$$SS_{E.C} = \sum \frac{Y_{.j}^2}{3} - N\bar{Y}_{..}^2$$

$$= \frac{1}{3}\left[17.4^2 + 24.3^2 + 12.4^2\right] - 9 \times (6.23)^2$$

$$= 24.220.$$

$$SS_{Tot} = \sum\sum y_{ij}^2 - N\bar{Y}_{..}^2$$

$$= 26.0339.$$

Source of variation	Sum sq.	D.f	Mean sq	F-rate
Treatments.	1.6805	2	0.8403	$F_0 = 25.1326$
Blocks.	24.220	2	12.11	$F_0' = 362.57$
Error.	0.1334	4	0.0334.	
Total	26.0339	8		

If,

$$\left.\begin{array}{l} F_0 > F_{(k-1),(k-1)(b-1),\alpha} \\ F_0' > F_{(b-1),(k-1)(b-1),\alpha} \end{array}\right\} F_{2,4,0.05} = 6.94.$$

Efficiency of Blocking.

* The relative efficiency (RE) of randomized block to CR (completely randomized) is,

i). $RE = \dfrac{(b-1)B + b(k-1)E}{(bk-1)E}$; b : no of blocks

k : no of treatments

B : estimate of the variance between blocks (MSB)

E : estimate of the experimental error (MSE).

2) $RE = \dfrac{MSE\ (CRD)}{MSE\ (RCBD)}$

If RE > 1 ⇒ Blocking was effective in reducing variability

RE = 1 ⇒ Blocking made no difference.

RE < 1 ⇒ Blocking is not effective. ∵ CRD is better than RCBD.

Example.

1) If $MSE\ (CRD) = 42$ (without blocking)

$MSE\ (RCBD) = 25$ (with blocking)

$RE = \dfrac{42}{25} = 1.68 > 1$

∴ Blocking made the experiment 68% more efficient compared to CRD.

Scheffe's method for RCBD.

* The method can be used to compare contrast treatment mean as well as block mean.

Block mean Contrast.

$$L'_u = d_{1u}\mu_1 + d_{2u}\mu_2 + \dots + d_{bu}\cdot\mu_b \quad ; u = 1, 2, \dots, m.$$

The corresponding contrast in the sample,

$$D_u = d_{1u}\cdot\bar{Y}_{\cdot 1} + d_{2u}\cdot\bar{Y}_{\cdot 2} + \dots + d_{ku}\cdot\bar{Y}_{\cdot b}$$

$$S'_{\alpha u} = \sqrt{(b-1)\cdot F_{\alpha,(b-1),(b-1)(k-1)}\cdot\hat{\sigma}^2\cdot\sum_{j=1}^{b}\frac{d_{ju}^2}{k}} \quad ; \hat{\sigma}^2 = MSE$$

No :

Date: __/__/__

If $|D_u| > S'_{\alpha u}$ $\Rightarrow$ null hypothesis rejected.
[$H_0 : L'_u = 0$ rejected].

Treatment mean contrast

$$L''_u = d_{1u} \cdot \mu_{1.} + d_{2u} \cdot \mu_{2.} + \dots + d_{ku} \cdot \mu_{k.} \quad ; \; u = 1, 2, \dots, m$$

The corresponding contrasts in the sample,

$$D'_u = d_{1u} \cdot \overline{X}_{1.} + d_{2u} \cdot \overline{X}_{2.} + \dots + d_{ku} \cdot \overline{Y}_{i.}$$

$$S''_{\alpha u} = \sqrt{(k-1) \cdot F_{\alpha, (k-1), (b-1)(k-1)} \cdot \hat{\sigma}^2 \cdot \sum_{i=1}^{k} \frac{d_{iu}^2}{b}} \quad ; \; \hat{\sigma}^2 = MSE$$

* If $|D'_u| > S''_{\alpha u}$ $\Rightarrow$ null hypothesis rejected.
[$H_0 : L''_u = 0$ is rejected].

Example.

To evaluate 3 file management systems A, B and C, a firm designed a test involving 5 different word processing operators. Since operator variability was believed to be a significant factor, each of the 5 operators was trained on each of the systems, in random order. The mean time (weeks) to learn a system is given below. Carry out ANOVA test ($\alpha = 0.05$) and draw the conclusions.

	System		
Operator	A	B	C
1	6	16	24
2	19	17	22
3	14	13	19
4	13	12	18
5	18	17	22.

Check

i) $\mu_1 = \mu_2 = \mu_3$

ii) $\mu_1 = \frac{\mu_2 + \mu_3}{2}$

using scheffe's method.

No : Date: __/__/__

21/10/2024.

