EECS 545 Lecture 2 Quiz Solutions

Question 1. $\mathbf{y = \frac{c+1}{3}}$.

Let $y = \alpha$. Then, the total least squared loss is $E = \alpha^2 + (c-\alpha)^2 + (1-\alpha)^2 = 3\alpha^2 - (2c+2)\alpha + (c^2+1)$. Minimizing for α, we get $\alpha = \frac{c+1}{3}$.

Question 2. $\mathbf{\frac{c^2+5}{2}}$

Plugging in values:

$$\Phi w = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

Then, the loss (in vectorized form) is

$$E(w) = \frac{1}{2}\|\Phi w - y\|^2 = \frac{1}{2}\left\| \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ c \\ 1 \end{bmatrix} \right\|^2 = \frac{1}{2}\left\| \begin{bmatrix} -1 \\ -c \\ -2 \end{bmatrix} \right\|^2 = \frac{c^2+5}{2}$$

Question 3. $\begin{bmatrix} c+3 \\ 1 \\ 2 \end{bmatrix}$

Plugging in:

$$\nabla E(w) = \Phi^\top(\Phi w - y) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -c \\ -2 \end{bmatrix} = \begin{bmatrix} -c-3 \\ -1 \\ -3 \end{bmatrix}$$

Gradient update:

$$w' = w - \eta \nabla E(w) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -c-3 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} c+3 \\ 1 \\ 2 \end{bmatrix}$$

Question 4. True.

Conceptually, we are fitting a quadratic polynomial to three points. We know from algebra there is always an optimal solution that fits the points perfectly.

Question 5. False.

Conceptually, we are fitting a linear equation to three points. However, if $c \neq \frac{1}{2}$, then there is no way to fit this equation without some error.