

# Stat 4202: Mathematical Statistics II

## Homework 2

Spring 2025

**Question 1** (10.38). Show that the estimator of  $\omega = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$  is consistent.

**Question 2.** To show that an estimator can be consistent without being unbiased or even asymptotically unbiased, consider the following estimation procedure: To estimate the mean of a population with the finite variance  $\sigma^2$ , we first take a random sample of size  $n$ . Then we randomly draw one of  $n$  slips of paper numbered from 1 through  $n$ , and if the number we draw is 2, 3, ..., or  $n$ , we use as our estimator the mean of the random sample; otherwise, we use the estimate  $n^2$ . Show that this estimation procedure is

(a) consistent;

(b) neither unbiased nor asymptotically unbiased;

**Question 3.** If  $X_1, X_2, \dots, X_n$  constitute a random sample of size  $n$  from an exponential population, show that  $\bar{X}$  is a sufficient estimator of the parameter  $\theta$ .

**Question 4.** If  $X_1$  and  $X_2$  constitute a random sample of size  $n = 2$  from a Poisson population, show that the mean of the sample is a sufficient estimator of the parameter of  $\lambda$ .

**Question 5.** Given a random sample of size  $n$  from a uniform population with  $\alpha = 0$ , find an estimator of  $\beta$  by the method of moments.

**Question 6.** If  $X_1, X_2, \dots, X_n$  constitute a random sample of size  $n$  from a population given by

$$g(x; \theta, \delta) = \begin{cases} \frac{1}{\theta} \cdot e^{-\frac{x-\delta}{\theta}} & \text{for } x > \delta \\ 0 & \text{elsewhere} \end{cases}$$

find estimators for  $\theta$  and  $\delta$  by the method of moments. This distribution is sometimes referred to as the two-parameter exponential distribution, and for  $\theta = 1$  it is the distribution of Example 3.

**Question 7.** Given a random sample of size  $n$  from a continuous uniform population, use the method of moments to find formulas for the parameters  $\alpha$ , and  $\beta$ .