Example 02 → RCBD

i) $|D_1| = 18.67 > S'_{\alpha/2} \Rightarrow H_0'$ is rejected

ii) $|D_2| = 2 > S''_{\alpha/2} = 1.9689 \Rightarrow H_0''$ is rejected.

ANOVA ⇒ H_0, H_0' is rejected for each.

Chapter 06 → Latin Square Design.

ex:- want to compare the effect of 3 fertilizers.

① F1, F2, F3

$\boxed{\text{CRD}}$

② F_1, F_2, F_3

Plots = P_1, P_2, P_3

$\boxed{\text{RCBD}}$

③ F_1, F_2, F_3 ⇐ fertilizer

P_1, P_2, P_3 ⇐ plots

T_1, T_2, T_3 ⇐ Time duration (month / week)

$\boxed{\text{LS}}$

* The LS method is a statistical design used in experiments to control for two outside variables. In addition to the treatment variable.
* It is perticqularly useful when the experimental units can be arranged in two different ways such as raws & columns.
* A Latin Square is an P×P matrix build with 3 different Symbols where each symbol appears exactly one in each row and each column. This ensures that the effect of the two outside variables are controlled leaving the treatment effect isolated for Analysis.

$4 \times 4 \Rightarrow P_1, P_2, P_3, P_4$: $P = 4$.

	1	2	3	4
1	P_1	P_2	P_3	P_4
2	P_2	P_3	P_4	P_1
3	P_3	P_4	P_1	P_2
4	P_4	P_1	P_2	P_3

* Rows are blocking factor.
* Colums are another blocking factor.
* Treatments are assigned s.t. each treatment appears exactly onece in every row & every column, ensuring the effect of row and column balanced out.

Advantages →

1). Controls for 2 nuisance ~~naisis~~ factors.
2) Can test more effects with the small number of observations.
3). Analysis is ~~the~~ simple

Disadvantages →

1). The no of threatments is limited to the no of rows or columns.
2) when degrees of freedom of SSE is less than the LS design may not be as efficient as RBCD or CRD. d.f of SSE = $(P-1).(P-2)$.

$\therefore$ when $P=1$ or $P=2$, $SSE=0$. $\therefore$ we need 3 or more treatments.

Model $\quad Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}$

where, $\quad i = 1,2,\dots,P$

$j = 1,2,\dots,P$

$k = 1,2,\dots,P$

No : Date: / /

Y_{ijk} = Observation of i^{th} treatment at j^{th} row & k^{th} column.

μ = overall mean.

α_i = i^{th} treatment effect.

β_j = j^{th} row effect.

γ_k = k^{th} column effect.

$\varepsilon_{i,j,k}$ = Random error.

- Assumptions →
  1) ε_{ijk} are independent & uncorrelated.
  2) $\varepsilon_{ijk} \sim N(0, \sigma^2)$.

- Constraints →
  1) $\sum_{i=1}^{p} \alpha_i = 0$
  2) $\sum_{j=1}^{p} \beta_j = 0$
  3) $\sum_{k=1}^{p} \gamma_k = 0$

- Hypothesis.

$H_0 : \alpha_i = 0 \; \forall i$

$H_1 : \alpha_i \neq 0$ for at least one i.

$H_0 : \beta_j = 0 \; ; \forall j$

$H_1 : \beta_j \neq 0$ for at least one j

$H_0 : \gamma_i = 0 \; \forall k$

$H_1 : \gamma_i \neq 0$ for at least one k.

Unbiased estimators.

$\hat{\mu} = \bar{Y}$

$\hat{\alpha}_i = \frac{Y_{i..}}{p} - \bar{Y}_{...}$

$\hat{\beta}_j = \frac{Y_{.j.}}{p} - \bar{Y}_{...}$

$\gamma_k = \frac{Y_{..k}}{p} - \bar{Y}_{...}$

Y = Total

$\bar{Y}_{...}$ = sample mean

$Y_{i..}$ = i^{th} treatment model

$Y_{.j.}$ = j^{th} row total

$Y_{..k}$ = k^{th} column total

we can prove,

$E[\hat{\mu}] = \mu$

$E[\hat{\mu}] = \mu$

$E[\hat{\alpha}_i] = \alpha_i$

$E[\hat{\beta}_j] = \beta_j$

$E[\hat{\gamma}_k] = \gamma_k$.

