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## **Binary Exponentiation:**

```
typedef long long ll;
ll power(ll a, ll b, ll m) {
    ll ans = 1;
    a %= m;
    while (b > 0) {
        if (b & 1) ans = (ans * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return ans;
}
```

If m is prime, calculate only  $x^{n \mod (m-1)}$  instead of  $x^n$ , which follows from Fermat's little theorem.

## **Modulo Inverse:**

```
int gcd(int a, int b, int& x, int& y)
{
    x = 1, y = 0;
    int x1 = 0, y1 = 1, a1 = a, b1 =
b;
    while (b1) {
        int q = a1 / b1;
        tie(x, x1) = make_tuple(x1, x
- q * x1);
        tie(y, y1) = make_tuple(y1, y
- q * y1);
        tie(a1, b1) = make_tuple(b1, a1 - q * b1);
    }
    return a1;
}
int modInverse(int a, int m) {
    int x, y;
```

```
int g = gcd(a, m, x, y);
while (x < 0) x += m;
return x;
}</pre>
```

```
Linear Diophantine Equation:
int gcd(int a, int b, int& x, int&
y) {
x = 1, y = 0;
    int x1 = 0, y1 = 1, a1 = a, b1 =
    while (b1) {
        int q = a1 / b1;
        tie(x, x1) = make tuple(x1, x
- a * x1);
        tie(y, y1) = make tuple(y1, y
- q * y1);
        tie(a1, b1) = make tuple(b1,
a1 - q * b1);
    return a1;
bool find any solution (int a, int b,
int c, int &x0, int &y0, int &g)
g = gcd(abs(a), abs(b), x0, y0);
if (c % q) {
return false;
x0 *= c / q; y0 *= c / q;
if (a < 0) \times 0 = -x0; if (b < 0) \times 0 =
-y0; return true;
void shift solution(int & x, int & y,
int a, int b, int cnt) {
x += cnt * b; y -= cnt * a;
int find all solutions (int a, int b,
int c, int minx, int maxx, int miny,
int maxy) {
```

```
int x, y, q;
if (!find any solution(a, b, c, x, y,
g))
return 0; a /= q;
     a /= q;
int sign a = a > 0? +1: -1; int
sign b = b > 0 ? +1 : -1;
shift solution (x, y, a, b, (minx - x)
/ b);
if (x < minx)
shift solution (x, y, a, b, sign b);
if (x > maxx)
return 0; int lx1 = x;
shift solution (x, y, a, b, (maxx - x)
/ b);
if (x > maxx)
shift solution(x, y, a, b,
-sign b);
int rx1 = x;
shift solution(x, y, a, b,
-(miny - y) / a); if (y < miny)
shift solution(x, y, a, b,
-sign a);
if (y > maxy)
return 0; int 1x2 = x;
shift solution(x, y, a, b,
-(maxy - y) / a); if (y > maxy)
shift solution (x, y, a, b, sign a);
int rx2 = x;
if (1x2 > rx2) swap(1x2, rx2);
int lx = max(lx1, lx2); int rx =
min(rx1, rx2);
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
```

## **Linear Sieve of Eratosthenes:**

```
// space optimized one is faster
though
const int MX = 1e7 + 5;
bitset<MX> isPrime;
vector<int> primes;
void sieve(int n) {
    isPrime.set();
    isPrime[0] = isPrime[1] = false;
    for (int i = 4; i \le n; i += 2)
isPrime[i] = false;
    for (int i = 3; i * i <= n; i +=
        if (!isPrime[i]) continue;
        for (int j = i * i; j <= n; j
+= i) isPrime[j] = false;
    primes.push back(2);
    for (int i = 3; i \le n; i += 2) {
        if (isPrime[i])
Primes.push back(i);
```

# **Segmented Sieve:**

```
vector<char> segmentedSieve(long long
L, long long R) {
   // generate all primes up to
sart(R)
    long long lim = sqrt(R);
   vector<char> mark(lim + 1,
    vector<long long> primes;
   for (long long i = 2; i <= lim;
++i) {
       if (!mark[i]) {
            primes.emplace back(i);
            for (long long j = i * i;
j <= lim; j += i) mark[j] = true;</pre>
```

```
vector<char> isPrime(R - L + 1,
true);
   // 0 denotes L, 1 denotes L + 1,
so on...
    for (long long i : primes)
       for (long long j = max(i * i,
(L + i - 1) / i * i); j <= R; j += i)
           isPrime[j - L] = false;
    if (L == 1) isPrime[0] = false;
    return isPrime;
```

## **Memory Optimized Block Sieve:**

```
Uses O(S + \sqrt{n}) memory only
// vector<int> prime generated;
int count primes(int n) {
    const int S = 10000;
    vector<int> primes;
    int nsqrt = sqrt(n);
    vector<char> is prime(nsqrt + 2,
true):
    for (int i = 2; i <= nsqrt; i++)
        if (is prime[i]) {
            primes.push back(i);
            for (int j = i * i; j <=
nsqrt; j += i)
                is prime[j] = false;
    int result = 0;
    vector<char> block(S);
    for (int k = 0; k * S <= n; k++)
        fill(block.begin(),
block.end(), true);
        int start = k * S;
        for (int p : primes) {
            int start idx = (start +
p - 1) / p;
            int j = max(start idx, p)
* p - start;
```

```
for (; j < S; j += p)
                block[j] = false;
        if (k == 0)
            block[0] = block[1] =
false;
        for (int i = 0; i < S &&
start + i <= n; i++) {
            if (block[i]) {
//prime generated.push back(start +
                result++;
   return result;
```

#### **Euler Phi:**

```
\Theta(n) = count of coprime numbers \leq n
 a^b \mod m = a^{b \mod \Theta(m)} \mod m, m and a coprime
int eulerPhi(int n) {
    int res = n;
    int sqrtn = sqrt(n);
    for (int i = 0; i < Primes.size()</pre>
&& Primes[i] <= sqrtn; i++) {
        if (n % Primes[i] == 0) {
             while (n % Primes[i] ==
0) n /= Primes[i];
             sqrtn = sqrt(n);
             res /= Primes[i];
             res *= (Primes[i] - 1);
    if (n != 1) {
         res /= n;
         res *= n - 1;
    return res;
```

#### **Miller Rabin Primality Test:**

Determinisitic test, guaranteed for upto 7 . 10<sup>18</sup>, complexity: 7 times the complexity of a<sup>b</sup> mod c.

```
typedef unsigned long long ull;
typedef long long 11;
const 11 \mod = 1000000007;
ll modpow(ll b, ll e) {
    ll ans = 1;
   for (; e; b = b * b % mod, e /=
2)
        if (e & 1) ans = ans * b %
mod;
    return ans;
ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M
* a * b);
    return ret + M * (ret < 0) - M *
(ret >= (ll)M);
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod),
        if (e \& 1) ans = modmul(ans,
b, mod);
    return ans;
bool isPrime(ull n) {
    if (n < 2 | | n % 6 % 4 != 1)
return (n | 1) == 3;
   ull A[] = \{2, 325, 9375, 28178,
450775, 9780504, 1795265022},
        s = builtin ctzll(n - 1), d
= n \gg s;
    for (ull& a : A) { // ^ count
trailing zeroes
        ull p = modpow(a % n, d, n),
i = s;
        while (p != 1 \&\& p != n - 1)
&& a % n && i--) p = modmul(p, p, n);
```

```
if (p != n - 1 && i != s)
return false;
}
return true;
}
```

## <u>Pollard's Rho Algorithm for Prime</u> Factorization:

Solves UVA – 11476 (Prime Factorization of 800 numbers upto  $10^{16}$ )

Range: 10^18 (tested), should be okay up to 2^63-1 miller\_rabin(n): returns 1 if prime, 0 otherwise Magic bases:

 $\begin{array}{l} n<4,759,123,141\ 3:2,7,61\\ n<1,122,004,669,633\ 4:2,13,23,1662803\\ n<3,474,749,660,383\ 6:2,3,5,7,11,13\\ n<2^64\ 7:2,325,9375,28178,450775,9780504,1795265022 \end{array}$ 

Identifies 70000 18 digit primes in 1 second on Toph pollard\_rho(n):

If n is prime, returns n, otherwise returns a proper divisor of n

Able to factorize ~120 18 digit semiprimes in 1 second on Toph

Able to factorize  $\sim$ 700 15 digit semiprimes in 1 second on Toph

Note: for factorizing large number, do trial division upto cubic root and then call pollard rho once.

```
#include <bits/stdc++.h>
#define LL long long
using namespace std;
LL mult(LL a, LL b, LL mod) {
    assert(b < mod && a < mod);
    long double x = a;
    uint64_t c = x * b / mod;
    int64_t r = (int64_t)(a * b - c *
mod) % (int64_t)mod;
    return r < 0 ? r + mod : r;
}
LL power(LL x, LL p, LL mod) {</pre>
```

```
LL s = 1, m = x;
   while (p) {
        if (p \& 1) s = mult(s, m,
mod);
        p >>= 1;
        m = mult(m, m, mod);
    return s;
bool witness (LL a, LL n, LL u, int t)
   LL x = power(a, u, n);
   for (int i = 0; i < t; i++) {
        LL nx = mult(x, x, n);
        if (nx == 1 && x != 1 && x !=
n - 1) return 1;
        x = nx;
   return x != 1;
vector<LL> bases = \{2, 325, 9375,
28178, 450775, 9780504, 1795265022};
bool miller rabin(LL n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
   LL u = n - 1;
   int t = 0;
    while (u % 2 == 0) u /= 2, t++;
   // n-1 = u*2^t
    for (LL v : bases) {
       LL a = v % (n - 1) + 1;
        if (witness(a, n, u, t))
return 0;
   return 1;
LL gcd(LL u, LL v) {
    if (u == 0) return v;
    if (v == 0) return u;
    int shift = builtin ctzll(u |
v);
    u >>= builtin ctzll(u);
```

