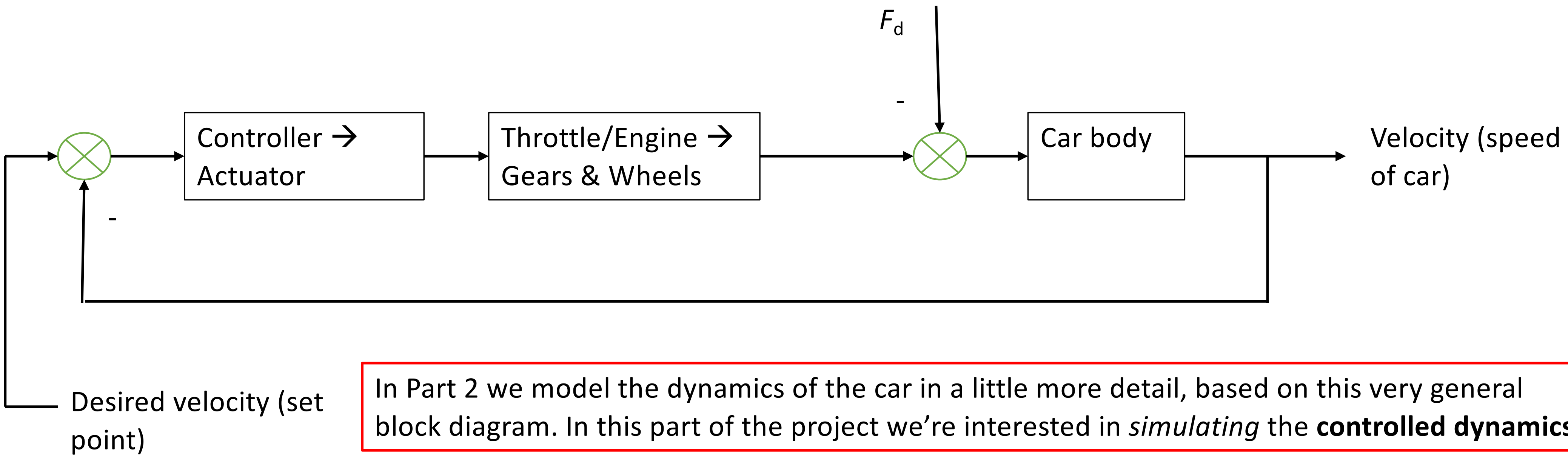
ME528 Control Systems Design

Research Project – Part 2

Automotive Cruise Control - Dynamic Analysis



In Part 2 we model the dynamics of the car in a little more detail, based on this very general block diagram. In this part of the project we're interested in *simulating* the **controlled dynamics**.

Starting with the car as a whole again, we can put,

$$m\ddot{x} = m\dot{v} = F - F_d$$

where F is the traction force which is developed by the engine and transmitted through the transmission (gears) to the wheels, and F_d is a composite term containing all the disturbance forces. These include the slope of the road, a gravitational force defined by F_g and a drag force due to rolling resistance defined by F_r, and an aerodynamic damping term given by F_a. So,

$$F_d = F_g + F_r + F_a$$

We already know that the slope of the road generates a gravitational term, as in Part 1, and we re-use that concept here again, so,

$$F_g = mg\sin\theta$$

Now we introduce the rolling resistance, and this is conventionally defined by the following,

$$F_r = mgC_r\mathrm{sgn}(\dot{x}) = mgC_r\mathrm{sgn}(v)$$

Finally we have the aerodynamic damping, and this is given by,

$$F_a = \frac{1}{2}\rho C_d A|\dot{x}|\dot{x} = \frac{1}{2}\rho C_d A|v|v$$

We'll return to the disturbance forces later – noting that the simple linear damping term that we used in Part 1 to cover all the various dissipative terms is now a more complicated term, represented by $F_r + F_a$.
The next thing is to consider the traction force F, and we deal with this by assuming that our car is driven by an IC engine and that the torque-speed characteristics of an IC engine can be quite adequately represented by,

$$T(\Omega) = T_m\left(1 - \beta\left(\frac{\Omega}{\Omega_m} - 1\right)^2\right)$$

where T_m is the maximum torque available at engine speed Ω_m (noting that Ω_m may not be the actual maximum speed that the engine is capable of and in fact may be a little lower than this), and β is a characteristic of the specific engine.