No : Date: / /

$$SS_{Tot} = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}(Y_{ijk}-\overline{Y}_{...})^2 = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} Y_{ijk}^2 - p^2\overline{Y}_{...}^2$$

$$SS_{Tr} = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}(\overline{Y}_{i..}-\overline{Y}_{...})^2 = \sum_{i=1}^{p}\frac{Y_{i..}^2}{p} - p^2\overline{Y}_{...}^2$$

$$SS_{row} = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}(\overline{Y}_{.j.}-\overline{Y}_{...})^2 = \sum_{j=1}^{p}\frac{Y_{.j.}^2}{p} - p^2\overline{Y}_{...}^2$$

$$SS_{col} = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}(\overline{Y}_{..k}-\overline{Y}_{...})^2 = \sum_{k=1}^{p}\frac{Y_{..k}^2}{p} - p^2.\overline{Y}_{...}^2$$

$$SS_{Tot} = SS_{Tr} + SS_{col} + SS_{row} + SS_E$$

ANOVA

S.V	SS	DF	MS	F ratio
Treatment	SS_{Tr}	$p-1$	$SS_{Tr}/(p-1)$	$F_0 = MS_{Tr}/MSE$
Rows	SS_{row}	$p-1$	$SS_{row}/(p-1)$	$F_0' = MS_{row}/MSE$
Columns	SS_{col}	$p-1$	$SS_{col}/(p-1)$	$F_0'' = MS_{col}/MSE$
Error	SS_E	$(p-1)(p-2)$	$SS_{Err}/(p-1)(p-2)$	
Total	SS_{Tot}	p^2-1		

$$F\text{-table} = F_{(p-1),(p-1)(p-2),\alpha} > F_0, \; F_0', \; F_0''$$

Example

A Researcher wants to evaluate the perfomers of 4 different financial forcasting models in crediting stock returns. The models which are testing model A - moving Average (MA)

model B - exponential smoothing.

model C - ARIMA.

model D - Nural Network.

The experiment is conducted over 4 time periods. week 1, week 2, 3, 4. Predictions are made under 4 different market conditions. Bull market, Bear Market, stable market & volatile market.

The performers of these models was Analyze using the mean absolute percentage error. (MAPE) To asset predicting accuracy across dif time periods and market conditions. The following table represents the results. Carry out the ANOVA for the experiment at the 0.1 significance level and state the relevant conclusions.

Time (week)	Market: 1) Bull	2) Bear	3). Stable	4). Volatile.	
1	Model A 0.056	B 0.072	C 0.061	D 0.048	$\Rightarrow Y_{.1.} = 0.23$
2	B 0.065	C 0.071	D 0.052	A 0.06.	$\Rightarrow Y_{.2.} = 0.24$
3.	C 0.07	D 0.063	A. 0.049	B 0.064	$\Rightarrow Y_{.3.} = 0.24$
4	D 0.054	A 0.069	B 0.073	C 0.061.	$\Rightarrow Y_{.4.} = 0.25$
	$Y_{..1} = 0.245$	$Y_{..2} = 0.275$	$Y_{..3} = 0.235$	$Y_{..4} = 0.233$	

- Model →

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}$$

- Hypothesis →

i - Treatment

j - row

k - column

$H_0 : \alpha_i = 0 , \forall i$

$H_1 : \alpha_i \neq 0$, for at least one i.

$H_0 : \beta_j = 0 , \forall j$

$H_1 : \beta_j \neq 0$ for at least one j

$H_0 : \gamma_k = 0 , \forall k$

$H_1 : \gamma_k \neq 0$ for at least one k.

Analyze using the
asset predicting
ket conditions. The
ry out the ANOVA
level and state

4). Volatile.

D
$0.048 \Rightarrow Y_{.1.} = 0.237$

A
$0.06. \Rightarrow Y_{.2.} = 0.242$

B
$0.064 \Rightarrow Y_{.3.} = 0.246$

C
$0.061. \Rightarrow Y_{.4.} = 0.25$

$Y_{..4} = 0.233$

i - Treatment
j - row
k - column

$$SS_{Tot} = \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{4} Y_{ijk}^2 - P^2 \bar{Y}_{...}^2$$

$$= 0.9761 - 4^2 \times 0.003813$$

$$= 0.9151.$$

$Y_{1..} = 0.234$ $\quad$ $Y_{3..} = 0.263$

$Y_{2..} = 0.274$ $\quad$ $Y_{4..} = 0.217.$

$$SS_{Tr} = \sum_{i=1}^{P} \frac{Y_{i..}^2}{P} - P^2 \cdot \bar{Y}_{...}^2$$

$$= \frac{1}{4}[0.24609] - 16 \times 0.003813.$$

$$SS_{Tr} = 0.0005145$$

No ______ Date: ___/___/______

ANOVA

S.V	SS	D.F	MS	F
Treatment	0.0006	3	0.0002	$F_0 = 0.0102 < F_{table}$
Rows	0.0002	3	0.00007	$F_0' = 0.0036 < F_{table}$
Columns	0.1354	3	0.0451	$F_0'' = 2.2893 < F_{table}$
Errors	0.1183	6	0.0197	
Total	0.2545	15		

$$F_{table} = F_{3,6,0.001}$$
$$= 9.7795$$

$\therefore H_0, H_0', H_0''$ are not rejected.