v >>= builtin ctz(v);

```
if (u > v) swap(u, v);
        v = v - u;
    } while (v);
    return u << shift;</pre>
mt19937 64
rng(chrono::steady clock::now().time
since epoch().count());
LL pollard rho(LL n) {
    if (n == 1) return 1;
    if (n \% 2 == 0) return 2;
    if (miller rabin(n)) return n;
    while (true) {
        LL x =
uniform int distribution<LL>(1, n -
1) (rng);
        LL y = 2, res = 1;
        for (int sz = 2; res == 1; sz
*= 2) {
            for (int i = 0; i < sz \&\&
res <= 1; i++) {
                x = mult(x, x, n) +
1;
                res = gcd(abs(x - y))
n);
            y = x;
        if (res != 0 && res != n)
return res:
   }
const int MX = 2.2e5 + 7;
vector<int> primes;
bool isp[MX];
void sieve() {
    fill(isp + 2, isp + MX, 1);
    for (int i = 2; i < MX; i++)
        if (isp[i]) {
            primes.push back(i);
            for (int j = 2 * i; j <
MX; j += i) isp[j] = 0;
```

```
vector<LL> factorize(LL x) {
    vector<LL> ans;
    for (int p : primes) {
        if (1LL * p * p * p > x)
break;
        while (x % p == 0) {
            x /= p;
            ans.push back(p);
    if (x > 1) {
        LL z = pollard rho(x);
        ans.push back(z);
        if (z < x) ans.push back(x / y)
z);
    return ans;
int main() {
    sieve();
    int t;
    cin >> t:
    while (t--) {
        long long x;
        if (!(cin >> x)) break;
        vector<LL> ans =
factorize(x);
        sort(ans.begin(), ans.end());
        vector<pair<LL, int>> ff;
        for (LL x : ans) {
            if (ff.size() &&
ff.back().first == x)
                ff.back().second++;
            else
                ff.push back(\{x, 1\});
        cout << x << " =";
        bool first = true;
        for (auto pr : ff) {
            if (!first) cout << " *";
            first = false;
            cout << " " << pr.first;</pre>
            if (pr.second > 1) cout
<< "^" << pr.second;
```

```
}
cout << endl;
}
</pre>
```

# <u>Matrix Exponentiation (with Fibonacci Finding):</u>

Random example of a recurrence relation:

$$\begin{aligned} F_n &= 2F_{n-1} + 3F_{n-4} + 1 \\ \begin{bmatrix} F_4 \\ F_3 \\ F_2 \\ F_1 \\ 1 \end{bmatrix} &= \begin{bmatrix} 2 & 0 & 0 & 3 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ F_2 \\ F_1 \\ F_0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
const int MOD = 1e9 + 7;
struct Matrix {
   int r, c;
   vector<vector<int>> mat;
   Matrix(int r, int c) {
        this->r = r, this->c = c;
        mat.assign(r, vector<int>(c,
0));
   Matrix(vector<vector<int>> &vec)
        r = vec.size();
        c = vec[0].size();
        mat.assign(r, vector<int>(c,
0));
        for (int i = 0; i < r; i++) {
            for (int j = 0; j < c;
j++) mat[i][j] = vec[i][j];
istream & operator >> (istream & in,
Matrix &mt) {
    for (int i = 0; i < mt.r; i++)
        for (int j = 0; j < mt.c;
j++) in >> mt.mat[i][j];
   return in;
```

```
ostream & operator << (ostream & out,
Matrix &mt) {
    for (int i = 0; i < mt.r; i++) {
        for (int j = 0; j < mt.c;
j++) out << mt.mat[i][j] << " ";</pre>
        out << endl;
    return out;
Matrix mul(Matrix &a, Matrix &b) {
    // assert(a.mat[0].size() ==
b.mat.size());
    Matrix product(a.mat.size(),
b.mat[0].size());
    for (int j = 0; j < a.mat.size();
j++) {
        for (int i = 0; i <
b.mat.size(); i++) {
            for (int k = 0; k <
b.mat[0].size(); k++) {
                product.mat[j][k] =
(product.mat[j][k] + OLL +
(a.mat[j][i] * 1LL * b.mat[i][k]) %
MOD) %
MOD;
    return product;
Matrix power(Matrix a, ll b) {
    // assert(a.mat.size() ==
a.mat[0].size());
    Matrix ans(a.mat.size(),
a.mat.size());
    for (int i = 0; i <
ans.mat.size(); i++) ans.mat[i][i] =
1;
    while (b) {
        if (b \& 1) ans = mul(ans, a);
        a = mul(a, a);
        b >>= 1;
```

```
return ans;
Gaussian Elimination:
#include <bits/stdc++.h>
using namespace std;
const double EPS = 1e-9;
int Gauss (vector<vector<double>> a,
vector<double> &ans) {
    // a is the coefficient matrix,
and the last column is
    // added from the rhs of the
equations
    int n = (int)a.size(), m =
(int)a[0].size() - 1;
    vector<int> pos(m, -1);
    double det = 1;
    int rank = 0;
    for (int col = 0, row = 0; col <
m && row < n; ++col) {
        int mx = row;
        for (int i = row; i < n; i++)
// for partial pivoting
            if (fabs(a[i][col]) >
fabs(a[mx][col])) mx = i;
        if (fabs(a[mx][col]) < EPS) {</pre>
            det = 0;
            continue;
        for (int i = col; i <= m;
i++) swap(a[row][i], a[mx][i]);
        if (row != mx) det = -det;
        det *= a[row][col];
        pos[col] = row;
        for (int i = 0; i < n; i++) {
            if (i != row &&
fabs(a[i][col]) > EPS) {
                double c = a[i][col]
/ a[row][col];
                for (int j = col; j
<= m; j++) a[i][j] -= a[row][j] * c;
```

```
++row;
        ++rank;
    ans.assign(m, 0);
    for (int i = 0; i < m; i++) {
        if (pos[i] != -1) ans[i] =
a[pos[i]][m] / a[pos[i]][i];
    for (int i = 0; i < n; i++) {
        double sum = 0;
        for (int j = 0; j < m; j++)
sum += ans[i] * a[i][i];
        if (fabs(sum - a[i][m]) >
EPS) return -1; // no solution
   for (int i = 0; i < m; i++)
        if (pos[i] == -1) return 2;
// infinte solutions
    return 1;
// unique solution
int main() {
   int n, m;
    cin >> n >> m;
    vector<vector<double>> v(n);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j \le m; j++)
            double x;
            cin >> x;
            v[i].push back(x);
    vector<double> ans;
    int k = Gauss(v, ans);
    if (k)
        for (int i = 0; i < n; i++)
            cout << fixed <<</pre>
setprecision(5) << ans[i] << ' ';</pre>
    else
        cout << "no solution\n";</pre>
```

return 0;

## **Sparse Table:**

Only done on immutable arrays, can answer queries in O(1) for idempotent operations like min, max, gcd etc. i.e. operations that can be done multiple times without affecting the final result.

```
vector<vector<int>>
range minimum(vector<int>& vec) {
    int n = vec.size();
    vector<vector<int>> sparse(n,
vector<int>( lg(n) + 1, INT MAX));
    for (int i = 0; i < n; i++)
sparse[i][0] = vec[i];
    for (int j = 1; j \le lg(n);
j++) {
        for (int i = 0; i + (1 << j)
-1 < n; i++)  {
            sparse[i][j] =
                min(sparse[i][j-1],
sparse[i + (1 << (j - 1))][j - 1]);
    return sparse;
int min query(vector<vector<int>>&
sparse, int 1, int r) { // minimum of
[l, r]
    int len = r - l + 1;
    return min(sparse[l][ lg(len)],
sparse[r - (1 << (lg(len))) +
1][ lq(len)]);
```

## **Binary Indexed Tree/Fenwick Tree:**

Range query and update both in O(logn), used mostly for range sum (with updates), can be done on other reversible functions as well. Much less constant factors than segment tree.

```
typedef long long ll;
// 1-based indexing
```

```
// point-update, range-query
const int MX = 1e5 + 5;
vector<ll> BIT(MX), a(MX);
int n;
void add(int idx, int delta) { //
increase idx'th element by delta
    for (; idx <= n; idx += idx & -
idx) BIT[idx] += delta;
11 query(int idx) { // returns sum
in range [1, x]
    11 \text{ sum} = 0;
    for (; idx > 0; idx -= idx & -
idx) sum += BIT[idx];
    return sum;
ll query(int l, int r) { return
query(r) - query(l - 1); }
2D Fenwick Tree:
sum(x1, y1, x2, y2) = sum(x2, y2) - sum(x1 - 1, y2) -
sum(x2, y1 - 1) + sum(x1 - 1, y1 - 1)
const int mx = 1e3 + 5;
int r, c;
vector<vector<ll>> bit(r,
vector<ll>(c));
ll sum(int x, int y) {
    11 \text{ ret} = 0;
    for (int i = x; i >= 0; i = (i & 
(i + 1)) - 1)
    for (int j = y; j >= 0; j = (j &
(j + 1)) - 1) ret += bit[i][j];
    return ret;
void add(int x, int y, int delta) {
    for (int i = x; i < bit.size(); i
= i | (i + 1)
    for (int j = y; j < 
bit[i].size(); j = j | (j + 1))
bit[i][j] += delta;
```

#### **Min Segment Tree:**

```
const int MX = 2e5 + 5;
vector<int> numbers(MX);
struct seg tree {
    vector<int> vec;
    const int NEUTRAL ELEMENT =
INT MAX;
    inline int lc(int x) { return (x
               // left child
<< 1); }
    inline int rc(int x) { return ((x
<< 1) | 1); } // right child
    seg tree(int n) { vec.assign(4 *
n, NEUTRAL ELEMENT); }
    inline int combine(int a, int b)
{ return min(a, b); }
   // call build(1, 1, n)
    void build (int at, int start, int
end) {
        if (start == end) {
            vec[at] = numbers[start];
            return;
        int mid = (start + end) >> 1;
        build(lc(at), start, mid);
        build(rc(at), mid + 1, end);
        vec[at] =
combine(vec[lc(at)], vec[rc(at)]);
        return;
    // for updating index i, call
update(1, 1, n, i)
    void update(int at, int start,
int end, int update index) {
        if (start == end && start ==
update index) {
            vec[at] =
numbers[update index];
            return;
        int mid = (start + end) >> 1;
        if (update index <= mid) {</pre>
```

## **Sum Segment Tree Lazy Propagation:**

};

