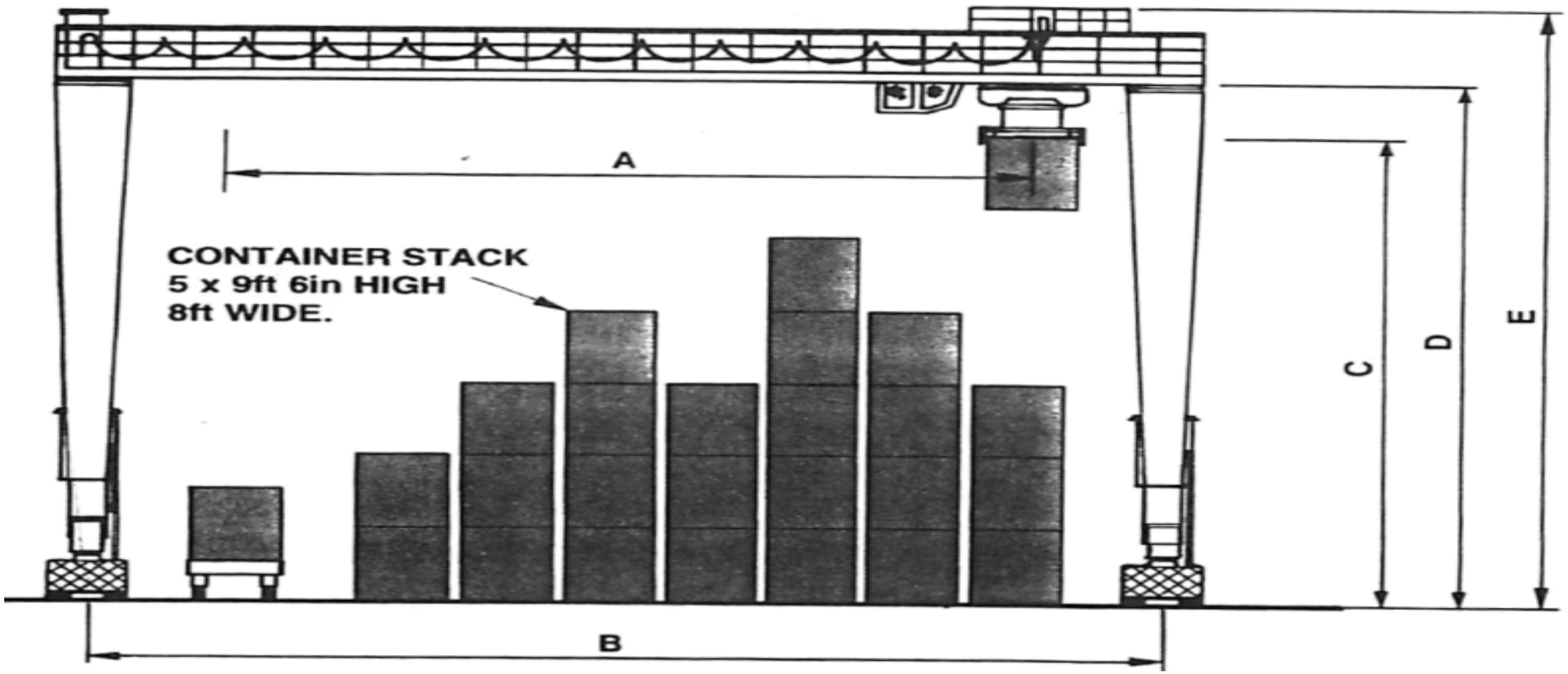
Feedback Linearisation Control

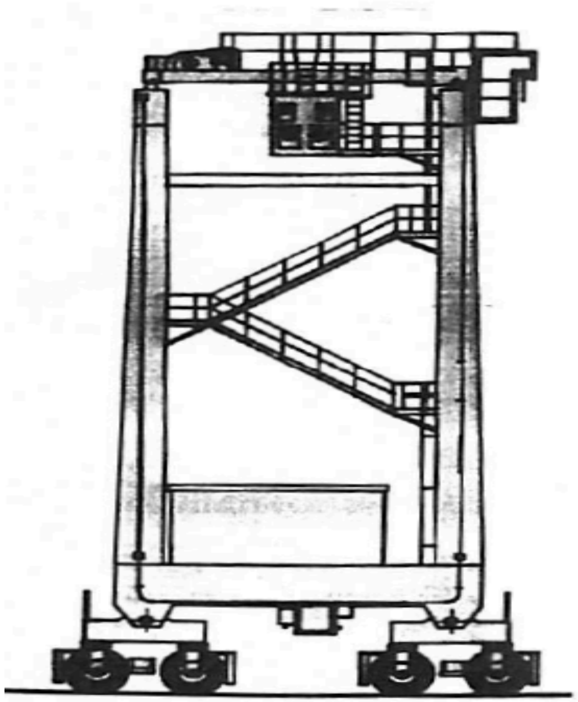
Application to the control of a mobile gantry crane

