Project One: Alternating Disks

Name: Anthony Maida

Email: amaida@csu.fullerton.edu

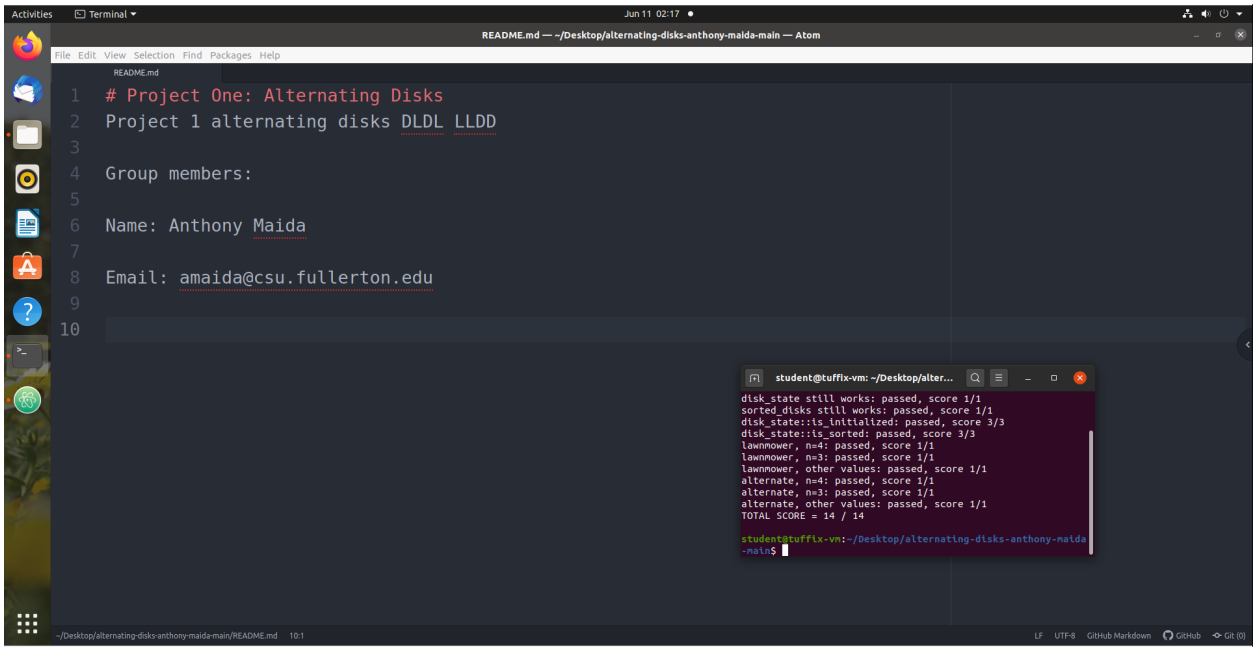
This is my project 1, which involves alternating disks of size 2n and then sorting using two different algorithms. The n is the number of light disks and dark disks, so 2n is all light and dark disks combined. The first algorithm is the lawnmower algorithm and the second is the alternate algorithm.

What I did for this project was I first had to add a check to see if the disks are initialized and if it is sorted. To check if it is initialized, I made sure that the even index disks, including 0, are dark disks or have a value of 1. I then made sure that the odd index disks are light or have a value of 0. To check if it is sorted, I made sure that the first half of the disks are all light disks or have a value of 0.

The next step was to add algorithms for the lawnmower and alternate. The lawnmower has a for loop that repeats (n+1)/2 times. In that loop, I have two more for loops. The first loop goes from left to right and makes swaps and the second for loop goes back to the beginning from right to left also making swaps. The alternate has a loop that runs n+1 times and there are two check conditions in that loop. The first check will see if the index of the parent loop is even, including 0, which will start the algorithm at the first disk going all the way to the right using a loop. The second check will see if the index of the parent loop is odd, which will then start the algorithm at the second disk and end it before the second to last disk using a loop. So this means it alternates between starting at the first disk and starting at the second disk and the amount of times it does it is determined by n, which is the number of either light or dark disks.

Also included in this report is a screenshot of atom within tuffix, the pseudocode for each algorithm, step count of each algorithm and the proofs showing that they are in $O(n^2)$.

