

Feedback Linearisation Control

**Application to the control of a mobile
gantry crane**

Examples Class 2



So, in the first part of this discussion we showed that the highly nonlinear dynamic problem of the RTG crane could be reduced to an error equation using the principles of feedback linearisation, as follows:

$$(\ddot{\underline{q}}_d - \ddot{\underline{q}}) + \{h\}(\dot{\underline{q}}_d - \dot{\underline{q}}) + \{g\}(\underline{q}_d - \underline{q}) = \ddot{\underline{\varepsilon}} + \{h\} \dot{\underline{\varepsilon}} + \{g\} \underline{\varepsilon} = 0$$

The desired quantities can be stated explicitly as vectors:

$$\underline{q}_d = \begin{bmatrix} x_d \\ y_d \\ l_d \\ \psi_d \\ \alpha_d \end{bmatrix}, \quad \dot{\underline{q}}_d = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{l}_d \\ \dot{\psi}_d \\ \dot{\alpha}_d \end{bmatrix}, \quad \ddot{\underline{q}}_d = \begin{bmatrix} \ddot{x}_d \\ \ddot{y}_d \\ \ddot{l}_d \\ \ddot{\psi}_d \\ \ddot{\alpha}_d \end{bmatrix},$$

These are actually the set points for the five generalised coordinates, in vectors of displacements, velocities, and accelerations. Many of them may well be set to zero, but some will be non-zero and may also be step or ramp functions, or more complicated functions of time rather than simple constant values.