```
inline ll combine(ll a, ll b) {
return a + b; }
    inline void push (int at, int
start, int end) {
       if (lazy[at] ==
NEUTRAL ELEMENT) return;
       vec[at] = combine(vec[at],
lazy[at] * (end - start + 1));
        if (start != end) {
            lazy[lc(at)] =
combine(lazy[lc(at)], lazy[at]);
            lazy[rc(at)] =
combine(lazy[rc(at)], lazy[at]);
        lazy[at] = NEUTRAL ELEMENT;
    inline void pull(int at) {
vec[at] = combine(vec[lc(at)],
vec[rc(at)]); }
   // call build(1, 1, n)
    void build(int at, int start, int
end) {
        lazy[at] = NEUTRAL ELEMENT;
        if (start == end) \{
            vec[at] = numbers[start];
            return;
        int mid = (start + end) >> 1;
        build(lc(at), start, mid);
        build(rc(at), mid + 1, end);
        pull(at);
        return;
    // for incrementing index [l, r]
by val, call update(1, 1, n, 1, r,
    void update(int at, int start,
int end, int q left, int q right, ll
val) {
        push(at, start, end);
        if (start > q right || end <
q left) return;
```

```
if (start >= q left && end <=
q right) {
            lazy[at] = val;
            push(at, start, end);
            return;
        int mid = (start + end) >> 1;
        update(lc(at), start, mid,
q left, q right, val);
        update(rc(at), mid + 1, end,
q left, q right, val);
        pull(at);
   // for query [l, r] call query(1,
1, n, l, r)
    ll query(int at, int start, int
end, int q left, int q right) {
        push(at, start, end);
        if (start > q right || end <
q left) return NEUTRAL ELEMENT;
        if (start >= q left && end <=
q right) return vec[at];
        int mid = (start + end) >> 1;
        return combine (query (lc (at),
start, mid, q left, q right),
                       query(rc(at),
mid + 1, end, q left, q right));
} ;
DSU:
const int MX = 1e6 + 5;
vector<int> size v(MX);
vector<int> parent(MX);
int find set(int v) \{ // O(1) \}
   if (v == parent[v])
        return v;
    else
        return parent[v] =
find set(parent[v]);
void make set(int v) {
```

parent[v] = v;

```
size v[v] = 1;
bool union sets(int a, int b) { //
0(1)
    // returns false if same set
    a = find set(a);
    b = find set(b);
    if (a != b) {
        if (size v[a] < size v[b])</pre>
swap(a, b);
        parent[b] = a;
        size v[a] += size v[b];
        return true;
        return false;
```

## **Ordered Set:**

```
#include <bits/stdc++.h>
#include
<ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace gnu pbds;
template <typename T>
using ordered set =
    tree<T, null type, less<T>,
rb tree tag,
tree order statistics node update>;
#define ll long long
int main() {
   int n, k;
    cin >> n >> k;
    ordered set<int> children;
    for (int i = 1; i \le n; i++)
children.insert(i);
   int startPosition = 0;
   while (children.size() > 0) {
        startPosition %=
(children.size());
```

```
int posToRemove =
(startPosition + k) %
children.size();
        startPosition = posToRemove;
        auto t =
children.find by order(posToRemove);
       cout << *t << " ";
       children.erase(t);
   return 0;
```

## **Square Root Decomposition:**

```
#include <bits/stdc++.h>
using namespace std;
int n, q;
vector<int> a(n);
int main() {
   // input data
   // preprocessing
    const int len =
        (int) sqrt(n + .0) + 1; //
size of the block and the number of
blocks
   vector<int> b(len);
    for (int i = 0; i < n; ++i) b[i /
len] += a[i];
   // answering the queries
   while (q--) {
       int l, r;
        // read input data for the
next query
        cin >> 1 >> r;
        int sum = 0;
        int c l = l / len, c r = r /
       if (c l == c r)
            for (int i = 1; i \le r;
++i) sum += a[i];
        else {
            for (int i = 1, end =
(c l + 1) * len - 1; i \le end; ++i)
```

sum += a[i];

```
for (int i = c l + 1; i
<= c r - 1; ++i) sum += b[i];
            for (int i = c r * len; i
<= r; ++i) sum += a[i];
```

## Mo's Algorithm for Highest Frequency:

```
#include <bits/stdc++.h>
using namespace std;
#define endl "\n"
typedef long long 11;
struct Query {
    int l, r, index, ans;
const int BLOCK SIZE = 350;
const int MX = 1e5 + 10;
int n, q, \max freq = 1;
int current left = 0, current right =
vector<int> numbers, frequency(MX),
freq frequency;
vector<Query> queries;
void add(int index) {
    if (index < 0 \mid | index >=
numbers.size()) return;
    int element = numbers[index];
    int previous frequency =
frequency[element];
    frequency[element]++;
    int current frequency =
frequency[element];
    if (previous frequency > 0)
freq frequency[previous frequency]--;
freq frequency[current frequency]++;
(freq frequency[current frequency] ==
1 && current frequency > max freq)
        max freq = current frequency;
void remove(int index) {
```

```
if (index < 0 || index >=
numbers.size()) return;
    int element = numbers[index];
    int previous frequency =
frequency[element];
    frequency[element] --;
    int current frequency =
frequency[element];
    if (current frequency > 0)
freq frequency[current frequency]++;
freq frequency[previous frequency]--;
    if (max freq ==
previous frequency &&
freq frequency[previous frequency] ==
        max freq = current frequency;
void processQueries(Query& q) {
    while (current left > q.1) {
        current left--;
        add(current left);
    while (current right < q.r) {</pre>
        current right++;
        add(current right);
    while (current left < q.1) {
        remove (current left);
        current left++;
    while (current right > q.r) {
        remove(current right);
        current right--;
    q.ans = max freq;
void solve() {
    cin >> n >> q;
    numbers.resize(n);
    freq frequency.assign(n + 2, 0);
    for (int i = 0; i < n; i++) cin
>> numbers[i];
```

```
queries.resize(q);
    for (int i = 0; i < q; i++) {
        cin >> queries[i].l >>
queries[i].r;
        queries[i].index = i;
    sort (queries.begin(),
queries.end(), [&] (const Query& p,
const Query& q) {
        if (p.l / BLOCK SIZE != q.l /
BLOCK SIZE)
            return make pair (p.1,
p.r) < make pair(q.l, q.r);
        return (p.l / BLOCK SIZE & 1)
? (p.r < q.r) : (p.r > q.r);
    });
    for (int i = 0; i < q; i++)
processQueries(queries[i]);
    sort(queries.begin(),
queries.end(),
         [&] (const Query& p, const
Query& q) { return p.index < q.index;
});
    for (int i = 0; i < q; i++) {
        cout << queries[i].ans <<</pre>
endl;
```

## **Lowest Common Ancestor (LCA):**

```
for (int j = 1; j < LOG; j++)
            up[other][i] =
up[up[other][j - 1]][j - 1];
        dfs(other, v);
// call dfs(root, -1) before any of
the following functions
int getKthAncestor(int u, int k) {
    // make sure dfs(root, -1) has
been called
    if (depth[u] < k) return -1;
    for (int j = LOG - 1; j >= 0; j--
) {
        if (k \& (1 << j)) u =
up[u][j];
    return u;
int getDistanceBetweenNodes(int a,
int b) {
    // make sure dfs(root, -1) has
been called
    if (depth[a] < depth[b]) swap(a,</pre>
b);
    int k = depth[a] - depth[b];
    int distance = k;
    a = getKthAncestor(a, k);
    if (a == b) return distance;
    for (int j = LOG - 1; j >= 0; j--
) {
        if (up[a][j] != up[b][j]) {
            a = up[a][j];
            b = up[b][j];
            distance += 2 * (1 << j);
    return distance + 2;
int get LCA(int a, int b) {
    // make sure dfs(root, -1) has
been called
```

```
if (depth[a] < depth[b]) swap(a,
b);
  int k = depth[a] - depth[b];
  a = getKthAncestor(a, k);
  if (a == b) return a;
  for (int j = LOG - 1; j >= 0; j--
) {
    if (up[a][j] != up[b][j]) {
        a = up[a][j];
        b = up[b][j];
    }
} return up[a][0];
}
```

## Dijkstra's Algorithm (SSSP):

```
typedef long long ll;
struct Edge {
    int to;
    ll weight;
    bool operator>(const Edge& other)
const. {
        if (weight != other.weight)
return weight > other.weight;
        return to > other.to;
} ;
const ll INF = 2e15;
vector<vector<Edge>> adj;
void dijkstra(int source, vector<11>&
dist, vector<int>& parent) {
    const int n = adj.size();
    dist.assign(n, INF);
    parent.assign(n, -1);
    vector<bool> visited(n + 2,
false);
    dist[source] = 0;
    priority queue < Edge,
vector<Edge>, greater<Edge>> pq;
    pq.push({source, 0});
    while (!pq.empty()) {
```

## Bellman Ford (SSSP):

```
bool bellman ford(int source,
vector<ll>& dist, vector<int>&
parent) {
    const int n = adj.size();
    dist.assign(n, INF);
    parent.assign(n, -1);
    for (int i = 1; i <= n; i++) {
        bool changed = false;
        for (int from = 0; from < n;
from++) {
            for (auto& edge :
adj[from]) {
                int to = edge.to;
                ll weight =
edge.weight;
                if (dist[from] < INF)</pre>
                    if (dist[to] >
dist[from] + weight) {
                        if (i == n)
return false;
                        dist[to] =
dist[from] + weight;
```

## Floyd Warshall (APSP):

If there exists a negative cycle between i and j, distfw[i][j] will be negative after the end of the iterations.

## **Matrix Multiplication (APSP):**

