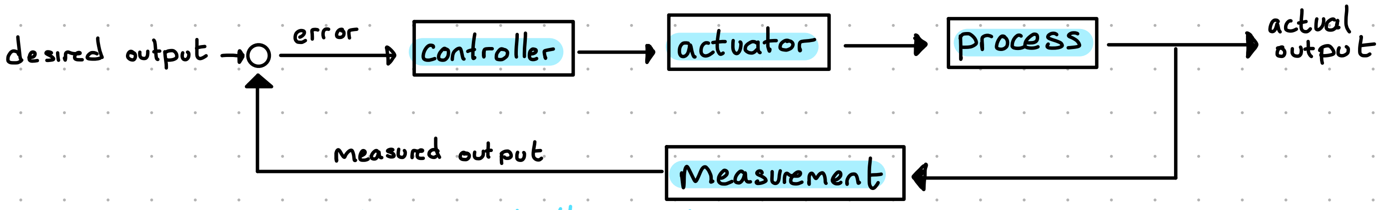
Control

Week 1

Summary

BLOCK DIAGRAMS



CONTROLLER - what is controlling the output

ACTUATOR - what is being used to alter the output

PROCESS - what it is being applied to

MEASUREMENT - what is measuring the output

LINEARISATION

DECIDE ON EQUILIBRIUM POINTS, x_0 (or a in lecture slides)

- this is the point that you linearise around

USE TAYLOR SERIES EXPANSION

- one variable

$$f'(\Delta x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0} \Delta x$$

- two variables

$$f'(\Delta x, \Delta y) = f(x_0, y_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0, y_0} \Delta x + \left.\frac{\partial f}{\partial y}\right|_{x_0, y_0} \Delta y$$

where $\Delta x = x - x_0$

$\Delta y = y - y_0$

Week 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

TO GET TRANSFER FUNCTION

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\text{OUTPUT/SYSTEM RESPONSE}}{\text{INPUT}}$$

1. FIND SYSTEM OF EQUATIONS

   eg. $m_1\ddot{y}_1 + b\dot{y}_1 + k(y_1 - y_2) = F(t)$

   $m_2\ddot{y}_2 + k_1(y_2 - y_1) = 0$

2. TAKE LAPLACE OF EACH EQ^N

3. PUT INTO MATRIX FORM

   eg. $\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$

   with a, b, c, d containing m, b, ks

4. REARRANGE TO MAKE EQUAL TO $Y_1(s)$, $Y_2(s)$

   eg. $\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} F(s) \\ 0 \end{bmatrix}$

