

Outline

- Generative Models Basics
- Autoregressive Models
- Autoencoder and Variational Autoencoder
- **Generative Adversarial Network**
- Diffusion Models

So far...

PixelCNNs define tractable density function, optimize likelihood of training data:

$$p_{\theta}(x) = \prod_{i=1}^n p_{\theta}(x_i|x_1, \dots, x_{i-1})$$

VAEs define intractable density function with latent \mathbf{z} :

$$p_{\theta}(x) = \int p_{\theta}(z)p_{\theta}(x|z)dz$$

Cannot optimize directly, derive and optimize lower bound on likelihood instead

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What if we give up on explicitly modeling density, and just want ability to sample?

GANs: don't work with any explicit density function!

Instead, take game-theoretic approach: learn to generate from training distribution through 2-player game

Generative Adversarial Networks

Ian Goodfellow et al., “Generative Adversarial Nets”, NIPS 2014

Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

Q: What can we use to represent this complex transformation?

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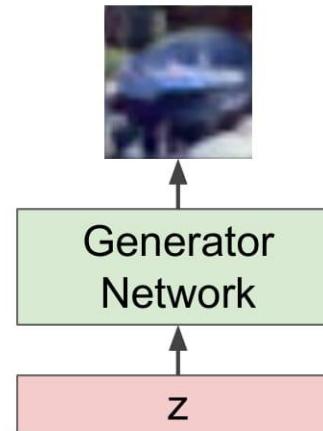
Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

Q: What can we use to represent this complex transformation?

A: A neural network!

Output: Sample from training distribution

Input: Random noise



Training GANs: Two-player game

Ian Goodfellow et al., “Generative Adversarial Nets”, NIPS 2014

Generator network: try to fool the discriminator by generating real-looking images

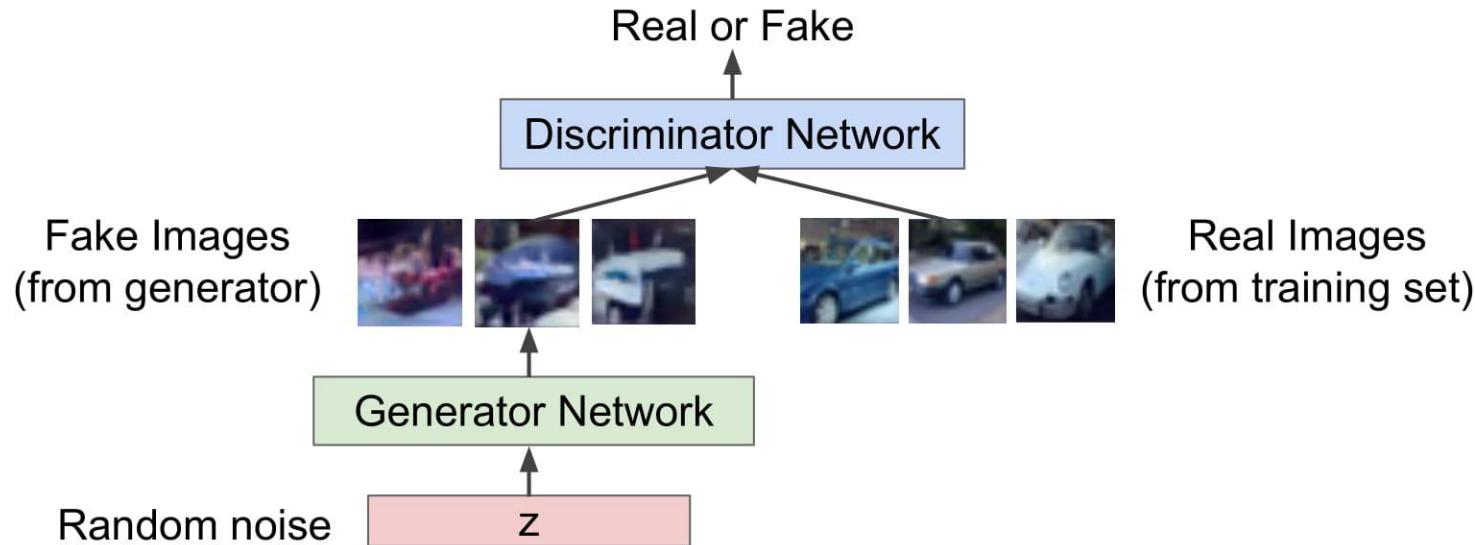
Discriminator network: try to distinguish between real and fake images