```
vector<vector<int>>
extend shortest path(vector<vector<in
t >> \& d,
   vector<vector<int>>& w) {
    vector<vector<int>> new d(n + 2,
vector < int > (n + 2));
    for (int i = 1; i \le n; i++) {
        for (int j = 1; j \le n; j++)
            new d[i][j] = INF;
            for (int k = 1; k \le n;
k++) {
                new d[i][j] =
\min(\text{new d[i][j]}, d[i][k] + w[k][j]);
    return new d;
void mat mul shortest path faster() {
    distmm = adj mat;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j \le n; j++)
distmm[i][i] = 0;
    }
    int b = n - 1;
    int m = 1;
    while (m < b) {
        distmm =
extend shortest path(distmm, distmm);
        m *= 2;
```

## Max Flow (Edmond Karp):

```
#include <bits/stdc++.h>
using namespace std;
int n;
const int INF = 2e8;
vector<vector<int>> capacity, adj;
int bfs(int s, int t, vector<int>&
parent) {
    fill(parent.begin(),
parent.end(), -1);
    parent[s] = -2;
    queue<pair<int, int>> q;
    q.push({s, INF});
    while (!q.empty()) {
        int cur = q.front().first;
        int flow = q.front().second;
        q.pop();
        for (int next : adj[cur]) {
            if (parent[next] == -1 \&\&
capacity[cur][next]) {
                parent[next] = cur;
                int new flow =
min(flow, capacity[cur][next]);
                if (next == t) return
new flow;
                q.push({next,
new flow});
    return 0;
int maxflow(int s, int t) {
    int flow = 0;
    vector<int> parent(n);
    int new flow;
```

```
11
    while (new flow = bfs(s, t,
parent)) {
        flow += new flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -=
new flow;
            capacity[cur][prev] +=
new flow;
            cur = prev;
    return flow;
MST (Kruskal):
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
struct Edge {
    int from, to;
    ll weight;
    bool operator<(const Edge& other)</pre>
const {
        if (weight != other.weight)
return weight < other.weight;</pre>
        return make pair(from, to) <
make pair(other.from, other.to);
};
int n;
vector<int> parent, size v;
vector<pair<int, int>> result; //
resulting MST
int find set(int v) \{ // O(1) \}
    if (v == parent[v])
        return v;
```

else

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return parent[v] =

```
find set(parent[v]);
bool union sets(int a, int b) { //
0(1)
    // returns false if same set
    a = find set(a);
    b = find set(b);
    if (a != b) {
        if (size v[a] < size v[b])</pre>
swap(a, b);
        parent[b] = a;
        size v[a] += size v[b];
        return true;
    } else
        return false;
ll kruskal(vector<Edge>& edges) {
    parent.resize(n + 2);
    size v.assign(n + 2, 1);
    for (int i = 0; i \le n; i++)
parent[i] = i;
    sort(edges.begin(), edges.end());
    11 \text{ total weight} = 0;
    for (int i = 0; i < edges.size();</pre>
i++) {
        int u = edges[i].from;
        int v = edges[i].to;
        ll w = edges[i].weight;
        if (find set(u) !=
find set(v)) {
            total weight += w;
result.push back(make pair(u, v));
            union sets(u, v);
    return total weight;
```

# **Topological Sorting:**

```
int n, d;
vector<vector<int>> adj;
vector<char> color;
stack<int> st;
bool dfs(int node) {
    color[node] = 'G';
   bool flag = true;
    for (auto& they : adj[node]) {
        if (color[they] == 'G')
return false;
        if (color[they] == 'W') flag
&= dfs(they);
    color[node] = 'B';
    st.push (node);
    return flag;
bool topSort(vector<int>& ans) {
    color.assign(n + 1, 'W');
    // if s1 is dependent on s2,
adj[s2].push back(s1)
    vector<int> ret;
    bool possible = true;
    for (int i = 1; i <= n &&
possible; i++)
        if (color[i] == 'W') possible
\&= dfs(i);
    if (!possible) {
        return false; // not
possible
   } else {
        while (!st.empty()) {
            ans.push back(st.top());
            st.pop();
        return true;
```

## SCC (Kosaraju's Algorithm):

```
vector<vector<int>> adj, adj rev;
vector<bool> visited;
stack<int> st;
void dfs1(int node) {
    visited[node] = true;
    for (auto& other : adj[node]) {
        if (!visited[other])
dfs1(other);
    st.push(node);
void dfs2(int node) {
    visited[node] = true;
    cout << node << " ";
    for (auto& other : adj rev[node])
        if (!visited[other])
dfs2(other);
int main() {
    int n, s;
    cin >> n >> s;
    visited.assign(n + 1, false);
    adj.resize(n + 1);
    adj rev.resize(n + 1);
    for (int i = 0; i < s; i++) {
        int a, b;
        cin >> a >> b;
        adj[a].push back(b);
        adj rev[b].push back(a);
    for (int i = 1; i <= n; i++)
        if (!visited[i]) dfs1(i);
    visited.assign(n + 1, false);
    while (!st.empty()) {
        int now = st.top();
        st.pop();
        if (visited[now]) continue;
        dfs2(now);
```

```
cout << endl;
}
```

## **Acyclicity Checking and Cycle Finding:**

Finds cycle in O(E), the following is for directed graph. For undirected graphs, gray color is not even needed.

```
#include <bits/stdc++.h>
using namespace std;
int n;
vector<vector<int>> adj;
vector<char> color;
vector<int> parent;
int cycle start, cycle end;
bool dfs(int v) {
    color[v] = 1;
    for (int u : adj[v]) {
        if (color[u] == 0) {
            parent[u] = v;
            if (dfs(u)) return true;
        } else if (color[u] == 1) {
            cycle end = v;
            cycle start = u;
            return true;
    color[v] = 2;
    return false;
void find cycle() {
    color.assign(n, 0);
    parent.assign(n, -1);
    cycle start = -1;
    for (int v = 0; v < n; v++) {
        if (color[v] == 0 && dfs(v))
break;
    if (cycle start == -1) {
```

```
cout << "Acyclic" << endl;
} else {
    vector<int> cycle;
    cycle.push_back(cycle_start);
    for (int v = cycle_end; v !=
cycle_start; v = parent[v])
        cycle.push_back(v);
    cycle.push_back(cycle_start);
    reverse(cycle.begin(),
cycle.end());

cout << "Cycle found: ";
    for (int v : cycle) cout << v
<< " ";
    cout << endl;
}

DAG Reachability:</pre>
```

```
DAG Reachability:
#include <bits/stdc++.h>
using namespace std;
const int N = 3e5 + 9;
/*add edge(s, t): insert edge (s,t)
to the network if it does not make a
cvcle
is reachable(s, t): return true iff
there is a path s --> t
Algorithm by G.F. ITALIANO
Complexity: amortized O(n) per
update*/
// 0-indexed
struct Italiano {
   int n;
    vector<vector<int>> par;
    vector<vector<int>>>
child:
    Italiano(int n)
       : n(n), par(n, vector<int>(n,
-1)), child(n,
vector<vector<int>>(n)) {}
    bool is reachable(int s, int t) {
return s == t || par[s][t] >= 0; }
```

```
bool add edge(int s, int t) {
        if (is reachable(t, s))
return false; // break DAG condition
       if (is reachable(s, t))
return true; // no-modification
performed
        for (int p = 0; p < n; ++p) {
            if (is reachable(p, s) &&
!is reachable(p, t) meld(p, t, s,
t);
        return 1;
   void meld(int root, int sub, int
u, int v) {
        par[root][v] = u;
        child[root][u].push back(v);
        for (int c : child[sub][v]) {
            if (!is reachable(root,
c)) meld(root, sub, v, c);
// add edges one by one. if it breaks
DAG law then print it
int main() {
   int n, m;
   cin >> n >> m;
   Italiano t(n);
   while (m--) {
       int u, v;
        cin >> u >> v;
        --u;
        --v;
        if (t.is reachable(v, u))
           cout << u + 1 << ' ' << v
+ 1 << '\n';
       else
            t.add edge(u, v);
   cout << 0 << ' ' << 0 << '\n';
   return 0;
```

# **String Hashing:** typedef long long ll; const int MX = 1e6 + 5; const int p1 = 137, p2 = 281; const int MOD1 = 1e9 + 7; const int MOD2 = 1e9 + 9; // Some back-up primes: 1072857881, 1066517951, 1040160883 // use double hashing for caution // if both hash values match, then same string // else different vector<int> POWER1 (MX), POWER2 (MX); vector<int> INV1(MX), INV2(MX); int gcd(int a, int b, int& x, int& y) x = 1, y = 0;int x1 = 0, y1 = 1, a1 = a, b1 =b; while (b1) { int q = a1 / b1;tie(x, x1) = make tuple(x1, x)- a \* x1);tie(y, y1) = make tuple(y1, y- q \* y1);tie(a1, b1) = make tuple(b1,a1 - q \* b1);return a1; int modInverse(int a, int m) { int x, y; int q = gcd(a, m, x, y);while (x < 0) x += m;return x; void precompute (const int p, const int MOD, vector<int>& POWER, vector<int>& INV) { POWER[0] = 1;INV[0] = 1;int inv of p = modInverse(p, MOD):

for (int i = 1; i < MX; i++) {

```
POWER[i] = (POWER[i - 1] *
1LL * p) % MOD;
        INV[i] = (INV[i - 1] * 1LL *
inv of p) % MOD;
int compute hash for a string(const
string& s, const int p, const int
MOD,
                              const
vector<int>& POWER) {
    // call precompute() first
    int hash value = 0;
    const int n = s.size();
    for (int i = 0; i < n; i++) {
        int val = ((s[i] - 'a' + 1) *
1LL * POWER[i]) % MOD;
        ll now = hash value + val;
        while (now \geq MOD) now -=
MOD;
        hash value = now;
    return hash value;
vector<int> compute prefix hash(const
string& s, const int p, const int
    const int n = s.size();
    int p power = 1;
    vector<int> hash values(n, 0);
    for (int i = 0; i < n; i++) {
        hash values[i] = (p power *
1LL * (s[i] - 'a' + 1)) % MOD;
        if (i > 0) {
            ll now = hash values[i] +
hash values[i - 1];
            while (now >= MOD) now -=
MOD;
            hash values[i] = now;
        p power = (p power * 1LL * p)
% MOD;
```

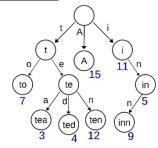
return hash values;

