

Segment Tree

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The Problem

Range Sum Query

You are given an integer array a . Answer q queries of the following form:

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$$\text{sum}(3, 6) = \text{sum}(0, 6) - \text{sum}(0, 2) = 3 - 6 = -3$$

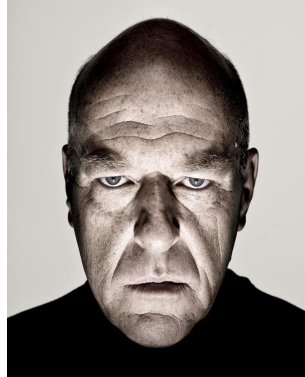
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Let's see a hard version



A More Challenging Problem

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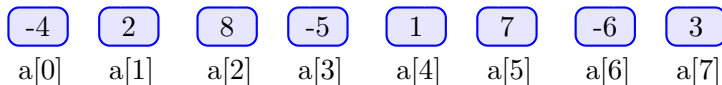
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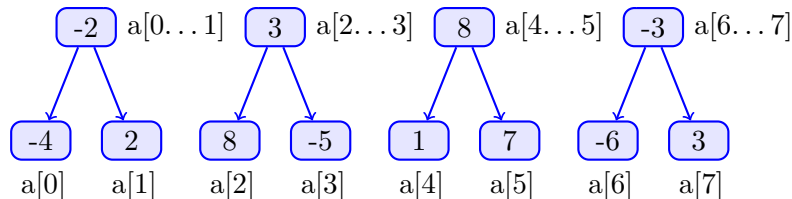
Will Prefix Sum Work Well Now?

No, update makes it $O(n)$. Look for something better!

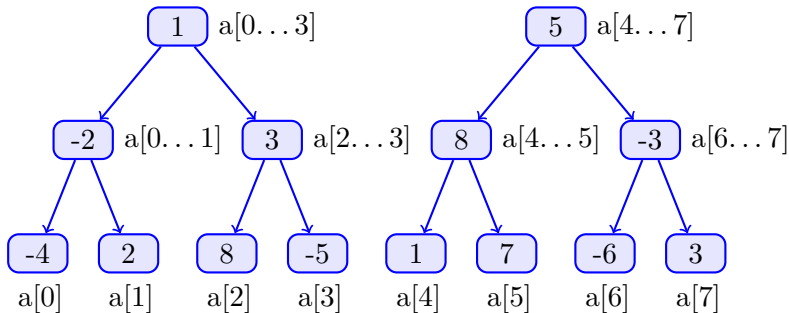
Construction of Segment Tree



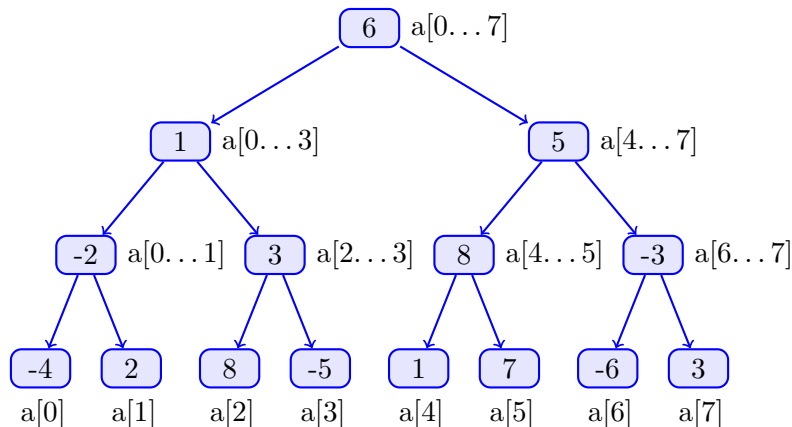
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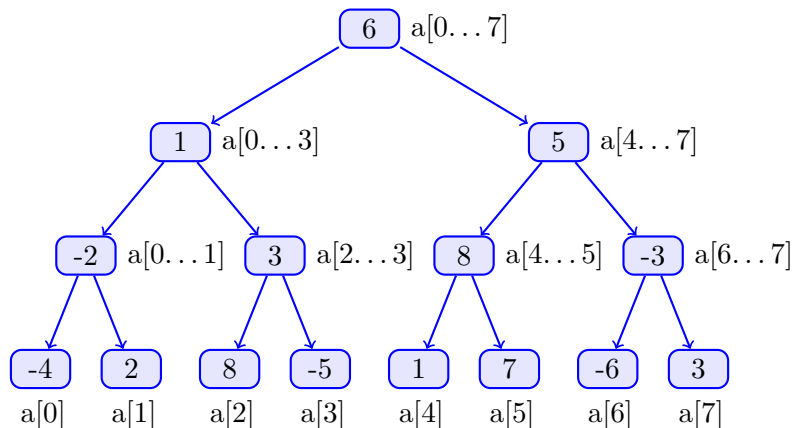
Construction of Segment Tree