5. MULTIPLY OUT TO GET $Y_1(s) = \times F(s)$

   eg $Y_1(s) = \frac{d}{ad-bc} F(s)$

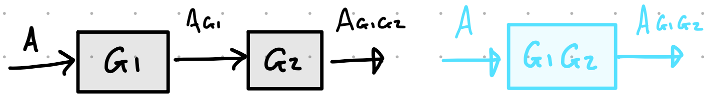
6. REARRANGE TO GET TRANSFER FUNCTION

   $\frac{Y_1(s)}{F(s)}$

Week 3

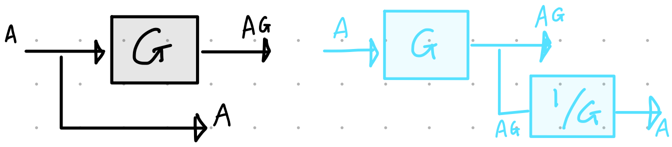
BLOCK DIAGRAM REDUCTION

1. COMBINING BLOCKS IN SERIES

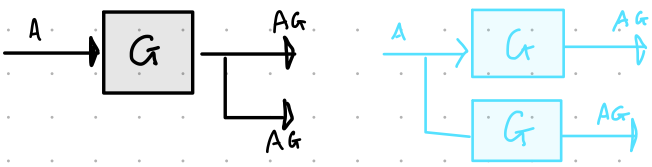


2. COMBINING BLOCKS IN PARALLEL

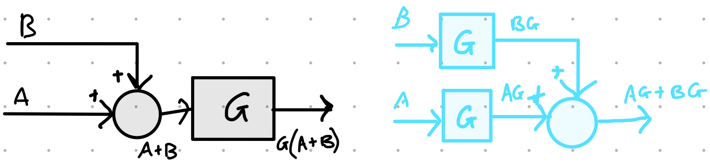
3. MOVING BRANCH POINT AHEAD OF BLOCK



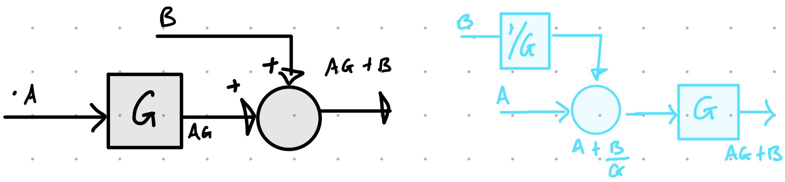
4. MOVING BRANCH POINT BEHIND THE BLOCK



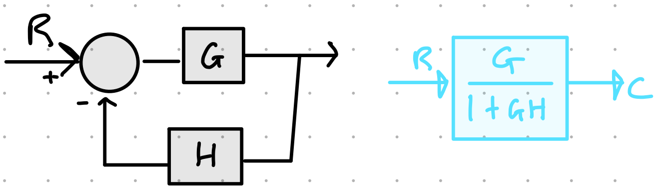
5 MOVING SUMMING POINT AHEAD OF THE BLOCK



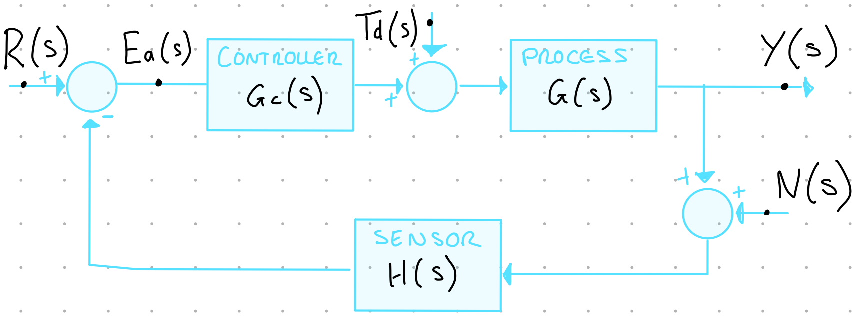
6. MOVING SUMMING POINT BEFORE THE BLOCK



7. ELIMINATION OF FEEDBACK LOOP



Week 1



$Y(s)$ – output
$R(s)$ – input
$Td(s)$ – disturbance
$N(s)$ – noise
$Ea(s)$ – error signal

LOOP GAIN

$$L(s) = G_c(s)\,G(s)$$

SENSITIVITY FUNCTION

$$S = \frac{1}{1+L} = \frac{\partial T}{\partial G}\frac{G}{T} = \frac{\partial \ln T}{\partial \ln G}$$

$\frac{\partial T}{\partial G} = \frac{gf' - fg'}{g^2}\left(\frac{f}{g}\right)$

$$S_c = \frac{L}{1+L}, \quad S + S_c = 1$$

UNITY FEEDBACK SYSTEM

$$H(s) = 1$$

$$Y(s) = \underbrace{\frac{G_c G}{1 + G_c G}}_{\text{input}} R + \underbrace{\frac{G}{1+G_c G}}_{\text{disturbance}} Td - \underbrace{\frac{G_c G}{1+G_c G}}_{\text{noise}} N$$

$$Y(s) = \frac{L}{1+L}R + \frac{G}{1+L}Td - \frac{L}{1+L}N$$

$$Y(s) = S_c R + SGTd - S_c N$$

FROM BLOCK DIAGRAM

$$Y(s) = G(T + G_c E)$$
$$E(s) = R - H(N + Y)$$

sub E into Y

$$Y(s) = GT + GG_cR - GG_c\cancel{H}N - GG_c\cancel{H}Y$$

H = 1

$$Y(1 + GG_c) = GT + GG_cR - GG_cN$$

ERROR SIGNAL

$$E(s) = R(s) - Y(s)$$

$$E(s) = \frac{1}{1+L}R - \frac{G}{1+L}Td + \frac{L}{1+L}N$$

$$E(s) = SR - SGTd + S_cN$$

To minimise noise → $L(s)$ must be small
To minimise disturbance → $L(s)$ must be big