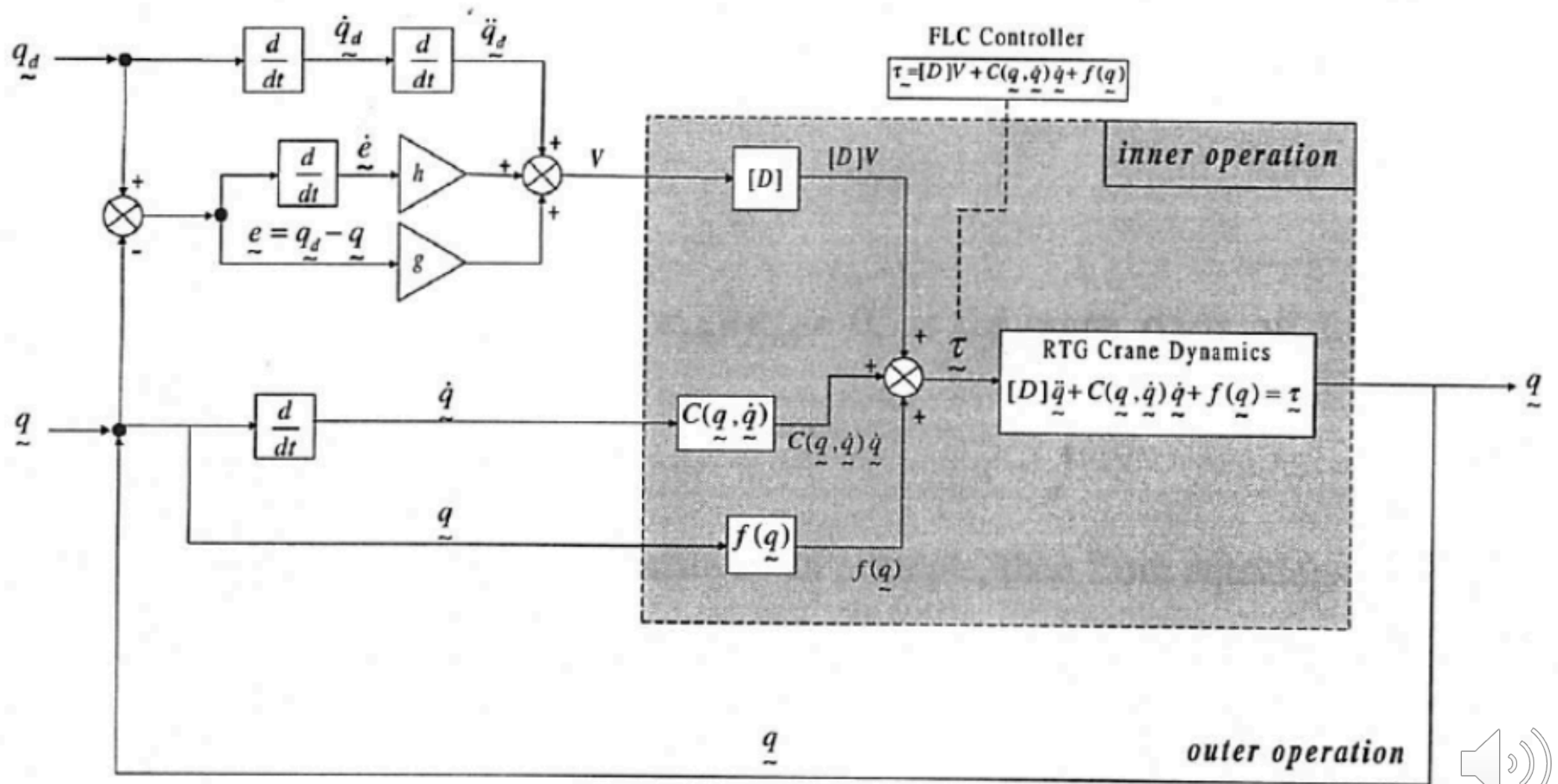


The system is only partially actuated, through the active gains

$\{h_x, h_y, h_l\}$ and $\{g_x, g_y, g_l\}$ and we see that there are no gains associated with coordinates Ψ and α .

This means that those two coordinates cannot be directly actuated, which makes sense physically. We can now show the time-domain simulation for the RTG problem, and this leads to a numerical solution of the error equation. In practice we would implement this in the s domain, using advanced Simulink features for vectorised processing to maximise the speed of controlled response.