Examples Class 1

A Mobile Gantry Container Crane – a huge flexible robot!
(sometimes called a 'Rubber Tyred Gantry Crane – or RTG crane)



These machines are usually to be found in docks and railway sidings, all over the world, and can stack ISO freight containers up to 5 or 6 high. Each container can be up to 40 Tonnes in mass. A long-term problem with RTG cranes is the flexible cabled suspension gear from which the lifting frame or 'spreader' is suspended. Large swaying motions can be generated under adverse operating and weather conditions.

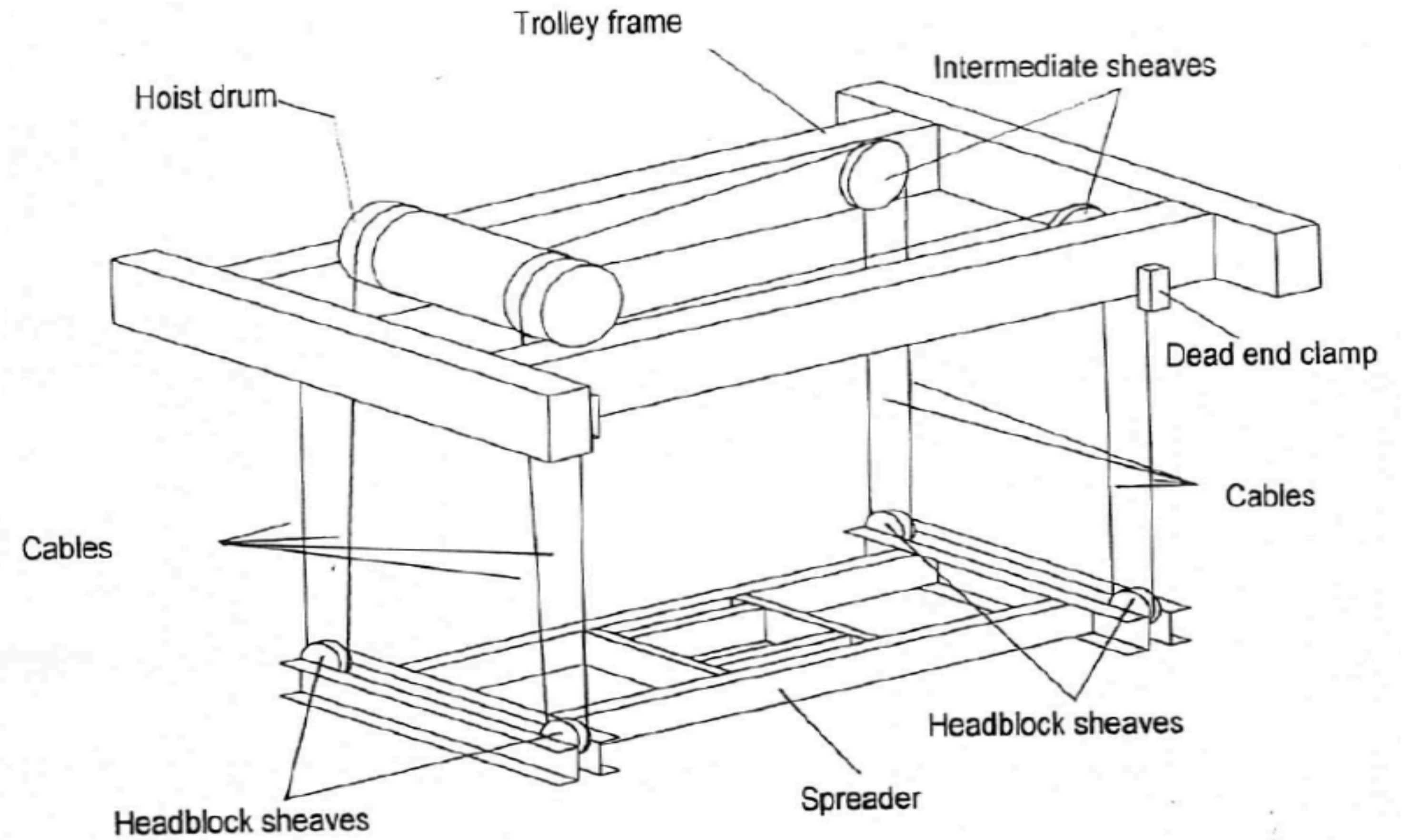


DIMENSIONS (mm) - typical

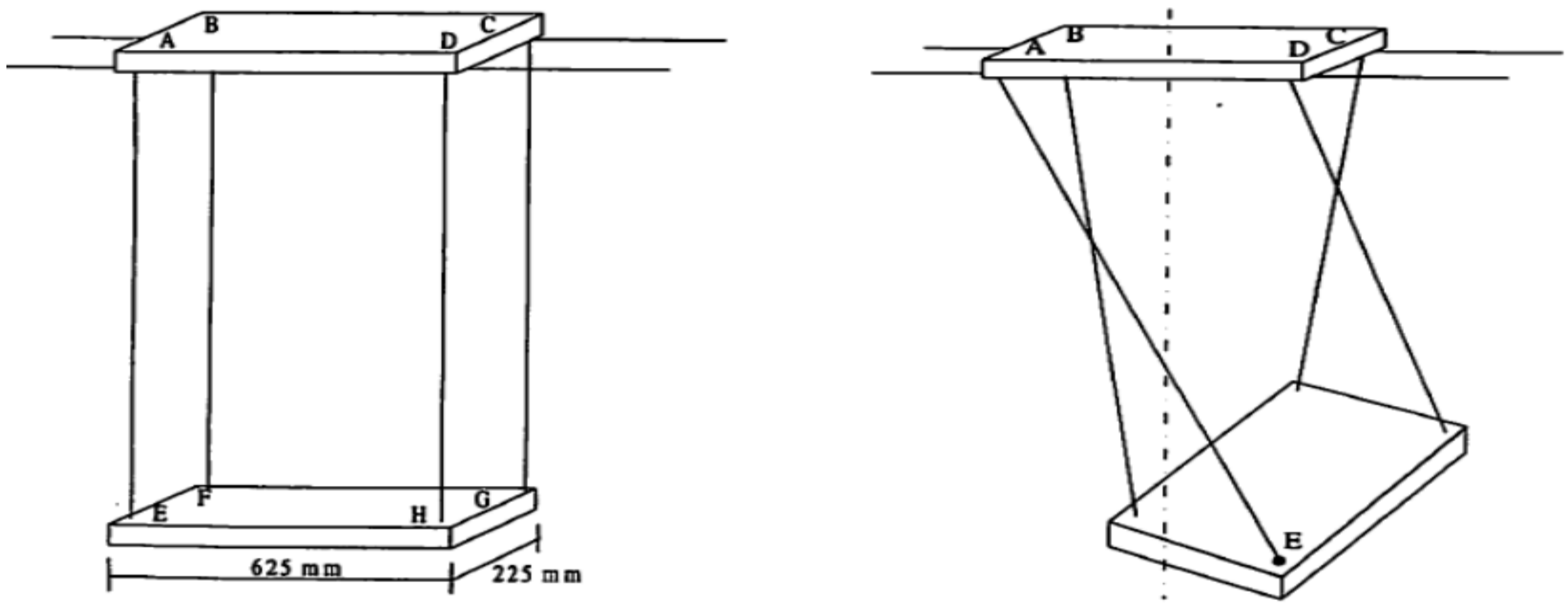
	7+1 Wide	6+1 Wide	
A	19354	16612	Trolley Travel
B	23977	21234	Inside Clear Width

	1 over 6	1 over 5	1 over 4	
C	20600	17704	14808	Under Spreader
D	22429	19533	16637	Underside Top Beam
E	25553	22657	19761	Overall Height

This is a side elevation of the RTG crane, showing the engine bay and the side structure. These vehicles are very large, > 25 m in overall height, and the spreader suspension cabling is highly susceptible to the effects of cross-winds, leading to significant 'sway' of the spreader and the container being hoisted. This can seriously affect the driver's ability to stack containers accurately and quickly. So, controlling the hoist dynamics of an RTG crane is an important aim. The dynamics are highly nonlinear, but can be encoded accurately in differential equation form. This is a good application for *feedback linearisation* control.

