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Multimodal optimization by using hybrid of artificial bee colony algorithm and BFGS algorithm

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Abstract. Optimization has become one of the important fields in Mathematics. Many problems in engineering and science can be formulated into optimization problems. They maybe have many local optima. The optimization problem with many local optima, known as multimodal optimization problem, is how to find the global solution. Several metaheuristic methods have been proposed to solve multimodal optimization problems such as Particle Swarm Optimization (PSO), Genetics Algorithm (GA), Artificial Bee Colony (ABC) algorithm, etc. The performance of the ABC algorithm is better than or similar to those of other population-based algorithms with the advantage of employing a fewer control parameters. The ABC algorithm also has the advantages of strong robustness, fast convergence and high flexibility. However, it has the disadvantages premature convergence in the later search period. The accuracy of the optimal value cannot meet the requirements sometimes. Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm is a good iterative method for finding a local optimum. Compared with other local optimization methods, the BFGS algorithm is better. Based on the advantages of the ABC algorithm and the BFGS algorithm, this paper proposes a hybrid of the artificial bee colony algorithm and the BFGS algorithm to solve the multimodal optimization problem. The first step is that the ABC algorithm is run to find a point. In the second step is that the point obtained by the first step is used as an initial point of BFGS algorithm. The results show that the hybrid method can overcome from the basic ABC algorithm problems for almost all test function. However, if the shape of function is flat, the proposed method cannot work well.

1. Introduction

Optimization has become one of the important fields in Mathematics. Many problems can be formulated into optimization problems. The optimization problems play an important role in industry and science fields. For example, optimization has been applied in industrial wastewater [4] and in tomotherapy for prostate cancer patients [9].

The optimization problem maybe has many local optima. The optimization problem with many local optima, known as multimodal optimization problem, is how to find a global solution from several optimum points. Several metaheuristic methods have been proposed to solve multimodal optimization problems such as Particle Swarm Optimization (PSO), Genetics Algorithm (GA), Artificial Bee Colony (ABC) algorithm, etc [2].

The ABC algorithm is one of the popular evolutionary algorithms that proposed by Karaboga in 2005 [5]. The ABC algorithm was successfully applied in many applications such as job shop scheduling. The ABC algorithm has several advantages compared with the other population-based algorithms. The first advantage is that it is easy to be used, high flexible and tardiness [7].

The second advantage is that it employs a fewer control parameters than the other algorithms such as Particle Swarm Optimization and Genetic Algorithm [6]. The third advantage is that the ABC Algorithm hybridizes easily with the other optimization algorithms [8]. The forth advantage is that the ABC Algorithm is strong robustness and fast convergence. The ABC algorithm also can handle stochastic cost objective function [3]. However, the ABC algorithm has the disadvantages that it easily meets premature convergence in the later search period. The optimum value cannot be found sometimes [10].

Quasi-Newton method is one of the good methods for solving the minimization problem. The size of the gradient change is constructed well enough to produce super linear convergence. This method has better convergence than gradient descent method. Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm is one of type Quasi Newton methods which is a good iterative method for finding a local optimum. This method has also been proven to be better than gradient descent when applied to the neural network [1] .

Based on the advantages of the ABC algorithm and the BFGS algorithm, this paper proposes a hybrid of the artificial bee colony algorithm and the BFGS algorithm to solve the multimodal optimization problem. The principle of a hybrid method utilizes the strengths and reduces the weaknesses possessed by each method, so it can optimize function successfully. The performance of the proposed method is evaluated by using 5 test functions.

2. The ABC Algorithm
The ABC algorithm is one of the popular evolutionary algorithms that is proposed by Karaboga in 2005 [5]. It is very simple, clear and adaptable. The ABC algorithm is inspired by the behavior of bee colonies in search for food sources. The search cycle of ABC algorithm consists of three rules. The first rule is that the employee bees are sent to a food source and the nectar quality is evaluated. The second rule is that onlookers choose the food sources after obtaining information from employee bees and calculating the nectar quality. The last rule is that determining the scout bees and sending them onto possible food sources. The ABC procedure could be represented in **Algorithm 1**.

Algorithm 1 : Schematic pseudo code of ABC procedure.