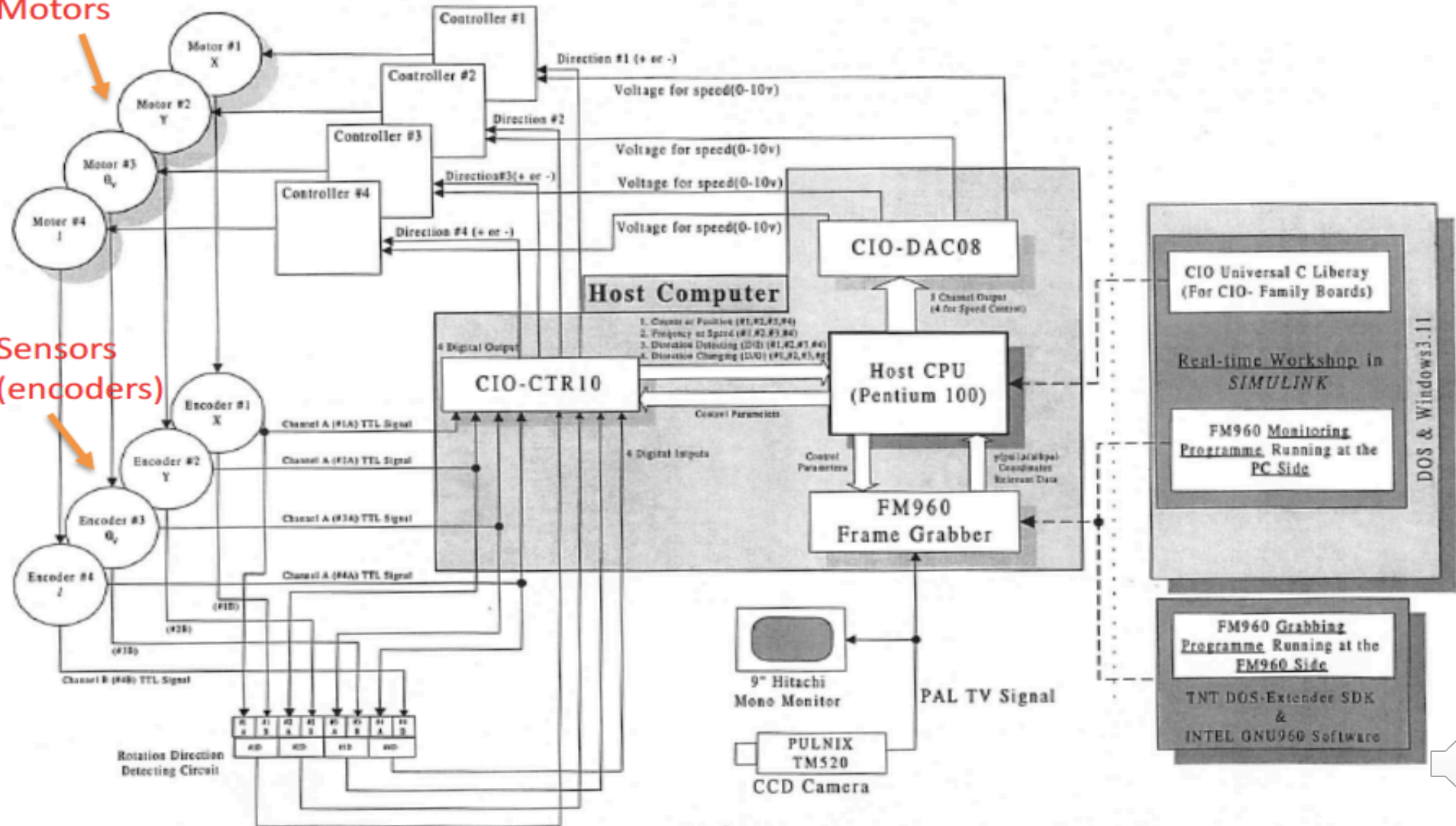


Actuation
Motors

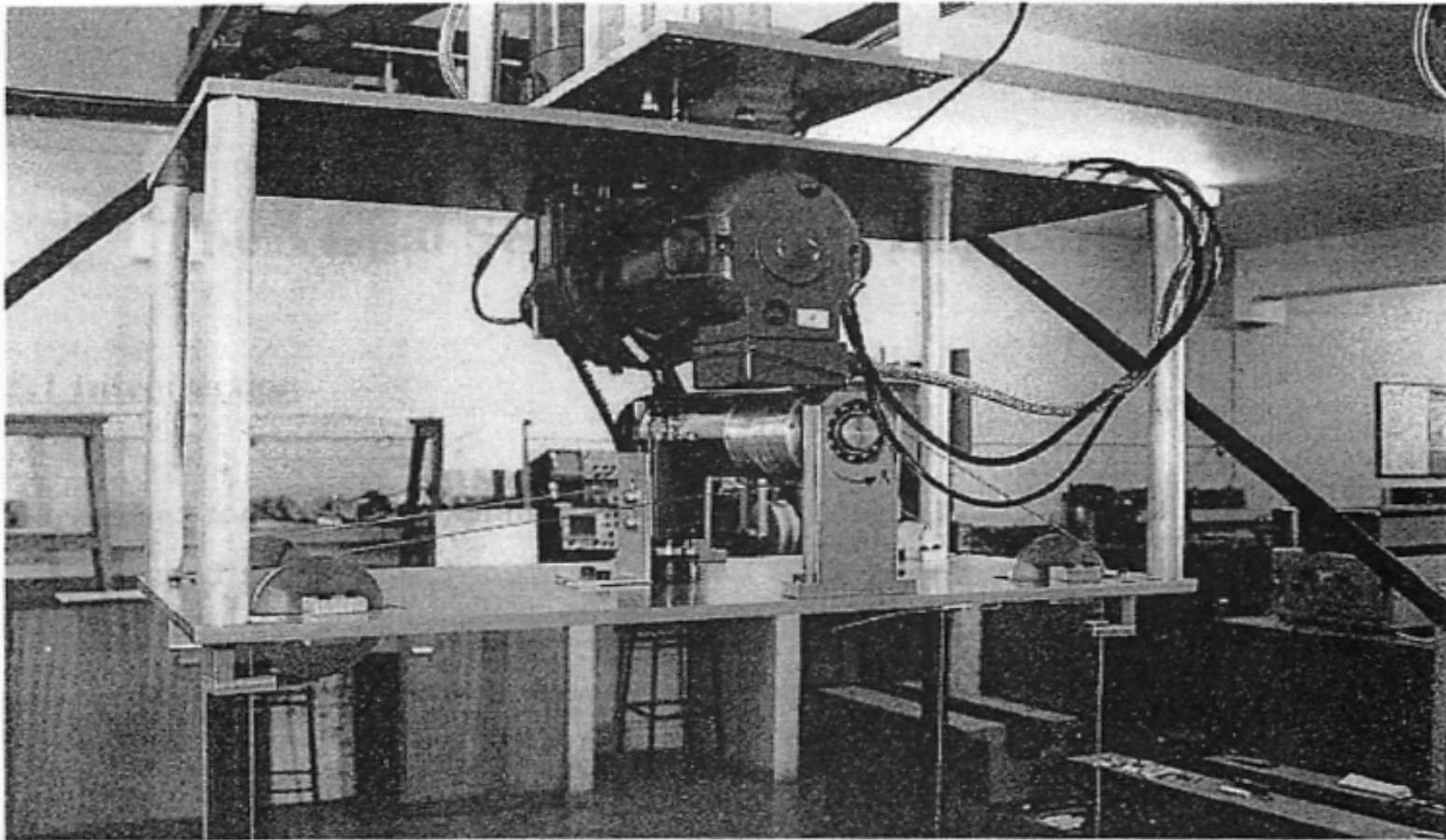
Sensors
(encoders)