Build Code

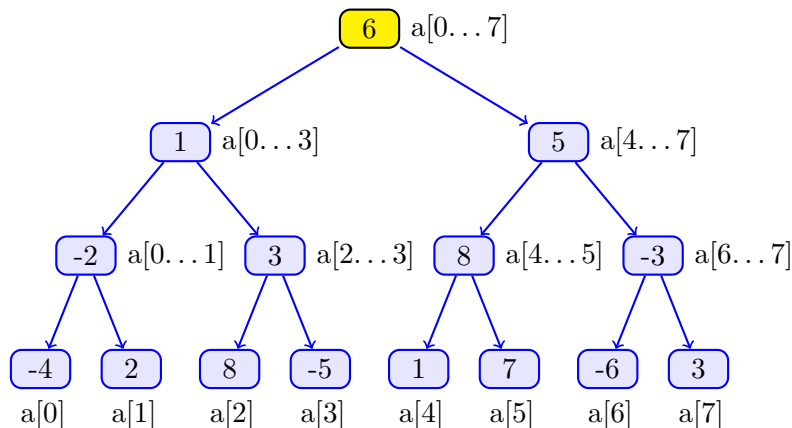
```
void build(int arr[], int idx, int l, int r) {  
    if (l == r) tree[idx] = arr[l];  
    else {  
        int mid = (l + r) / 2;  
        build(arr, idx * 2, l, mid);  
        build(arr, idx * 2 + 1, mid + 1, r);  
        tree[idx] = tree[idx * 2] + tree[idx * 2 + 1];  
    }  
}
```

