Physics 2301: Intermediate Mechanics II

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1 January 7, 2025

1.1 Course Introduction

Dr. Boveia talked about the course and we will head towards relativistic mechanics. In this course, we will master the concepts from Physics 1250 but more broadly. **Tuesday** is the lecture day and **Wednesday,** Thursday, and Friday are the problem-solving days. The grading scale would be on the **Standard Ohio State** grading scale. There might be a curve but it will only curve up, not down. Our grade would rely on **Quizzes (20%), Midterm (20%), Final (20%), Homework (30%)**, and **Participation (10%)**. The exams are **open-book, open-notes, and open-internet**.

1.2 Success Tips

Here are some tips to succeed in Physics 2301:

- **Attend all lectures and problem-solving sessions:** Regular attendance will help you understand the material better and keep up with the course pace.
- **Stay organized:** Keep track of all assignments, quizzes, and exam dates. Use a planner or digital calendar to manage your time effectively.
- **Participate actively:** Engage in class discussions and ask questions whenever you have doubts. Participation counts towards your grade.
- **Form study groups:** Collaborate with your peers to discuss concepts and solve problems. Group study can provide different perspectives and enhance understanding.
- **Utilize office hours:** Take advantage of Dr. Boveia's office hours to seek clarification on topics you find challenging.
- **Practice regularly:** Consistently work on homework and additional problems to reinforce your understanding of relativistic mechanics.
- **Review notes:** Regularly review your lecture notes and summarize key points to aid retention.

- **Use available resources:** Make use of the textbook, online resources, and any supplementary materials provided by Dr. Boveia.
- **Stay healthy:** Ensure you get enough rest, eat well, and manage stress to maintain your overall well-being.

1.3 Review of Vectors and Matrices

Definition 1

A **scalar quantity** is a quantity with only magnitude. Examples include mass, temperature, and time.

Definition 2

A **vector quantity** is a quantity with both magnitude and direction. Examples include displacement, velocity, and force. Vectors follow a different set of rules compared to scalars. For example, there are two multiplication rules for vectors: **Dot Product** and **Cross Product**. The dot product results in a scalar quantity while the cross product results in a vector quantity.

$$|\vec{v}| = \text{Length and} \quad \vec{v} = \frac{|\vec{v}|}{\sqrt{3}}(V_x, V_y, V_z) \text{ if } \vec{v} = \langle V_x, V_y, V_z \rangle$$

Scalar × Vector = Vector

Vector · Vector = Scalar (Dot Product or Inner Product)

$$\vec{v} \cdot \vec{v} = V_x \cdot V_x + V_y \cdot V_y + V_z \cdot V_z = |\vec{v}|^2$$

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z = |\vec{A}||\vec{B}| \cos\theta$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$\hat{x}$ = 'Unit Vector' $|\hat{x}| = 1$

$$\vec{A} \cdot \vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) \cdot (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = A_xB_x + A_yB_y + A_zB_z$$

$\hat{x}, \hat{y}, \hat{z}$ form an orthogonal normal basis, meaning that $\hat{x} \cdot \hat{y} = 0$

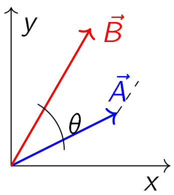
$$\hat{x} \cdot \hat{y} = 0, \quad \hat{y} \cdot \hat{z} = 0, \quad \hat{z} \cdot \hat{x} = 0$$

Here, Ortho means that $\hat{x} \cdot \hat{y} = 0$ and Normal means that $\hat{x} \cdot \hat{x} = 1$

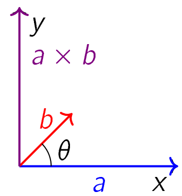
Vector × Vector = Cross Product = Vector

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \hat{n}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Dot Product
$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$



Cross Product
$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

Figure 1: Geometric interpretation of dot and cross products

Dot product is just the magnitude of the projection of $\vec{A}$ onto $\vec{B}$.

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \hat{n}$$

Dot product is the component of $\vec{B}$ along $\vec{A}$. The dot product is a scalar quantity.

The cross product is a vector that is perpendicular to both $\vec{A}$ and $\vec{B}$ ($\vec{A} \perp \vec{B}$). The magnitude of the cross product is the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$. The direction of the cross product follows the right-hand rule.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

1.4 Matrices

A **matrix** is a rectangular array of numbers arranged in rows and columns. Matrices are used to represent linear transformations and solve systems of linear equations.

Definition 3

A matrix A with m rows and n columns is denoted as $A \in \mathbb{R}^{m \times n}$. Each element of the matrix is denoted as a_{ij}, where i is the row index and j is the column index.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \tag{1}$$

Matrix Addition: Two matrices A and B of the same dimension can be added by adding their corresponding elements.

$$(A + B)_{ij} = a_{ij} + b_{ij} \tag{2}$$

Matrix Multiplication: The product of two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$ is a matrix $C \in \mathbb{R}^{m \times p}$

where each element is given by the dot product of the corresponding row of A and column of B.

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \tag{3}$$

Identity Matrix: The identity matrix I is a square matrix with ones on the diagonal and zeros elsewhere. It acts as the multiplicative identity for matrices.

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{4}$$

Transpose of a Matrix: The transpose of a matrix A, denoted A^T, is obtained by swapping its rows and columns.

$$(A^T)_{ij} = a_{ji} \tag{5}$$

Determinant of a Matrix: The determinant is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the matrix. For a 2×2 matrix, the determinant is given by:

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \tag{6}$$

Inverse of a Matrix: The inverse of a square matrix A, denoted A^{-1}, is the matrix such that $AA^{-1} = A^{-1}A = I$. A matrix is invertible if and only if its determinant is non-zero.

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A) \tag{7}$$

where $\text{adj}(A)$ is the adjugate of A.

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Figure 2: Example of a matrix

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