The University of Michigan
Electrical Engineering & Computer Science
EECS 281: Data Structures and Algorithms
Fall 2024

Lab 4: Priority Queues, Heaps, and Union Find

Instructions:
Work on this document with your group, then enter the answers on the canvas quiz.

Note:
Be prepared before you meet with your lab group, and read this document so that you know what you must submit for full credit. You can even start it ahead of time and then ask questions during any lab section for help completing it.

You MUST include the following assignment identifier at the top of every file you submit to the autograder as a comment. This includes all source files, header files, and your Makefile (if there is one). If there is not autograder assignment, you may ignore this.

Project Identifier: 15C1680EE94C640EC35E1694295A3625C3254CBA

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1 Priority Queues & Heaps

1. What does it mean for a tree to be complete?
   A. every node has either 0 or 2 children
   B. every level except the last is necessarily filled, and in the last level, nodes are filled in from left to right
   C. every node in the left subtree must be of lower priority than that of the root, and every node in the right subtree must be of greater priority than that of the root
   D. none of the above

2. Which of the following statements is/are true about a binary heap? Select all that apply.
   A. heapsort has worst-case $\Theta(n \log n)$ time complexity
   B. fix-up has an average-case time complexity of $\Theta(\log n)$
   C. insertion is done by inserting the element at the end, and then calling fix-up
   D. deletion is done by simply removing the root
   E. none of the above

3. Which of the following represents a min-heap? Select all that apply.
   A. [2, 13, 8, 16, 13, 10, 40, 25, 17]
   B. [47, 9, 12, 9, 2, 10, 10, 4, 3, 1]
   C. [3, 5, 6, 7, 12, 15, 14, 9, 10, 11]
   D. [59, 58, 60, 57, 85, 49, 32, 21, 5]
   E. none of the above

4. Consider an empty min-heap priority queue. If you insert the elements 12, 4, 9, 27, 13, 2, and 6 into the heap (in that order, following the algorithm specified in lecture) and remove the most extreme element twice, what are the possible array representations of the heap? Select all that apply.
   A. [6, 9, 12, 13, 27]
   B. [6, 9, 27, 12, 13]
   C. [6, 12, 9, 27, 13]
   D. [27, 13, 12, 9, 6]
   E. none of the above

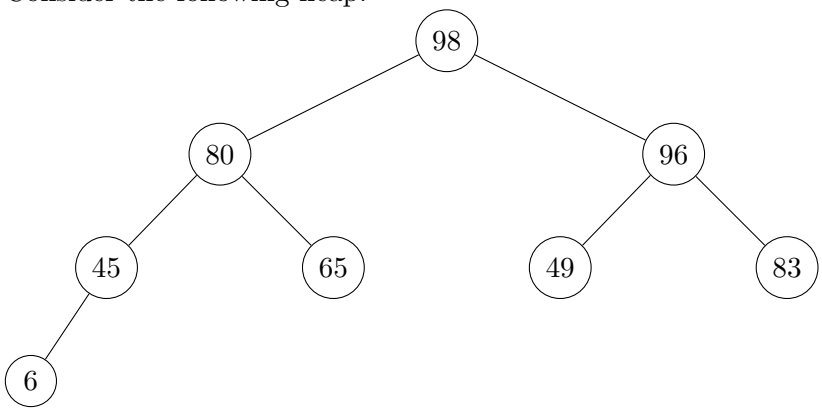
5. Four approaches to heapify are shown below:
   I. Proceeding from the bottom of the heap to the top, while repeatedly calling fixUp()
   II. Proceeding from the bottom of the heap to the top, while repeatedly calling fixDown()
   III. Proceeding from the top of the heap to the bottom, while repeatedly calling fixUp()
   IV. Proceeding from the top of the heap to the bottom, while repeatedly calling fixDown()

   Which of the above approaches will always build a valid heap?
   A. I and II only
   B. I and IV only
   C. II and III only

D. III and IV only

E. I, II, III, and IV

6. Consider the following heap:



You insert the value of 81 into the heap. After the heap invariant is restored, what is the final array representation of this max heap?