LET $Td = 0$ $N = 0$

$$E + \Delta E = \frac{1}{1 + G_c(G + \Delta G)}R$$

$$\Delta E = \frac{-G_c\Delta G}{(1 + G_cG + G_c\Delta G)(1 + G_cG)}R$$

$G_cG >> G_c\Delta G$

$$\Delta E \approx \frac{-G_c\Delta G}{(1+L)^2}R$$

For Large L, $1 + L \approx L$

$$\Delta E \approx -\frac{1}{L}\frac{\Delta G}{G}R$$

TRACKING ERROR

$$E(s) = R(s) - Y(s) \qquad (Y(s) = T(s)R(s))$$

$$E(s) = R(s)(1 - T(s))$$

STEADY STATE ERROR

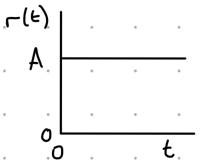
$$E_{ss}(\infty) = \lim_{s \to 0} s\,E(s)$$

In laplace domain as $s \to 0$
In time domain as $t \to \infty$

Week 5

INPUT SIGNALS

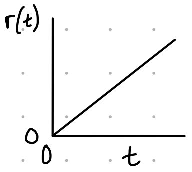
STEP



$$r(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$R(s) = \frac{A}{s}$$

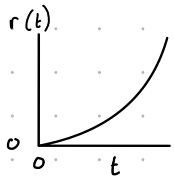
RAMP



$$r(t) = \begin{cases} At, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$R(s) = \frac{A}{s^2}$$

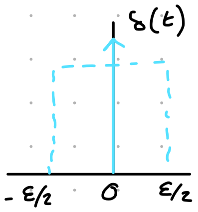
PARABOLIC



$$r(t) = \begin{cases} \frac{At^2}{2}, & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \frac{2A}{s^3}$$

IMPULSE



$$f_\varepsilon(t) = \begin{cases} 1/\varepsilon, & -\frac{\varepsilon}{2} \leq t \leq \frac{\varepsilon}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(t) = \lim_{\varepsilon \to 0} f_\varepsilon(t)$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

and

$$\int_{-\infty}^{\infty} \delta(t-a) g(t)\, dt = g(a)$$

$$\mathcal{L}\{\delta(t)\} = 1$$

SECOND ORDER SYSTEMS

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

with unit step input $1/s$

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

inverse laplace

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta)$$

where

$$\beta = \sqrt{1-\zeta^2}, \quad \theta = \cos^{-1}(\zeta)$$

PERFORMANCE OF 2ND ORDER

RISE TIME, t_r - time taken to reach input value for first time

for overdamped, $0.3 \leq \zeta \leq 0.8$

$$T_{r1} = \frac{2.16\zeta + 0.6}{\omega_n}$$

PEAK TIME, t_p - time to reach first peak of the overshoot

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

PEAK RESPONSE, M_{pt}

$$M_{PT} = 1 + e^{-\zeta T_P}$$

SETTLING TIME, t_s - time to reach steady state within tolerance

$$e^{-\zeta\omega_n T_s} < 0.02 \quad \text{decay}$$

2% tolerance standard

$$\zeta\omega_n T_s \approx 4$$

$$T_s \approx 4\tau = \frac{4}{\zeta\omega_n} \quad \left(\tau = \frac{1}{\zeta\omega_n}\right)$$

