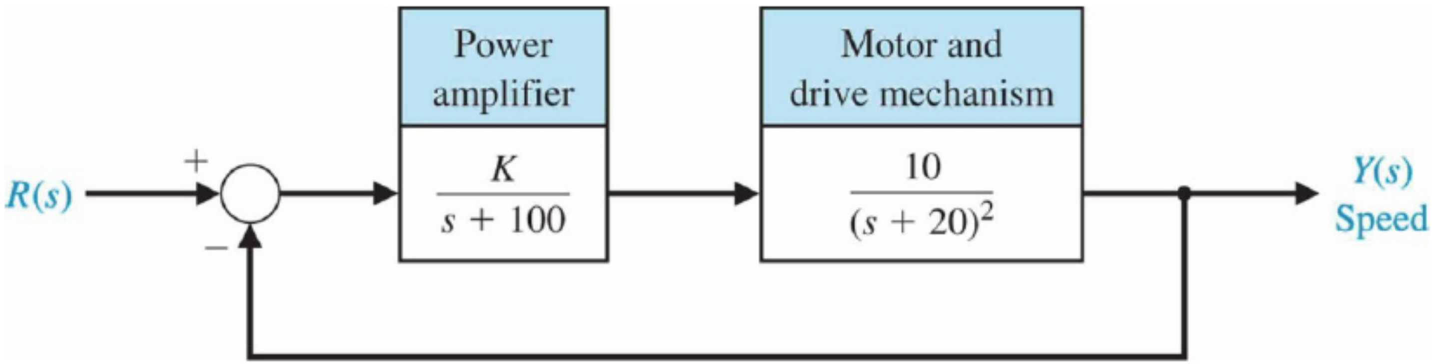
Routh-Hurwitz Array Calculations – effect of gain on the roots of characteristic equation

Solution

Q1. A magnetic tape drive has been designed for back-up mass-storage. It is necessary to control the velocity of the tape accurately. The speed control of the tape drive is represented by the system shown in Figure 1.

Determine the limiting gain for a stable system and consider the effect of the sign and the magnitude of the gain on the roots – and what this means for the overall qualitative response of the system.



(a) The overall system transfer function is given by:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{10K}{(s+100)(s+20)^2\left(1 + \frac{10K}{(s+100)(s+20)^2}\right)}$$

Leading to:

$$T(s) = \frac{10K}{(s+100)(s+20)^2\left(\frac{(s+100)(s+20)^2 + 10K}{(s+100)(s+20)^2}\right)}$$

Which simplifies to:

$$T(s) = \frac{10K}{(s+100)(s+20)^2 + 10K} = \frac{10K}{(s+100)(s^2+40s+400) + 10K}$$

So the denominator, which forms the characteristic equation, is:

$$s^3 + 140s^2 + 4400s + 40000 + 10K = 0$$

The Routh-Hurwitz array is:

$$\begin{array}{lcc} s^3 & 1 & 4400 \\ s^2 & 140 & 40000 + 10K \\ s^1 & b & \\ s^0 & 40000 + 10K & \end{array}$$

where

$$b = \frac{140(4400) - (40000 + 10K)}{140}$$

Examining the first column and requiring all the terms to be positive, we see that the system is stable if $-4000 < K < 57600$.

Roots - if we start by taking $K = -4000$ we get the following roots:

```
K = -4000
Solve[(s^3) + (140 * (s^2)) + (4400 * s) + 40 000 + (10 * K) == 0, s]
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-4000

$\{\{s \to 0\}, \{s \to 10 \left(-7 - \sqrt{5}\right)\}, \{s \to 10 \left(-7 + \sqrt{5}\right)\}\}$

The roots are real and unequal for $K = -4000$ so this value of K predicts an over-damped system.

If we then take $K = 20000$ then we get a different situation:

```
K = 20000
Solve[(s^3) + (140 * (s^2)) + (4400 * s) + 40000 + (10 * K) == 0, s]
Simplify[%]
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20000

$\{\{s \to -120\}, \{s \to 10\,(-1 - i\sqrt{19})\}, \{s \to 10\,(-1 + i\sqrt{19})\}\}$

$\{\{s \to -120\}, \{s \to -10 - 10\,i\sqrt{19}\}, \{s \to 10\,i\,(i + \sqrt{19})\}\}$

The roots are real and complex conjugates so we are transiting to an under-damped system.