 (i) Initialize the ABC and problem parameters
 (ii) Initialize the Food Source Memory (FSM)
 (iii) Repeat
     (a) Send the employee bees to the food sources.
     (b) Send the onlookers to select a food source.
     (c) Send the scouts to search possible new food.
     (d) Memorize the best food source.
    Until (termination criterion are met)

2.1. Initialization of the ABC and problem parameter.
In the optimization problem, if function $f(\mathbf{x})$ will be minimized and $\mathbf{x}$ is decision variable, then the optimization problem is formulated as follows:

$$\min_{\mathbf{x} \in \mathbf{X}} f(\mathbf{x}) \tag{1}$$

If $\mathbf{x}$ has N dimensions then $\mathbf{x}$ can be written as follows $\mathbf{x} = \{x_1, x_2, ..., x_N\}$ where $x_i \in [LB_i, UB_i]$. LB_i and UB_i are the lower and upper bound values for the variable x_i. The parameters of the ABC algorithm are the following:

(i) Population size (SN) is the number of food sources (candidate of solutions) in the population. SN is equal to the number of employee bees or onlooker bees.

(ii) Maximum Cycle Number (MCN) is representation of the maximum number of generations.

(iii) Limit is used to diversify the search, to determine the number of allowable generations for which each non improved food source is to be abandoned.

2.2. Initialization of the Food Source Memory (FSM)
A food source is a candidate solution to the optimization problem. Food Source Memory (FSM) is a memory allocation consisting of food resources and may be determined by SN in equation (2).

$$FSM = \begin{bmatrix} x_1(1) & x_1(2) & ... & x_1(N) \\ x_2(1) & x_2(2) & ... & x_2(N) \\ ... & ... & ... & ... \\ x_N(1) & x_N(2) & ... & x_N(N) \end{bmatrix} \tag{2}$$

with each vector generated from:

$$\begin{aligned} x_j(i) &= LB_i \pm (UB_i - LB_i) \times r \\ \forall j &\in \{1, 2, ..., SN\} \forall i \in \{1, 2, ..., N\} \end{aligned} \tag{3}$$

Note: $r \sim (0, 1)$ is uniform random number between 0 and 1.

2.3. Sending of the employee bees to the food sources
In this step, each employee bee is assigned to its food source and in turn, a new one is generated from its neighbouring solution, using equation (4).

$$\begin{aligned} x'_j(i) &= x_j(i) \pm (x_j(i) - x_k(i)) \times r \\ \forall k &\in \{1, 2, ..., SN\} k \neq j \text{ and } r \sim (0, 1) \end{aligned} \tag{4}$$

In this step, the quality nectars on food sources ($x_j(i)$ and $x'_j(i)$) also are evaluated. If the quality nectar of $x'_j(i)$ is better than the quality nectar of $x_j(i)$ then $x_j(i)$ will be replaced by $x'_j(i)$.

2.4. Sending of the onlookers to select a food source
The onlooker bee has the same number of food sources as the employee. In this step the probability of each source of food from the previous step is calculated food sources are chosen by bees onlooker by using Roulette Wheel Selection.

2.5. Sending of the scouts to search possible new food
Scout bees will randomly searching for new food sources to replace the abandoned food source by using equation (3). The abandoned food source is a food source that can not be updated on a particular experiment. These food sources will be determined by the parameter limit [10].

3. The BFGS Algorithm
Quasi-Newton method is an alternative method that can be used for solving optimization problems. This method was successfully used to minimize errors on artificial neural networks [1]. Quasi-Newton method is method that can be used if the calculation of the Hessian matrix is difficult or time consuming. This method has a rapid convergence when compared with the method of gradient descent. One of Quasi Newton method types is the BFGS method. The BFGS algorithm can be seen in **Algorithm 2**.
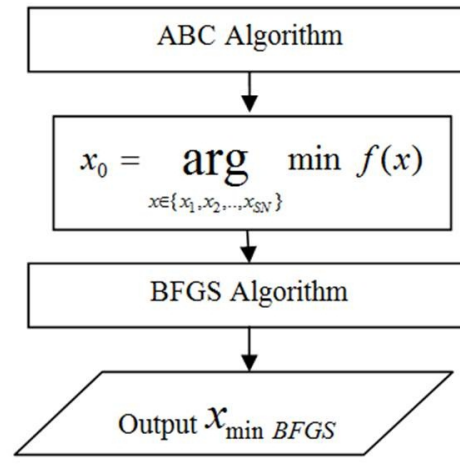
Figure 1. Flowchart of the proposed method.

