0.9495 is 1.64 and z-score for 0.9505 is 1.65. So, z-score for 0.9500 will be average of the z-scores

$$z_c = \frac{1.64+1.65}{2} = 1.645$$

Step 2: Calculate Critical Value

Critical Value $= \mu + z_c \frac{\sigma}{\sqrt{N}}$

$= 200 + 1.645 \times \left(\frac{65}{\sqrt{100}} \right)$

$= 200 + 1.645 \times (6.5)$

$= 200 + 10.6925$

$= 210.6925$

$= 210.69$

Step 3: Make Decision on the basis of Critical Value

Since, 207 < 210.69; $\bar{x}$ lies in the acceptance region and we fail to reject the Null Hypothesis.

P Value Method

Step 1: Calculate value of z-score

z score $= \frac{\bar{x}-\mu}{(\sigma/\sqrt{N})}$

$= \frac{207-200}{(65/\sqrt{100}\,)}$

$= \frac{7}{6.5}$

$= 1.0769$

$= 1.08$

Step 2: Calculate p-value

The value in z-table corresponding to 1.08 is 0.8599. Since, the sample is on right side of the distribution, and it is a one-tailed test

p-value = 1-0.8599

p-value = 0.1401

Step 3: Make decision on basis of p-value

Since, p-value is more than the significance level (0.1401 > 0.05), we fail to reject Null Hypothesis.

Question 3.b)

You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current hypothesis test conditions (sample size, mean, and standard deviation), the value of α and β come out to 0.05 and 0.45 respectively.

Now, a different sampling procedure is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other.

Answer (3.b.)

Null Hypothesis: Time of effect $\leq$ 200 sec. Drug performs a satisfactory job if the time of effect is at most 200 sec.

Type-1 Error:

- Drugs are able to cure the pain, but take more than 200 sec, wherein actually they take less than 200 sec

Type-2 Error:

- Drugs are able to cure the pain, within at most 200 sec, wherein actually they take more than 200 sec.

Assumption: The drug is able to cure the pain; only the time of effect is of concern to judge the effectiveness and usability of drug.

Scenario 1: $\alpha = 0.05$ and $\beta = 0.45$

This value of alpha and beta will be more preferred in situation where the pharma company is doing a Quality test of drug before launch. Type-I error will be more critical for us because on the basis of type-I error we may reject drugs which could have cured pain in less time for patients. Type -II error is less critical as drug may take more than 200 seconds to show results and in that case we could wait for more time for drug effectiveness.
Also, this is suitable in a condition where the pharma company is working on identifying the correct testing techniques for the drug to reduce the number of false negatives.

Scenario 2: $\alpha = 0.15$ and $\beta = 0.15$

Increasing alpha will reduce beta value and vice versa. Usually it is not possible to have both alpha and beta smaller in value. We can only achieve this by increasing our sample size. So at the time of market launch we can increase our sample size to get less value of alpha and beta because at the time of launch we want to make sure that the type-I and type-II error probabilities are less.

Question 4

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign for its existing subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.
Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer

Why

A/B testing is entirely based on the two-sample proportion test. A/B testing is essentially an experiment where the two versions are shown to users at random, and statistical analysis is used to determine which variation performs better for a given conversion goal. Here, we have similar scenario as two taglines were proposed for the campaign and we have to decide which one is better. Using A/B testing we can determine which tagline has performed better or has no effect on visitor behavior.

How

The following steps are involved in A/B testing

1) We start with defining our goal on which we would measure the success criteria. This may depend on number of hits, number of downloads, number of users who filled the form, etc.
2) Then, we decide where the online ad campaign should be started. For this we should choose platform where existing subscribers visit is maximum as there we can gather more data in a faster manner.
3) On the chosen platforms, we place both variation of tagline in a manner that they are similar in placements and in user access, for an unbiassed analysis.

4) We then divide the population into two parts based on user click on tagline. Let say p1 is the population who has clicked on tagline 1 and p2 is the population who has clicked on tagline 2.
5) From both population p1 and p2 some people will convert positively (it depends on our goal) and let's call it $\overline{p1}$ and $\overline{p2}$.
6) Now aim is to calculate difference between $\overline{p1}$ and $\overline{p2}$ to see if there is significant difference between them.
7) Based on this difference we can decide which tagline has performed better for online ad campaign.