Hardware Setup

Software Setup



This low resolution photograph shows a 1/8 scale laboratory hardware implementation of this control system.



Rotation (very top) and hoist (central) electro-mechanical hardware.



Running the scaled system under test:

Setup:

$$\{h\} = [1.8122 \ 1.359 \ 2.3918 \ 0 \ 0]^T, \quad \{g\} = [1.631 \ 0.4617 \ 1.4302 \ 0 \ 0]^T$$

$$q_i = [0.0 \ -0.5 \ 0.7 \ 0.0 \ 0.0]^T, \quad q_d = [1.5 \ 0.5 \ 1.2 \ 0.0 \ 0.0]^T$$

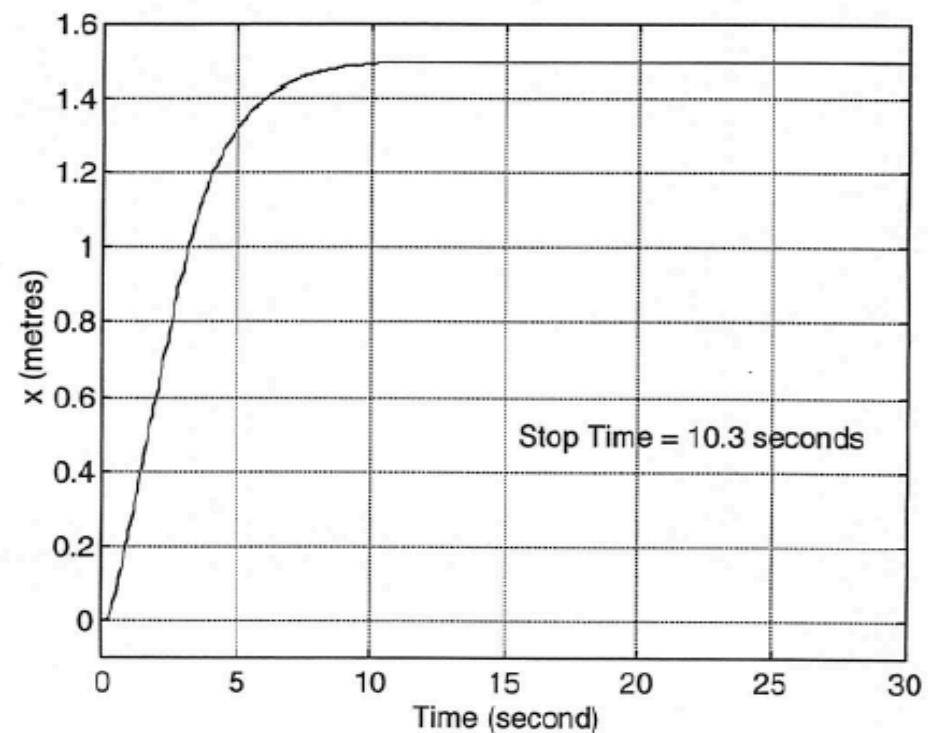
Test results:

$$t_{xs} \text{ (stop time, with } \dot{x} = 0) = 10.2 \text{ seconds}$$

$$t_{ys} \text{ (stop time, with } \dot{y} = 0) = 12.3 \text{ seconds}$$

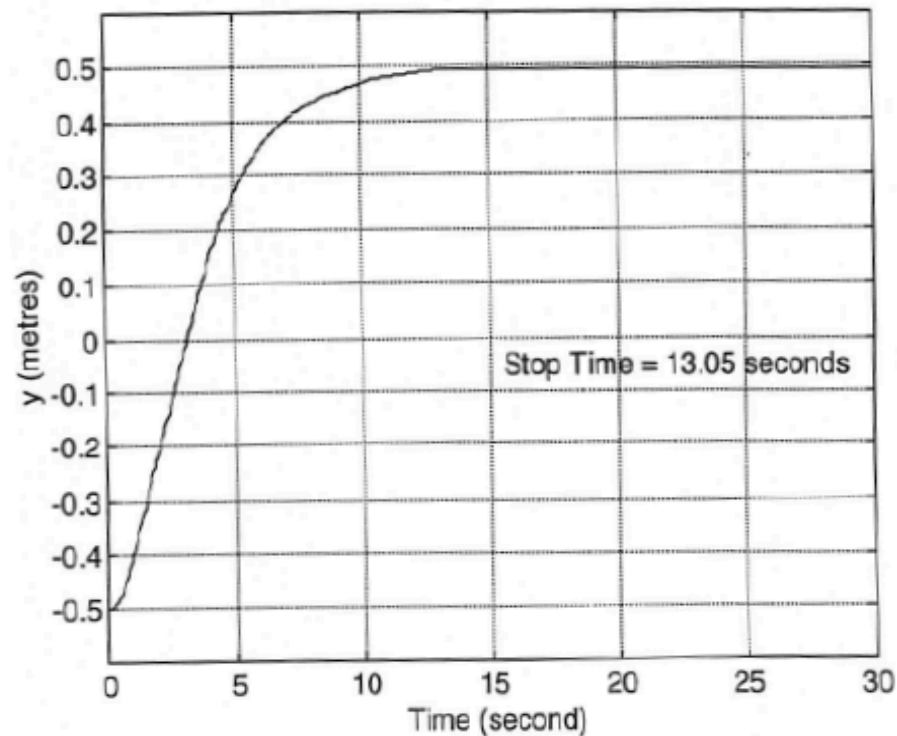
$$t_{ls} \text{ (stop time, with } \dot{l} = 0) > 30 \text{ seconds}$$