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Train jointly in **minimax game**

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

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Discriminator outputs likelihood in (0,1) of real image

Discriminator output for generated fake data $G(z)$

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- Discriminator (θ_d) wants to **maximize objective** such that $D(x)$ is close to 1 (real) and $D(G(z))$ is close to 0 (fake)
- Generator (θ_g) wants to **minimize objective** such that $D(G(z))$ is close to 1 (discriminator is fooled into thinking generated $G(z)$ is real)

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Alternate between:

1. **Gradient ascent** on discriminator

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. **Gradient descent** on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

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Putting it together: GAN training algorithm

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
- Sample minibatch of m examples $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\mathbf{x})$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D_{\theta_d}(\mathbf{x}^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)}))) \right]$$

end for

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
- Update the generator by ascending its stochastic gradient (improved objective):

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)})))$$

end for

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- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D_{\theta_d}(\mathbf{x}^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(\mathbf{z}^{(i)}))) \right]$$

Some find $k=1$
more stable,
others use $k > 1$,
no best rule.

Recent work (e.g.
Wasserstein GAN)
alleviates this
problem, better
stability!

end for

- Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$.
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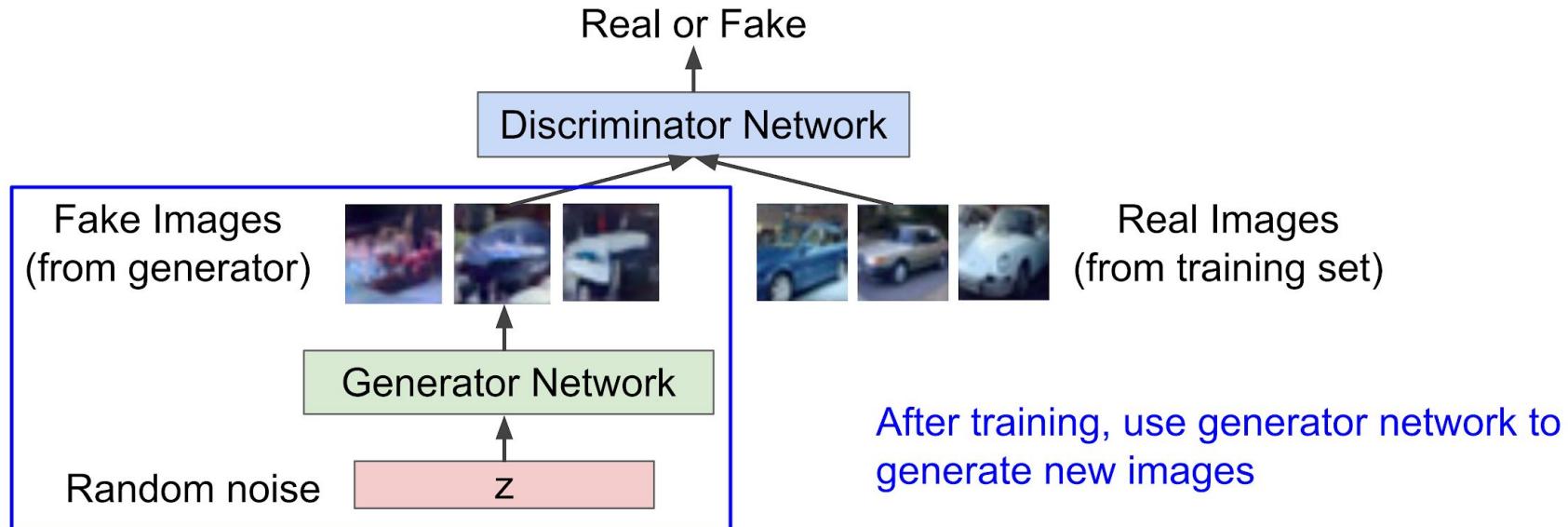
end for

