Stat 4202: Mathematical Statistics II

Homework 2

Spring 2025

Question 1. *Show that the estimator of* $\omega = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$ *is consistent.*

Question 2. *To show that an estimator can be consistent without being unbiased or even asymptotically unbiased, consider the following estimation procedure: To estimate the mean of a population with the finite variance* σ^2*, we first take a random sample of size* n*. Then we randomly draw one of* n *slips of paper numbered from 1 through* n*, and if the number we draw is 2, 3,* $ldots$*, or* n*, we use as our estimator the mean of the random sample; otherwise, we use the estimate* n^2*. Show that this estimation procedure is*

(a) *consistent;*

(b) *unbiased but not asymptotically unbiased;*

Question 3. *If* $X_1, X_2, \ldots, X_n$ *constitute a random sample of size* n *from an exponential population, show that* $\overline{X}$ *is a sufficient estimator of the parameter* θ*.*

Question 4. *If* X_1 *and* X_2 *constitute a random sample of size* $n = 2$ *from a Poisson population, show that the mean of the sample is a sufficient estimator of the parameter of* λ*.*

Question 5. *Show that the mean of a random sample of size* n *from an exponential population is a minimum variance unbiased estimator of the parameter* θ*.*

Question 6. *Given a random sample of size* n *from a uniform population with* $\alpha = 0$*, find an estimator of* β *by the method of moments.*

Question 7. *If* $X_1, X_2, \ldots, X_n$ *constitute a random sampmle of size* n *from a population given by*

$$g(x;\theta,\delta) = \begin{cases} \frac{1}{\theta} \cdot e^{-\frac{x-\delta}{\theta}} & \text{for } x > \delta \\ 0 & \text{elsewhere} \end{cases}$$

Question 8. *Given a random sample of size* n *from a continous uniform population , use the method of moments to find formulas for the parameters* α*, and* $beta$*.*