$\therefore$ Effect of treatment = 0

$\therefore$ Effect " time = 0

$\therefore$ " " market condition = 0.

Graeco - Latin Square Design.

	1	2	3	4
1	A, α	B, β	C, γ	D, δ
2	B, γ	C, δ	D, α	A, β
3	C, δ	D, α	A, β	B, γ
4	D, β	A, γ	B, δ	C, α.

A ~~grade~~ graeco- Latin square is a type of experimental design that combine two ladin squares in such a way that no treatment combination repeats across the design. It is used to control for two external factors while testing for the effect of the 3rd factor or another treatment.

Two latina square are superrimposed on each other one using latin letter and other using greek letters such that each latting letter is peared with each greek letter exactly once.

(Condit^{n}s) There are P treatmends where P is the size of the square.

One square uses latin - tetters to represent the treatments.

The other square uses greek-letters to represents different treatments.

Each latting letter appear exactly once in each row and each columns and each greek letter does the same.

every pair of a latting and a greek letter occurs exacly once in entire region. ensur~~i~~ing no repititions.

18/11/2024

2-treatments → Graeco latin square.

* Should be a balanced design [Same no of rows & columns].

	1	2	3	4
1	A,α	B,β	C,γ	D,δ
2	B,γ	A,δ	D,α	C,β
3	C,δ	D,α	A,β	B,α
4	D,β	C,α	B,δ	A,γ

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

α	β	γ	δ
γ	δ	α	β
δ	γ	β	α
β	α	δ	γ

$$Y_{ijkl} = \mu + \theta_i + J_j + \omega_k + \psi_l + \varepsilon_{ijkl}.$$

$i = \overline{1,p}$, $j =$

Y_{ijkl} : Observation of i^{th} row at j^{th} latin letter treatment k^{th} greek letter treatment and l^{th} column.

μ = Overall mean.

θ_i = i^{th} row effect.

J_j = j^{th} Lattin Letter treatment effect

ω_k = k^{th} greak letter treatment effect.

ψ_l = l^{th} column effect.

Assumptions.

ε_{ijkl} are independent and uncorrelated.

$\varepsilon_{ijkl} \sim N(0, \sigma^2)$

No : Date: / /

Constraints.

1) $\sum_{i=1} \theta_i = 0$

2) $\sum \tau_j = 0$

3) $\sum \omega_k = 0$

4) $\sum \psi_l = 0.$

Hypothesis.

$H_0 : \theta_i = 0 \quad \forall i$

$H_a : \theta_i \neq 0$ for at least one i.

$H_0' : J_j = 0 \quad \forall j$

$H_a' : J_j \neq 0$ for at least one j.

$H_0'' : \omega_K = 0 \quad \forall K$

$H_a'' : \omega_K \neq 0$ for at least one K.

$H_0''' : \psi_l = 0 \quad \forall l$

$H_a''' : \psi_l \neq 0$ for at least one l.

Unbiased estimators

$\hat{\mu} = \bar{Y}_{....} \longrightarrow E[\hat{\mu}] = \mu.$

$\hat{\theta}_i = \frac{Y_{i...}}{P} - \bar{Y}_{....} \longrightarrow E[\hat{\theta}_i] = \theta_i$

$\hat{J}_j = \frac{Y_{.j..}}{P} - \bar{Y}_{....} \longrightarrow E[\hat{J}_j] = J_j$

$\hat{\omega}_k = \frac{Y_{..k.}}{P} - \bar{Y}_{....} \longrightarrow E[\hat{\omega}_k] = \omega_k$

$\hat{\psi} = \frac{Y_{...l}}{P} - \bar{Y}_{....} \longrightarrow E[\hat{\psi}_l] = \psi_l.$

$y_{\cdots}$

$y_{i\cdots}$

$y_{\cdot j\cdot\cdot}$

$y_{\cdot\cdot k\cdot}$ - k^{th} greek letter treatment total

$y_{\cdots l}$ - l^{th} column total

$$SS_T = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}\sum_{l=1}^{p}(y_{ijkl} - \bar{y}_{\cdots\cdot})^2 = \sum\sum\sum\sum y_{ijkl}^2 - p^2 \cdot \bar{y}_{\cdots\cdot}^2$$

$$SS_{row} = \sum\sum\sum\sum(\bar{y}_{i\cdots} - \bar{y}_{\cdots\cdot})^2 = \sum_{i=1}^{p}\frac{y_{i\cdots}^2}{p} - p^2\bar{y}_{\cdots\cdot}^2$$

