EECS545 Lecture 15 Quiz Solutions

1. (T/F) The objective function of K-means decreases monotonically.

Solution: True.

2. Which of the following about EM algorithm is true? Select all that apply.

   (a) The EM algorithm can be used for MLE (maximum likelihood) estimate problems involving latent variables.

   (b) The EM algorithm monotonically increases the lower bound of the log-likelihood $L(q, \theta)$.

   (c) If the posterior $P(\mathbf{Z} \mid \mathbf{X}; \theta)$ is tractable, the EM algorithm always monotonically increases the observed data log-likelihood of the data.

   (d) For all complex models where $P(\mathbf{Z} \mid \mathbf{X}; \theta)$ is not tractable, EM monotonically decreases the log-likelihood of data.

   (e) The EM algorithm can find the global maximum data likelihood if ran sufficiently long.

Solution: (a), (b), and (c). See the lecture notes.

3. Which of the following is true about the E-step of the EM algorithm? Select all that apply.

   (a) E-step computes complete data log-likelihood

   (b) E-step computes the posterior probability of the latent variables

   (c) E-step updates the parameters of the model

   (d) In a single E-step, the log-likelihood of the observed data is increased.

   (e) In a single E-step, the lower bound $L(q, \theta)$ of the log-likelihood of the observed data is increased.

Solution: (b) and (e). In E-step, we compute the posterior $P(\mathbf{Z} \mid \mathbf{X}; \theta)$ and set it as $q(\mathbf{Z})$ given fixed parameters θ of the model. This increases the lower bound on the log-likelihood of the observed data, but the log-likelihood of the observed data is kept constant because the parameter does not change.

4. (T/F) q(Z) is fixed during M-step.

Solution: True.

5. In the latent variable models we discussed in the class, what does each of the following terms mean, respectively (in the same order)?

- complete likelihood
- posterior
- observed data likelihood

(a) $p(\mathbf{X}, \mathbf{Z} \mid \theta)$ $\quad p(\mathbf{Z} \mid \mathbf{X}, \theta)$ $\quad p(\mathbf{X} \mid \theta)$

(b) $p(\mathbf{X} \mid \theta)$ $\quad p(\mathbf{X}, \mathbf{Z} \mid \theta)$ $\quad p(\mathbf{X} \mid \mathbf{Z}, \theta)$

(c) $p(\mathbf{X}, \mathbf{Z} \mid \theta)$ $\quad p(\mathbf{X} \mid \mathbf{Z}, \theta)$ $\quad p(\mathbf{X} \mid \theta)$

(d) $p(\mathbf{X}, \mathbf{Z} \mid \theta)$ $\quad p(\mathbf{X} \mid \theta)$ $\quad p(\mathbf{X} \mid \mathbf{Z}, \theta)$

(e) $p(\mathbf{X}, \mathbf{Z} \mid \theta)$ $\quad p(\mathbf{X} \mid \mathbf{Z}, \theta)$ $\quad p(\mathbf{Z} \mid \mathbf{X}, \theta)$

Solution: (a)

- complete likelihood: $p(\mathbf{X}, \mathbf{Z} \mid \theta)$.
- posterior: $p(\mathbf{Z} \mid \mathbf{X}, \theta)$.
- observed data likelihood: $p(\mathbf{X} \mid \theta)$.

Note that

- $p(\mathbf{X} \mid \mathbf{Z}, \theta)$ is called the conditional data likelihood.