Typical values for a saloon car are:

$T_m = 190\ Nm$ (140.1 lbft)
$\Omega_m = 420\ rad/s$ (4010.7 rpm)
$\beta = 0.4$

Now, we need to introduce the gearing of the car and also the wheel radius to relate the engine speed to the vehicle speed. This is done by putting,

$$\Omega = \frac{n}{r}v = \alpha_n v$$

where n is the gear ratio and r is the wheel radius (excluding the tyre tread profile).

Now, we recall that $u(t)$ is the control input, by general definition. Here we'll think of it as an actuation parameter controlling the torque, so that the engine traction force is now defined by,

$$F = \left(\frac{n}{r}\right) u(t) T(\Omega)$$

This works because $\frac{n}{r} = \frac{\Omega}{v}$ (see previous slide) and so, after rearrangement, the equation above reads $F = \frac{uT\Omega}{v}$ (dropping the arguments for simplicity), so $Fv = uT\Omega$. We know from first principles that 'force x translational velocity' and 'torque x angular velocity' are both measures of power, and noting that the u is introduced as a proportionality controlling the magnitude of the traction force via the torque, then we see that both sides balance.

The traction force is often expressed using the α_n quantity which we introduced in the previous slide, and if restoring the argument for the torque we also tend to use that quantity so that the vehicle velocity appears explicitly, like this,

$$F = \alpha_n u T(\alpha_n v)$$

It can be very instructive to plot graphs of engine torque against engine (crankshaft) speed and engine torque against road speed for different gear ratios. We can also substitute the last equation for traction force into our general dynamic equation for the car (slide 2) to get,

$$m\dot{v} = \alpha_n u T(\alpha_n v) - mgC_r \operatorname{sgn}(v) - \frac{1}{2}\rho C_d A|v|v - mg\sin\theta$$

where $T(\alpha_n v) = T(\Omega) = T_m\left(1 - \beta\left(\frac{\Omega}{\Omega_m} - 1\right)^2\right)$. The car's velocity v is the output and is also the 'state' of the system.

Possible gear ratios, defined generally by α_n, could be $\alpha_1 = 40, \alpha_2 = 25, \alpha_3 = 16, \alpha_4 = 12, \alpha_5 = 10$.

In terms of controller effectiveness we know from Part 1 that PI control can provide a necessary and sufficient control for the simpler linearised system, so it is worth starting with this control here too.

So, we have,

$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt$$

where $e(t) = v_r - v$.

Now, a controller can be seen as a dynamic system in its own right, and in that context we think of the controller's 'state', z, which is effectively the integral of the error, so we get this,

$$\frac{dz}{dt} = \dot{z} = e(t) = v_r - v$$

So we can re-state the controller in terms of the controlled velocity, the set-point, the controller state, and the two PI constants, as shown,

$$u = k_p(v_r - v) + k_i z$$

In full we have this,

$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt = k_p(v_r - v) + k_i z$$

Now we can start to assemble some plausible data for a simulation.

m can be typically somewhere between 750 kg and 1250 kg [1000 kg to start]
α_n depends on the gear ratio: 40, 25, 16, 12, 10 [16 chosen for *n*=3]
k_p and k_i set to 0.5 and 0.1 respectively.
T_m can typically be around 190 Nm.
Ω_m is therefore around 420 rad/s, and engine speed is $\Omega = 400$ rad/s.
β = 0.4, and set-point speed v_r set to 20 m/s (44.7 mph, 72 kph).
g = 9.81 m/s^2.
C_r = 0.01, $C_d = 0.32$.
ρ = 1.3 kg/m^3.
A is typically around 2.4 m^2.
θ is set for a flat road at 0°.