Algorithm 2 : **BFGS algorithm method**

(i) Input $\mathbf{x_0}, \epsilon$ (stopping criteria) and k_{max}.

(ii) Set initial iteration $k = 0$ and $\mathbf{B} = \mathbf{I}$ where $\mathbf{I}$ is an indentity matrix.

(iii) Calculate $f(\mathbf{x}_k)$

(iv) While ($\|\nabla(\mathbf{x}_k)\|\| > \epsilon$) or ($k <= k_{max}$) do

    (a) Calculate $\mathbf{d}_k = -\mathbf{B}_k \nabla f(\mathbf{x}_k)$, where $\mathbf{d}_k$ generating direction.

    (b) Select α that is able to minimize $f(\mathbf{x}_k + \alpha_k \mathbf{d}_k)$

    (c) $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$

    (d) Calculate $\mathbf{B}_{k+1}$ by using equation (5).

$$\mathbf{B}_{k+1} = \mathbf{B}_k - \frac{\mathbf{B}_k \mathbf{s}_k (\mathbf{B}_k \mathbf{s}_k)^T}{\mathbf{s}_k^T \mathbf{B}_k \mathbf{s}_k} + \frac{\mathbf{y}_k \mathbf{y}_k^T}{\mathbf{y}_k^T \mathbf{s}_k} + \phi_k [\mathbf{s}_k^T \mathbf{B}_k \mathbf{s}_k] \mathbf{v}_k \mathbf{v}_k^T \tag{5}$$

    where $\phi_k \in [0,1], \mathbf{s}_k = \mathbf{x}_{k+1} - \mathbf{x}_k$ and
$\mathbf{v}_k = \frac{\mathbf{y}_k}{\mathbf{y}_k^T \mathbf{s}_k} - \frac{\mathbf{B}_k \mathbf{s}_k}{\mathbf{s}_k^T \mathbf{B}_k \mathbf{s}_k}$.

    (e) $k = k + 1$

    End while

4. Proposed Method

The ABC Algorithm is a global optimization technique that is well known, inspired by the behavior of bee colonies. The ABC algorithm has advantages of memory, local search and solution improvement mechanism over the other meta-heuristic algorithms. The scout bee in the ABC algorithm controls the exploration process to find the area of global solution point. The onlooker bee and employee bee controls the exploitation process to find global solution point in the area which found by the scout bee. However, the ABC algorithm does not guarantee to give the optimal solution and often has premature convergence. To overcome this problem, in this paper proposes a hybrid of the ABC algorithm and the BFGS algorithm.

Hybridization schemes of ABC algorithm and BFGS algorithm as shown in Figure 1. In the proposed method, the best solution of the ABC algorithm is used as an initial value for the

BFGS method. Figure 2 illustrates how to the proposed method works. $\mathbf{x}_0$ is the best solution of the ABC algorithm which is represented by a yellow point. It is used to be an initial point of the BFGS algorithm. Furthermore, the BFGS method will find the solution that is represented by a green point. The point is very close to the optimal solution that represented by a red point.

5. Experimental Results and Discussion
The numerical solution of the proposed method is evaluated by using a computer system with the following specifications: processor: Intel(R) Core i5 3317U CPU 1.7G.Hz, memory: 4 GB, and operating system: Windows 7 Home Basic, Service Pack 1. The hybrid of the ABC algorithm and the BFGS algorithm is implemented using software Matlab 2008.