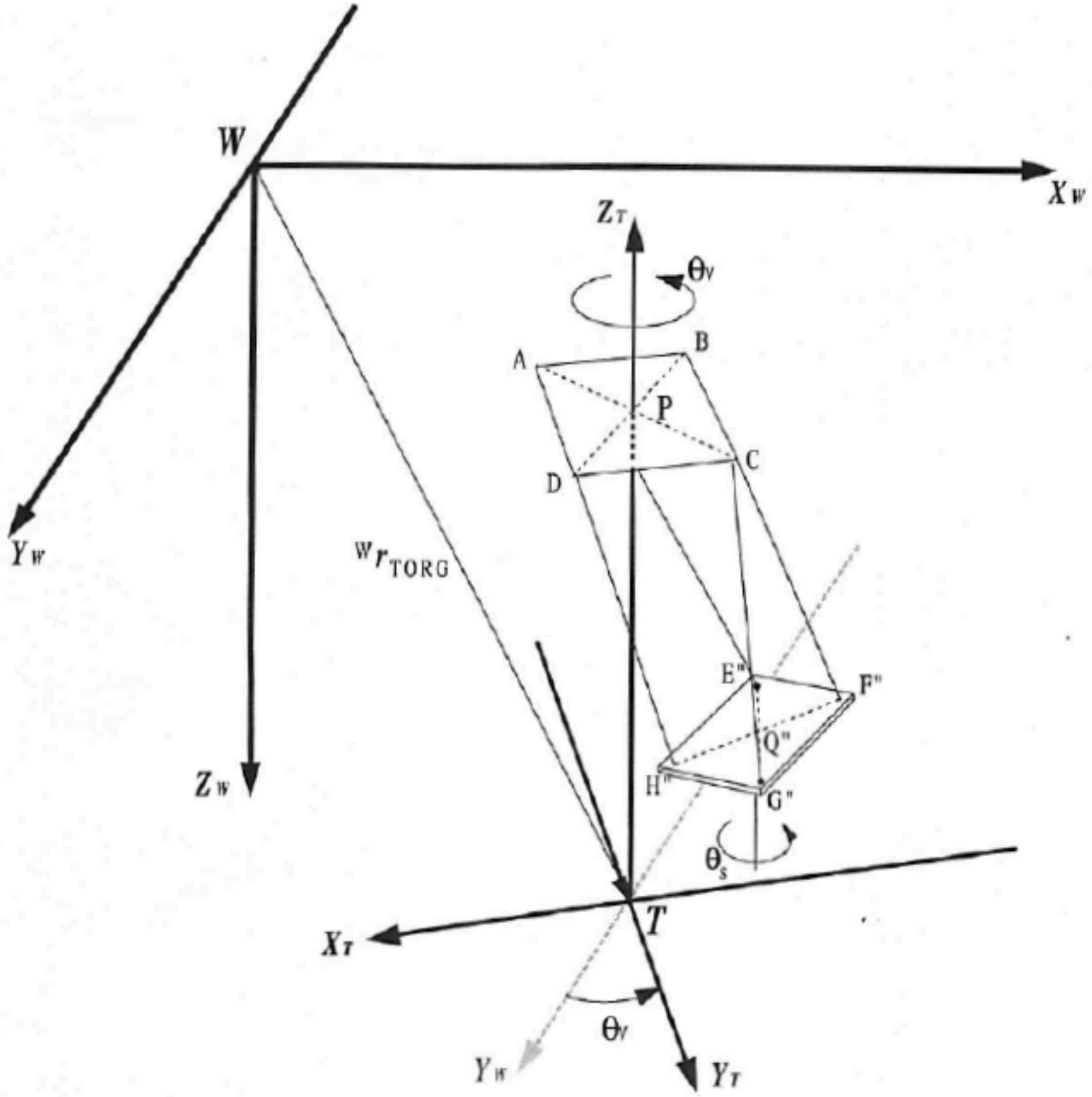


This graphic gives a good idea of the cabling typically used in an RTG spreader suspension system. The trolley frame is a fabricated structure that runs along the top of the crane, laterally, from side to side. It contains the hoisting drums and associated sheaves (pulleys) and drive motors. The spreader is the upper part of the fabricated structure that sits onto the top of the container. The spreader has corner ‘twistlocks’ that lock into holes in the corners of the top of the container. This allows the spreader to lock on to the container to hoist it up.

