Model Paper

Question 1 (Set Theory)   **20 marks**

1. Fill in the blanks with $\in$, $\notin$, $\subseteq$, = or $\neq$. Note that each symbol can be used more than once. (5 marks)

    i.   1   ----------   {2, 4, 5, 6}
    ii.   {1, 4}   ----------   {2, {1,4}, 7}
    iii.   {2, 3}   ----------   {1, 2, 3, 4}
    iv.   4   ----------   {1, {2, 3}, 4, 5}
    v.   {0.5,-2}----------   { $x|2x^2+3x-2=0$ and x is a real number}

2. Represent the given information in a Venn diagram. U is the universal set.   (5 marks)

    U= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, L= {2, 4, 6, 7, 9}, M= {1, 2, 3, 4, 5, 6, 8},   N= {2, 3, 4, 5}

3. Given the universal set U= {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}, A= {1, 9, 17}, B= {3, 5, 7, 11, 13, 15, 19}, C= {3, 7, 9, 15, 17}, D={2, 4} find the following.

    i.   All subsets of A   (2 marks)
    ii.   B $\cap$ (A $\cup$ C')   (2 marks)
    iii.   Represent the relation R = {(2,1), (4,9)} in an arrow diagram considering the two sets A and D.   (2 marks)

4. Among 200 candidates who were interviewed for a position at a company, 100 had a degree, 70 had a professional qualification and 140 had working experience. 40 of them had a degree and experience. 20 had a professional qualification and experience. 60 had a degree and a professional qualification and 10 had all three. How many candidates did not have any sort of qualification including working experience?   (4 marks)

Question 2 (Propositional Logic)   **20 marks**

1. Construct the truth table of [($\neg$p $\lor$ $\neg$q) $\land$ (p $\lor$ $\neg$q) $\land$r] => ($\neg$q $\land$r)]   (3 marks)
2. Show $\neg$(p $\rightarrow$ q) is equivalent to p $\land$ $\neg$q   (3 marks)
3. Show that (p $\rightarrow$ q) $\land$ (q $\rightarrow$ p) is logically equivalent to p $\leftrightarrow$ q.   (3 marks)
4. Write a logical statement which is equivalent to PVQ   (3 marks)
5. Identify the types of the following composite propositions
    i. $\neg$(p $\lor$ (p $\land$ q))
    ii.   ($\neg$p $\rightarrow$ $\neg$r) $\lor$ q

iii.   q → (p → r)

iv.   (p →q) V (q → p)   (2*4=8 marks)

**Question 3 (Matrix Algebra)**   **20 marks**

If $A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

1. Find the following:

     i.   A + B   (2 marks)

     ii.   AB   (2 marks)

     iii. B^{-1} (3 marks)

2. The following diagram shows the flow of traffic in one way roads (indicated by arrows) in a city. The flow of traffic in and out of the city is measured in terms of vehicles per hour (vph). The average number of vehicles entering and leaving each intersection per hour appears on each road. Answer the questions given below.
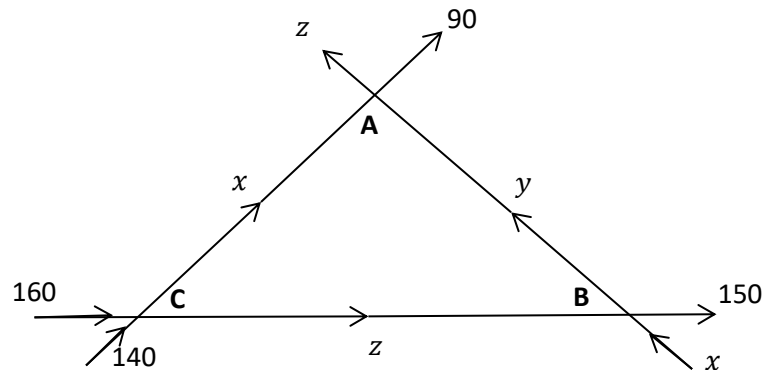
     i.   Construct a mathematical model that describes the traffic flow in the city.