```
int prefix hash(vector<int>& pref,
vector<int>& INV, int 1, int r, const
int p,
                const int MOD) {
    // call precompute() first
    assert(1 \le r);
    if (l == 0) return pref[r];
    int inv = INV[1];
    ll\ now = pref[r] - pref[l - 1];
    while (now < 0) now += MOD;
    while (now >= MOD) now -= MOD;
    return (inv * 1LL * now) % MOD;
pair<int, int>
getBothPrefixHashes(vector<int>&
pref1, vector<int>& pref2,
int l, int r) {
    pair<int, int> ans;
    ans.first = prefix hash(pref1,
INV1, 1, r, p1, MOD1);
    ans.second = prefix hash(pref2,
INV2, 1, r, p2, MOD2);
    return ans;
vector<int> compute suffix hash(const
string& s, const int p, const int
MOD) {
   const int n = s.size();
   int p power = 1;
   vector<int> hash values(n, 0);
   for (int i = n - 1; i \ge 0; i--)
        hash values[i] = (p power *
1LL * (s[i] - 'a' + 1)) % MOD;
        if (i < n - 1) {
            ll now = hash values[i] +
hash values[i + 1];
            while (now \geq MOD) now -=
MOD;
            hash values[i] = now;
```

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```
p power = (p power * 1LL * p)
% MOD;
    return hash values;
int suffix hash(vector<int>& suf,
vector<int>& INV, int 1, int r, const
int p,
                const int MOD) {
   // call precompute() first
    const int n = suf.size();
   assert(1 \le r);
   if (r == n - 1) return suf[1];
   int inv = INV[n - 1 - r];
   ll now = suf[l] - suf[r + 1];
   while (now < 0) now += MOD;
   while (now \geq MOD) now -= MOD;
    return (inv * 1LL * now) % MOD;
pair<int, int>
getBothSuffixHashes(vector<int>&
suf1, vector<int>& suf2, int 1,
int r) {
   pair<int, int> ans;
   ans.first = prefix hash(suf1,
INV1, 1, r, p1, MOD1);
    ans.second = prefix hash(suf2,
INV2, 1, r, p2, MOD2);
    return ans;
```

#### Trie (Prefix Tree):



Sometimes also helpful for maximum xor problems.

```
struct Trie {
    const int MX = 1e5 + 5;
    int root, n node;
    vector<vector<int>> node;
    vector<bool> isWord;
    Trie() {
        root = 0, n node = 0;
        // assuming only small
letters
        node.assign(MX,
vector<int>(26, 0));
        isWord.assign(MX, false);
        //-1 means no edge for ('a' +
i)th character
        fill(node[root].begin(),
node[root].end(), -1);
    void insert(const string& s) {
        const int len = s.size();
        int. now = root:
        for (int i = 0; i < len; i++)
            if (node[now][s[i] - 'a']
== -1) {
                node[now][s[i] - 'a']
= ++n node;
fill(node[n node].begin(),
node[n node].end(), -1);
            now = node[now][s[i] -
'a'];
        isWord[now] = true;
};
```

## **CSES Word Combination with Trie:**

```
#include <bits/stdc++.h>
using namespace std;

template <typename... T>
#define error(args...)
```

```
string s = \#args;
        replace(s.begin(), s.end(),
',', ' '); \
        stringstream ss(s);
        istream iterator<string>
it( ss);
        err( it, args);
void err(istream iterator<string> it)
template <typename T, typename...
void err(istream iterator<string> it,
T a, Args... args) {
   cerr << *it << "=" << a << ", ";
   err(++it, args...);
#define int long long
#define F first
#define S second
const long long inf = 1LL << 62;
const int md = 1000000007;
int trie[1000005][26];
bool stop[1000005];
int dp[5005];
int ct = 0;
void insert(const string& s) {
   int node = 0;
   for (int i = 0; i < s.size();
i++) {
        if (!trie[node][s[i] - 'a'])
trie[node][s[i] - 'a'] = ++ct;
        node = trie[node][s[i] -
'a'];
   stop[node] = 1;
```

```
string s;
int search(int x) {
    int node = 0, ans = 0;
    for (int i = x; i < s.size();
i++) {
        if (!trie[node][s[i] - 'a'])
return ans;
        node = trie[node][s[i] -
'a'];
        if (stop[node]) {
            (ans += dp[i + 1]) %= md;
    return ans;
void solve() {
    cin >> s;
    int k;
    cin >> k;
    while (k--) {
        string x;
        cin >> x:
        insert(x);
    dp[s.size()] = 1;
    for (int i = s.size() - 1; i >=
0: i--) {
        dp[i] = search(i);
    cout << dp[0];
```

## **Aho-Korasick Algorithm:**

```
const int K = 26;
struct Vertex {
   vector<int> next, go;
   // leaf will store which index's
string it is leaf of
   int leaf = -1, p = -1, link = -1;
   char pch;
   Vertex(int p = -1, char ch = '$')
: p(p), pch(ch) {
```

```
next.assign(K, 0);
        qo.assign(K, 0);
        fill (begin (next), end (next),
-1);
        fill (begin (go), end (go), -1);
    }
};
vector<Vertex> trie(1);
void add string(string const& s, int
index) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (trie[v].next[c] == -1) {
            trie[v].next[c] =
trie.size();
            trie.emplace back(v, ch);
        v = trie[v].next[c];
    trie[v].leaf = index;
int go (int v, char ch);
int get link(int v) {
    if (trie[v].link == -1) {
        if (v == 0 || trie[v].p == 0)
            trie[v].link = 0;
        else
            trie[v].link =
go(get link(trie[v].p), trie[v].pch);
    return trie[v].link;
int qo(int v, char ch) {
    int c = ch - 'a';
    if (trie[v].go[c] == -1) {
        if (trie[v].next[c] != -1)
            trie[v].go[c] =
trie[v].next[c];
        else
            trie[v].qo[c] = v == 0 ?
0 : go(get link(v), ch);
    return trie[v].go[c];
```

```
vector<int> patternCount(const
string& text, const vector<string>&
patterns) {
   // assuming one string once
   // if one string multiple times,
do some mapping
   // so that one string is
considered once in the trie
   int cur = 0:
   queue<int> q;
   for (int i = 0; i < text.size();
i++) {
        cur = go(cur, text[i]); //
go to the next node from current
        a.push(cur);
    // vector<bool>
visited(trie.size() + 2, false);
   vector<int> ok(patterns.size() +
   while (!q.empty()) {
       int now = q.front();
        q.pop();
       // visited[now] = true;
       if (trie[now].leaf != -1)
ok[trie[now].leaf]++;
        if (trie[now].link != -1 /*&&
!visited[trie[now].link]*/)
            q.push(trie[now].link);
   // so ok stores the number of
occurences of each pattern in the
text
   return ok;
```

## Manachar's Algorithm:

Works with palindromic substrings.
#include <bits/stdc++.h>
using namespace std;
#define endl "\n"
int main() {
 string s;

```
cin >> s;
    const int n = s.size();
    vector<int> d1(n, 0);
    // radius of maximum odd length
palindrome centered at i
    // here d1[i] = the palindrome
has d1[i] - 1 right characters from i
   // e.g. for aba, d1[1] = 2;
    for (int i = 0, l = 0, r = -1; i
< n; i++) {
        int k = (i > r) ? 1 :
min(d1[l + r - i], r - i);
        while (0 \le i - k \&\& i + k \le
n \&\& s[i - k] == s[i + k]) k++;
        d1[i] = k--;
        if (i + k > r) {
           l = i - k;
            r = i + k;
    vector<int> d2(n, 0);
    // radius of maximum even length
palindrome centered at i
    // here d2[i] = the palindrome
has d2[i] - 1 right characters from i
    // e.g. for abba, d2[2] = 2;
    for (int i = 0, l = 0, r = -1; i
< n; i++) {
        int k = (i > r) ? 0 :
min(d2[1 + r - i + 1], r - i + 1);
        while (0 \le i - k - 1 \&\& i +
k < n \&\& s[i - k - 1] == s[i + k]
k++;
        d2[i] = k--;
        if (i + k > r) {
           1 = i - k - 1;
            r = i + k;
    for (int i = 0; i < n; i++) cout
<< d1[i] << ' ';
    cout << endl;</pre>
    for (int i = 0; i < n; i++) cout
<< d2[i] << ' ';
```

```
cout << endl;
  // number of palindromes
  long long ans = 0;
  for (int i = 0; i < n; i++) ans
+= d1[i] + d2[i];
  cout << ans << endl;
  return 0;
}</pre>
```

#### **KMP Algorithm:**

lps[i] is the length of the longest proper prefix that is also a suffix of S.

```
vector<int> getLPS(const string&
pattern) {
    const int n = pattern.size();
    vector<int> lps(n, 0);
    for (int i = 1; i < n; i++) {
        if (pattern[i] ==
pattern[lps[i - 1]])
            lps[i] = lps[i - 1] + 1;
        else {
            int k = lps[i - 1];
            while (k > 0 \&\&
pattern[i] != pattern[k]) k = lps[k -
            if (pattern[i] ==
pattern[k]) k++;
            lps[i] = k;
    return lps;
int KMPSearch (const string& text,
const string& pattern,
              const vector<int>& lps)
    // returns number of matches
    const int m = pattern.size();
    const int n = text.size();
    // lps array for pattern
    int i = 0, j = 0, match = 0;
    while (i < n) {
        if (pattern[j] == text[i])
i++, j++;
```

#### **Z-Algorithm:**

z[i] is the length of the longest substring from i-th position that is also a prefix of S

```
vector<int> z_function(const string&
s) {
    const int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, 1 = 0, r = 0; i <
n; ++i) {
        if (i <= r) z[i] = min(r - i);
        while (i + z[i] < n &&
s[z[i]] == s[i + z[i]]) ++z[i];
        if (i + z[i] - 1 > r) l = i,
r = i + z[i] - 1;
    }
    return z;
}
```