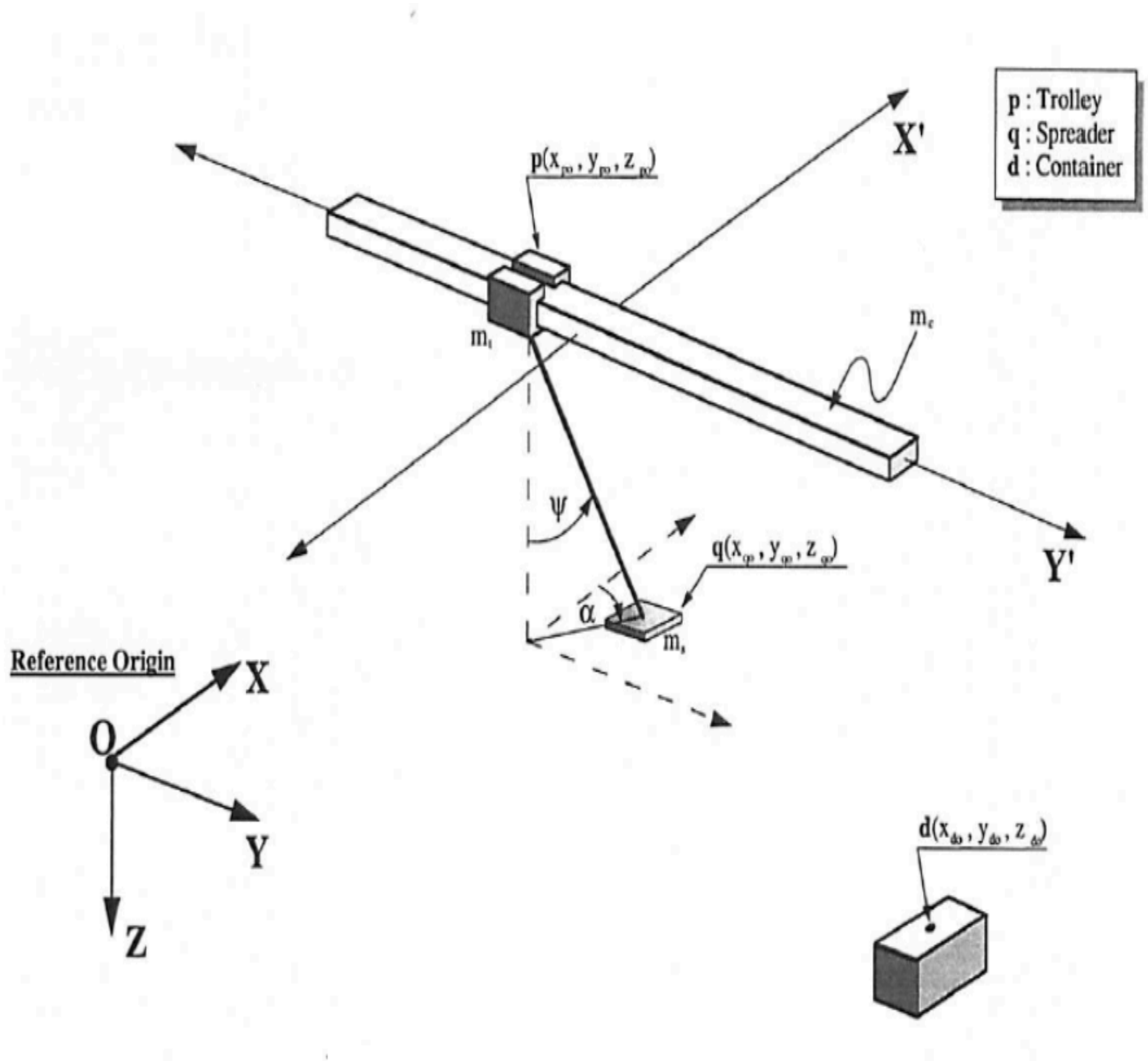


The dimensions shown above are to 1/8 scale with respect to a typical RTG crane

These graphics show a simplified geometry for a scale model of an RTG crane trolley and spreader suspension system. In this case the scale is 1/8 and the four pairs of parallel cables in each corner are replaced with four single cables. The global geometrical behaviour of this configuration is very close to the real thing but a little simpler to analyse. This configuration can be used to build up an understanding of the geometry and hence the kinematics of the suspension. This is needed to be able to get representative differential equations of motion for the vehicle.



This is an important diagram. It contains a schematic of the general geometry of the key parts of the crane. Coordinate frame $WX_VY_VZ_V$ is a vehicle-fixed frame and the main frame of reference on the machine. The other frame $TX_TY_TZ_T$ is attached to the trolley, (which actually moves along a very constrained path along the top of the vehicle from side to side). In this theoretical configuration any position of the trolley relative to the vehicle-fixed frame is permitted (through position vector $\mathbf{W}_{rTorg}$). In practice it is much more limited. The spreader can rotate relative to the trolley, and in principle at least the trolley can rotate and translate relative to the vehicle. This sort of schematic allows the engineer to start to think clearly about the possible motions within the system, and what the practical constraints are, and the to assign generalised coordinates.



We can use the previous schematic representation of the geometry of the vehicle to define a more practical set-up; one based on real-life constraints. Here we see that the trolley moves laterally along a simplified top-beam, and we start with the simplest flexible suspension system of all, a single cable pendulum suspension. We also include the target, the container onto which we want the spreader to align with, as quickly and directly as possible. In this simplest case we have identified three generalised coordinates for the spreader relative to the trolley:

Length of suspension cable l, swing angle Ψ, and polar spreader coordinate α.

We also have coordinates x and y. These are, respectively, the fore-aft motion of the vehicle itself (in the X direction) and the combined lateral motion of the trolley and vehicle in the Y direction. So, the minimal set of generalised coordinates for this problem is l, Ψ, α, x, y.

Differential equations of motion for the simplified ‘single cable’ RTG crane dynamics. You can see the five generalised coordinates here and also how they are coupled. This is a highly nonlinear problem. If we were to linearise this by removing the nonlinear terms we would lose ALL the dynamics, and so the control problem would be completely trivialised.

$$\left(1+\frac{m_t+m_c}{m_s}\right)\ddot{x} + (\ddot{l}\sin\psi\cos\alpha + l\ddot{\psi}\cos\psi\cos\alpha - l\ddot{\alpha}\sin\psi\sin\alpha - l\dot{\psi}^2\sin\psi\cos\alpha$$

$$-l\dot{\alpha}^2\sin\psi\cos\alpha + 2\dot{l}\dot{\psi}\cos\psi\cos\alpha - 2\dot{l}\dot{\alpha}\sin\psi\sin\alpha - 2l\dot{\alpha}\dot{\psi}\cos\psi\sin\alpha) = \frac{F_x}{m_s}$$

$$\left(1+\frac{m_t}{m_s}\right)\ddot{y} + (\ddot{l}\sin\psi\sin\alpha + l\ddot{\psi}\cos\psi\sin\alpha + l\ddot{\alpha}\sin\psi\cos\alpha - l\dot{\psi}^2\sin\psi\sin\alpha$$

$$-l\dot{\alpha}^2\sin\psi\sin\alpha + 2\dot{l}\dot{\psi}\cos\psi\sin\alpha + 2\dot{l}\dot{\alpha}\sin\psi\cos\alpha + 2l\dot{\psi}\dot{\alpha}\cos\psi\cos\alpha) = \frac{F_y}{m_s}$$

$$\ddot{l} + \ddot{x}\sin\psi\cos\alpha + \ddot{y}\sin\psi\sin\alpha - l\dot{\psi}^2 - l\dot{\alpha}^2\sin^2\psi + g(1-\cos\psi) = \frac{F_l}{m}$$

$$l\ddot{\psi} + \ddot{x}\cos\psi\cos\alpha + \ddot{y}\cos\psi\sin\alpha + 2\dot{l}\dot{\psi} - l\dot{\alpha}^2\sin\psi\cos\alpha + g\sin\psi = 0$$

$$(l\sin\psi)^2\ddot{\alpha} + (-l\sin\psi\sin\alpha)\ddot{x} + (l\sin\psi\cos\alpha)\ddot{y} + 2\dot{\alpha}l\dot{l}\sin^2\psi + 2\dot{\alpha}l^2\dot{\psi}\sin\psi\cos\psi = 0$$

Five nonlinear ODEs representing the simplified dynamics of the Crane, assuming a single cable suspension system for the spreader.