PERCENT OVERSHOOT

$$PO = \frac{M_{pt} - f_{ss}}{f_{ss}} \%$$

$M_{pt} \to$ peak value
$f_{ss} \to$ final response

$$PO = 100 e^{-\zeta T_P}$$

* For a given ζ the response is faster for a larger ω_n

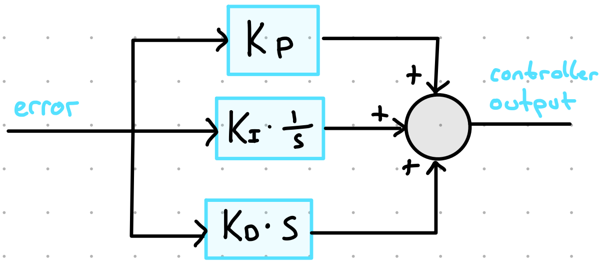
* Overshoot is not dependant on ω_n

Week 6

PID CONTROLLERS

P - proportional, K_P
I - integral, K_I
D - derivative, K_D

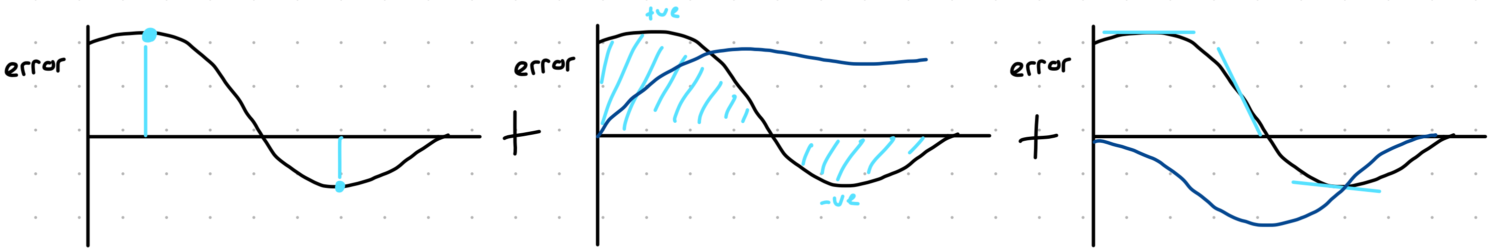
} describe how error signal is handled



Proportional — present error

Integral — accumulated error

Derivative — future - derivative of error



Can have P or PI or PID or PD by setting gain to zero

In time domain

$$\boxed{u(t) = K_P \cdot e(t) + K_I \cdot \int e(t)\, dt + K_D \cdot \frac{de(t)}{dt}}$$

In laplace

$$\boxed{G_c(s) = K_P + \frac{K_I}{s} + K_D s}$$

Week 7

TRANSFER FUNCTION $\Longrightarrow$ DIFF EQUATION

$$G(s) = 3\frac{(s+4)}{s^2+2s+5} \qquad \ddot{y} + 2\dot{y} + 5y = 3\dot{u} + 12u$$

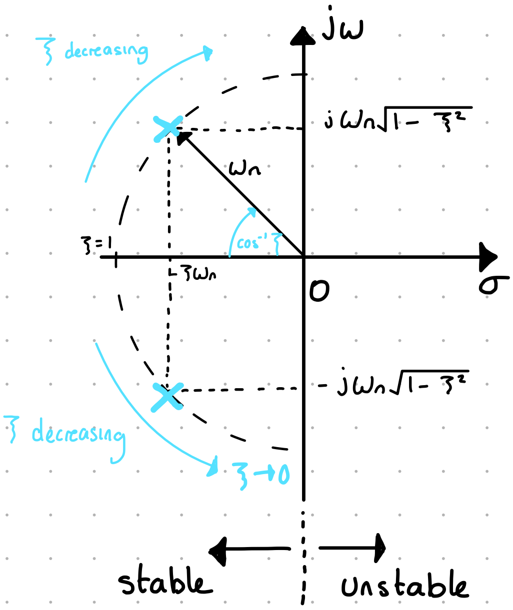
REAL POLE $\rightarrow$ $C_1 e^{-xt}$

COMPLEX CONJUGATE POLE $\rightarrow$ $Ae^{-xt}\sin(yt + \phi)$

Time constant $\tau = \frac{1}{x}$

Time to decay, $= 4\tau$

S-PLANE



- Radius is natural frequency
- Angle related to damping ratio
- Damped frequency component on imaginary axis
- Stable vs unstable depending on +/- real axis component