# Geometry: (entirely from ACM-Library by BUET-Gifted-Hypocrites)

```
#include <bits/stdc++.h>
using namespace std;

const double pi = 4 * atan(1);
const double eps = 1e-6;
```

#### BUET QWERTY - Bangladesh University of Engineering & Technology

```
inline int dcmp (double x) { if
(fabs(x) < eps) return 0; else return
x < 0 ? -1 : 1; 
double fix acute (double th) {return
th<-pi ? (th+2*pi): th>pi ? (th-2*pi)
: th;}
inline double getDistance (double x,
double v) { return sqrt(x * x + v *
v); }
inline double torad(double deg) {
return deg / 180 * pi; }
struct Point {
    double x, y;
    Point (double x = 0, double y =
0): x(x), y(y) \{ \}
   void read () { scanf("%lf%lf",
&x, &v); }
    void write () { printf("%lf %lf",
x, y); }
    bool operator == (const Point& u)
const { return dcmp(x - u.x) == 0 \&\&
dcmp(y - u.y) == 0; 
    bool operator != (const Point& u)
const { return !(*this == u); }
    bool operator < (const Point& u)</pre>
const { return dcmp(x - u.x) < 0 | |
(dcmp(x-u.x) == 0 && dcmp(y-u.y) < 0);
    bool operator > (const Point& u)
const { return u < *this: }</pre>
    bool operator <= (const Point& u)</pre>
const { return *this < u || *this ==</pre>
u; }
    bool operator >= (const Point& u)
const { return *this > u || *this ==
u; }
    Point operator + (const Point& u)
{ return Point(x + u.x, y + u.y); }
    Point operator - (const Point& u)
{ return Point(x - u.x, y - u.y); }
```

```
Point operator * (const double u)
{ return Point(x * u, y * u); }
    Point operator / (const double u)
{ return Point(x / u, y / u); }
    double operator * (const Point&
u) { return x*u.y - y*u.x; }
};
typedef Point Vector;
typedef vector<Point> Polygon;
struct Line {
    double a, b, c;
    Line (double a = 0, double b = 0,
double c = 0: a(a), b(b), c(c) {}
struct Segment{
    Point a:
    Point b;
    Seament(){}
    Segment(Point aa, Point bb)
{a=aa,b=bb;}
} ;
struct DirLine {
    Point p;
    Vector v;
    double ang;
    DirLine () {}
    DirLine (Point p, Vector v):
p(p), v(v) { and = atan2(v.y, v.x); }
    bool operator < (const DirLine&</pre>
u) const { return ang < u.ang; }</pre>
namespace Punctual {
    double getDistance (Point a,
Point b) { double x=a.x-b.x, y=a.y-
b.v; return sqrt(x*x + y*y); }
namespace Vectorial {
    double getDot (Vector a, Vector
b) { return a.x * b.x + a.y * b.y; }
```

```
double getCross (Vector a, Vector
b) { return a.x * b.v - a.v * b.x; }
    double getLength (Vector a) {
return sqrt(getDot(a, a)); }
    double getPLength (Vector a) {
return getDot(a, a); }
    double getAngle (Vector u) {
return atan2(u.y, u.x); }
    double getSignedAngle (Vector a,
Vector b) {return getAngle(b) -
getAngle(a);}
    Vector rotate (Vector a, double
rad) { return Vector(a.x*cos(rad) -
a.y*sin(rad),
a.x*sin(rad)+a.y*cos(rad)); }
    Vector ccw (Vector a, double co,
double si) {return Vector(a.x*co-
a.y*si, a.y*co+a.x*si);}
   Vector cw (Vector a, double co,
double si) {return
Vector(a.x*co+a.y*si, a.y*co-
a.x*si);}
   Vector scale(Vector a, double s =
1.0) {return a / getLength(a) * s;}
    Vector getNormal (Vector a) {
double 1 = getLength(a); return
Vector(-a.y/l, a.x/l); }
namespace ComplexVector {
    typedef complex<double> Point;
    typedef Point Vector;
    double getDot(Vector a, Vector b)
{ return real(conj(a)*b); }
    double getCross(Vector a, Vector
b) { return imag(conj(a)*b); }
   Vector rotate (Vector a, double
rad) { return a*exp(Point(0, rad)); }
};
namespace Linear {
    using namespace Vectorial;
```

```
Line getLine (double x1, double
y1, double x2, double y2) { return
Line (y2-y1, x1-x2, y1*x2-x1*y2);
   Line getLine (double a, double b,
Point u) { return Line(a, -b, u.y * b
- u.x * a); }
   bool getIntersection (Line p,
Line q, Point& o) {
        if (fabs(p.a * q.b - q.a *
p.b) < eps)
            return false;
       o.x = (q.c * p.b - p.c * q.b)
/ (p.a * q.b - q.a * p.b);
        o.y = (q.c * p.a - p.c * q.a)
/ (p.b * q.a - q.b * p.a);
        return true;
   bool getIntersection (Point p,
Vector v, Point q, Vector w, Point&
0) {
        if (dcmp(getCross(v, w)) ==
0) return false;
        Vector u = p - q;
        double k = getCross(w, u) /
getCross(v, w);
        o = p + v * k;
        return true;
    double getDistanceToLine (Point
p, Point a, Point b) { return
fabs(getCross(b-a, p-a) /
getLength(b-a)); }
    double getDistanceToSegment
(Point p, Point a, Point b) {
        if (a == b) return
getLength(p-a);
       Vector v1 = b - a, v2 = p -
a, v3 = p - b;
        if (dcmp(getDot(v1, v2)) < 0)
return getLength(v2);
```

```
else if (dcmp(getDot(v1, v3))
> 0) return getLength(v3);
        else return fabs(getCross(v1,
v2) / getLength(v1));
    double getDistanceSegToSeg (Point
a, Point b, Point c, Point d) {
        double Ans=INT MAX;
Ans=min(Ans, getDistanceToSegment(a, c,
Ans=min(Ans, getDistanceToSegment(b, c,
Ans=min(Ans, getDistanceToSegment(c, a,
Ans=min(Ans, getDistanceToSegment(d, a,
b));
        return Ans:
    Point getPointToLine (Point p,
Point a, Point b) { Vector v = b-a;
return a+v*(getDot(v, p-a) /
getDot(v,v)); }
    bool onSegment (Point p, Point a,
Point b) { return dcmp(getCross(a-p,
b-p)) == 0 && dcmp(getDot(a-p, b-p))
<= 0; }
    bool haveIntersection (Point al,
Point a2, Point b1, Point b2) {
        if (onSegment (a1,b1,b2))
return true;
        if (onSegment (a2,b1,b2))
return true;
        if(onSegment(b1,a1,a2))
return true;
        if (onSegment (b2, a1, a2))
return true; //Case of touch
```

```
double c1=getCross(a2-a1, b1-
a1), c2=getCross(a2-a1, b2-a1),
c3=getCross(b2-b1, a1-b1),
c4=getCross(b2-b1,a2-b1);
        return dcmp(c1)*dcmp(c2) < 0
&& dcmp(c3)*dcmp(c4) < 0;
   bool onLeft(DirLine 1, Point p) {
return dcmp(l.v * (p-l.p)) >= 0;}
namespace Triangular {
   using namespace Vectorial;
   double getAngle (double a, double
b, double c) { return acos((a*a+b*b-
c*c) / (2*a*b)); }
   double getArea (double a, double
b, double c) { double s = (a+b+c)/2;
return sqrt(s*(s-a)*(s-b)*(s-c)); }
   double getArea (double a, double
h) { return a * h / 2; }
   double getArea (Point a, Point b,
Point c) { return fabs(getCross(b -
a, c - a)) / 2; }
   double getDirArea (Point a, Point
b, Point c) { return getCross(b - a,
c - a) / 2;}
   //ma/mb/mc = length of median
from side a/b/c
   double getArea (double ma, double
mb, double mc) {double s=(ma+mb+mc)/2;
return 4/3.0 * sqrt(s*(s-ma)*(s-
mb) * (s-mc));}
   //ha/hb/hc = length of
perpendicular from side a/b/c
   double get Area (double ha, double
hb, double hc) {
       double H=(1/ha+1/hb+1/hc)/2;
double A = 4 * sqrt(H * (H-
1/ha) * (H-1/hb) * (H-1/hc)); return
1.0/ A ;
```

```
bool pointInTriangle(Point a,
Point b, Point c, Point p) {
        double s1 = getArea(a,b,c);
        double s2 = getArea(p,b,c) +
getArea(p,a,b) + getArea(p,c,a);
        return dcmp(s1 - s2) == 0;
    }
} ;
namespace Polygonal {
    using namespace Vectorial;
    using namespace Linear;
    using namespace Triangular;
    double getSignedArea (Point* p,
int n) {
        double ret = 0;
        for (int i = 0; i < n-1; i++)
            ret += (p[i]-p[0]) *
(p[i+1]-p[0]);
        return ret/2;
    int getConvexHull (Point* p, int
n, Point* ch) {
        sort(p, p + n);
        // preparing lower hull
        int m = 0;
        for (int i = 0; i < n; i++) {
            while (m > 1 \&\&
dcmp(getCross(ch[m-1]-ch[m-2], p[i]-
ch[m-1])) <= 0) m--;
            ch[m++] = p[i];
        // preparing upper hull
        int k = m;
        for (int i = n-2; i >= 0; i--
) {
```

```
while (m > k \&\&
dcmp(getCross(ch[m-1]-ch[m-2], p[i]-
ch[m-2])) <= 0) m--;
            ch[m++] = p[i];
        if (n > 1) m--;
        return m;
    int isPointInPolygon (Point o,
Point* p, int n) {
        int wn = 0;
        for (int i = 0; i < n; i++) {
            int j = (i + 1) % n;
            if (onSegment(o, p[i],
p[j]) \mid\mid o == p[i]) return 0;
            int k =
dcmp(getCross(p[j] - p[i], o-p[i]));
            int d1 = dcmp(p[i].y -
o.y);
            int d2 = dcmp(p[j].y -
\circ.y);
            if (k > 0 && d1 <= 0 &&
d2 > 0) wn++;
            if (k < 0 && d2 <= 0 &&
d1 > 0) wn--;
        return wn ? -1 : 1;
    void rotatingCalipers(Point *p,
int n, vector<Segment>& sol) {
        sol.clear();
        int j = 1; p[n] = p[0];
        for (int i = 0; i < n; i++) {
            while (getCross(p[j+1]-
p[i+1], p[i]-p[i+1]) > getCross(p[j]-
p[i+1], p[i]-p[i+1])
                j = (j+1) % n;
sol.push back(Segment(p[i],p[j]));
            sol.push back(Segment(p[i
+ 1],p[j + 1]));
        }
```

```
void rotatingCalipersGetRectangle
(Point *p, int n, double& area,
double& perimeter) {
        p[n] = p[0];
        int l = 1, r = 1, j = 1;
        area = perimeter = 1e20;
        for (int i = 0; i < n; i++) {