Three of the Coordinates are actuated (x, y, l) and two are un-actuated (α, ψ).

There is cross-coupling between all five coordinates.

The questions are:

1. Can classical linear control theory be used for a system like this?
2. If not, how do we control a nonlinear system of this sort?
3. Also, how do we deal with the fact that it is under-actuated?

Conclusions: There is no way that linear classical control can be used on this problem without a great deal of simplification, and consequent loss of information. We could use several nonlinear control techniques: fuzzy logic, neural networks, H_∞, feedback linearisation, sliding mode control, and centre manifold control. We will choose feedback linearisation as a good candidate control methodology for this system.

Feedback Linearisation is a useful technique for nonlinear copled systems such as this.
We generate a torque control vector which is a function of desired and actual states, and then use this to create an equivalent system in the form of a simple closed loop linear model.

The first step is to take the governing equations of motion (in whatever form they may take) and write them in matrix form:

$$[D]\ddot{\underset{\sim}{q}}+C(\underset{\sim}{q},\dot{\underset{\sim}{q}})\dot{\underset{\sim}{q}}+f(\underset{\sim}{q})=\underset{\sim}{\tau}$$

$[D]$ is a configuration dependent inertia matrix, C contains centripetal and Coriolis accelerations (here), and $f(q)$ represents gravitational PE terms.

You can see that this implementation of feedback linearisation is slightly more complicated than we used for the parametrically excited pendulum, by the presence of the term $C(q,\dot{q})\dot{q}$. This term contains nonlinear terms which are functions of both displacement and velocity. The term $f(q)$ contains configurational nonlinearities only.

So this is what we get:

$$\underset{\sim}{q} = \begin{bmatrix} x \\ y \\ l \\ \psi \\ \alpha \end{bmatrix}, \qquad \underset{\sim}{\dot{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{l} \\ \dot{\psi} \\ \dot{\alpha} \end{bmatrix}, \qquad \underset{\sim}{\ddot{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{l} \\ \ddot{\psi} \\ \ddot{\alpha} \end{bmatrix},$$

$$[D]=\begin{bmatrix} 1+\dfrac{m_t+m_c}{m_s} & 0 & \sin\psi\cos\alpha & l\cos\psi\cos\alpha & -l\sin\psi\sin\alpha \\ 0 & 1+\dfrac{m_t}{m_s} & \sin\psi\sin\alpha & l\cos\psi\sin\alpha & l\sin\psi\cos\alpha \\ \sin\psi\cos\alpha & \sin\psi\sin\alpha & 1 & 0 & 0 \\ l\cos\psi\cos\alpha & l\cos\psi\sin\alpha & 0 & l^2 & 0 \\ -l\sin\psi\sin\alpha & l\sin\psi\cos\alpha & 0 & 0 & (l\sin\psi)^2 \end{bmatrix}$$

$$C(\underset{\sim}{q},\underset{\sim}{\dot{q}}) =$$

$$\begin{bmatrix} 0 & 0 & 2\dot{\psi}\cos\psi\cos\alpha - 2\dot{\alpha}\sin\psi\sin\alpha & -l\dot{\psi}\sin\psi\cos\alpha & -l\dot{\alpha}\sin\psi\cos\alpha - 2l\dot{\psi}\cos\psi\sin\alpha \\ 0 & 0 & 2\dot{\psi}\cos\psi\sin\alpha + 2\dot{\alpha}\sin\psi\cos\alpha & -l\dot{\psi}\sin\psi\sin\alpha & -l\dot{\alpha}\sin\psi\sin\alpha + 2l\dot{\psi}\cos\psi\cos\alpha \\ 0 & 0 & 0 & -l\dot{\psi} & -l\dot{\alpha}\sin^2\psi \\ 0 & 0 & 2l\dot{\psi} & 0 & -l^2\dot{\alpha}\sin\psi\cos\psi \\ 0 & 0 & 0 & 0 & l\sin\psi(2\dot{l}\sin\psi + 2l\dot{\psi}\cos\psi) \end{bmatrix}$$