                                                                               (3 marks)

     ii.   Find the respective flows of traffic by applying matrix inversion on the linear system comprising three equations that describe traffic flows at intersections A, B and C.   (10 marks)

Question 4 (Coordinate Geometry)   **20 marks**

1. The general form of an equation of a straight line (L_1) is 6x-2y-4=0.
   - i.   What is the slope and the intercept?   (2 marks)
   - ii.   Find the line equation parallel to L_1 going through the point (4, 2)   (2 marks)

2. Suppose the lines **L₁**: $x - y + 2 = 0$ **L₂**: $3x + 5y - 10 = 0$ **L₃**: $3x + y - 14 = 0$

   - i.   Find the vertices of the triangle.   (3 marks)
   - ii.   Graph the above lines on a xy plane.   (3 marks)
   - iii.   Find the equation of the circle that goes through the above three vertices.
                                                                        (5 marks)
   - iv.   Find the area between the circle and the triangle.   (5 marks)

Question 5 (Logarithms)   **20 marks**

1. Solve for x in the following equations
   - i. $\log_{10} x = 10$
   - ii. $log_2 32 = x$
   - iii. $log_x 16 = 4$
   - iv. $log_4 \frac{1}{64} = \frac{1}{x}$

                                                      (1*4 = 4 marks)

2. Combine the following in to simple logarithm.
   - i.   4 log x + ½ log(x + 2)
   - ii.   3 log s + 1/4 log t – 2 log(t2 + 1)

                                                      (2*2 = 4 marks)

3. Solve the following equations
   - i. $\log_2(3x + 6) = \log_2(27)$ (2 marks)
   - ii. $\log_5(x + 2) - \log_5 x = \log_5(2x - 1) - \log_5(3x - 12)$ (3 marks)
   - iii. 2 log 3 + 1/2 log 16 - log 3 = $\log x$ (3 marks)

4. According to the change of base rule: $\log_b x = \frac{\log_a x}{\log_a b}$

   Solve the equation $10^{2x} - 12(10^x) + 35 = 0$ for 3 decimal places.   (4 marks)

Question 6 (Statistics)   **20 marks**

1. Define the following terminologies used in statistics
   i.   Element
   ii. Variable
   iii. Observation
   iv. Population   (1*4 = 4 marks)

2. Following table consists the mathematics marks of 8 students.

85	55	60	55	75	90	68	70

   Find the following measurements of the data set.
   i.   Mean
   ii. Median
   iii. Mode
   iv. Q1
   v.   Q3
   vi. Inter Quartile Range   (1*6 = 6 marks)

3. Following table shows the number of hours that students have walked in NSBM Walk 2019.

Number of Hours	Number of Students
0.5 - 1	2
1 - 1.5	4
1.5 - 2	7
2 - 2.5	10
2.5 - 3	16
3 - 3.5	8
3.5 - 4	3

   i.   Find the mean of the data set
   ii. Find the median of the data set
   iii. Find the mode of the data set
   iv. Find the variance of the data set
   v.   Find the standard deviation of the data set   (2*5 =10 marks)

Formulae: Measures of central tendency and dispersion

$$Mean = \frac{\sum X_i}{n}$$

$$Mean = \frac{\sum f_i X_i}{\sum f_i}$$

$$Mode = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) \times c$$

$$Median = L_1 + \left(\frac{n/2 - F_1}{f_m}\right) \times c$$

$$Variance\left(S^2\right) = \frac{\sum\left(X - \overline{X}\right)^2}{n - 1}$$

$$s^2 = \frac{\sum f_i\left(X_i - \overline{X}\right)^2}{\sum f_i - 1}$$

$$s = \sqrt{\frac{\sum f_i\left(X_i - \overline{X}\right)^2}{\sum f_i - 1}}$$