$$\psi(t=30) \approx 3.0^\circ$$

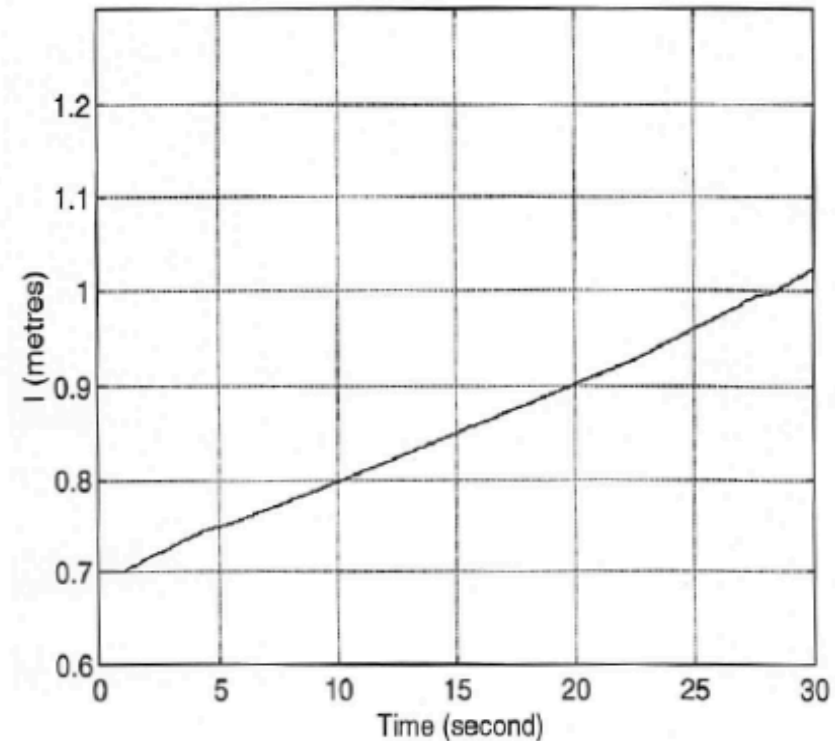


Position of the trolley in the x- direction





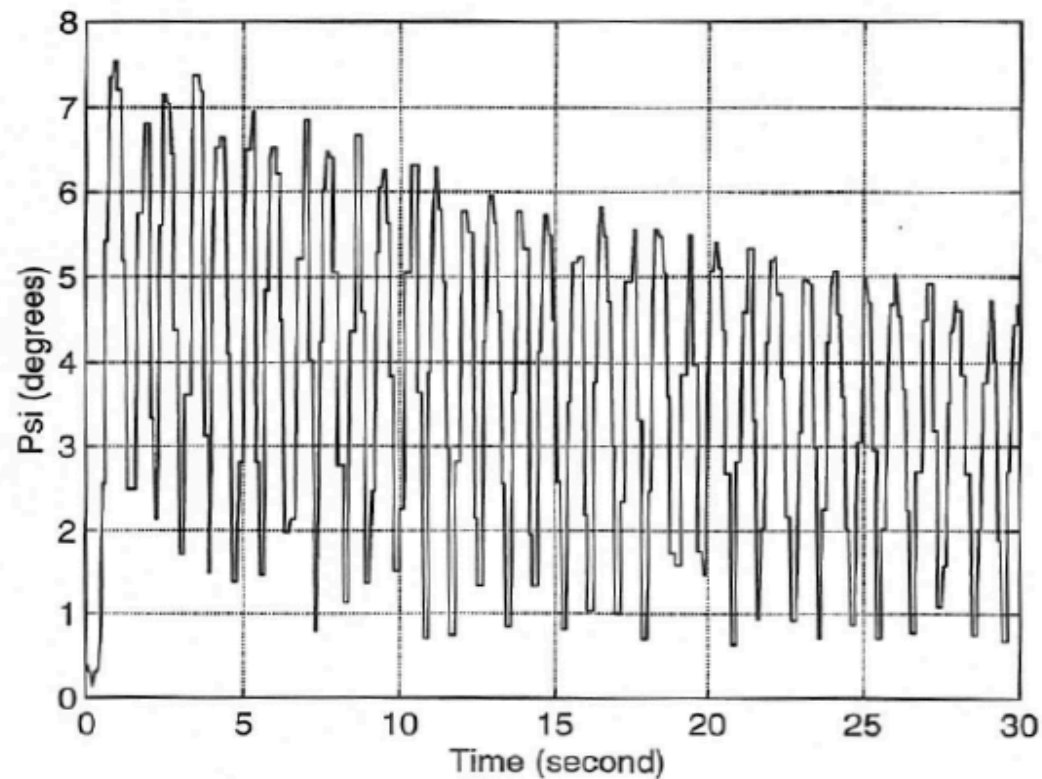
Position of the trolley in the y- direction



The cable length of the spreader l

x and y for the trolley are actuated and hit their set-points easily. Cable length l is also directly actuated and takes a bit longer for this test (not quite at set point of 1.2 m yet)

