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COMP9418 Assignment 2 - Smart Building Lighting

Jordan Huang (z5418948) and Samuel Hodgson (z5416863)

General Model Overview

Our model follows the structure of a Kalman Filter. Each ‘area’ (room, corridor and outside) is associated with a normal distribution that describes its number of occupants. We model these distributions as naturally changing over time in line, as well as adjusting our beliefs accordingly in response to observed emissions.

Every 15 second period, our existing beliefs about these distributions changes, according to a state transition matrix (these transitions were learnt from the provided datasets). This transition adjusts the distributions based on historically seen movements between the rooms, and additionally increases the variance in each of the rooms to account for the uncertainty of movement. The observations of the sensors update our beliefs, also affecting the distributions of populations in each area.

After these steps are applied to the model, we are left with our best estimates as to the distribution of people within the room. Our choice to turn on a light or not is determined by expected value - we choose whichever choice we expect to cost the least.

Transition Probabilities

A transition probability matrix was created to simulate the natural transitions a person could take between rooms in one time step of 15 seconds. Initially these transition probabilities were constant throughout the entire day, however the model was strengthened by splitting the day into two hour periods. This decision was justified by the movement patterns of individuals not being the same at various times throughout the day (i.e entering/leaving the office, lunch break, morning and afternoon work blocks).

There is however the simplifying assumption that movement patterns remain relatively constant within the two hour periods. Reducing the lengths of these periods would increase flexibility and potentially allow for better fitting, but it would further reduce the quantity of data available to find each transition probability, increasing the variance and chance for overfitting.

The probabilities were generated using the two days of provided data. For each time step, the rooms with decreased population were observed and this decrease was distributed amongst possible neighbouring rooms based upon their relative increase in the same time step. After finding probabilities from each timestep, they were averaged over the specified two hour periods.

Afterwards we had a post-processing step where improbable transitions, such as those across the entire office space, were removed and redistributed towards feasible rooms - there was an assumption made of how far an individual could travel in 15 seconds however the assumption was not too restrictive, and multiple room travel was found to be very feasible.

Evidencing + Sensor Assumptions

Applying evidence was done separately and sequentially, with each sensor modifying the distributions of its associated rooms. This is another limitation of the Kalman Filter model - by factorising the state into an

expression of 37 areas rather than storing the state as a single 37-area vector, we lose out on the spatial dependencies between rooms in the same time step. Our compromise was that after applying all the evidence, the vector of means would be renormalised to sum up to 40 again. That way, if, for example, a room was evidenced to have a much larger population than expected, this observation can still filter through and affect the other rooms through the normalisation.
We judged this to be acceptable, as the alternative would again introduce an infinitude of states into the model (as it would now require storing all 37 normal distributions jointly). Not only would this increase the time complexity by exponential factors, but there would be far too little data to effectively utilise the increased expressive power.

Camera Sensor and Robot Sensor

The camera and robot sensors output a count of the number of people in the rooms they were in, which provided a very natural way of evidencing. We modelled this count value as normally distributed, with mean equal to the population in the sensor's respective room. Hence, to update our belief of the room's distribution, we join the two Gaussian Factors and set evidence.

Data analysis of the sensors revealed the camera sensor to be very slightly negatively biased - it sometimes outputted a count 1 less than the true count, but was otherwise correct. The robot sensor, however, did not make a mistake in the 2 days of data, and hence we set its distribution to have no bias and a very low variance.

Motion Sensor

Motion sensors posed a problem to our model structure - applying evidence in a Gaussian Kalman Filter requires normal distributions, otherwise the distributions within the nodes are changed to no longer be normal. Looking at the data, a reading of 'no motion' largely indicates that the room is empty, with a few scenarios where there were 1-2 people still in the room. Hence, we interpreted a reading of 'no motion' as a camera-sensor-like observation that there were 0 people in the room, allowing us to convert the emission into a Gaussian Factor and apply evidence.

Regarding 'motion' readings: If our prior distribution had a large mean (for e.g. 5 people), we expect that reading and hence our distribution shouldn't change much. If our prior distribution had a mean close to 0, we don't expect this result, and should skew up our mean accordingly. To account for this cleanly, we converted this reading into an observation that there were $\mu_{prev} + \lambda e^{-\mu_{prev}}$ people, where λ is some parameter we tuned - the negative exponential function cleanly meets the above requirements. Again, this allows us to create, and evidence on, a Gaussian factor to update our beliefs.

Door Sensor

Door sensors measured a different sort of reading to the others, in that their data reflects the number of people who passed through the doorway over the 15 seconds previous to the tick. Hence, this evidence was applied before the matrix transition.

