Lecture 3 – Part 1

Block Diagrams

- Block Diagrams
- Generalised Solution for Feedback Systems

Open & Closed loop systems

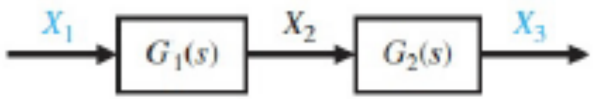
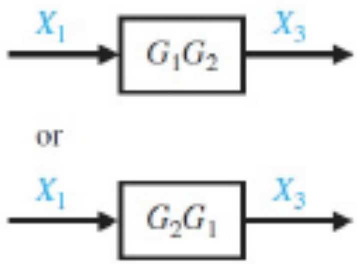
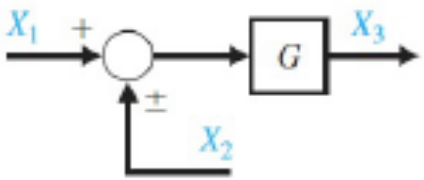
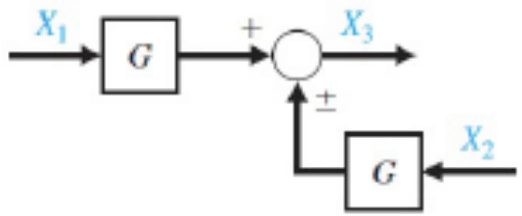
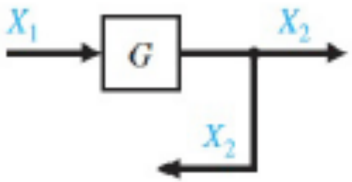
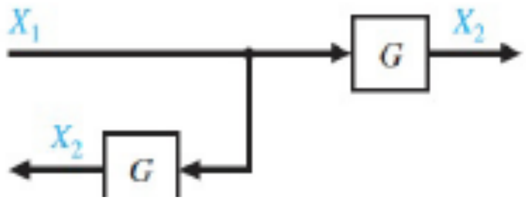
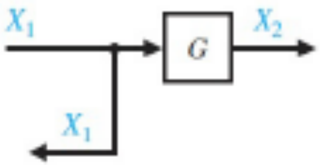
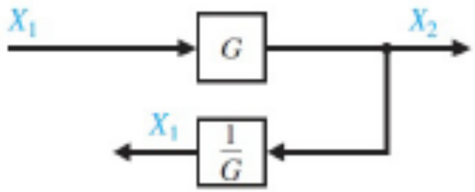
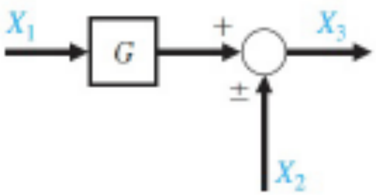
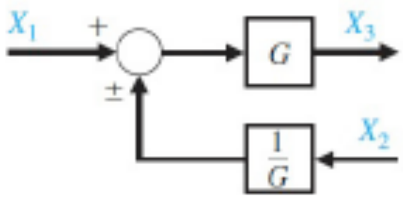
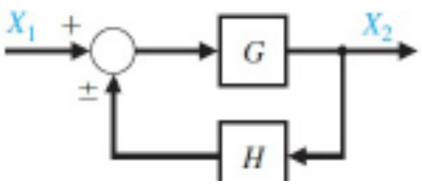
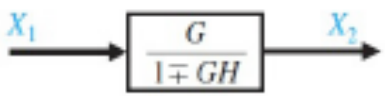
- **Open loop controllers do not contain feedback loops**, they proceed directly from input to output and rely on accurate modelling and internal callibrations in order to work properly.

- **Closed loop controllers contain at least one feedback loop.** The feedback can be negative in which case the controller is effectively driven by a difference (or error) signal. If the feedback is positive then the controller is driven by the additive signal. We can generalise for both types of feedback.

- We have examined both types of controller already - in the specific context of the DC electric motor. The *field control* variant is based on open loop control. The *armature control* variant is based on closed loop control with negative feedback.

- **All types of controller are represented by means of *block diagrams*.** It is vital that we can understand basic block diagrams and therefore build control systems from them.

