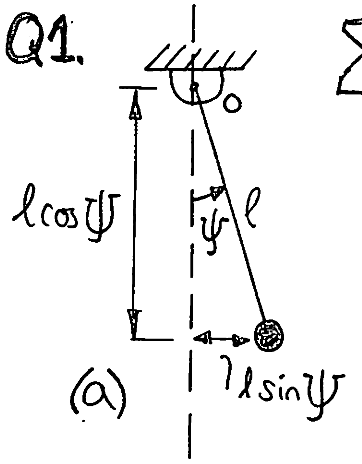
Introductory Dynamics Revision - Tutorial Sheet 1 Solutions

Q1.



(a)

$$\sum T_{\circlearrowright}: \quad \underbrace{-}_{}(mg)\, l\sin\psi = I_o\ddot{\psi} \qquad \text{where } I_o = ml^2$$

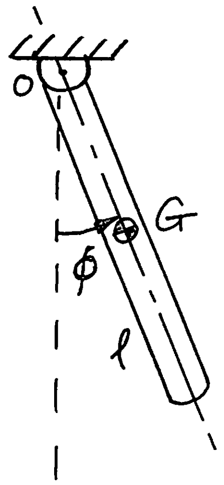
($l \equiv$ radius of gyr.)

$-$ve because this torque is due to mg and it resists motion through ψ, defined +ve as $\circlearrowright$.

So we get: $$ml^2\ddot{\psi} + mgl\sin\psi = 0$$

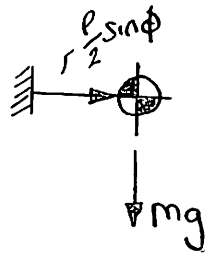
ie $$l\ddot{\psi} + g\sin\psi = 0$$

$\therefore$ we end up with: $$\ddot{\psi} + \frac{g}{l}\sin\psi = 0$$



Now, Parallel axes theorem gives:

$$I_O = I_G + m\left(\frac{l}{2}\right)^2 \quad \text{and} \quad I_G = \frac{ml^2}{12}$$



$$\sum T_{\circlearrowright}: \quad -(mg)\frac{l}{2}\sin\phi = I_o\ddot{\phi}$$

ie $$I_o\ddot{\phi} + mg\frac{l}{2}\sin\phi = 0$$

(in terms of I_o)

Alternatively we have:
(by applying // axes theorem)

$$\left(I_G + \frac{ml^2}{4}\right)\ddot{\phi} + mg\frac{l}{2}\sin\phi = 0$$

Finally if $I_G = mk^2 = m\dfrac{l^2}{12}$ (where k - given - is $\sqrt{\dfrac{l^2}{12}}$)

then we have $$I_o = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

So, now we have: $$\frac{ml^2}{3}\ddot{\phi} + mg\frac{l}{2}\sin\phi = 0$$

giving: $$\frac{l}{3}\ddot{\phi} + \frac{g}{2}\sin\phi = 0$$