Training GANs: Two-player game

Ian Goodfellow et al., "Generative Adversarial Nets", NIPS 2014

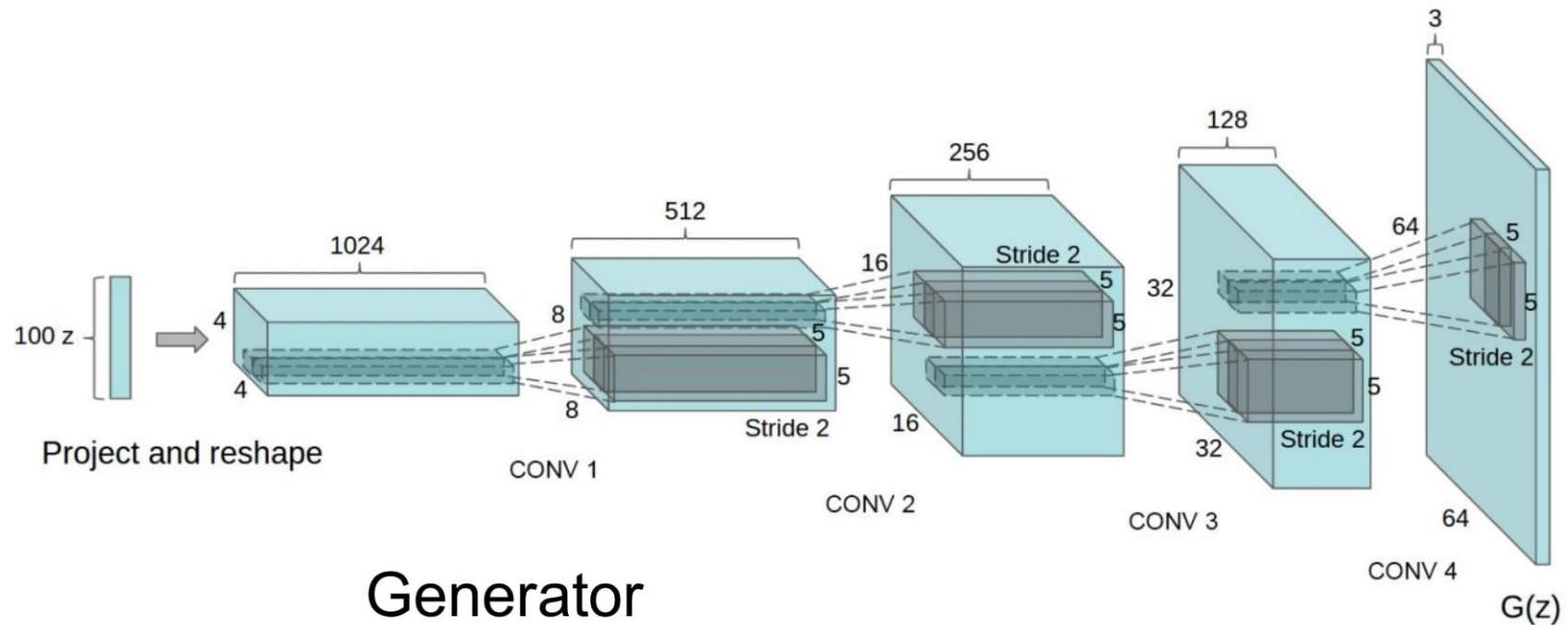
Generator network: try to fool the discriminator by generating real-looking images

Discriminator network: try to distinguish between real and fake images



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Generative Adversarial Nets: Convolutional Architectures



Radford et al, "Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks", ICLR 2016

Generative Adversarial Nets: Convolutional Architectures

Samples
from the
model look
much
better!



