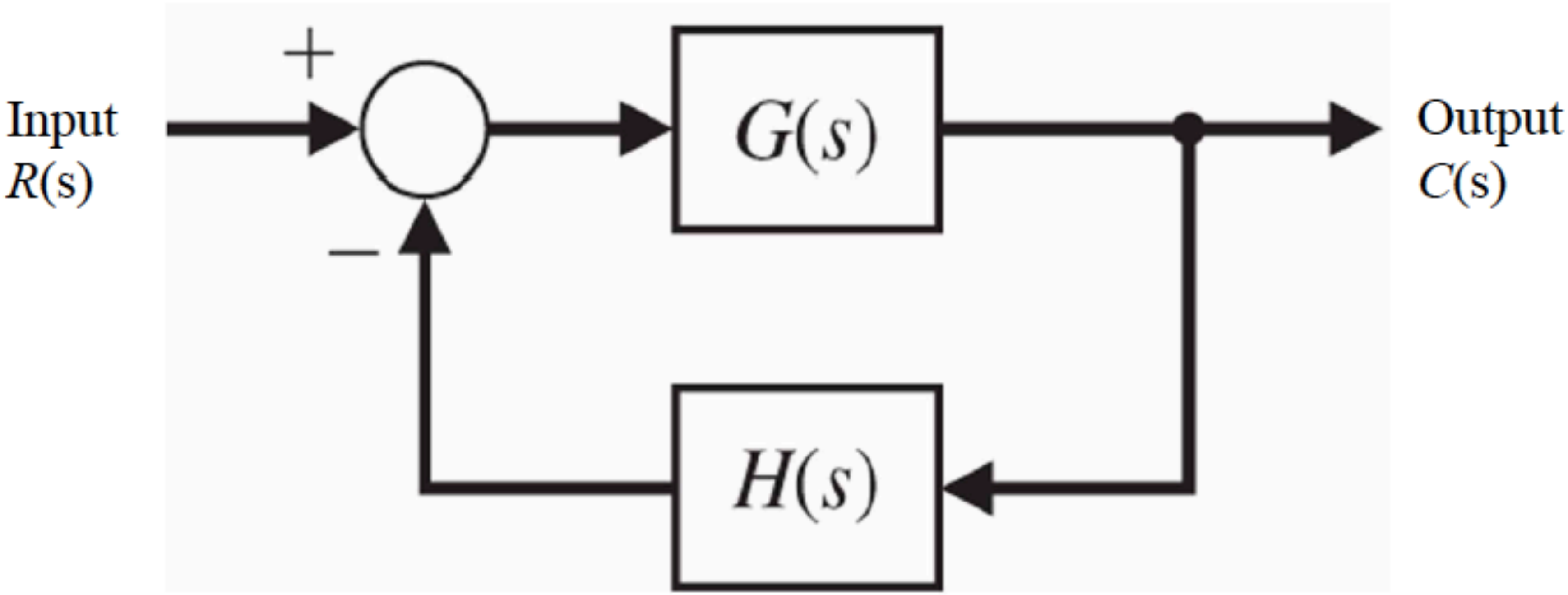
Lecture 7 – part 2

Sensitivity of controllers to parameter variations

Error in closed-loop systems



Since $E_a(s) = \frac{C(s)}{G(s)}$ then we have this for the actuating error signal:

$$E_a(s) = \frac{\frac{G(s)}{(1+G(s)H(s))}R(s)}{G(s)} = \frac{1}{(1+G(s)H(s))}R(s)$$

This tells us that if we want to reduce the error we need $(1+G(s)H(s)) > 1$ for the range of s under consideration.

This factor generally plays a very important role in the characteristics of feedback control systems.

The *sensitivity* of a control system to parameter variations is also really important. A great advantage of the closed loop system is its ability to reduce the system's sensitivity. We look next at the effect of parameter variations in the closed loop system.

The sensitivity of a closed-loop system to system parameters

System Sensitivity is classically defined as follows:

$$S = \frac{\%\ change\ in\ T(s)}{\%\ change\ in\ G(s)}$$

Recalling that *T*(s) is the *overall system transfer function* (C(s) / R(s)) and *G*(s) is the *process transfer function*. We can refer to the closed loop block diagram as a reminder of these definitions. The formal term system sensitivity is usually simplified down to sensitivity, with no loss of generality.