- In this lecture we will look more closely at using block diagrams, and in *reduction techniques, (*as summarised in the next slide).

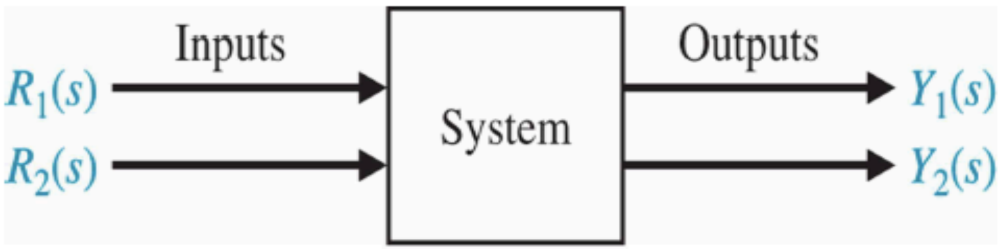
Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

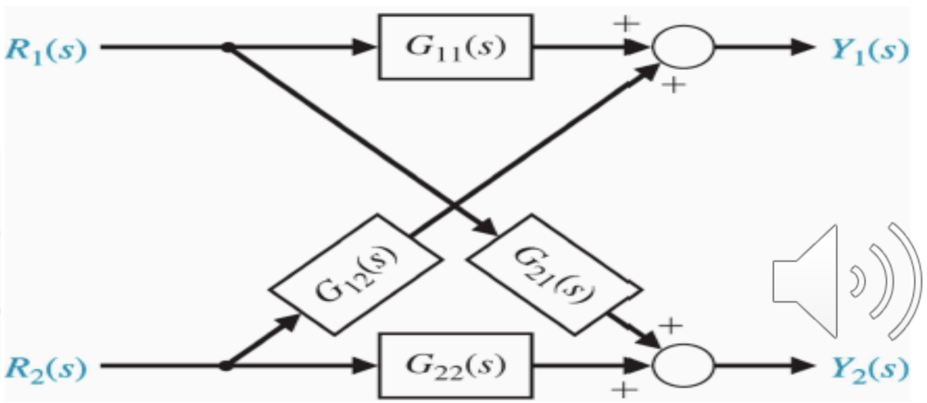
What do block diagrams actually represent?

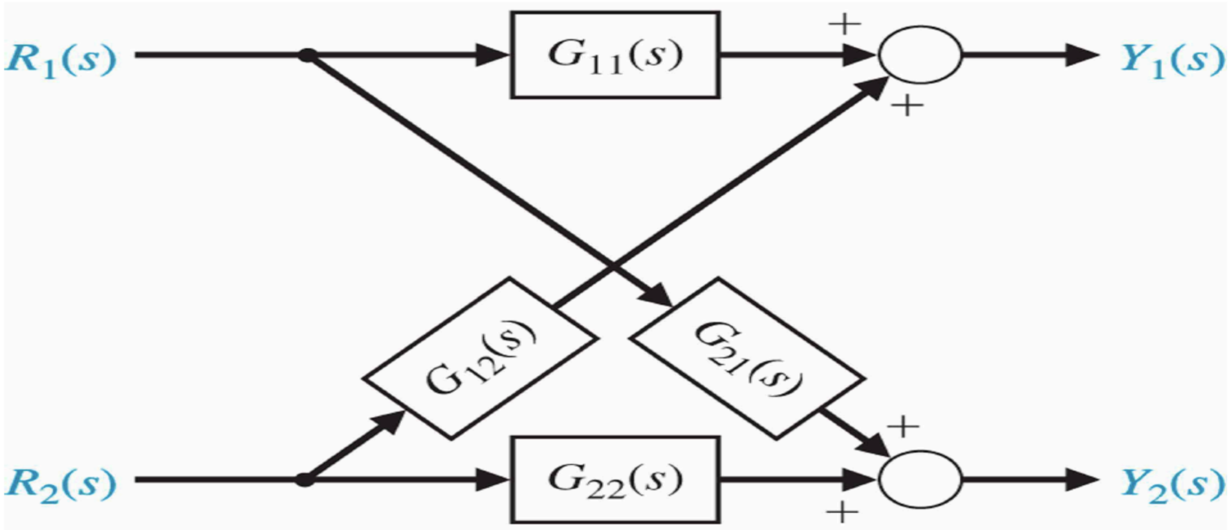
1. They show *causality* – a representation of the causes and effects embedded within the inter-relationships of the controlled variables in a system.