So all is in place and we could solve the system dynamics equation (in red on slide 4) by building a *Simulink* simulation. We can plot time responses of the output velocity, v, and also the controller state, z, and then plot the ***phase space*** defined by (v, z).

Using the data on the previous slide allows us to plot the phase space for the combined dynamical systems of the car and the controller. This is a useful technique for exploring some aspects of stable operation, as it accommodates the dynamics of the linear controller and those of the nonlinear car.

Once the phase space has been plotted you'll be able to see that there is a very obviously defined *equilibrium point* which we can denote symbolically as (v_e, z_e). Referring to the time responses will greatly aid your understanding of where this point must be in the phase space.

It's also instructive to change gear, n, and see how the phase space changes, and then plot the phase space for the car reaching the set-point velocity in all five gears (by showing the phase for all the gears on one plot).

It's important to understand that what we have here is a simulation of the dynamics of a linear PI controller controlling a nonlinear system (the car) and that this is not the same thing as building a control system based on transfer functions from linearised dynamics. We note that the phase space in this context is represented by the controller state z plotted as a function of car (output) velocity v. The phase space shows you that the set-point can be reached for a specific controller state, according to the gear that the car is in.

We should also note that we have deliberately not introduced saturation into the PI controller, so as to explore the phase space without physical restrictions. As we noted in Part 1, in practice most controllers employ some form of saturation to keep them operating within a realistic physical space.

Finally, it is interesting and potentially very useful to examine the phase space for other combinations of data, to see what happens when the controller is pushed hard by setting challenging dynamic conditions for the car.

Instructions:

1. Use Simulink to build a simulation of the dynamics of the nonlinear differential equation in v shown in red on slide 4. You should now be proficient enough in Simulink having completed Part 1 to be able to do this unaided, and to generate time response plots of the output velocity v and the controller state z, and then a plot of the phase space, parameterised by (z, v). Set the simulation up so that you can vary the velocity set-point, and assign values to all the system constants that you can conveniently change. Use the data on slide 6 to plot the phase space for third gear ($n = 3$).

2. Determine the equilibrium point for the system. Use the time responses to get an idea of what this is in practice, and then locate it on the phase space plot. Then plot the phase space for all five gear ratios on one graph, and after that repeat these five simulations for an incline. You should then have two graphs, each containing five phase plots ($n = 1, 2, .., 5$) of the controlled dynamics of the car, first running on a flat road and then running up an incline. Given the insights that you've developed during your project so far, consider **two more** interesting scenarios of your own choice and then plot two more graphs of the phase space (each graph showing plots for all five gears). Explain why you chose these scenarios.

3. Then, discuss your findings from the phase space plots, referring to the time responses as well if that is helpful.

4. Now go back to the equation in red on slide 4 and re-write this as a *second order* differential equation where the dependent variable is no longer velocity v but instead it is displacement x. This is the differential equation of the nonlinear dynamics of the car, *in terms of its displacement*, and it is still under the influence of the engine torque T and the applied PI control u. By considering the problem as one of feedback linearisation, proceed to derive the expression for the control torque τ_c. Show that by applying this control torque to the system the dynamics then degenerate into a homogeneous differential equation in terms of the error, and proceed to obtain this error equation and then also simulate the *control torque* and the *error in velocity* in Simulink, and output them.

Instructions:

5. Now return to the lecture and tutorial material supplied on MyPlace for the whole of ME528 and write a further critique of how your project has applied the remainder of these topics. Discuss what you would have to do to the project problem statement to apply all of it.

6. Write up your work so far so that you have a formal record of all the work done for Part 2 of the project. You should not produce more than 15 A4 pages for Part 2.

7. Put the two parts of the project together, making sure that you edit them into one continuous document, integrating the numbering and captioning of diagrams, graphs, tables, and references. Make sure that the report conforms to Departmental norms for project reports, starting with an abstract and finishing with a conclusions section and references.