Update Segment Tree



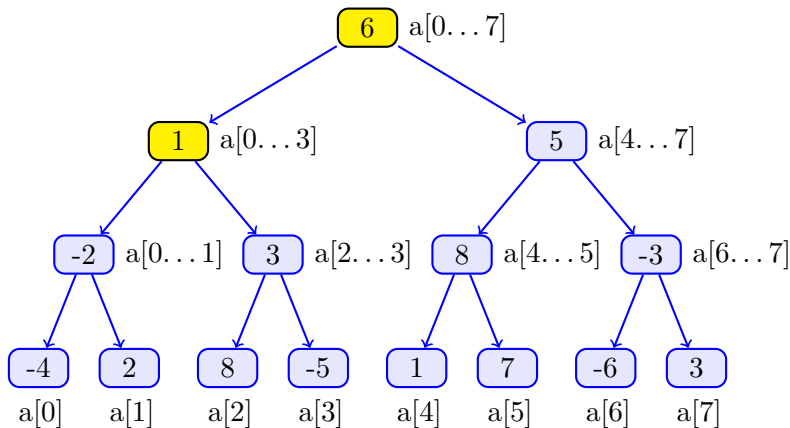
$update(2, -7) : a[2] = -7$

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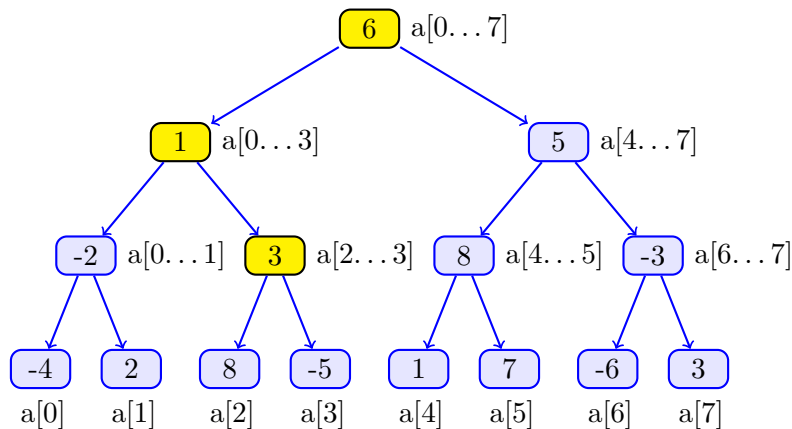
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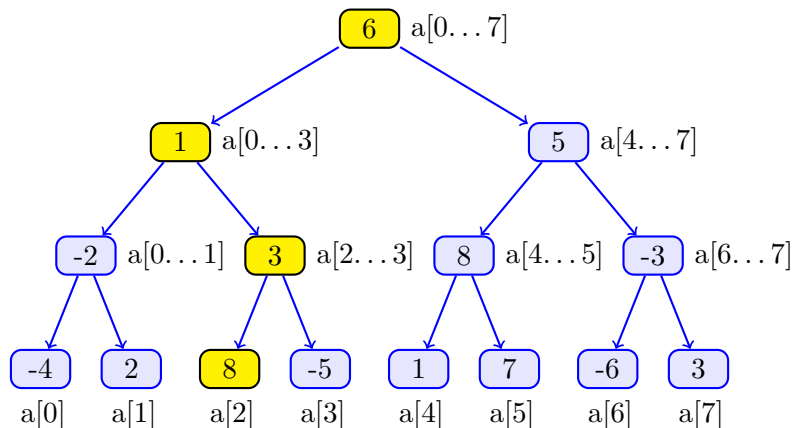
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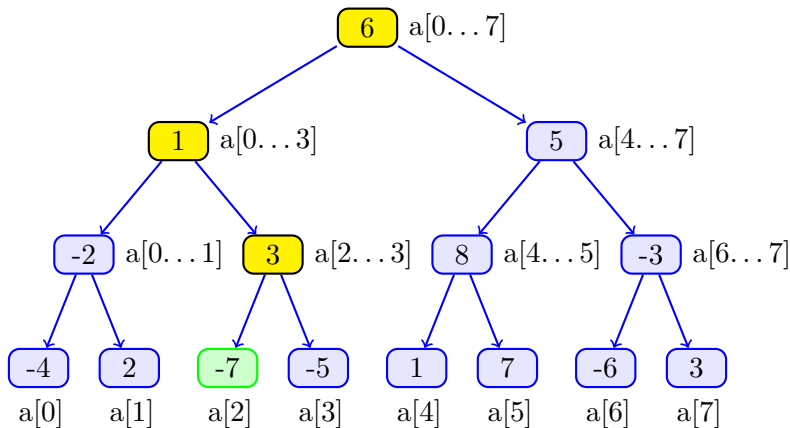