Atom Screenshot in Tuffix:



Pseudocode:

Lawn Mower Algorithm:

STEP COUNT = 1+1+3+((n+3)/2)*(((2n-1)*(2+max(2+1,0)))+((2n-1)*(2+max(2+1,0))))+1 = $10n^2$+25n-10

```
sorted lawn = array of 2n values                                    // 1 tu
swap num = 0                                                        // 1 tu
repeat num = (n+1) /2                                               //  3 tu
for i = 0 to repeat num do                                          // ((n+1)/2)+1) times
        for lTr = 0 to 2n-2 do                                      // (2n-1) times
                if sorted lawn[lTr] > sorted lawn[lTr+1]            // 2 tu
                        swap(sorted lawn[lTr], sorted lawn [lTr+1]) // 2 tu
                        swap num += 1                               // 1 tu
                endif                                               // 0 tu
        endfor                                                      // 0 tu
        for rTl = 2n-1 down to 1 do                                 // (2n-1) times
                if sorted lawn[rTl] < sorted lawn[rTl-1]            // 2 tu
```

```
                    swap(sorted lawn[rTl], sorted lawn[rTl-1])      // 2 tu
                    swap num += 1                                   // 1 tu
                endif                                               // 0 tu
        endfor                                                      // 0 tu
return sorted lawn and swap num                                     // 1 tu
endfor                                                              // 0 tu
```

Alternate Algorithm:

STEP COUNT = 1+1+2+(n+2)*(2+max(((2n-1)*(2+max(2+1,0)), ((2n-3)*(2+max(2+1,0)))+1
= $10n^2+17n-1$

```
sorted alter = array of 2n values                                   // 1 tu
swap num = 0                                                        // 1 tu
repeat num = n + 1                                                  // 2 tu
for i = 0 to repeat num do                                          // (n+2) times
        if (i%2==0)                                                 // 2 tu
                for ind = 0 to 2n-2 do                              // (2n-1) times
                        if sorted alter[ind] > sorted alter[ind+1]  // 2 tu
                                swap(sorted alter[ind], sorted alter[ind+1])    // 2 tu
                                swap num += 1                       // 1 tu
                        endif                                       // 0 tu
                endfor                                              // 0 tu
        else                                                        // 0 tu
                for ind = 1 to 2n-3 do                              // (2n-3) times
                        if sorted alter[ind] > sorted alter[ind+1]  // 2 tu
                                swap(sorted alter[ind], sorted alter[ind+1])    // 2 tu
                                swap num += 1                       //1 tu
                        endif                                       // 0 tu
                endfor                                              // 0 tu
        endif                                                       // 0 tu
return sorted alter and swap num                                    // 1 tu
endfor                                                              // 0 tu
```

Proofs:

Mathematical Definition of $O(n^2)$:

$O(n^2)$ = set of all functions f(n) such that

∃ n0 >= 0 and c > 0 such that

$f(n) <= c * n^2 \ \forall \ n>= n0$

Lawnmower Proof:

$10n^2+25n-10 \in O(n^2)$
Find c > 0 and n0 >= 0 such that
$10n^2+25n-10 <= c * n^2 \ \forall \ n>= n0$

Good choice for c = 10 + 25 +10 = 45 c = 45
$10n^2+25n-10 <= 45n^2$
$45n^2-10n^2-25n+10 >= 0$
$(10+25+10)n^2-10n^2-25n+10 >= 0$
$10n^2-10n^2+25n^2-25n+10n^2+10 >= 0$
$10n^2-10n^2 \rightarrow = 0$
$25n^2-25n \rightarrow >= 1$
$10n^2+10 \rightarrow >= 0$
n0 = 1

Alternate Proof:

$10n^2+17n-1 \in O(n^2)$
Find c > 0 and n0 >= 0 such that
$10n^2+17n-1 <= c * n^2 \ \forall \ n>= n0$

Good choice for c = 10 + 17 + 1 = 28 c = 28
$10n^2+17n-1 <= 28n^2$
$28n^2-10n^2-17n+1 >= 0$
$(10+17+1)n^2-10n^2-17n+1 >= 0$
$10n^2-10n^2+17n^2-17n+1n^2+1 >= 0$
$10n^2-10n^2 \rightarrow = 0$
$17n^2-17n \rightarrow >= 1$
$1n^2+1 \rightarrow >= 0$
n0 = 1