Week 8

For n particles, in a m-DOF system, there are mn 2nd order ODES

STATE VARIABLE MODELS

STATE SPACE REPRESENTATION = 2 matrix equations

STATE DIFFERENTIAL

$$\dot{x} = Ax + Bu$$

OUTPUT EQUATION

$$\dot{y} = Cx + Du$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

need to define state variables and amount.

eg. 2DOF SYSTEM

$x_1 = q \quad x_2 = y \quad x_3 = \dot{q} \quad x_4 = \dot{y}$

TYPES OF MODELS

Continuous time-invariant

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Continuous time-variant

$$\left.\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned}\right\}$$

eg car losing mass as fuel burns

Week 9 - STABILITY

STABILITY THEORY - study of stability of solutions to diff equations and dynamical systems

Only the denominator affects stability

LTI SYSTEMS

-linear time - invariant

- relationship between output & input is a linear map

$$\sum_k C_k r_k(t) = \sum C_k y_k(t)$$

- Output of system does not depend explicitly on time

$r(t) \rightarrow y(t)$ $\quad$ $r(t+\tau) \rightarrow y(t+\tau)$

BIBO STABILITY

Bounded output, bounded input. - STABLE

-only applies to LTI systems

STABILITY TYPES

IF SYSTEM IS DISTURBED - will it stay close to original state
- will it converge back to target state

ABSOLUTE STABLE SYSTEM

- SYSTEM IS STABLE FOR ALL RANGE INPUTS
- ALL POLES ARE LOCATED ON LEFT HALF OF S-PLANE

CONDITIONALLY STABLE SYSTEM

- SYSTEM IS ONLY STABLE FOR CERTAIN RANGE OF INPUTS

MARGINALLY STABLE SYSTEM

- SYSTEM IS STABLE BY PRODUCING OUTPUT WITH CONSTANT AMPLITUDE AND CONSTANT FREQUENCY FOR BOUNDED INPUT
- ANY TWO POLES OF TF ARE ON IMAGINARY AXIS
- DOESN'T DIVERGE OR CONVERGE

LAGRANGIAN POINTS - in orbit are gravitationally stable points existing in any 2-body system

ROUTH-HURWITZ CRITERION

This states that the number of poles on the right hand side of the s-plane (unstable) is equal to the number of sign changes in the first column of the Routh array.

ROUTH ARRAY

s^n	a_n	a_{n-2}	a_{n-4}
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}
s^{n-2}	(b_{n-1})	b_{n-3}	0
s^{n-3}	c_{n-1}	c_{n-3}	0
s^{n-4}	d_{n-1}	0	0
s^0	e_{n-1}	0	0

$$q(s) = a_n^{n} + a_{n-1}^{n-1} + a_{n-2}^{n-2} + a_{n-3}^{n-3} + a_{n-4}^{n-4} + a_{n-5}^{0}$$

a - taken from characteristic eq^n
b, c... - calculated

$$b_{n-1} = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \qquad b_{n-3} = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$c_{n-1} = \frac{-1}{b_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix} \qquad c_{n-3} = \frac{-1}{b_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_{n-1} & 0 \end{vmatrix}$$

$$d_{n-1} = \frac{-1}{c_{n-1}}\begin{vmatrix} b_{n-1} & b_{n-3} \\ c_{n-1} & c_{n-3} \end{vmatrix} \qquad e_{n-1} = \frac{-1}{d_{n-1}}\begin{vmatrix} c_{n-1} & c_{n-3} \\ d_{n-1} & 0 \end{vmatrix}$$

ONCE CALCULATED, IF THERE ARE ANY SIGN CHANGES IN FIRST COLUMN - SYSTEM IS UNSTABLE

CASE 1 - first element of a row is zero

1. set to ε
2. calc rest of array
3. If sign changes are consistent whether ε is +ve or -ve
   - can determine if stable or unstable
4. If sign changes are NOT consistent
   - stability cant be determined

CASE 2 - all elements of a row are zero

1. Find auxilary polynomial (use coefficients of row above)
2. Differentiate aux eq^n
3. Replace zero row with coeff from differential
4. Continue array

CASE 3 - repeated roots of characteristic eq^n on im axis

1. Simple roots on im axis
   - marginally stable
2. Repeated roots on im axis
   - System is unstable

Array will cause false positive