Date

Sample Paper - Mathematics.

Question 1 (set theory)

1.

(i) $2 \in \{2, 4, 7\}$

(ii) $7 \in \mathbb{Z}^+$

(iii) $\{1, -2, 8\} \subseteq \mathbb{Z}$

(iv) $21 \in \{168, 147, 126, \ldots\}$

(v) $\{16\} \neq \{4, 8, 12\}$

2. $U = \{1, 2, 3, 4, 5, 7, 9, 11, 13, 15\}$

$A = \{1, 3, 9, 15\}$

$B = \{3, 9, 11\}$

$D = \{2, 4\}$

(i) $A \cap B = \{3, 9\}$

(ii) $(A \cup D)' = \{5, 7, 11, 13\}$

(iii) $D \times B = \{2, 4\} \times \{3, 9, 11\} = \{(2,3), (2,9), (2,11), (4,3), (4,9), (4,11)\}$

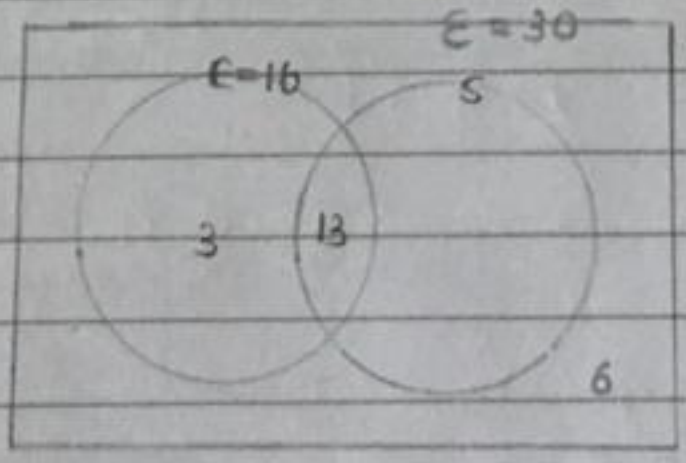
(iv) Find all subsets of $A = 2^{|A|}$

$= 2^4$

$= 16$

$\{1\}, \{3\}, \{9\}, \{15\}, \{1,3\}, \{1,9\}, \{1,15\}, \{3,9\}, \{3,15\}, \{9,15\}, \{1,3,9\}, \{1,9,15\}$
$\{3,9,15\}$ $\{1,3,15\}, \{1,3,9,15\}, \emptyset$

3.



$|E \cup S| = |E| + |S| - |E \cap S|$

$30 - 6 = 16 + |S| - 13$

$24 = 3 + |S|$

$24 - 3 = |S|$

$|S| = 21$

Those who like only science $= 21 - 13$

$= 8$ //

Sonly $= S - E$

$= S \cap E'$

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Question 2 (Propositional logic)

1.

(i) $\sim(p \vee q) = \sim p \wedge \sim q$

RHS

p	q	p∨q	~(p∨q)
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	F

L.H.S

p	q	~p	~q	~p∧~q
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	F	F	F

∴ R.H.S = L.H.S.

(ii) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

R.H.S

p	q	r	q∧r	p∧(q∧r)
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	T	F
T	F	F	F	F
T	F	T	F	F
T	T	F	F	F
T	T	T	T	T

L.H.S

p	q	r	p∧q	(p∧q)∧r
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	F	F
T	T	F	F	F
T	T	T	T	T

∴ R.H.S = L.H.S

(iii) $\sim((p \wedge q) \wedge r) = (\sim p \wedge \sim q) \vee \sim r$

R.H.S

p	q	r	$(p \vee q) \wedge r$	$\sim((p \vee q) \wedge r)$
F	F	F	F	T
F	F	T	F	T
F	T	F	F	T
F	T	T	T	F
T	F	F	F	T
T	F	T	T	F
T	T	F	F	T
T	T	T	T	F

L.H.S

p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p \wedge \sim q$	$(\sim p \wedge \sim q) \vee \sim r$
F	F	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	T	F	T	F	T	F	T
F	T	T	T	F	F	F	F
T	F	F	F	T	T	F	T
T	F	T	F	T	F	F	F
T	T	F	F	F	T	F	T
T	T	T	F	F	F	F	F

$\therefore$ R.H.S = L.H.S.

2

(i) $p \wedge (q \wedge \sim p)$

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
F	F	T	F	F
F	T	T	T	F
T	F	F	F	F
T	T	F	F	F

This compound proposition is contradictions proposition.

(ii) $p \Leftrightarrow (\sim p \wedge q)$

p	q	$\sim p$	$\sim p \wedge q$	$p \Leftrightarrow (\sim p \wedge q)$
F	F	T	F	T
F	T	T	T	F
T	F	F	F	F
T	T	F	F	F

This compound proposition is contingent proposition.