$$SS_{Latin} = \sum\sum\sum\sum(\bar{y}_{\cdot j\cdot\cdot} - \bar{y}_{\cdots\cdot})^2 = \sum_{j=1}^{p}\frac{y_{\cdot j\cdot\cdot}^2}{p} - p^2\bar{y}_{\cdots\cdot}^2$$

$$SS_{greek} = \sum\sum\sum\sum(\bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot})^2 = \sum_{k=1}^{p}\frac{y_{\cdot\cdot k\cdot}^2}{p} - p^2 \cdot \bar{y}_{\cdots\cdot}^2$$

$$SS_{col} = \sum\sum\sum\sum(\bar{y}_{\cdots l} - \bar{y}_{\cdots\cdot})^2 = \sum_{l=1}^{p}\frac{y_{\cdots l}^2}{p} - p^2 \cdot \bar{y}_{\cdots\cdot}^2$$

$$SS_{Tot} = SS_{row} + SS_{col} + SS_{Latin} + SS_{greek} + SS_E$$

- ANOVA table.

S.V.	SS	D.f	M.S.	F
row	SS_{row}	$p-1$	$SS_{row}/(p-1)$	$F_0 = MS_{row}/MSE$
latin	SS_{latin}	$p-1$	$SS_{latin}/(p-1)$	$F_0' = MS_{latin}/MSE$
greek	SS_{greek}	$p-1$	$SS_{greek}/(p-1)$	$F_0'' = MS_{greek}/MSE$
col	SS_{col}	$p-1$	$SS_{col}/(p-1)$	$F_0''' = MS_{col}/MSE$
error	SS_E	$(p-1)(p-3)$	$SS_E/(p-1)(p-3)$	
Total		(p^2-1)		

If F-ratio $> F_{(p-1),(p-1)(p-3),\alpha}$ $\Rightarrow$ reject null

(Q1). An industrial angineer is investigating the effect of 4 essembly methods. A, B, C, D on the essembly time for a color television component. Four Operators are selected for the study. Further the ingineer knows that each essembly method produces such fating that the time required for the last assembly may be greater than the time required for the first. regardless for the method. That is a treand develops in the required assembly time. The engineer suspects that the work places used by the four operators may represent and additional source of radiation Fourth factor workplace ($\alpha, \beta, \gamma, \delta$) may be introduced and building the graco - latin square that follows. Analyz the data from these experiment & draw conclusions..

Order of Assembly	Order 1		2		3		4		
1	$C\beta$	11	$B\gamma$	10	$D\delta$	14	$A\alpha$	8	$Y_{1\cdots} = 43$
2	$B\alpha$	8	$C\delta$	12	$A\gamma$	10	$D\beta$	12	$Y_{2\cdots} = 42$
3	$A\delta$	9	$B\alpha$	11	$D\beta$	7	$C\gamma$	15	$Y_{3\cdots} = 42$
4	$D\gamma$	9	$A\beta$	8	$C\alpha$	18	$B\delta$	6	$Y_{4\cdots} = 41$
	$Y_{\cdots 1} = 37$		$Y_{\cdots 2} = 41$		$Y_{\cdots 3} = 49$		$Y_{\cdots 4} = 41$		

- Model →

$$Y_{ijkl} = \mu + \theta_i + \tau_j + \omega_k + \psi_l + \varepsilon_{ijkl}$$

- Assumptions →

ε_{ijkl} are independent and uncorrelated.

$\varepsilon_{ijkl} \sim N(0, \sigma^2)$

- ANOVA Table →

$Y_{\cdot A \cdot\cdot} = 35$ $\qquad Y_{\cdot\cdot\alpha\cdot} = 45$ $\qquad \bar{Y}_{\cdots\cdot} = 10.5$

