16361 Dynamics

Tutorial Sheet 1

Introductory Dynamics Revision

Q1. For the simple pendulum shown in Fig Q1a apply Newton's Second Law in order to derive the equation of motion. You can assume that the pendulum arm is massless and that all the mass is concentrated in the lump at the end. Note that the mass moment of inertia of the pendulum, about the pivot, can be taken to be $I_o = ml^2$. Explain why the equation is nonlinear and suggest an assumption which will allow you to linearise the equation.

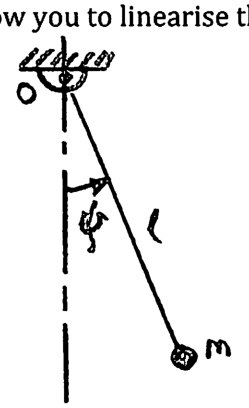


Fig Q1a

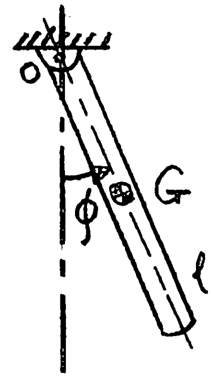


Fig Q1b

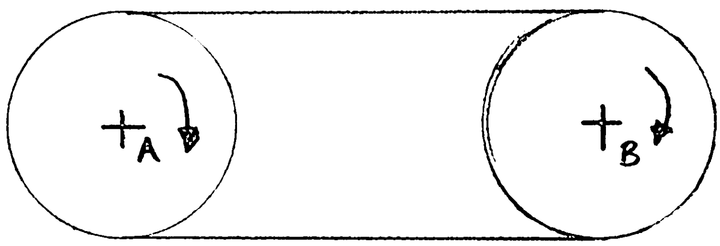
Figure Q1b shows a different case in which the mass is uniformly distributed within the pendulum arm. In this case the mass moment of inertia of the pendulum, about the pivot, is given by $I_o = I_G + m\left(\frac{l}{2}\right)^2$ (by application of the parallel axes theorem). Find the appropriate expression for I_G for a slender uniform rod or beam and then proceed to derive the equation of motion for the system.

[Answers: $\ddot{\psi} + \frac{g}{l} sin\psi = 0$, $\ddot{\phi} + \frac{3g}{2l}\phi = 0$.]

Q2. A short heavy shaft is being turned in a lathe which is driven by a motor running at 1400 rpm, and the speed reduction between the motor and the lathe spindle is 10 to 1. The motor torque can be taken to be constant at 15.35 Nm and the friction torque at the lathe spindle is a constant 17.5 Nm. The mass moment of inertia of the rotating parts within the motor is 0.08 kgm^2, and that of the lathe face-plate, spindle, and work-piece being turned is a total of 1.2 kgm^2. Assume that the turning tool is given an excessively heavy cut that stops the work-piece in one revolution, and then proceed to calculate the total force on the tip of the tool if it is cutting at a radius of 140 mm. You can assume the tool tip force to be uniform during the deceleration period.

[Answer: 2.1 kN]

Q3. Two shafts, A and B, lie parallel, their axes being 50 cm apart. Each carries a pulley of diameter 30 cm and the pulleys are connected by a belt so that power can be transmitted from one shaft to the other. If each pulley and shaft has a combined mass moment of inertia about its axis of 0.05 kgm^2 and if the mass per unit length of the belt is 0.4 kg/m, determine the kinetic energy when the angular velocity of the shaft is 100 rpm. If the system is initially at rest, what constant torque must be applied about A to accelerate the system to the above angular velocity after 10 revolutions?



[Answers: 6.445 J, 0.102 N]

Q4. A single aircraft wheel is of mass 125 kg, diameter 0.9 m, and radius of gyration 0.375 m. As it approaches the runway the wheel is completely without any angular velocity, but once contact is made the friction between the tyre and the runway causes it to rotate. Exactly one second after the initial contact the wheel rolls without slipping at an angular velocity which conforms to the landing speed of the aircraft. If the forward speed of the landing aircraft is 50 m/s at the instant when slipping between the tyre and the runway ceases, determine the following for the single wheel:

(a) its kinetic energy,
(b) the angular momentum about its axis of rotation,
(c) the magnitude of the frictional force between the runway and the tyre, assuming this to be constant during the interval between the initial touchdown and the cessation of slipping.

[Answers: 264.8 kJ, 1953 kgm^2/s, 4.34 kN]

Q5. In Figure Q5 disc 1 has a mass of m_1, a radius of r_1, and a moment of inertia about its central axis of $\frac{1}{2}m_1r_1^2$. Disc 2 has a mass of m_2, a radius of r_2, and a moment of inertia about its central axis of $\frac{1}{2}m_2r_2^2$. Both discs can rotate and slide up and down in the frictionless slot in the fixed frame as shown. Disc 1, having an angular velocity of ω rad/s clockwise, is dropped from a negligible height onto disc 2, which is stationary. If no external torques act and disc 1 does not bounce, show that the angular velocity of disc 1 after the slipping stops is given by $\frac{\omega}{\left(1+\frac{m_2}{m_1}\right)}$ in the clockwise direction. Write down a similar expression for the angular velocity of disc 2 at the same instant. If the coefficient of friction between the two discs is μ, show that the time t until slipping stops is given by $t = \frac{r_1\omega}{2\mu g\left(\frac{m_1}{m_2}+1\right)}$.

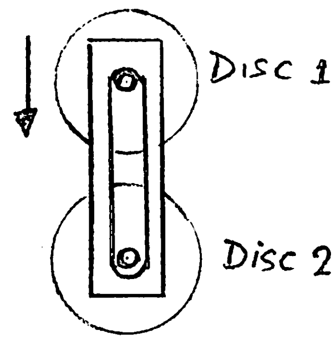


Figure Q5

Q6. A mass of 33 kg, initially at rest, falls 2 m onto a pile of mass 250 kg. Ignoring air resistance, and assuming that the two bodies move together after impact, find:

(a) the velocity with which the pile begins to move,
(b) the energy lost during impact,
(c) the mean resistance of the ground to penetration if the pile is driven 20 mm into the ground by the impact.

[Answers: 3.56 m/s, 2791 J, 190 N]

Q7. A wheel of radius r, mass m, and radius of gyration about its centre k_o rolls without slipping along a horizontal surface. The angular velocity is ω until it hits a step of height h ($h < r$) and climbs it. Find the angular velocity after impact.

[Answer: $\omega_2 = \frac{(k_o^2+r^2-rh)\omega}{k_o^2+r^2}$]

Q8. A uniform bar of length l is dropped at an angle θ to the vertical, and both ends have a velocity of v as end A hits the ground. If end A pivots about its contact point during the remainder of its motion, find the velocity with which end B hits the ground.

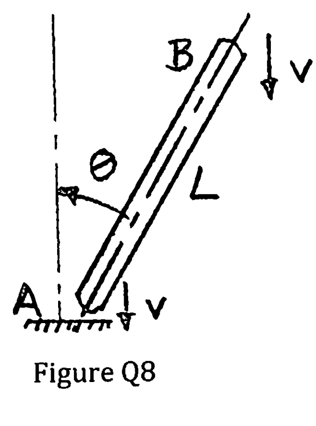


Figure Q8

[Answer: $\sqrt{\frac{9v^2}{4}sin^2\theta + 3gLcos\theta}$]

M.P.Cartmell 2016