            Vector v = (p[i+1]-p[i])
/ getLength(p[i+1]-p[i]);
            while (dcmp(getDot(v,
p[r%n]-p[i]) - getDot(v, p[(r+1)%n]-
p[i])) < 0) r++;
            while (j < r | |
dcmp(getCross(v, p[j%n]-p[i]) -
getCross(v,p[(j+1)%n]-p[i])) < 0)
j++;
            while (1 < j | |
dcmp(getDot(v, p[1%n]-p[i]) -
getDot(v, p[(l+1)%n]-p[i])) > 0) l++;
            double w = getDot(v,
p[r%n]-p[i])-getDot(v, p[l%n]-p[i]);
            double h =
getDistanceToLine (p[j%n], p[i],
p[i+1]);
            area = min(area, w * h);
            perimeter =
min(perimeter, 2 * w + 2 * h);
    Polygon cutPolygon (Polygon u,
Point a, Point b) {
        Polygon ret;
        int n = u.size();
        for (int i = 0; i < n; i++) {
            Point c = u[i], d =
u[(i+1)%n];
            if (dcmp((b-a)*(c-a)) >=
0) ret.push back(c);
            if (dcmp((b-a)*(d-c)) !=
0) {
```

```
Point t;
                getIntersection(a, b-
a, c, d-c, t);
                if (onSegment(t, c,
d))
                    ret.push back(t);
        return ret;
    int
halfPlaneIntersection(DirLine* li,
int n, Point* poly) {
        sort(li, li + n);
        int first, last;
        Point* p = new Point[n];
        DirLine* q = new DirLine[n];
        q[first=last=0] = li[0];
        for (int i = 1; i < n; i++) {
            while (first < last &&
!onLeft(li[i], p[last-1])) last--;
            while (first < last &&
!onLeft(li[i], p[first])) first++;
            q[++last] = li[i];
            if (dcmp(q[last].v *
q[last-1].v) == 0) {
                last--;
                if (onLeft(q[last],
li[i].p)) q[last] = li[i];
            if (first < last)</pre>
getIntersection(g[last-1].p, g[last-
1].v, q[last].p, q[last].v, p[last-
1]);
        while (first < last &&
!onLeft(q[first], p[last-1])) last--;
```

```
if (last - first <= 1) {
delete [] p; delete [] q; return 0; }
        getIntersection(q[last].p,
g[last].v, g[first].p, g[first].v,
p[last]);
        int m = 0;
        for (int i = first; i <=</pre>
last; i++) poly[m++] = p[i];
        delete [] p; delete [] q;
        return m;
    Polygon simplify (const Polygon&
poly) {
        Polygon ret;
        int n = poly.size();
        for (int i = 0; i < n; i++) {
            Point a = poly[i];
            Point b = poly[(i+1)%n];
            Point c = poly[(i+2)%n];
            if (dcmp((b-a)*(c-b)) !=
0 && (ret.size() == 0 || b !=
ret[ret.size()-1]))
                ret.push back(b);
        return ret;
    Point ComputeCentroid( Point*
p, int n) {
        Point c(0,0);
        double scale = 6.0 *
getSignedArea(p,n);
        for (int i = 0; i < n; i++) {
            int j = (i+1) % n;
            c = c +
(p[i]+p[j])*(p[i].x*p[j].y -
p[j].x*p[i].y);
        return c / scale;
```

```
// Tested :
https://www.spoj.com/problems/INOROUT
    // pt must be in ccw order with
no three collinear points
    // returns inside = 1, on = 0,
outside = -1
    int pointInConvexPolygon(Point*
pt, int n, Point p) {
        assert (n >= 3);
        int lo = 1 , hi = n - 1 ;
        while (hi - lo > 1) {
            int mid = (lo + hi) / 2;
            if(getCross(pt[mid] -
pt[0], p - pt[0]) > 0) lo = mid;
            else hi = mid;
       bool in =
pointInTriangle(pt[0], pt[lo],
pt[hi], p);
        if(!in) return -1;
        if (getCross(pt[lo] - pt[lo-
1], p - pt[lo-1]) == 0) return 0;
        if(getCross(pt[hi] - pt[lo],
p - pt[lo]) == 0) return 0;
        if(getCross(pt[hi] -
pt[(hi+1)%n], p - pt[(hi+1)%n]) == 0)
return 0:
        return 1;
   // Tested :
https://toph.co/p/cover-the-points
   // Calculate [ACW, CW] tangent
pair from an external point
    #define CW
                      -1
    #define ACW
    int direction (Point st, Point ed,
Point a)
               {return
dcmp(getCross(ed - st, q - ed));}
```

```
bool isGood(Point u, Point v,
Point O, int dir) {return
direction(Q, u, v) != -dir;}
    Point better (Point u, Point v,
Point Q, int dir) {return
direction(Q, u, v) == dir ? u : v;}
    Point tangents (Point* pt, Point
Q, int dir, int lo, int hi) {
        while (hi - lo > 1) {
            int mid = (lo + hi)/2;
            bool pvs =
isGood(pt[mid], pt[mid - 1], Q, dir);
            bool nxt =
isGood(pt[mid], pt[mid + 1], Q, dir);
            if (pvs && nxt) return
pt[mid];
            if(!(pvs || nxt)){
                Point p1 =
tangents (pt, Q, dir, mid+1, hi);
                Point p2 =
tangents (pt, Q, dir, lo, mid - 1);
                return better (p1, p2,
Q, dir);
            if(!pvs){
                if (direction (O,
pt[mid], pt[lo]) == dir) hi = mid -
1;
                else
if(better(pt[lo], pt[hi], Q, dir) ==
pt[lo]) hi = mid - 1;
                else lo = mid + 1;
            if(!nxt){
                if (direction (Q,
pt[mid], pt[lo]) == dir) lo = mid +
1;
                else
if (better(pt[lo], pt[hi], Q, dir) ==
pt[lo]) hi = mid - 1;
                else lo = mid + 1;
```

```
Point ret = pt[lo];
        for(int i = lo + 1; i <= hi;
i++) ret = better(ret, pt[i], Q,
dir);
        return ret;
    // [ACW, CW] Tangent
    pair<Point, Point>
get tangents (Point* pt, int n, Point
0){
        Point acw tan = tangents(pt,
Q, ACW, 0, n - 1);
        Point cw tan = tangents(pt,
Q, CW, 0, n - 1);
        return make pair (acw tan,
cw tan);
} ;
struct Circle {
    Point o;
    double r;
    Circle () {}
    Circle (Point o, double r = 0):
o(o), r(r) {}
   void read () { o.read(),
scanf("%lf", &r); }
    Point point(double rad) { return
Point(o.x + cos(rad)*r, o.y +
sin(rad)*r); }
    double getArea (double rad) {
return rad * r * r / 2; }
    //area of the circular sector cut
by a chord with central angle alpha
    double sector (double alpha)
{return r * r * 0.5 * (alpha -
sin(alpha));}
} ;
namespace Circular {
```

```
using namespace Linear;
   using namespace Vectorial;
   using namespace Triangular;
   int getLineCircleIntersection
(Point p, Point q, Circle O, double&
t1, double& t2, vector<Point>& sol) {
       Vector v = q - p;
       //sol.clear();
       double a = v.x, b = p.x -
0.0.x, c = v.y, d = p.y - 0.0.y;
        double e = a*a+c*c, f =
2*(a*b+c*d), q = b*b+d*d-0.r*0.r;
        double delta = f*f - 4*e*q;
        if (dcmp(delta) < 0) return
0;
        if (dcmp(delta) == 0) {
           t1 = t2 = -f / (2 * e);
            sol.push back(p + v *
t1);
           return 1;
        t1 = (-f - sqrt(delta)) / (2
* e); sol.push back(p + v * t1);
        t2 = (-f + sqrt(delta)) / (2
* e); sol.push back(p + v * t2);
       return 2;
   // signed area of intersection of
circle(c.o, c.r) and
   // triangle(c.o, s.a, s.b)
[cross(a-o, b-o)/2]
   double
areaCircleTriIntersection(Circle c,
Segment s) {
       using namespace Linear;
        double OA = getLength(c.o -
s.a);
        double OB = getLength(c.o -
s.b);
       // sector
```

```
(dcmp(getDistanceToSegment(c.o, s.a,
s.b) - c.r) >= 0)
            return
fix acute(getSignedAngle(s.a - c.o,
s.b - c.o)) * (c.r*c.r) / 2.0;
        // triangle
        if (dcmp(OA - c.r) \le 0 \&\&
dcmp(OB - c.r) \le 0)
            return getCross(c.o-
s.b, s.a-s.b) / 2.0;
        // three part: (A, a) (a, b)
(b, B)
        vector<Point>Sect; double
t1,t2;
getLineCircleIntersection(s.a, s.b,
c, t1, t2, Sect);
        return
areaCircleTriIntersection(c,
Segment(s.a, Sect[0]))
areaCircleTriIntersection(c,
Segment(Sect[0], Sect[1]))
areaCircleTriIntersection(c.
Segment(Sect[1], s.b));
    // area of intersecion of
circle(c.o, c.r) and simple
polyson(p[])
    // Tested : ZOJ 2675 - Little
Mammoth
    double areaCirclePolygon(Circle
c, Polygon p) {
        double res = .0;
        int n = p.size();
        for (int i = 0; i < n; ++ i)
            res +=
areaCircleTriIntersection(c,
Segment(p[i], p[(i+1)%n]);
```

```
return fabs(res);
   // interior
                         (d < R - r)
---> -2
   // interior tangents (d = R - r)
---> -1
   // concentric
                         (d = 0)
    // secants
                         (R - r < d <
R + r) ----> 0
   // exterior tangents (d = R + r)
---> 1
   // exterior
                         (d > R + r)
----> 2
    int getPos(Circle o1, Circle o2)
        using namespace Vectorial;
        double d = getLength(o1.o -
02.0);
        int in = dcmp(d - fabs(o1.r -
o2.r)), ex = dcmp(d - (o1.r + o2.r));
        return in<0 ? -2 : in==0? -1
: ex==0 ? 1 : ex>0? 2 : 0;
    int getCircleCircleIntersection
(Circle o1, Circle o2, vector<Point>&
sol) {
        double d = getLength(o1.o -
02.0);
        if (dcmp(d) == 0) {
           if (dcmp(o1.r - o2.r) ==
0) return -1:
           return 0;
       if (dcmp(o1.r + o2.r - d) <
0) return 0;
        if (dcmp(fabs(o1.r-o2.r) - d)
> 0) return 0;
        Vector v = o2.o - o1.o;
        double co = (o1.r*o1.r +
getPLength(v) - o2.r*o2.r) / (2 *
```