$Y_{\cdot B \cdot\cdot} = 35$ $\qquad Y_{\cdot\cdot\beta\cdot} = 38$

$Y_{\cdot C \cdot\cdot} = 56$ $\qquad Y_{\cdot\cdot\gamma\cdot} = 44$

$Y_{\cdot D \cdot\cdot} = 42$ $\qquad Y_{\cdot\cdot\delta\cdot} = 41$

$$SS_T = \sum\sum\sum\sum Y_{ijkl}^2 - P^2 \bar{Y}_{\dots}^2$$

$$= 1914 - 4^2 \times 10.5^2$$

$$= 150$$

$$SS_{row} = \sum_{i=1}^{4} \frac{Y_{i\dots}^2}{P} - P^2 . \bar{Y}_{\dots}^2$$

$$= \frac{1}{4} \times 7058 - 4^2 \times 10.5^2$$

$$SS_{row} = 0.5$$

$$SS_{Latin} = \sum_{j=1}^{4} \frac{Y_{.j..}^2}{P} - P^2 . \bar{Y}_{\dots}^2$$

$$= \frac{1}{4} \times 7850 - 4^2 \times 10.5^2$$

$$= 73.5 \quad [95.5]$$

$$SS_{greek} = \sum_{k=1}^{4} \frac{Y_{..k.}^2}{P} - P^2 \bar{Y}_{\dots}^2$$

$$= \frac{1}{4} \times 7086 - 4^2 \times 10.5^2$$

$$= 7.5$$

$$SS_{col} = \sum_{l=1}^{4} \frac{Y_{\dots l}^2}{P} - P^2 \bar{Y}_{\dots}^2$$

$$= \frac{1}{4} [7132] - 4^2 \times 10.5^2$$

$$= 19$$

$$SS_E = 27.5$$

S. V.	SS	D.F
Row	0.5	3
Latin		3
Greek	7.5	3
Column	19	3
Error		
Total		

30/12/2024.

07. Factorial Experimental Design.

* Factorial experiments study the effects of two or more factors simultaniously. These experiments help to identify the main effect (individul effect) of each factor on the responce variable. And interaction effects that means how factor interact with one another to influence the responce.

ex:- Testing the effect of fertilizer type and watering frequency on plant growth.

Factors : Fertilizer type (A) — Organic (a_1), Chemical (a_2)

Watering frequency (B) — Daily (b_1), Weekly (b_2)

Treatment combinations : 2x2 = 4 combinations.

$a_1 b_1$ - Organic + daily
$a_1 b_2$ - Organic + weekly
$a_2 b_1$ - chemical + daily
$a_2 b_2$ - chemical + weekly.

3 plants. (P_1 P_2 P_3)

	Plant height (cm)		Plants			
	A	B	1	2	3	
1	a_1	b_1	30	32	31	$\bar{Y}_1 = 31$
2	a_1	b_2	20	22	23	$\bar{Y}_2 = 21.67 \simeq 22$
3	a_2	b_1	40	42	41	$\bar{Y}_3 = 41$
4	a_2	b_2	25	27	26	$\bar{Y}_4 = 26.$

No : Date: / /

- Main effects:
  - Fertilizer (A) : Is there a difference in plant growth between Organic and chemical fertilizers?
  - Watering (B) : Is there a difference in plant growth between daily and weekly watering.

- Interaction effect:
  does the effect of fertilizer depend on watering frequecy?

Main effect of A : Average of a_2 - Average of a_1

$= 33.5 - 26.5$

$= 7$

Main effect of B : Average of b_1 - Average of b_2

$= 36 - 24$

$= 12$

Interaction effect : $\dfrac{a_1b_1 + a_2b_2}{2} - \dfrac{a_2b_1 + a_1b_2}{2}$

$= \dfrac{31 + 26}{2} - \dfrac{41 + 22}{2}$

$= 28.5 - 31.5$

$= (-)3$

* When we consider the design of factorial experiments we have to think about,
  1) what factor shall we included?
  2) At what levels shall these factors be taken.?
  3) How many experimental units shall we used?

- Model:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

$i = 1, 2, \ldots, a$

$j = 1, 2, \ldots, b$

$k = 1, 2, \ldots, n$

Y_{ijk} = Observation relevent to the i^{th} level of factor A, j^{th} level of factor B in the k^{th} replicate.

μ : overall mean.

α_i : effect of i^{th} level of factor A.

β_j : effect of j^{th} level of factor B.

$(\alpha\beta)_{ij}$: effect of the interaction between i^{th} level of factor A & j^{th} level of factor B.

ε_{ijk} : random error.

- Hypothesis :

H_0 : $\alpha_1 = \alpha_2 = \dots = \alpha_a = 0$

H_1 : $\alpha_i \neq 0$ for at least one i.

H_0' : $\beta_1 = \beta_2 = \dots = \beta_b = 0$

H_1' : $\beta_j \neq 0$ for at least one j.

H_0'' : $(\alpha\beta)_{ij} = 0$ for i and j. (NO interaction between A and B)

H_1'' : $(\alpha\beta)_{ij} \neq 0$

- Assumptions :

$\varepsilon_{ijk} \sim N(0, \sigma_\varepsilon^2)$

ε_{ijk}'s are independent.