The proposed method is compared with the basic ABC algorithm to evaluate the performace of the proposed method. In order to evaluate the performance of the proposed method, several benchmark functions are used for evaluating. The benchmark functions are the following:

(i) **Schaffer function**

Schaffer function is defined by equation (6). It is also multimodal function. $\mathbf{x}$ is in the interval of [-100, 100]. The global optimum value is 0 and the optimum solution is $\mathbf{x}_{opt} = (0, 0, \ldots, 0)$.

$$f_1(\mathbf{x}) = 0.5 + \frac{sin^2\left(\sqrt{x_1^2 + x_2^2}\right) - 0.5}{(1 + 0.001(x_1^2 + x_2^2))^2} \tag{6}$$

(ii) **Sphere function**

Sphere function is defined by equation (7). It is unimodal function, convex and continous. $\mathbf{x}$ is in the interval of [-100, 100]. The global optimum value is 0 and the optimum solution is $\mathbf{x}_{opt} = (0, 0, \ldots, 0)$.

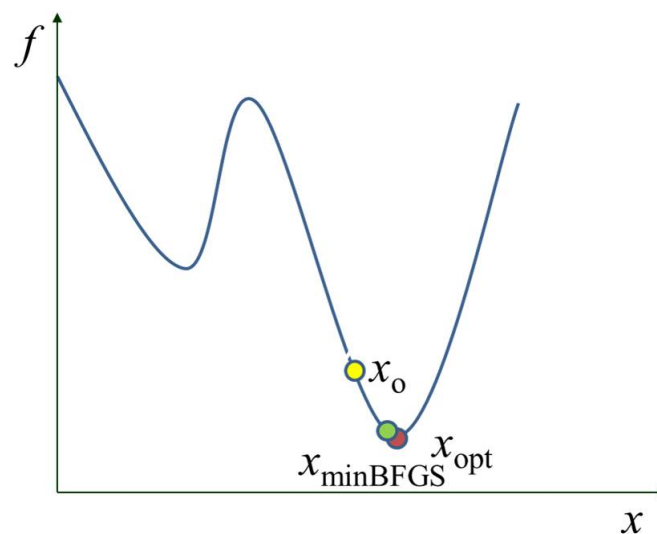$$f_2(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 \tag{7}$$

Figure 2. Ilustration of the proposed method.
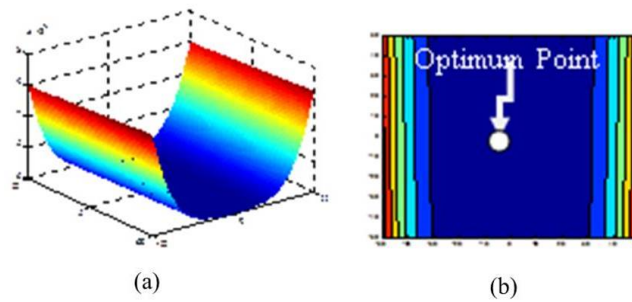
(a)　　　　　　　　　　　　　　　(b)

Figure 3. (a) Rosenbrock function view. (a) Rosenbrock function contour.

(iii) **Griewank function**

Griewank function is generally defined by equation (8). $\mathbf{x}$ is in the interval of [-200,200]. The global minimum value for Griewank function is 0 and the corresponding with global optimum solution $\mathbf{x}_{opt} = (100, 100, \ldots, 100)$. Because the number of local optima increases with the dimentionality, Griewank function is strongly multimodal function.

$$f_3(\mathbf{x}) = \frac{1}{4000}\left(\sum_{i=1}^{n}(x_i - 100)^2\right) - \left(\prod_{i=1}^{n} cos\left(\frac{x_i-100}{\sqrt{i}}\right)\right) + 1 \tag{8}$$

(iv) **Rastrigin function**

Rastrigin function is defined by equation (9). It is multimodal function. The difficult part about finding local optimal solution is that an optimization easily can be trapped in local optimum on its way toward global optimum. $\mathbf{x}$ is in the interval of [-5.12, 5.12]. The global optimum value is 0 and the optimum solution is $\mathbf{x}_{opt} = (0, 0, \ldots, 0)$.

$$f_4(\mathbf{x}) = \sum_{i=1}^{n}\left(x_i^2 - 10cos(2\pi x_i) + 10\right) \tag{9}$$