2. Blocks are unidirectional, operational, entities that can represent physical components, partial or even whole plants, or mathematical/computational operations. There is one thing that unifies ALL blocks and that is the Transfer Function. Each block, whatever it represents, **must** be definable mathematically by a transfer function. This is why all blocks must be unidirectional.

3. Control systems do not have to be restricted to one control variable as in the DC motor problems; they can be much more complicated than this and contain many variables which are all coupled and all controlled. Here is an example of two-input two-output system (left) and an interconnected system (right):







Re-stating the block diagram for the interconnected system again. We can extract the underlying equations directly from the block diagram. Note, $C_{1,2}$(s) replaces $Y_{1,2}$(s) here.

$$C_1(s) = G_{11}(s)R_1(s) + G_{12}(s)R_2(s)$$

$$C_2(s) = G_{21}(s)R_1(s) + G_{22}(s)R_2(s)$$

This is a two-input two-output system which can be generalised further to a J-input I-output system, for which we need to use matrix form:

$$\begin{bmatrix} C_1(s) \\ \vdots \\ C_I(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) \ldots G_{1J}(s) \\ \vdots \\ G_{I1}(s) \ldots G_{IJ}(s) \end{bmatrix} \begin{bmatrix} R_1(s) \\ \vdots \\ R_J(s) \end{bmatrix}$$

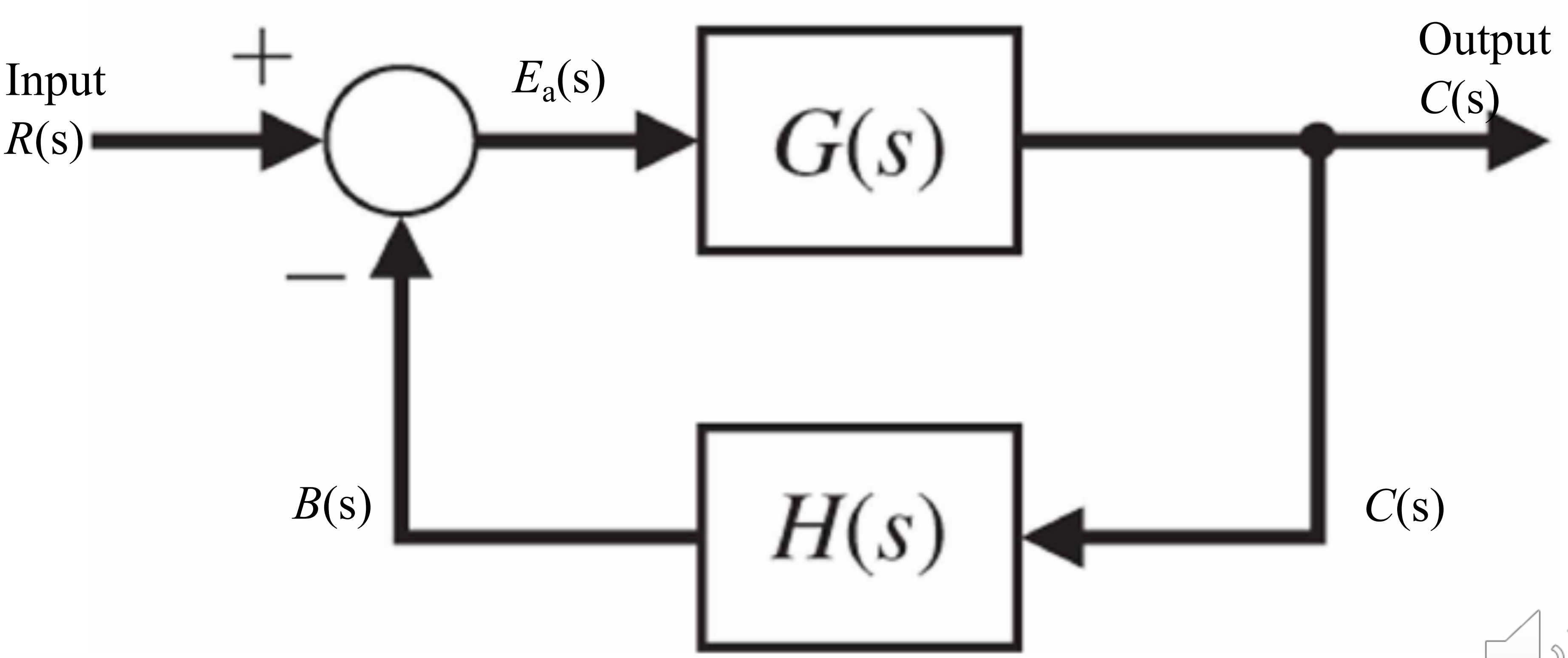
This can be simplified, without loss of generality, down to:

$$\mathbf{C} = \mathbf{G}\mathbf{R}$$

where the bold upright font represents the column vector of outputs **C**, the transfer matrix **G**, and the column vector of inputs **R**.

Generalisation of the Control System with Feedback

We start with what is familiar already and then generalise that – the closed loop controller with negative feedback.



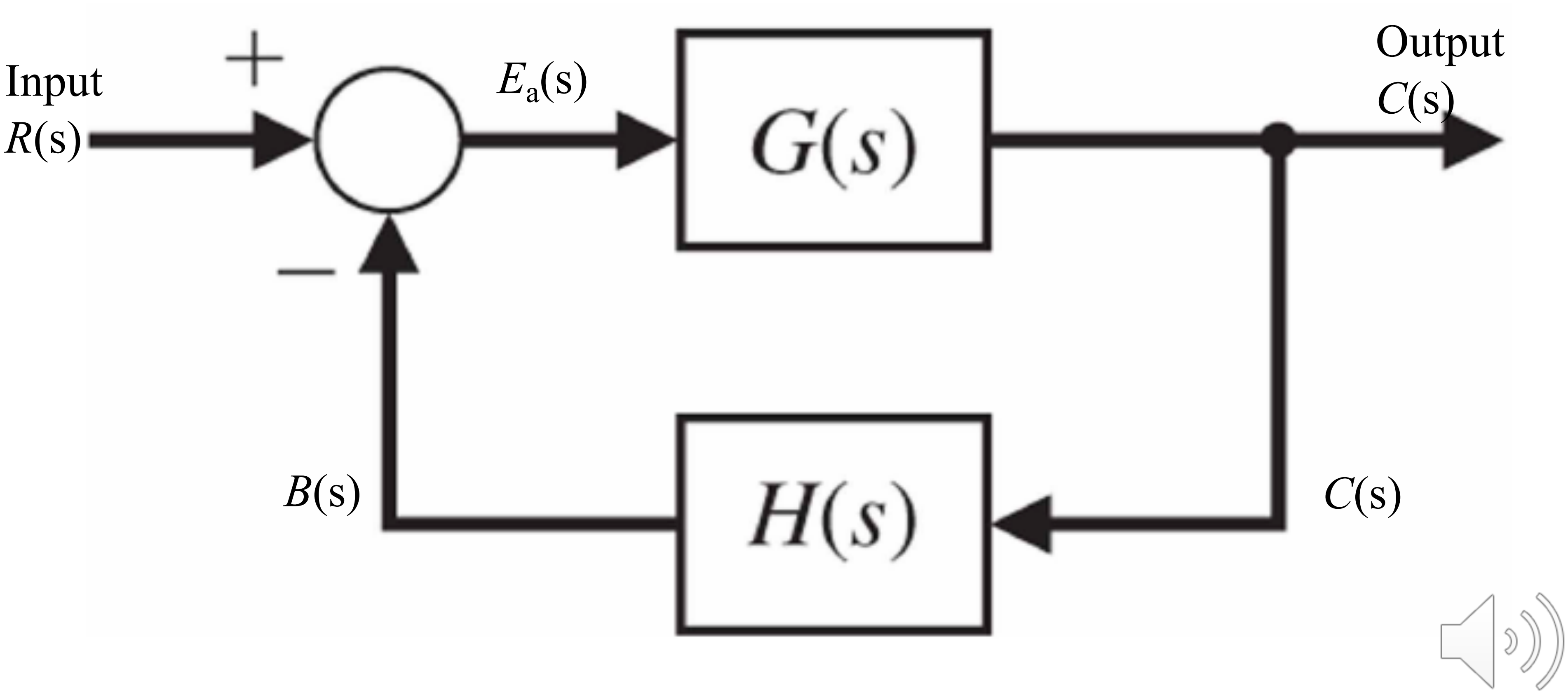
The signal that drives the plant is $E_a(s)$ and this is known formally as the *actuation signal*. It is the difference between the input and the signal that is fed back:

$$E_a(s) = R(s) - B(s)$$

As we know already, $B(s) = H(s)C(s)$ so $E_a(s) = R(s) - H(s)C(s)$.

We also know that the output is related to the actuation signal by the plant transfer function $G(s)$, so:

$$C(s) = G(s)E_a(s)$$



So, now we can substitute in here for $E_a(s)$ to get:

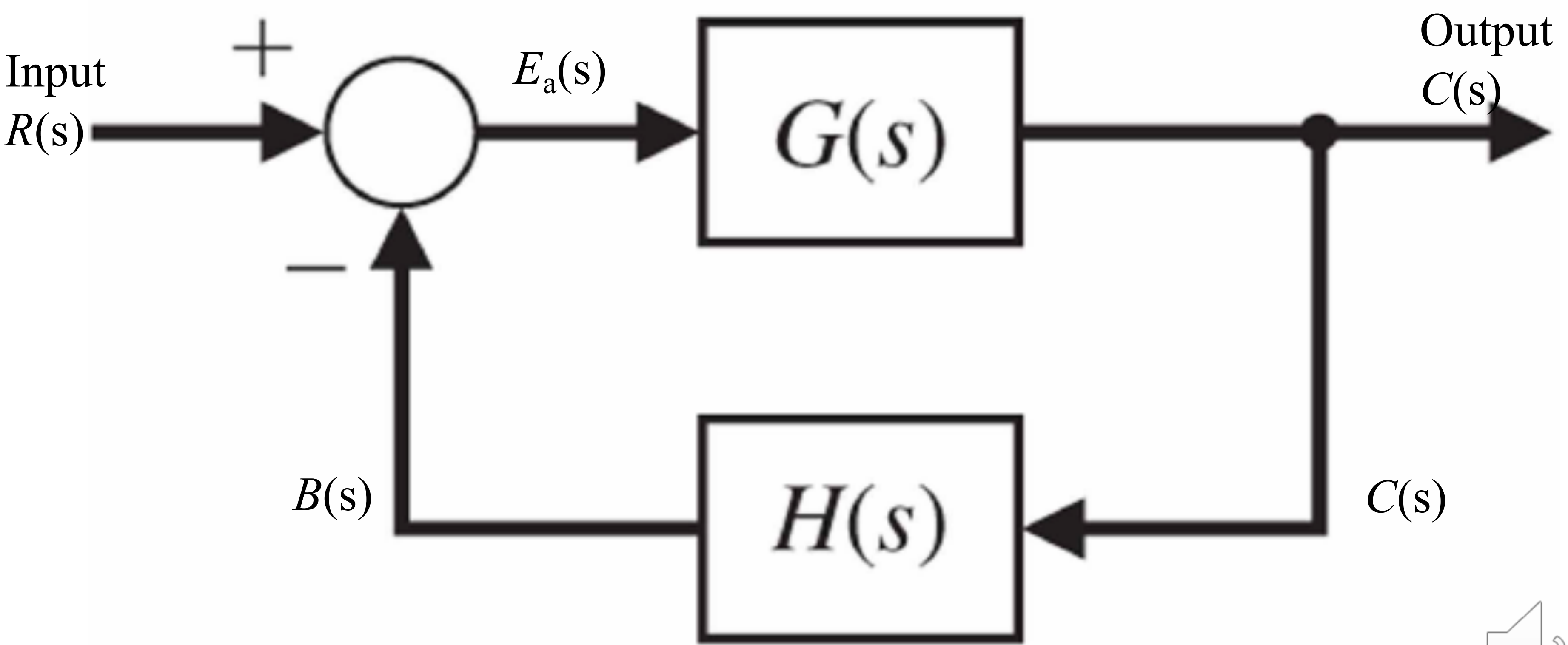
$C(s) = G(s)(R(s) - B(s))$

But we already know $B(s)$ in terms of $H(s)$ and $C(s)$, so we end up with:

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

Solving this for $C(s)$ leads to:

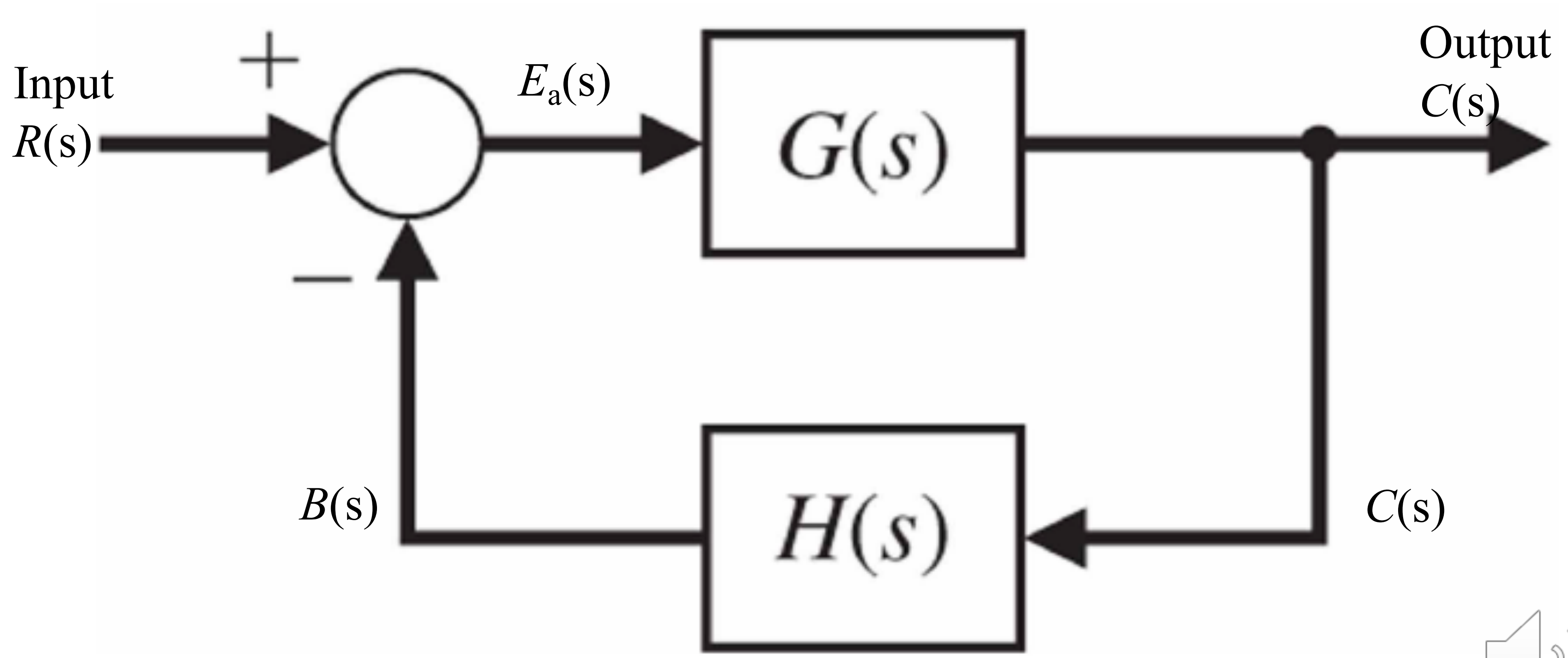
$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$



Rearranging this gives:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

This is the overall system transfer function, relating the output $C(s)$ to the input $R(s)$.



Clearly we could extend this to a positive feedback system where:

$$E_a(s) = R(s) + B(s)$$

so we get a different overall system transfer function of:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

So, in general we have this:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

where the feedback line is either subtractive or additive (i.e. negative or positive feedback).

This is a key result because so many systems can be reconciled into this form