A. 98, 80, 96, 45, 81, 49, 83, 6, 65

B. 98, 96, 81, 83, 65, 80, 49, 45, 6

C. 98, 81, 96, 80, 65, 49, 83, 45, 6

D. 98, 81, 96, 80, 65, 49, 83, 6, 45

E. none of the above

7. Consider a min-heap represented by the following array:

[50, 53, 61, 57, 70, 79, 68, 71]

Perform the following operations using the algorithms for binary heaps discussed in lecture. Ensure the heap property is restored at the end of each individual heap operation.

1. Push the value 40 into the min-heap
2. Push the value 54 into this min-heap
3. Update element 71 to have value 49
4. Update element 61 to have value 90

What does the array representation look like after all four operations are completed?

A. [40, 49, 79, 50, 54, 90, 68, 53, 57, 70]

B. [40, 49, 68, 50, 54, 79, 90, 53, 57, 70]

C. [40, 49, 50, 53, 70, 79, 54, 57, 90, 68]

D. [40, 49, 50, 53, 54, 90, 68, 79, 57, 70]

E. None of the above

8. We want to create a k-priority queue, which is a priority queue inspired data structure that allows inserting and removing elements while checking the k-th largest element in O(1) time. What is the optimal time complexity of inserting an element? (**HINT**: think about how you could implement this functionality with two priority queues: a min priority queue and a max priority queue)

A. O(log(n-k))

B. O(2k+log(n-k))

C. O(log(k*k*(n-k)))
D. O(log(k)*log(k)*log(n-k))
E. O(k*k*log(n-k))

9. Consider the following snippet of code:

```
#include <iostream>
#include <queue>
#include <priority_queue>

using namespace std;

class LegoBrick {
    private:
        double width, height;
        string color;
    public:
        LegoBrick(double width_in, double height_in)
            : width(width_in), height(height_in) {}

        void printLegoBrick() const {
            cout <<    (    << width << "," << height << ") ";
        }

        const double getArea() {
            return width * height;
        }
};

struct LegoBrickCompare {
    bool operator()(LegoBrick& a, LegoBrick& b) const {
        return a.getArea() < b.getArea();
    }
};

int main()
{
    vector<LegoBrick> bag_o_legos = { LegoBrick{1,2}, LegoBrick{2,4},
    LegoBrick{1,1}, LegoBrick{4,2},
    LegoBrick{3,3}, LegoBrick{1,5},
    LegoBrick{2,2},  LegoBrick{2,5}};

    priority_queue<LegoBrick, vector<LegoBrick>, LegoBrickCompare>
        my_fav_legos(bag_o_legos.begin(), bag_o_legos.end());

    for(int i = 0; i < 3; ++i){
        my_fav_legos.top().printLegoBrick();
        my_fav_legos.pop();
    }
}
```

You are searching through an old bag of legos and want to pick your three favorite pieces. What is the output to the code above?

A. (1,1) (1,2) (1,5)
B. (3,3) (4,2) (2,4)
C. (3,3) (2,4) (4,2)
D. (2,5) (3,3) (1,5)
E. (2,5) (3,3) (4,2)

2 Union Find

10. Consider the union-find container given below and suppose you used path compression when implementing union-find.

Item	1	2	3	4	5	6	7	8	9	10
Representative	1	8	1	2	5	4	6	10	3	5

I. After calling find on item 7, 4's representative would be 8.

II. After calling find on item 6, 8's representative would be 5.

III. Calling find on 3 would change one or more representatives in the table.

IV. There are 2 disjoint sets represented here.

V. After calling find on item 2, 6's representative would be 5.

Which of the following is true given these 10 elements and their representatives?

A. I only

B. II, IV

C. IV only

D. I, III, IV

E. II, IV, V