o1.r \* getLength(v));

```
double si = sgrt(fabs(1.0 -
co*co));
        Point p1 = scale(cw(v, co,
si), o1.r) + o1.o;
        Point p2 = scale(ccw(v, co,
si), o1.r) + o1.o;
        sol.push back(p1);
        if (p1 == p2) return 1;
        sol.push back(p2);
        return 2;
   double areaCircleCircle(Circle
o1, Circle o2) {
       Vector AB = 02.0 - 01.0;
       double d = getLength(AB);
       if(d \ge 01.r + 02.r) return
0;
       if(d + o1.r \le o2.r) return
pi * o1.r * o1.r;
       if(d + o2.r \le o1.r) return
pi * o2.r * o2.r;
        double alpha1 = acos((o1.r *
o1.r + d * d - o2.r * o2.r) / (2.0 *
01.r * d));
        double alpha2 = acos((o2.r *
o2.r + d * d - o1.r * o1.r) / (2.0 *
o2.r * d));
       return o1.sector(2*alpha1) +
o2.sector(2*alpha2);
   int getTangents (Point p, Circle
o, Vector* v) {
       Vector u = 0.0 - p;
        double d = getLength(u);
        if (d < o.r) return 0;
        else if (dcmp(d - o.r) == 0)
            v[0] = rotate(u, pi / 2);
            return 1:
       } else {
```

```
double ang = asin(o.r /
d);
            v[0] = rotate(u, -ang);
            v[1] = rotate(u, ang);
            return 2;
    int getTangentPoints (Point p,
Circle o, vector<Point>& v) {
        Vector u = p - o.o;
        double d = getLength(u);
        if (d < o.r) return 0;
        else if (dcmp(d - o.r) == 0)
            v.push back(o.o+u);
            return 1;
        } else {
            double ang = acos(o.r /
d);
            u = u / getLength(u) *
o.r;
            v.push back(o.o+rotate(u,
-ang));
            v.push back(o.o+rotate(u,
ang));
            return 2;
   int getTangents (Circle o1,
Circle o2, Point* a, Point* b) {
        int cnt = 0;
        if (dcmp(o1.r-o2.r) < 0) {
swap(o1, o2); swap(a, b); }
        double d2 = getPLength(o1.o -
02.0);
        double rdif = o1.r - o2.r,
rsum = o1.r + o2.r;
        if (dcmp(d2 - rdif * rdif) <
0) return 0;
        if (dcmp(d2) == 0 &&
dcmp(o1.r - o2.r) == 0) return -1;
```

```
double base = getAngle(o2.o -
01.0);
        if (dcmp(d2 - rdif * rdif) ==
0) {
            a[cnt] = o1.point(base);
b[cnt] = o2.point(base); cnt++;
            return cnt;
        double ang = acos((o1.r -
o2.r) / sgrt(d2) );
        a[cnt] = o1.point(base+ang);
b[cnt] = o2.point(base+ang); cnt++;
        a[cnt] = o1.point(base-ang);
b[cnt] = o2.point(base-ang); cnt++;
        if (dcmp(d2 - rsum * rsum) ==
0) {
            a[cnt] = o1.point(base);
b[cnt] = o2.point(pi+base); cnt++;
        else if (dcmp(d2 - rsum *
rsum) > 0) {
            double ang = acos((o1.r)
+ o2.r) / sqrt(d2) );
            a[cnt] =
ol.point(base+ang); b[cnt] =
o2.point(pi+base+ang); cnt++;
            a[cnt] = o1.point(base-
ang); b[cnt] = o2.point(pi+base-ang);
cnt++;
        return cnt;
    Circle CircumscribedCircle(Point
p1, Point p2, Point p3) {
        double Bx = p2.x - p1.x, By =
p2.y - p1.y;
        double Cx = p3.x - p1.x, Cy =
p3.y - p1.y;
        double D = 2 * (Bx * Cy - By
* Cx);
```

```
double cx = (Cv * (Bx * Bx +
By * By) - By * (Cx * Cx + Cy * Cy))
/ D + p1.x;
        double cy = (Bx * (Cx * Cx +
Cy * Cy) - Cx * (Bx * Bx + By * By))
/ D + p1.v;
        Point p = Point(cx, cy);
        return Circle(p, getLength(p1
- p));
   Circle InscribedCircle(Point p1,
Point p2, Point p3) {
        double a = getLength(p2 -
p3);
        double b = getLength(p3 -
p1);
        double c = getLength(p1 -
p2);
        Point p = (p1 * a + p2 * b +
p3 * c) / (a + b + c);
        return Circle(p,
getDistanceToLine(p, p1, p2));
   //distance From P : distance from
Q = rp : rq
   Circle getApolloniusCircle(const
Point& P, const Point& Q, double rp,
double rq ) {
        rq *= rq ;
        rp *= rp ;
        double a = rq - rp;
        assert (dcmp(a));
        double q = rq * P.x - rp *
Q.x ; q /= a ;
        double h = rq * P.y - rp *
Q.y ; h /= a ;
        double c = rq*P.x*P.x-
rp*Q.x*Q.x+rq*P.y*P.y-rp*Q.y*Q.y;
        c /= a ;
        Point o(q,h);
        double R = g*g + h*h - c;
        R = sqrt(R);
```

```
return Circle(o,R);
   }
} ;
struct Star{
    int n;
             // number of side of
the star
    double r; // radius of the
circum-circle
    Star(int n,double r) {this->n=n;
this->r=r; }
    double getArea(){
        double theta=pi/n;
        double s=2*r*sin(theta);
        double R=0.5*s/tan(theta);
        double a=0.5*n*s*R;
        double
a2=0.25*s*s/tan(1.5*theta);
        return a-n*a2;
    }
};
```

## **Starting Template:**

Create "debug.h" in the same directory:

```
#include <bits/stdc++.h>
using namespace std;
void print(int x) { cerr << x; }</pre>
void print(long x) { cerr << x; }</pre>
void print(long long x) { cerr <<</pre>
x; }
void print(unsigned x) { cerr << x;</pre>
void print(unsigned long x) { cerr
<< x; }
void print(unsigned long long x) {
cerr << x; }
void print(float x) { cerr << x; }</pre>
void print(double x) { cerr << x; }</pre>
void print(long double x) { cerr <<</pre>
x; }
void print(char x) { cerr << '\''</pre>
<< x << '\''; }
```

```
void print(const char *x) { cerr <<</pre>
'\"' << x << '\"'; }
void print(const string &x) { cerr
<< '\"' << x << '\"'; }
void print(bool x) { cerr << (x ?</pre>
"true": "false"); }
template <typename T, typename V>
void print(const pair<T, V> &x) {
    cerr << '{';
    print(x.first);
    cerr << ',';
    print(x.second);
    cerr << '}';
template <typename T>
void print(const T &x) {
    int f = 0;
    cerr << '{';
    for (auto &i : x) cerr << (f++ ?
"," : ""), print(i);
    cerr << "}";
void print() { cerr << "]\n"; }//one</pre>
template <typename T, typename... V>
void print(T t, V... v) { //one
    print(t);
    <u>if</u> (sizeof...(v)) cerr << ", ";
    print(v...); //one
\#define debug(x...)
    cerr << "[" << #x << "] = ["; \
    print(x) //one
// add a newline at the end
Now the main template:
#include <bits/stdc++.h>
using namespace std;
// q++ -DLOCAL -03 -std=c++14 .\a.cpp
#ifdef LOCAL
#include "debug.h"
#define debug(...)
```

```
#endif
#define endl "\n"
typedef long long 11;
void solve() {}
int main() {
   ios base::sync with stdio(false);
   cin.tie(NULL);
#ifdef LOCAL
   freopen("in.txt", "r", stdin);
   freopen("out.txt", "w", stdout);
#endif
   int t; cin >> t;
   for (int tc = 1; tc <= t; tc++) {
       // cout << "Case " << tc <<
": ";
       solve();
```

#### **Bit Hacks:**

- x = (1 << b) sets bit b.
- $x \&= \sim (1 << b)$  resets bit b.
- $x \stackrel{=}{} (1 \stackrel{<<}{} b)$  toggles bit b.
- x & -x is the LSB in x.
- if (x && (!((x-1) & x)) checks whether x is a power of 2.
- x = x & (x 1) turns off the lowest set bit.
- cout << bitset<8>(x) prints a number in binary format.
- \_builtin\_popcount(x) returns number of ones in x's binary.
   For long long, use builtin popcountll(x).
- \_\_builtin\_clz (leading 0),
   builtin ctz(trailing 0)