- Constraints :

$$\sum_{i=1}^{a} \alpha_i = 0 \qquad \sum_{i=1}^{a} (\alpha\beta)_{ij} = 0$$

$$\sum_{j=1}^{b} \beta_j = 0 \qquad \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0.$$

No : Date: / /

- ANOVA table →

S.V.	S.S.	M.S	D.F.	F
A factor treatment.	SS_A	$\frac{SS_A}{(a-1)}$	$(a-1)$	$F_0 = \frac{MS_A}{MS_E}$
B factor treatment	SS_B	$\frac{SS_B}{(b-1)}$	$(b-1)$	$F_0' = \frac{MS_B}{MS_E}$
Interaction.	SS_{AB}	$\frac{SS_{AB}}{(a-1)(b-1)}$	$(a-1)(b-1)$	$F_0'' = \frac{MS_{AB}}{MS_E}$
Error.	SS_E	$\frac{SS_E}{ab(n-1)}$	$ab(n-1)$	
Total.	SS_T		$abn-1$	

ex:. The distance travelled (in miles) between recharges of JCY batteries used ~~to~~ for electric cars appears to depend on charging time and the speed at which the car is run. A pilot study of the proto type ICY battery was carried out at 3 preset charging period as well as three operating speeds. The batteries were tested in a car running on rails withing the factory and robots were used as drivers. The experiment was replicated twise and the responces in Km achieved between charges were presented below. Analyze the data from this experiment & draw conclutions. ($\alpha = 0.05$).

B

	Operating speed	Charging time (h).			
		18	24	30.	$a=3$, $b=3$, $n=2$
A	56	330, 309 (639)	355, 370 (725)	380, 360 (740)	$Y_{1..} = 2104$
	72	335, 327 (662)	377, 381 (758)	390, 385 (775)	$Y_{2..} = 2195$
	88	326, 356 (682)	390, 386 (776)	388, 396 (784)	$Y_{3..} = 2242$
		$Y_{.1.} = 1983$	$Y_{.2.} = 2259$	$Y_{.3.} = 2299$	$Y_{...} = 6541$

speed	time	Y
S_{56}	T_{8}	317.5
S_{72}	T_{24}	379.
S_{88}.	T_{30}	392.
S_{96}	T_{24}	362.5
S_{72}	T_{30}	387.5
S_{88}	T_{18}	
S_{56}	T_{30}	
S_{72}	T_{18}	
S_{88}.	T_{24}	

$$SS_{Total} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} Y_{ijk}^2 - \frac{Y_{...}^2}{abn}$$

$$= 2,389,543 - \frac{6541^2}{3 \times 3 \times 2}$$

$$= 12,616.27$$

$$SS_A = \sum_{i=1}^{a} \frac{Y_{i..}^2}{bn} - \frac{Y_{...}^2}{abn}$$

$$SS_A = 1,640.78$$

SS

No :

Date: __/__/__

06/01/2025.

- When factorial design are combining with to other experimental arrange-ment. They are reffer to by the combine design structure.

Arragngement in a completely randomized design.

* In CRD the treatments (factor combinations) are randomly assigned to experimental units. Each factor combinations (Treatment) has an equal chance of being applied to any unit.

ex:- Consider the factorial experiment with 4 factors, A, B, C and D of 3 levels E1, E2 and E_3. There are 12 factor combinations AE_1, BE_1, CE_1, AE_2, ..., DE_3. Which can be considered as treatments in the factorial design Suppose there are 2 replicates for each treatment combination.

The treatment combinations can be arranged in a CRD as follows,

DE_3	AE_1	BE_1	CE_1	DE_1	AE_2	BE_2	CE_2	DE_2	AE_3	BE_3	CE_3
48	44	46	50	46	49	51	53				
45	46	45	52	45	48	47	55				

Arrangement in a completely randomized block design.

* In RCBD in experimental units are grouped in to blocks based on a characteristic to reduce variability.
* Each block recives all the treatments in random order.

ex:- Consider an experiment of two factors A and B. Each is of two levels. A_1, A_2 and B_1, B_2. Experiment

	A_1	A_2
B_1	40 30 25	18 42 33
B_2	42 29 28	33 45 43

RBD ▷

A_1B_1	A_1B_2	A_2B_1	A_2B_2
40	42	18	33
30	29	42	45
25	28	33	43.

No : Date: / /

Arronging Latin - Square

* In a Latin - Square design treatments are arranged in a ~~Such that~~ Square s.t.
* Each treatment appears once in each row.
* " " " " " column.
* It is typically used to control 2 sources of variability.

Ex:- If there are 4 treatments, a 4×4 LS can arrange the treatments.

2^k Full factorial design.

* A 2^k full factorial design is a type of experimental design used to study K factors, each with two levels including all the combinations.
* The two levels generally quoted as, $(-1) \rightarrow$ low level (20°C)
  $(+1) \rightarrow$ high level (40°C)
* This design is commonly applied when the objective is to determine the effects of multiple factors and their interactions on a responce variable.
* Number of runs for a 2^k full factorial designs.
  $K \rightarrow$ no of factors (2 levels for each).