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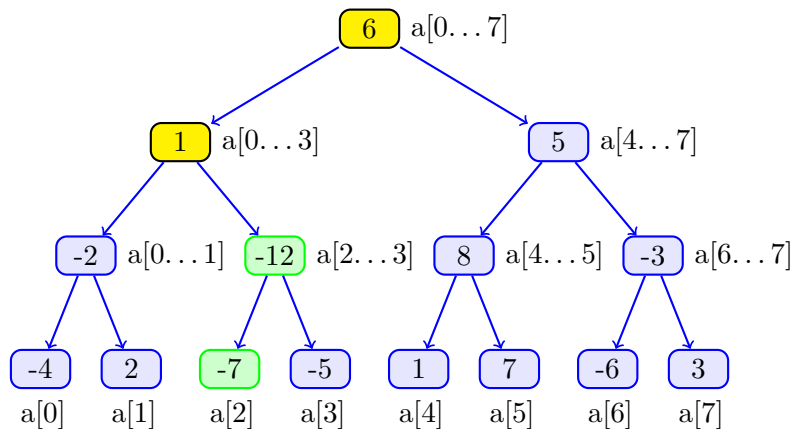
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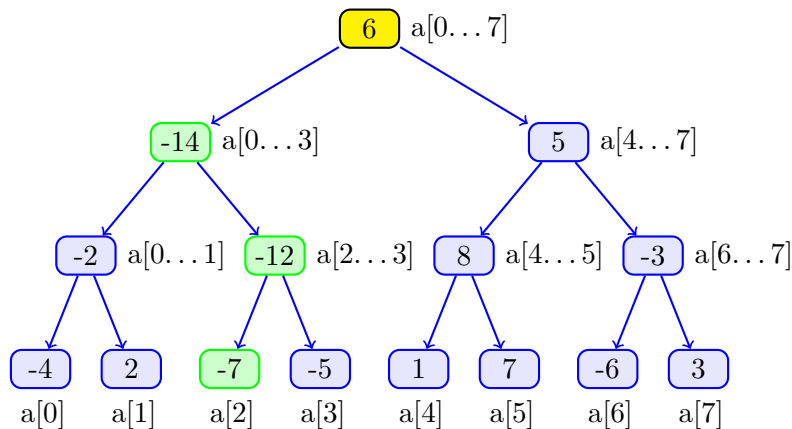
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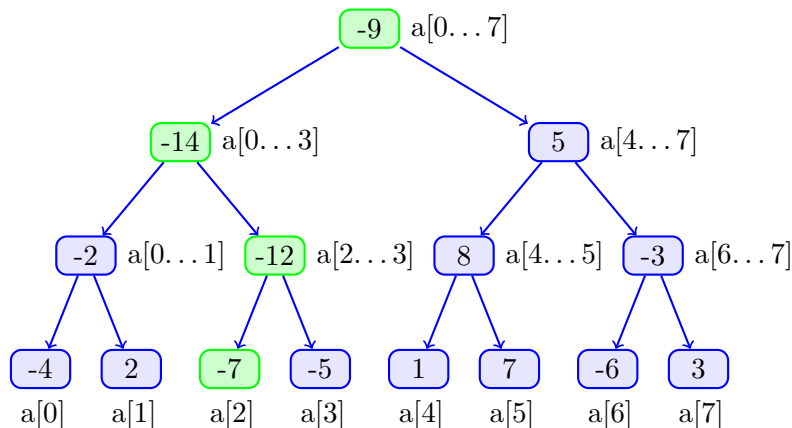
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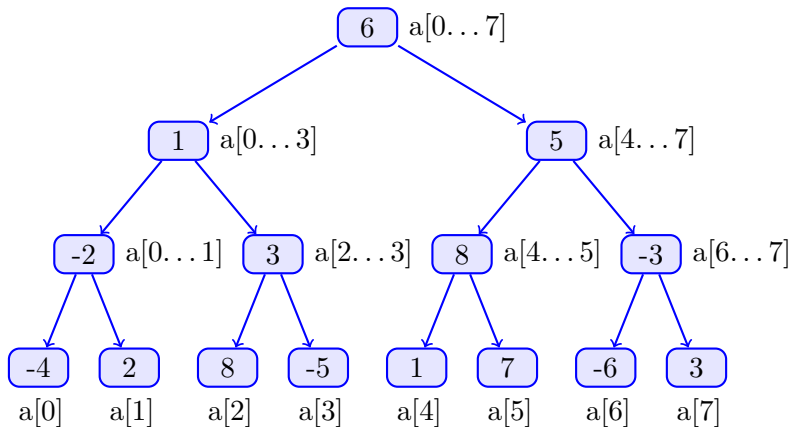