Radford et al,
ICLR 2016

Generative Adversarial Nets: Convolutional Architectures

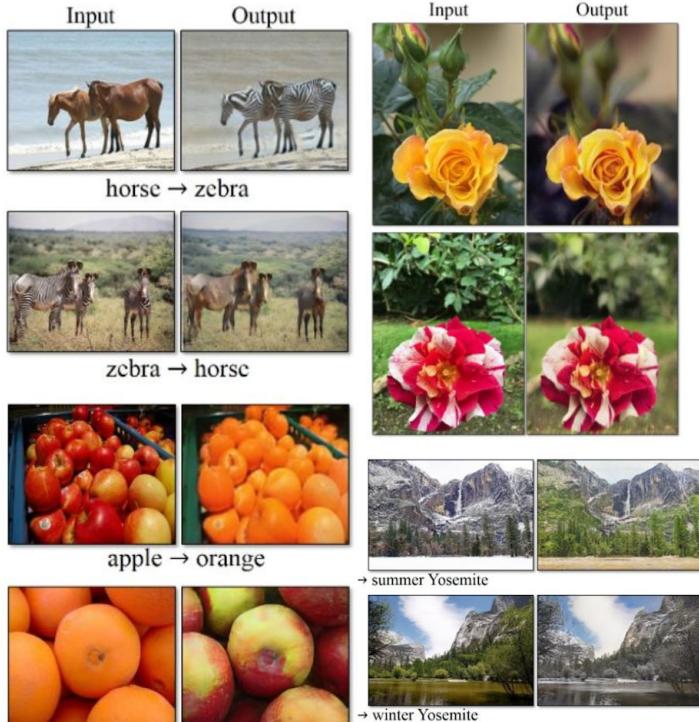
Interpolating
between
random
points in
latent space



Radford et al,
ICLR 2016

2017: Explosion of GANs

Source->Target domain transfer



CycleGAN. Zhu et al. 2017.

Text -> Image Synthesis

this small bird has a pink breast and crown, and black primaries and secondaries.



this magnificent fellow is almost all black with a red crest, and white cheek patch.



Reed et al. 2017.

Many GAN applications



Pix2pix. Isola 2017. Many examples at <https://phillipi.github.io/pix2pix/>

GAN Improvements: Higher Resolution

256 x 256 bedrooms



1024 x 1024 faces



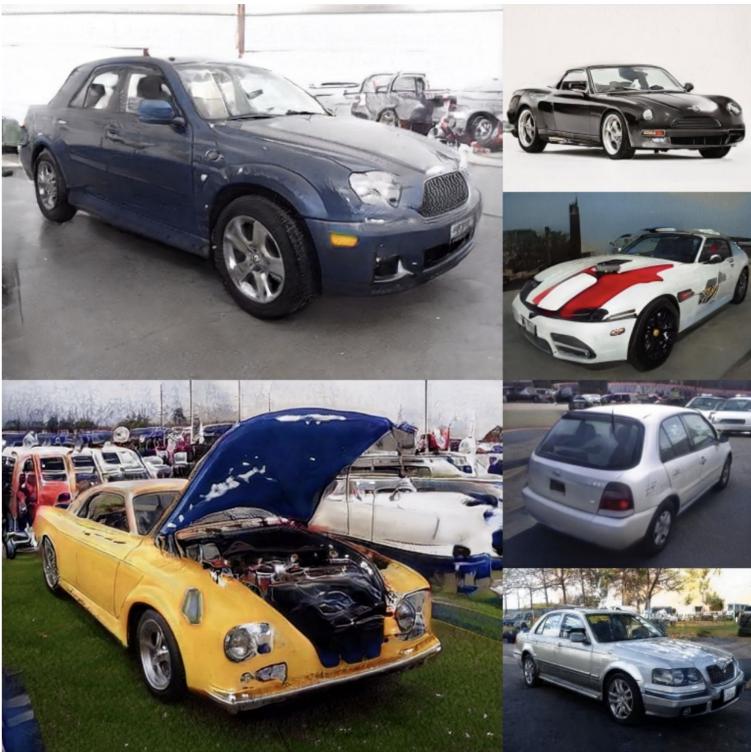
Karras et al, "Progressive Growing of GANs for Improved Quality, Stability, and Variation", ICLR 2018

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Slide credit: Justin Johnson

GAN Improvements: Higher Resolution

512 x 384 cars



1024 x 1024 faces



GANs: Summary

Don't work with an explicit density function

Take game-theoretic approach: learn to generate from training distribution through 2-player game

Pros:

- Beautiful samples!

Cons:

- Trickier / more unstable to train
- Can't solve inference queries such as $p(x)$, $p(z | x)$

Further Extensions:

- Better loss functions, more stable training (Wasserstein GAN, LSGAN, CycleGAN, many others)
- Conditional GANs, GANs for all kinds of applications