Number of factors		No of runs.
2	$\longrightarrow$	4
3	$\longrightarrow$	8
4	$\longrightarrow$	16
⋮		⋮
K	$\longrightarrow$	2^k

* In 2^k design in addition to main effects, many types of interactions between effects of factors can be defined

and measured

* 2^2 factorial design (one replicate)

		B low	B high
A	low.	(1)	b
	high	a	ab

(1), a, b, ab are factor combinations / runs.

$$\left.\begin{array}{l} \text{The main effect of } A = \frac{1}{2}[ab + a - b - (1)] \\ \text{,, ,, ,, ,, } B = \frac{1}{2}[ab + b - a - (1)] \\ \text{Interaction effect of A with B} = \frac{1}{2}[ab - a - b + (1)]. \end{array}\right\} \text{when there is 1 replicate}$$

~~more than~~

- when there are r replicates, devide by r.

* The main effect of a is the changing the responce variable cuased by moving A from its low level to its high level average over the levels of b.
* The main effect of b is the change in the responce variable cause by moving B from its low level to its high level. Average over the levels of A.
* The interaction effect measures how the two factors work together to influence the responce variable. beyond their main effects.

[sign - table]

Factor combination (runs).

Effects	(1)	a	b	ab	Divisor.
M	+	+	+	+	4
A	−	+	−	+	2
(B)	−	−	+	+	2
(AB)	+	−	−	+	2

General mean $= \frac{1}{4}[ab + a + b + (1)]$

No: Date: / /

Factorial design of 2 factors with 2 levels (2^2)

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}.$$

where, $i = 1, 2$, $j = 1, 2$ $k = 1, 2, \dots, n$.

Y_{ijk} = Observation relevent to the i^{th} level of factor A j^{th} level of factor B in k^{th} replicate.

μ = Overall mean.

α_i = effect of i^{th} level of factor A.

β_j = effect of j^{th} level of factor B.

$(\alpha\beta)_{ij}$ = " " the interpredation between i^{th} level of factor A & j^{th} level of factor B.

ε_{ijk} = random error.

Assumptions.

$\varepsilon_{ijk} \sim N(0, \sigma_\varepsilon^2)$

ε_{ijk}'s are independent

- Constraints. →

$$\sum_{i=1}^{2} \alpha_i = 0 \qquad \sum_{i=1}^{2} (\alpha\beta)_{ij} = 0$$

$$\sum_{j=1}^{2} \beta_j = 0 \qquad \sum_{j=1}^{2} (\alpha\beta)_{ij} = 0.$$

- Hypothesis →

$H_0 : \alpha_i = 0$ for $i = 1, 2$

$H_1 : \alpha_i \neq 0$ for at least one.

$H_0' : \beta_j = 0$ for $j = 1, 2$

$H_1' : \beta_j \neq 0$ for at least one j.

$H_0'' : (\alpha\beta)_{ij} = 0$ for i and j

$H_1'' : (\alpha\beta)_{ij} \neq 0$

$$SS_A = (A)^2 \cdot r.$$
$$SS_B = (B)^2 r.$$
$$SS_{AB} = (AB)^2 \cdot r$$
$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} Y_{ijk}^2 - \frac{Y_{...}^2}{abn.}$$

$$(A) = \frac{1}{2r} [ab + a - b - (1)]$$

Example

for 1 replicated design.

		A low	A high
B	low	18	20
	high	19	24

Find SS_T, SS_A, SS_B, SS_{AB}, SSE

* When it takes long time to run the experiment or process is expencive we may run the experiment at only one replicate. Then what would be the d.f of the error. (d.f = 0). In such situations we get 0 degrees of freedom of SSE.

ex:- Consider 2^2 factorial design with 2 replicates as given below.

		A low	A high
B.	low	18	20
		22	19
	high	14	20
		18	24.

$\Rightarrow$

		A low	A high
B	low	(1) 40	a 39
	high.	b 32	ab 44

~~$Y_{...}^2 = 19.375$~~

$Y_{...}^2$

Find SS_T, SSA, SSB, $SSAB$, SSE and construct the ANOVA table. Draw the relevent conclutions ($\alpha = 0.05$)

(B) ~~(A)~~ $= \frac{1}{2r}[ab + a - b - (1)]$

$= \frac{1}{2\times2}[44 + 32 - 39 - 40]$

$= (-)\ 0.75$

(A) ~~(B)~~ $= \frac{1}{2r}[ab + b - a - (1)]$

$= \frac{1}{2\times2}[44 + 39 - 32 - 40]$

$= 2.75$

Interaction effect of A with B $\Big\} = \frac{1}{2\times2}[44 - 32 - 39 + 40]$

$= 3.25$

$SS_A = (-0.75)^2 \times 2$

$= 1.125$

$SS_B = (2.75)^2 \times 2$

$= 15.125$

$SS_{AB} = (3.25)^2 \times 2$

$= 21.125$

$SS_T = 3065 - \frac{1}{2\times2\times2}$

$= 61.875$