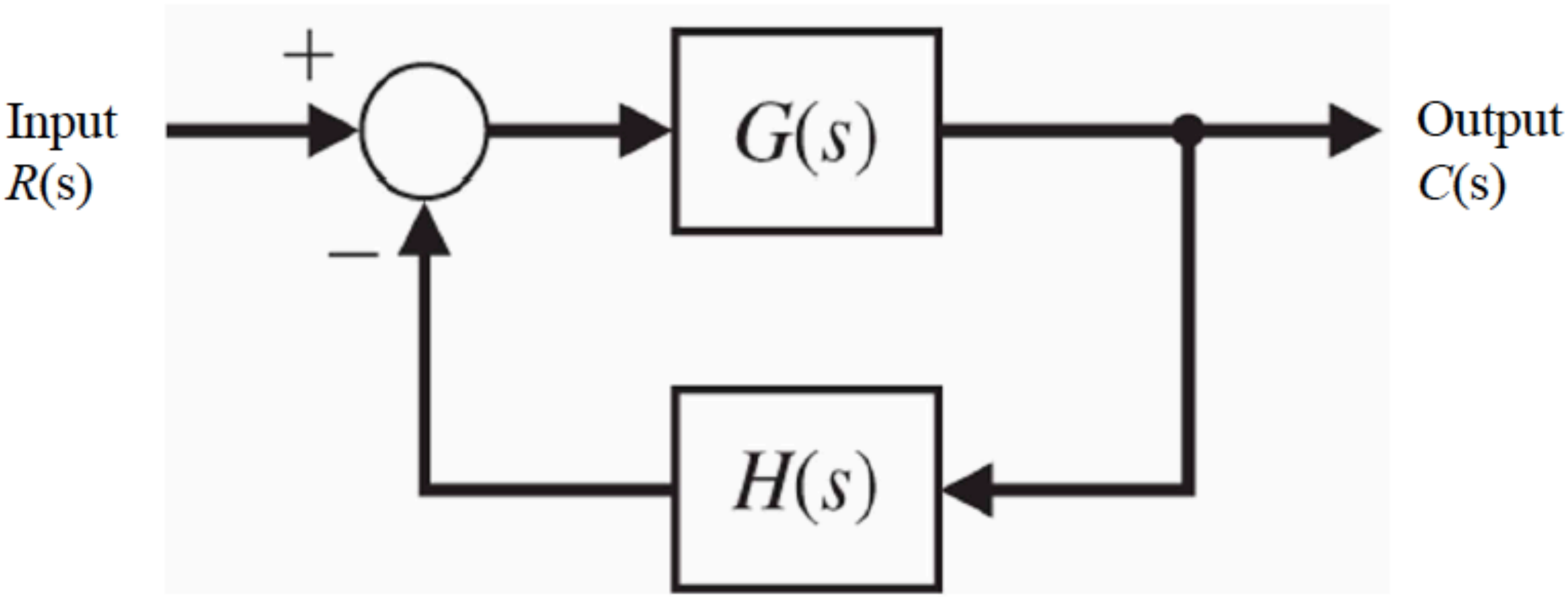
A main objective of closed loop controller design is to minimise its sensitivity.

We will drop the argument *s* in what follows, for easier notation, and then we can restore it at the end.

Returning to the definition of sensitivity we have:

$$S = \frac{\%\ change\ in\ T(s)}{\%\ change\ in\ G(s)} = \frac{\Delta T/T}{\Delta G/G} \approx \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\partial T}{\partial G} . \frac{G}{T}$$

Closed-loop system block diagram again:



So, we start with $T(s)$:

$$T = \frac{G}{1 + GH} = G(1 + GH)^{-1}$$

So, differentiating this with respect to G leads to:

$$\frac{\partial T}{\partial G} = 1.(1 + GH)^{-1} - 1.GH(1 + GH)^{-2}$$

So, if we multiply this result by $\frac{G}{T}$ then we can obtain an expression for S:

$$S = \frac{\partial T}{\partial G}.\frac{G}{T} = \frac{(1 + GH)^{-1}G(1 + GH)}{G} - GH(1 + GH)^{-2}G\frac{(1 + GH)}{G}$$

Cancelling out appropriately in the RHS above leads to:

$$S = 1 - \frac{GH}{(1 + GH)} = \frac{1 + GH - GH}{(1 + GH)} = \frac{1}{(1 + GH)} = \frac{1}{(1 + G(s)H(s))}$$

If we examine this expression we see that the way *to minimise the sensitivity of the controller*, S, is *to maximise the product* $G(s)H(s)$. We can do this by design.

Conclusions for closed loop systems:

1. For the reduction of closed loop error we want $(1 + G(s)H(s)) > 1$

2. To minimise system sensitivity we want to maximise $G(s)H(s)$

These can be made to be entirely compatible conditions in practice.