Lecture 10

Further notes on the Routh Hurwitz Stability criterion

(1) Threshold Stability

We can examine a more complicated system to see how we might explore the first condition in the notes of Lecture 9, when an element in the first column is equal to zero but other elements in that row are not equal to zero. In this situation we need to introduce the small parameter.

We start by taking this characteristic equation. It is a sixth degree polynomial and representative of a practical but more complicated physical system.

$$s^6 + s^5 + 3s^4 + 3s^3 + 2s^2 + s + 1 = 0$$

$n = 6$

The Routh-Hurwitz array looks like this:

s^6	1	3	2	1
s^5	1	3	1	
s^4	$0 \to \epsilon$	1	1	
s^3	$\frac{3\epsilon - 1}{\epsilon}$	$\frac{\epsilon - 1}{\epsilon}$		
s^2	$\frac{4\epsilon - 1 - \epsilon^2}{3\epsilon - 1}$	1		
s^1	e_1			
s^0	1			

Two points to note:

In the first column of row s^4 we see the zero, and we replace that with the small parameter ϵ . This generates subsequent nonzero entries for rows s^3, s^2, and s^1

Entry e_1 is too complicated to fit easily into the array so we state it below:

$$e_1 = \frac{\left(\frac{4\epsilon - 1 - \epsilon^2}{3\epsilon - 1}\right)\left(\frac{\epsilon - 1}{\epsilon}\right) - \left(\frac{3\epsilon - 1}{\epsilon}\right)}{\left(\frac{4\epsilon - 1 - \epsilon^2}{3\epsilon - 1}\right)}$$

We can now use a computer to calculate *the first column entries* for different values of ϵ, as follows:

	$\epsilon = 0.1$	$\epsilon = 0.01$	$\epsilon = 0.001$	$\epsilon = 0.0001$	$\epsilon = 0.00001$
s^6	1	1	1	1	1
s^5	1	1	1	1	1
s^4	0.1	0.01	0.001	0.0001	0.00001
s^3	-7	-97	-997	-9997	-99997
s^2	0.871	0.989	0.998	0.999	0.999
s^1	-0.967	-0.999	-0.999	-1	-1
s^0	1	1	1	1	1

So, it is reasonably clear (at least from this partial numerical test) that we get this general situation in the first column:

+
+
+
-
+
-
+

Therefore, the system appears to be generally unstable as $\epsilon \to 0$.

This technique can be used for system where when an element in the first column is equal to zero but other elements in that row are not equal to zero.