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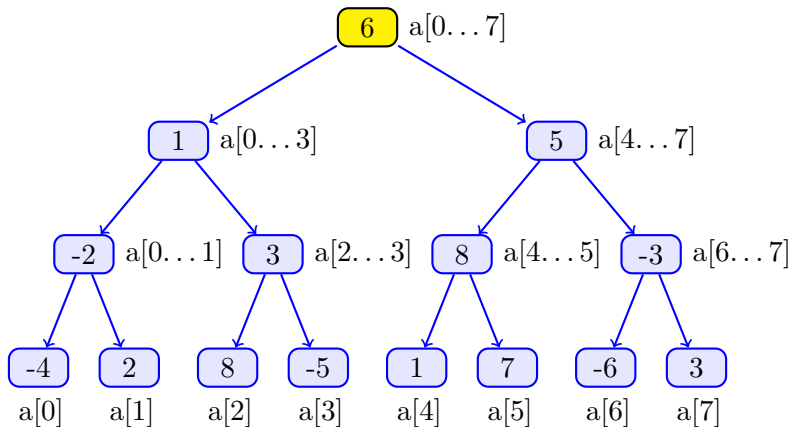
```
void update(int idx, int l, int r, int pos, int new_val) {  
    if (l == r)  
        tree[idx] = new_val;  
    else {  
        int mid = (l + r) / 2;  
        if (pos <= mid)  
            update(idx * 2, l, mid, pos, new_val);  
        else  
            update(idx * 2 + 1, mid + 1, r, pos, new_val);  
        tree[idx] = tree[idx * 2] + tree[idx * 2 + 1];  
    }  
}
```