(v) **Rosenbrock function**

Rosenbrock function is defined by equation (10). It is also multimodal function. $\mathbf{x}$ is in the interval of [-50, 50]. The global optimum value is 0 and the optimum solution is $\mathbf{x}_{opt} = (1, 1, \ldots, 1)$. The global optimum is inside a long, narrow, parabolic-shaped flat valley. Since it is difficult to converge to the global optimum.

$$f_5(\mathbf{x}) = \sum_{i=1}^{n-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \tag{10}$$

Each of the experiment was repeated 5 times with different random seeds. The mean and the deviation standard of the function value, and the mean and the deviation standard of the computational time obtained by the basic ABC algorithm and the hybrid of the ABC algorithm and BFGS algorithm for under the same condition are given in Table 1 and Table 2, respectively. The objective function value obtained by proposed method is better than the objective function value obtained by the basic ABC algorithm for almost all test function. However, the proposed method cannot works well for Rosenbrock function. The BFGS algorithm cannot work well because the Rosenbrock function shapes is very flat as shown in Figure 3 (a) and the global optimum of the Rosenbrock function is inside a long, narrow and parabolic-shaped flat valley which is shown by the Rosenbrock function contours in Figure 3 (b). The BFGS algorithm is failed to find a true step size that is able to minimize $f(\mathbf{x}_k + \alpha_k \mathbf{d}_k)$. The computational time of the proposed method is better than the basic ABC algorithm as shown in Table 2.

Table 1. The objective function value and optimum point obtained by the basic ABC algorithm and the proposed method.

f	D	ABC Algorithm Mean (Std)	Proposed Method Mean (Std)
f_1	2	5.3×10^{-3} (4.8×10^{-3})	$\mathbf{3.9 \times 10^{-3}}$ $(\mathbf{5.0 \times 10^{-3}})$
f_2	2	5.1×10^{-17} (2.2×10^{-17})	$\mathbf{1.77 \times 10^{-36}}$ $(\mathbf{7.7 \times 10^{-37}})$
	4	1.47×10^{-5} (4.25×10^{-5})	$\mathbf{3.2 \times 10^{-35}}$ (1.3×10^{-35})
f_3	2	2.2×10^{-3} (3.6×10^{-3})	$\mathbf{1.5 \times 10^{-3}}$ $(\mathbf{3.1 \times 10^{-3}})$
	4	3.48×10^{-2} (2.18×10^{-2})	$\mathbf{1.48 \times 10^{-2}}$ $(\mathbf{6.0 \times 10^{-3}})$
f_4	2	0 (0)	0 (0)
	4	4.05×10^{-1} (1.28)	$\mathbf{7.11 \times 10^{-15}}$ $(\mathbf{2.3 \times 10^{-15}})$
f_5	2	$\mathbf{4.4 \times 10^{-3}}$ $(\mathbf{6.4 \times 10^{-3}})$	9.49×10^{-1} (1.301)
	4	$\mathbf{5.86 \times 10^{-1}}$ $(\mathbf{8.97 \times 10^{-1}})$	7.38×10^{2} (1.44×10^{-3})

Table 2. The computational time of the basic ABC algorithm and the proposed method corresponding optimum point search in Tabel 1.

f	D	ABC Algorithm Mean	ABC Algorithm Std	Proposed Method Mean	Proposed Method Std
f_1	2	11.7836	0.8247	**9.5275**	**0.6212**
f_2	2	10.4157	0.7202	**9.3355**	**0.3464**
	4	10.4156	0.7037	**9.6526**	**0.3244**
f_3	2	12.1211	1.0269	**9.8939**	**0.2869**
	4	12.0705	1.0471	**10.0757**	**0.3712**
f_4	2	12.0267	0.4039	**10.207**	**0.4329**
	4	12.0263	0.3856	**10.5047**	**0.4673**
f_5	2	12.9441	1.6708	**10.0831**	**0.6389**
	4	12.1936	0.8299	**10.0019**	**0.6749**

6. Conclusion

We have proposed a method for multimodal function optimization by using a hybrid of the ABC algorithm and the BFGS method. The results show that the hybrid method can overcome from the basic ABC algorithm problems for almost all test function. The numerical results show the objective function value produced by the proposed method is very closed to global minumum value for almost all test functions. However, if the shape of function is flat, the proposed method cannot work well.

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