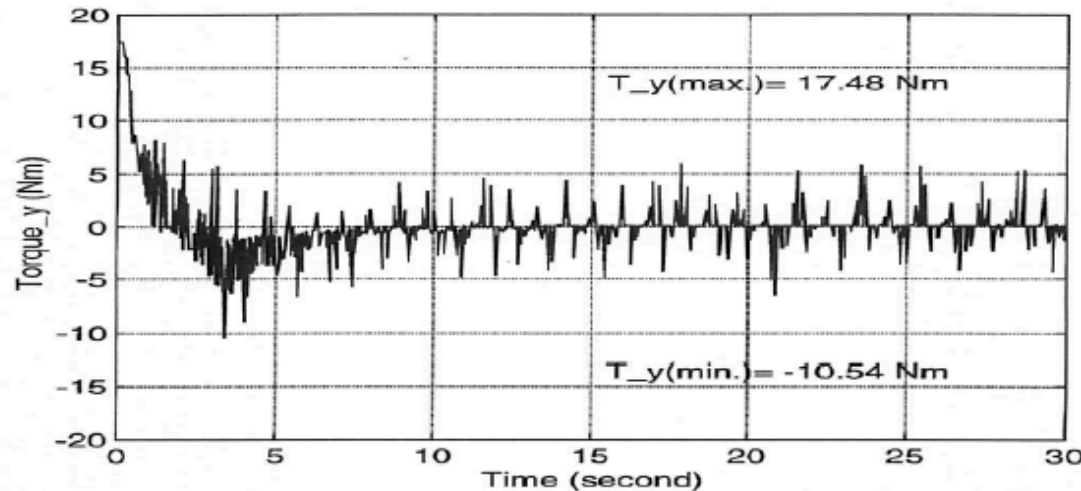




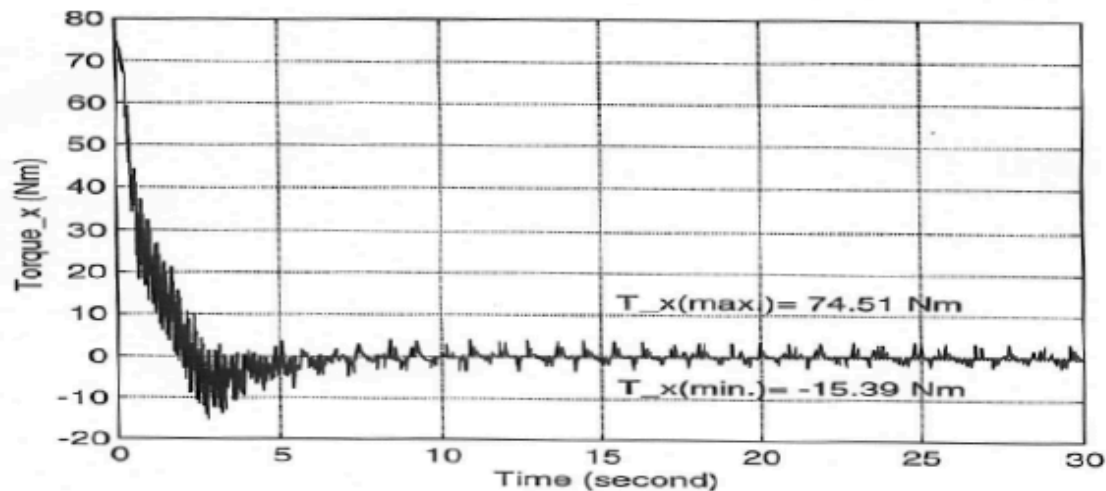
This is the 'swing' response of the spreader – the most notoriously difficult part of RTG crane control – and as we see the oscillations are of low amplitude, and decreasing, as the length control comes to an end – but taking ages because the system, as modelled, is very underdamped. This is not the case in practice.



Control actuation torques - effectively: $\underline{\tau} = [D] \underline{\ddot{V}} + C(\underline{q}, \underline{\dot{q}}) \underline{\dot{q}} + f(\underline{q})$



The required actuating torque of the trolley in the y- direction



The required actuating torque of the trolley in the x- direction

The control torque for the x and y trolley positioning continues to act throughout, and so this controller is quite computationally demanding for this application. The control for l has no obvious transient but otherwise has the same characteristic over time.



Conclusions:

1. Feedback linearisation can work effectively for coupled dynamic systems which are also under-actuated.
2. It is invariably much more computationally demanding than a PI or PID controller.
3. Successful implementation can be very dependent on instrumentation quality (good noise rejection is a must).
4. Stability calculations are frequently done in parallel with gain optimisation studies, and by using dynamical systems theories as applicable to nonlinear dynamics.