Answering Query From Segment Tree



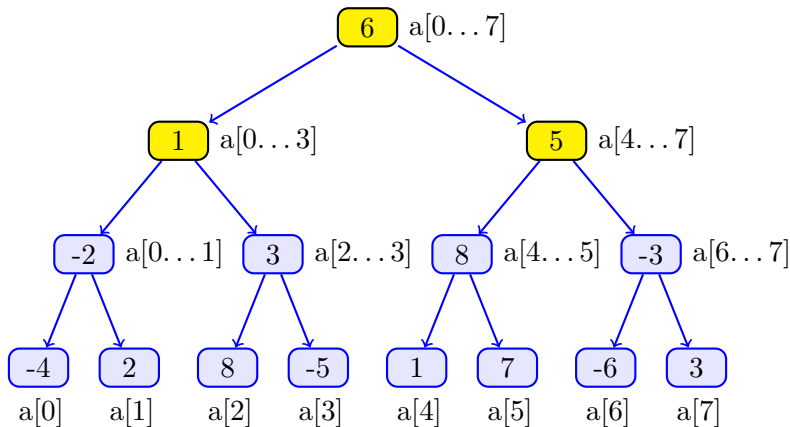
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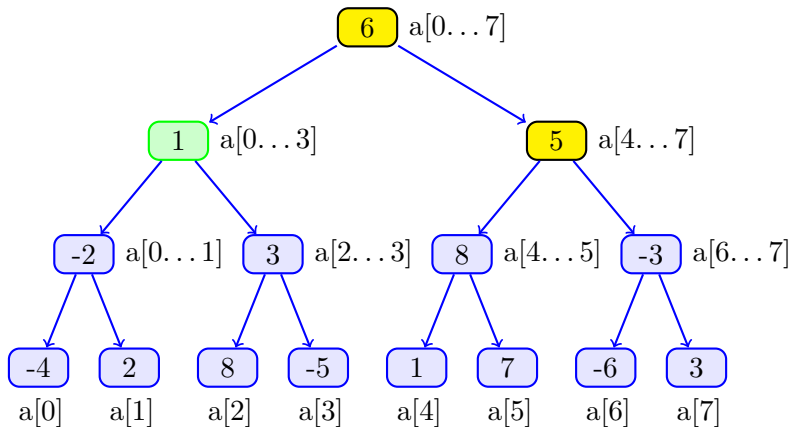
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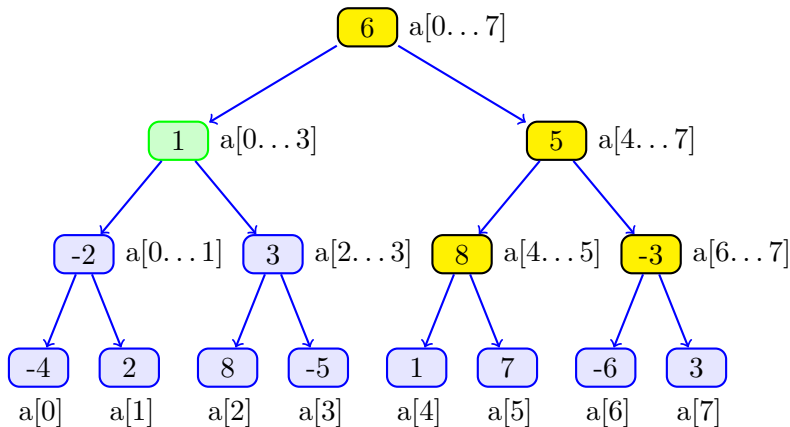
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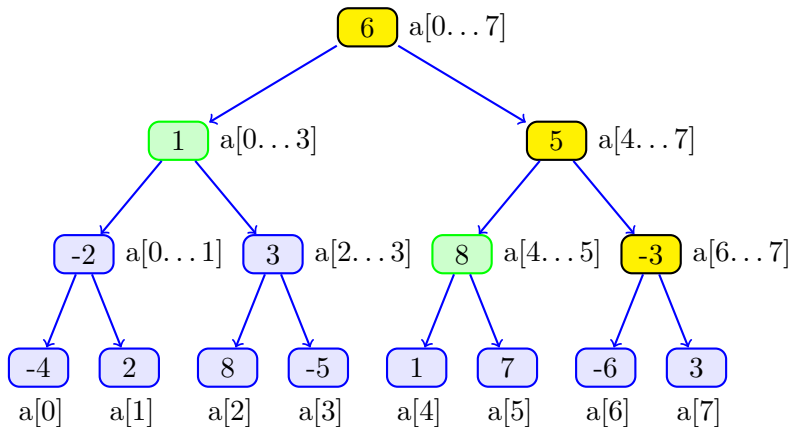
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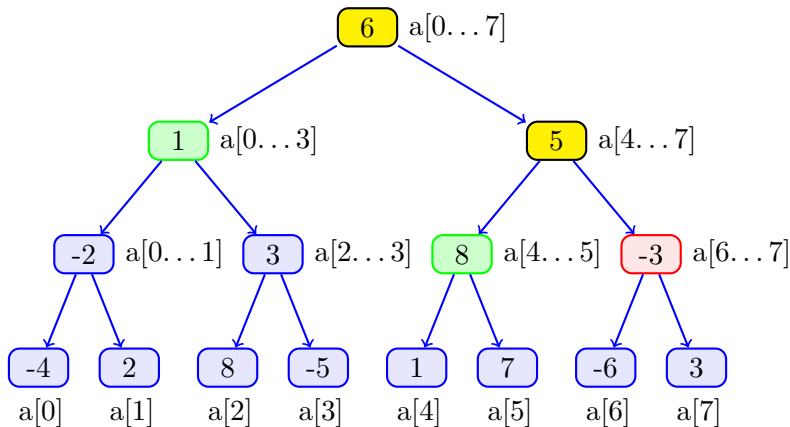
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Answering Query From Segment Tree



$$\text{sum}(0, 5) = 1 + 8 + \dots$$

Answering Query From Segment Tree



$$\text{sum}(0, 5) = 1 + 8 = 9$$

Query Code

```
int query(int idx, int l, int r, int ql, int qr) {  
    if (ql > qr) return 0;  
    if (ql == l && qr == r) return tree[idx];  
    int mid = (l + r) / 2;  
    return sum(idx * 2, l, mid, ql, min(qr, mid))  
        + sum(idx * 2 + 1, mid + 1, r, max(ql, mid + 1), qr);  
}
```

Complexity Comparison

Data Structure	Preprocessing	Query	Update
Segment Tree	$O(n)$	$O(\log n)$	$O(\log n)$
Prefix Sum Array	$O(n)$	$O(1)$	$O(n)$
Sqrt Decomposition	$O(n)$	$O(\sqrt{n})$	$O(\sqrt{n})$

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 - Least Common Multiple
- Range Update : Lazy Propagation

Thanks For Your Time
and Attention
Keep Using Segment Trees...