EECS545 Lecture 3 Quiz Solutions

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1. Let $D = \{(x_i, y_i)\}_{i=1,\ldots,3} = \{(-1, -0.5), (0, 0), (1, 1)\}$. Suppose that we want to fit a linear regression model with a 1-degree polynomial function with $\hat{y} = w_0 + w_1 x$. After convergence, what would be the solution of w_0 and w_1? Hint: write down the objective function and set the derivative with respective to w_0 and w_1 to zero to find the optimal solution.

> **Solution:** Plugging in for the objective function:
>
> $$E(w) = \frac{1}{2}\left[(w_0 - w_1 + 0.5)^2 + (w_0)^2 + (w_0 + w_1 - 1)^2\right]$$
>
> Partial derivatives:
>
> $$\frac{\partial E(w)}{\partial w_0} = (w_0 - w_1 + 0.5) + w_0 + (w_0 + w_1 - 1) = 3w_0 - 0.5$$
>
> $$\frac{\partial E(w)}{\partial w_1} = -(w_0 - w_1 + 0.5) + (w_0 + w_1 - 1) = 2w_1 - 1.5$$
>
> Setting these to zero, we get $w_0 = \frac{1}{6}, w_1 = \frac{3}{4}$.

2. Continued. Suppose instead we used ridge linear regression with $\lambda = 1$. Find w_0 and w_1 after convergence.

> **Solution:** Plugging in for the objective function:
>
> $$E(w) = \frac{1}{2}\left[(w_0 - w_1 + 0.5)^2 + (w_0)^2 + (w_0 + w_1 - 1)^2\right] + \frac{1}{2}(w_0^2 + w_1^2)$$
>
> Partial derivatives:
>
> $$\frac{\partial E(w)}{\partial w_0} = (w_0 - w_1 + 0.5) + w_0 + (w_0 + w_1 - 1) + w_0 = 4w_0 - 0.5$$
>
> $$\frac{\partial E(w)}{\partial w_1} = -(w_0 - w_1 + 0.5) + (w_0 + w_1 - 1) + w_1 = 3w_1 - 1.5$$
>
> Setting these to zero, we get $w_0 = \frac{1}{8}, w_1 = \frac{1}{2}$.

3. Consider polynomial regression by optimizing the least-squares objective function with regularization term λ. **Choose all options that apply**:

   (a) As λ increases, the curve formed by polynomial regression will become flatter.

(b) Best practice indicates that we should keep tuning λ until we find a final value λ^* that minimizes error on the test set.

(c) It is not valid to use the $L1$ norm in place of the $L2$ norm in the regularized objective function.

Solution:

(a) True.

(b) False.

(c) False.