A summary of the Root Locus Method for defining stability

The angle condition is generally defined by:

$$\angle F(s) = (2n + 1)\pi = -1$$

defines that a point of interest lies on the Root Locus (i.e. that it is a root of the characteristic equation for the system you are interested in.

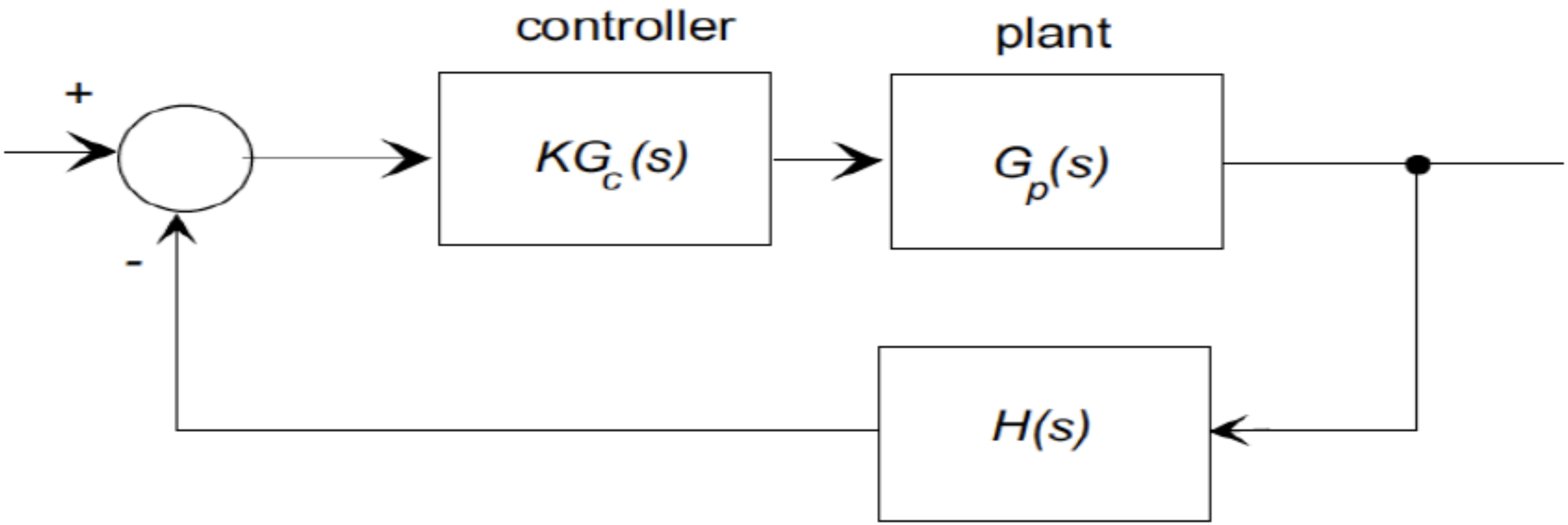
The magnitude condition is expressed as:

$$K = \frac{1}{|G(s)|}$$

this being (in words) the reciprocal of the magnitude of the open loop transfer function (OLTF) of the system, where:

$$F(s) = KG(s)$$

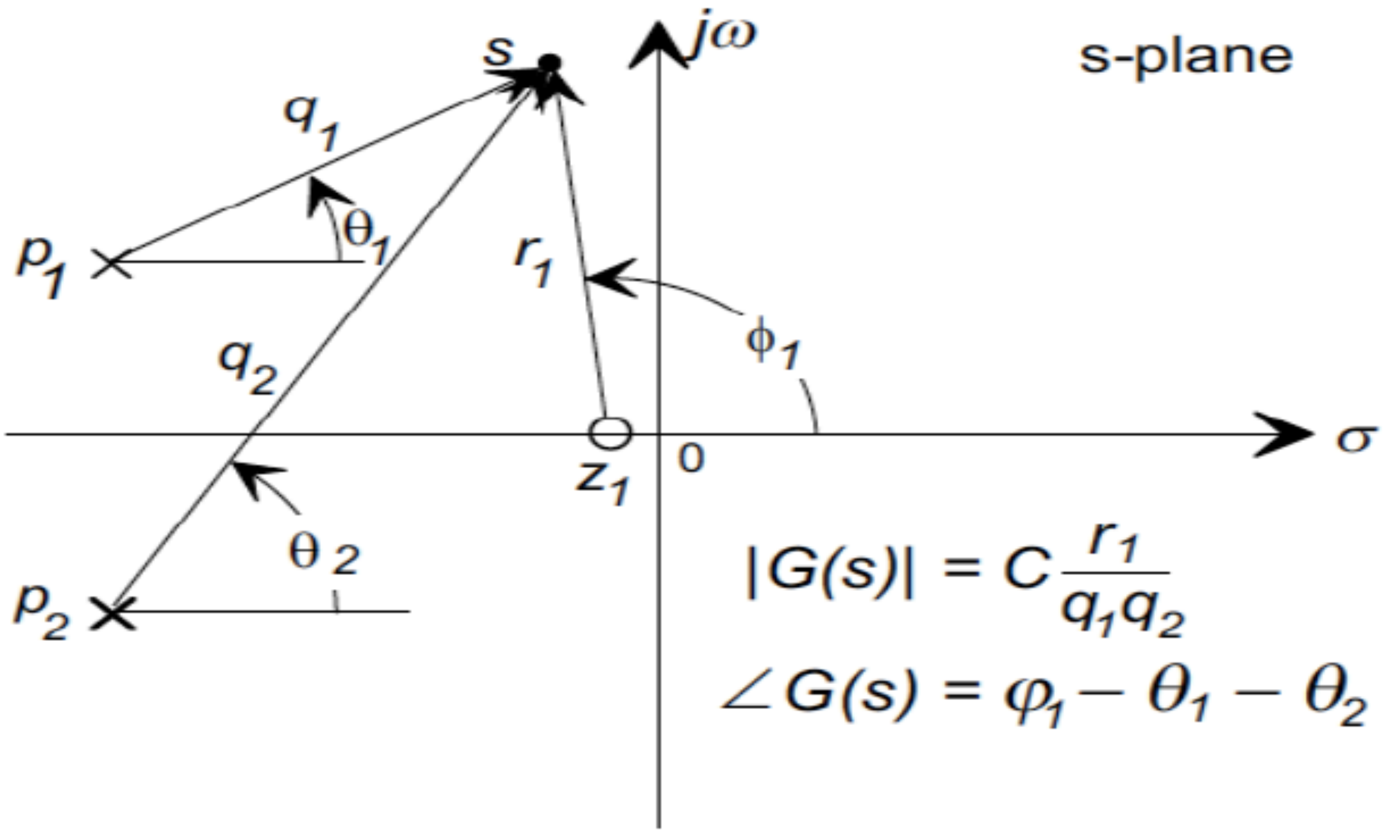
and the system is generalised as a closed loop system with negative feedback, as below, for which the OLTF is: $KG_C(s)G_P(s)H(s)$ and where $G_C(s)G_P(s)H(s) = G(s)$.



The angle condition can be used **in practice** in this form (see Lecture 7):

$$\angle G(s) = \sum_{i=1}^{m} \angle(s - z_i) - \sum_{i=1}^{n} \angle(s - p_i) = \sum_{i=1}^{m} \phi_i - \sum_{i=1}^{n} \theta_i$$

This diagram in Lecture 8 gives a very useful graphical interpretation of the above equation:



We can extract some generalisations from these bits of analysis:

- RL segments begin at a pole and end at a zero

- The number of poles generally exceeds the number of zeros for a characteristic equation

- The number of separate segments of a RL plot equals the number of poles.

The angle and magnitude conditions are based on the theory of the characteristic equation, and that the roots of this have negative real parts. This is identical to the premise on which the Routh-Hurwitz criterion is based.

The angle and magnitude conditions which underpin the RL theory suggest that the gain associated with points that can be shown to be on the RL are associated with stability – if lying on RL segments on the left hand side of the imaginary axis. See Lecture 9 for specific details of RL plots.