$$f(\underset{\sim}{q}) = \begin{bmatrix} 0 \\ 0 \\ g(1-\cos\psi) \\ gl\sin\psi \\ 0 \end{bmatrix} \qquad \underset{\sim}{\tau} = \begin{bmatrix} \frac{F_x}{m_s} \\ \frac{F_y}{m_s} \\ \frac{F_l}{m_s} \\ 0 \\ 0 \end{bmatrix}$$

The torque vector shows that there are three actuated coordinates and two that are not.

Note that the notation in the analysis that follows is a little different to what we used for the parametrically excited pendulum, to keep it in line with typical notation used for this type of industrial control problem.

$$\underset{\sim}{\tau} = [D]\,\underset{\sim}{V} + C(\underset{\sim}{q}, \underset{\sim}{\dot{q}})\,\underset{\sim}{\dot{q}} + f(\underset{\sim}{q})$$

The actual 'control' is denoted by $\underline{V}$. If we substitute $\underline{\tau}$ into the RHS of the general matrix form of the system dynamics, which was:

$$[D]\,\underset{\sim}{\ddot{q}} + C(\underset{\sim}{q}, \underset{\sim}{\dot{q}})\,\underset{\sim}{\dot{q}} + f(\underset{\sim}{q}) = \underset{\sim}{\tau}$$

then we get this:

$$[D]\,\underset{\sim}{\ddot{q}} = [D]\,\underset{\sim}{V}$$

Since it is reasonable to assume that the inertia matrix $[D]$ is invertible then we find that: $\underset{\sim}{\ddot{q}} = \underset{\sim}{V}$ Equation (A)

The ‘desired’ quantities below are the set-points, and the actual quantities are the responses of the generalised coordinates. Note that we use q as the generalised vector of ccordinates, as this is a multi-degree of freedom problem.

The controller is defined by the vector $\underline{V}$ where this is usually stated in terms of gain-controlled *actual* quantities and gain controlled *desired* quantities, like this:

$$\underset{\sim}{V} = -\{h\}\dot{\underset{\sim}{q}} - \{g\}\underset{\sim}{q} + r \qquad \text{Equation (B)}$$

where: $r = \ddot{\underset{\sim}{q}}_d + \{h\}\dot{\underset{\sim}{q}}_d + \{g\}\underset{\sim}{q}_d$

The {h} and {g} are diagonal matrices for velocity and position gains:

$$\{h\} = \begin{bmatrix} h_x & 0 & 0 & 0 & 0 \\ 0 & h_y & 0 & 0 & 0 \\ 0 & 0 & h_l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \{g\} = \begin{bmatrix} g_x & 0 & 0 & 0 & 0 \\ 0 & g_y & 0 & 0 & 0 \\ 0 & 0 & g_l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Inserting equation (B) into (A) leads to:

$$\underset{\sim}{\ddot{q}} + \{h\}\underset{\sim}{\dot{q}} + \{g\}\underset{\sim}{q} = r \qquad r = \underset{\sim}{\ddot{q}_d} + \{h\}\underset{\sim}{\dot{q}_d} + \{g\}\underset{\sim}{q_d}$$

By combining these two equations and defining the tracking error as the difference between the desired and actual values then we finally arrive at a simple error equation, as below:

$$\underset{\sim}{\varepsilon} = \underset{\sim}{q_d} - \underset{\sim}{q}$$

$$(\underset{\sim}{\ddot{q}_d} - \underset{\sim}{\ddot{q}}) + \{h\}(\underset{\sim}{\dot{q}_d} - \underset{\sim}{\dot{q}}) + \{g\}(\underset{\sim}{q_d} - \underset{\sim}{q}) = \underset{\sim}{\ddot{\varepsilon}} + \{h\}\underset{\sim}{\dot{\varepsilon}} + \{g\}\underset{\sim}{\varepsilon} = 0$$

So, we've ended up with the error equation again, in exactly the same form that we had for the parametrically excited pendulum. The next step is to implement this controller and test it out in practice. This will be continued in Part 2.