

Stat 4202: Mathematical Statistics II

Homework 2

Spring 2025

Question 1. Show that the estimator of $\omega = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$ is consistent.

Question 2. To show that an estimator can be consistent without being unbiased or even asymptotically unbiased, consider the following estimation procedure: To estimate the mean of a population with the finite variance σ^2 , we first take a random sample of size n . Then we randomly draw one of n slips of paper numbered from 1 through n , and if the number we draw is 2, 3, ..., or n , we use as our estimator the mean of the random sample; otherwise, we use the estimate n^2 . Show that this estimation procedure is

(a) consistent;

(b) neither unbiased nor asymptotically unbiased;

Question 3. If X_1, X_2, \dots, X_n constitute a random sample of size n from an exponential population, show that \bar{X} is a sufficient estimator of the parameter θ .

Question 4. If X_1 and X_2 constitute a random sample of size $n = 2$ from a Poisson population, show that the mean of the sample is a sufficient estimator of the parameter of λ .

Question 5. Given a random sample of size n from a uniform population with $\alpha = 0$, find an estimator of β by the method of moments.

Question 6. If X_1, X_2, \dots, X_n constitute a random sample of size n from a population given by

$$g(x; \theta, \delta) = \begin{cases} \frac{1}{\theta} \cdot e^{-\frac{x-\delta}{\theta}} & \text{for } x > \delta \\ 0 & \text{elsewhere} \end{cases}$$

find estimators for θ and δ by the method of moments. This distribution is sometimes referred to as the two-parameter exponential distribution, and for $\theta = 1$ it is the distribution of Example 3.

Question 7. Given a random sample of size n from a continuous uniform population, use the method of moments to find formulas for the parameters α , and β .