Now, we talk generally about a door, placed between two areas a1 and a2. The door sensor count could increment from four main factors:

1. A person moving from a1-a2

2. A person moving from a2-a1
3. A person who passes completely through both a1 and a2 and is not in either room before, afterwards, or both.
4. A person who moves between a1 and a2 multiple times in the same tick, hence being counted multiple times.

3 and 4 we judged unlikely and hard to incorporate into the model, so we made the simplifying assumption that the count came only from 1 and 2. From these simplifying assumptions we can express the mean door sensor value as $\mu_1 T_{a1a2} + \mu_2 T_{a2a1}$, where μ_1, μ_2 represent the mean value in a1 and a2 before the transition, and T represents the transition matrix. Additionally, we expressed the variance as proportional to this mean value - intuitively, the more people around the area we expect to be using the doorway, the less clear the reading the door sensor should be.

Thus, we again can create a Gaussian Factor for the door sensor, this time with 2 parents. Joining these factors and evidencing on the door sensor gives us the means and variances of its parents. We note that this does theoretically lead to dependencies between the parents, but analysis of the covariance matrices revealed that these dependencies were minimal (non-diagonal values were very small negative numbers), and we chose not to pursue this further, as it would greatly complicate the model.

Lighting Decisions

After the transition and evidencing steps, we are left with normal distributions that illustrate our belief of the distribution of people in each room. We are left with two options:

1. Turn on the light (which will cost 1c)
2. Leave the light off (which will cost 4c per person in the room)

To choose between these, we compare the expected value of the two choices. Option 1 clearly has an expected cost of 1c. Option 2, however, has us spend the value $max(X, 0)$, where X represents the number of people in the room. Given that the distribution of number of people is normal (i.e. $X \sim N(\mu, \sigma^2)$), we can calculate this expected value as $\int_0^\infty 4x\, f(x)\, dx$, where $f(x)$ represents the density of that normal distribution. This integral cannot be calculated in a closed form for non-zero values of μ. However it can be expressed in terms of the cumulative distribution function and as such can be approximated to high precision fairly quickly. Using vectorization further speeds up this process, and hence we can make the most cost-effective choice in terms of expected value, given the distributions of the room populations.

It is interesting to note how this decision-process behaves: given a sufficiently high variance, the expected value integral can exceed 1 even if the mean is close to 0 - which means if we are very unsure about the number of people in a room, we choose to turn the lights on even if our mean guess is that the room is empty. Thus, uncertainty will increase the chance that the building will turn the light on, which we note to be desired behaviour.

Time Complexity

As a whole, the model runs fairly quickly - there are no single expensive algorithms to run. The most expensive operation, in absolute terms, would be the transition matrix calculation. This was sped up using the scipy.sparse package, which allows for faster multiplication by only storing non-zero terms, so our matrix-vector multiplications for the time transitions ran in $O(nk)$ time, where n is the number of rooms and k is the number of non-zero elements in the transition matrix.

Since applying evidence was done separately, the largest factor created in each tick is a 3-dimensional normal distribution, used by the 4 door sensors, each of the others only creating 2D normal distributions. Thus, the matrix multiplications here involve at most 9 numbers, and with only 16 sensors this adds a negligible amount of time per tick.

Finally, as mentioned above, the lighting decision is done using vectorised calculations. Hence, it is still $O(n)$, but with a small constant coefficient.

General Assumptions

By its structure the Kalman Filter assumes the Markov property - that is, the next state (in our case, what we predict as the distributions of each area) is independent of all previous states given the current state. While this may not be entirely true (e.g. a movement trend could be inferred if we see a group of people move from, say, room 6 -> room 14 -> room 22), trying to account for such dependencies in our model would create a very complicated structure that is impractical to build, train and use. Furthermore, with the added number of parameters such a structure would need, it is unclear if such a model would even perform better.

As previously mentioned, the transition probabilities were assumed to be constant across different days and within two hour periods. While this is likely untrue in reality, not enough data was provided to create more specific transitions, and general trends (i.e. movement of people as they first come into the office in the morning, movement of people as they enter lunch break) should exist and be better captured with a changing transition matrix throughout the day. Furthermore, individuals were assumed to be identical to each other such that everyone has the same chance of transitioning from one room to another, since the data did not give us the ability to discern who was who within the transitions.

There was also the challenge of being forced to assume how far an individual could travel within 15 seconds. It was found that in some time steps only one room would decrease in people while another on the complete opposite side of the office increased. This was assumed to be a result of a chain of movements where the middle rooms in the chain did not have a net change as the same number of people entered and left within the 15 seconds. Thus we limited the room transitions to only include distances reasonable to travel within 15 seconds, a difficult task without any measure of scale.