Question 3 (coordinate Geometry)

1.

(i) $8x - 2y - 6 = 0$

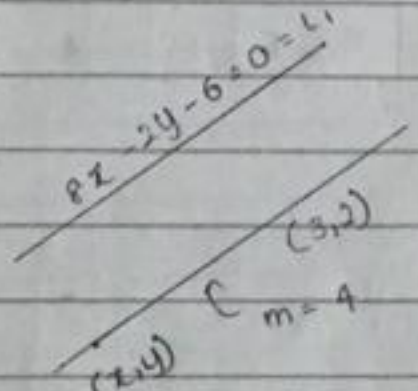
$4x - y - 3 = 0$

$y = 4x - 3$

(ii) slope $(m) = 4$

(iii) intercept $(c) = -3$

(iv)

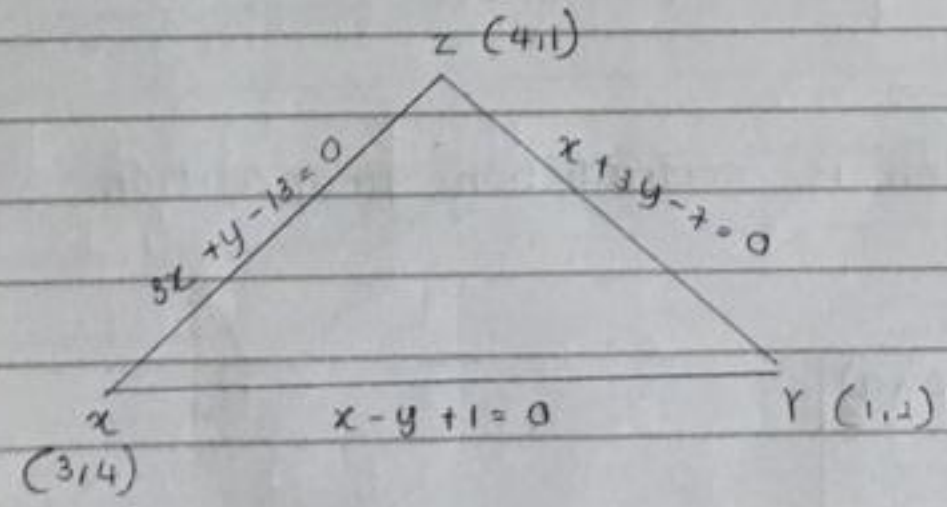


$$\frac{y-2}{x-3} = 4$$

$y - 2 = 4x - 12$

$\underline{4x - y - 10 = 0}$

2. (i)



L_{XY} & L_{YZ}

$x - y + 1 = 0 \rightarrow ①$

$x + 3y - 7 = 0 \rightarrow ②$

② - ① ⇒ $3y - 7 + y - 1 = 0$

$4y = 8$

$\underline{y = 2}$

① ⇒ $x - 2 + 1 = 0$

$\underline{x = 1}$

L_{xy} & L_{xz}

$x - y + 1 = 0 \rightarrow (3)$

$3x + y - 13 = 0 \rightarrow (4)$

$(3) + (4) \Rightarrow 4x - 12 = 0$

$4x = 12$

$\underline{x = 3}$

$\underline{y = 4}$

L_{xz} & L_{zy}

$3x + y - 13 = 0 \rightarrow (5)$

$x + 3y - 7 = 0 \rightarrow (6)$

$(6) \times 3 \Rightarrow 3x + 9y - 21 = 0 \rightarrow (7)$

$(7) - (5) \Rightarrow 9y - y - 21 + 13 = 0$

$8y - 8 = 0$

$\underline{y = 1}$

$\underline{x = 4}$

(ii) $x^2 + y^2 + 2gx + 2fy + c = 0$

$(4,1)$

$16 + 1 + 2g(4) + 2f(1) + c = 0$

$17 + 8g + 2f + c = 0 \rightarrow (1)$

$(1,2)$

$1^2 + 2^2 + 2g(1) + 2f(2) + c = 0$

$5 + 2g + 4f + c = 0 \rightarrow (2)$

$(3,4)$

$9 + 16 + 2g(3) + 2f(4) + c = 0$

$25 + 6g + 8f + c = 0 \rightarrow (3)$

$17 + 8g + 2f + c = 0 \rightarrow (1)$

$(1) \times 2 \Rightarrow 34 + 16g + 4f + 2c = 0 \rightarrow (4)$

$5 + 2g + 4f + c = 0 \rightarrow (2)$

$(4) - (2) \Rightarrow 29 + 14g + c = 0 \rightarrow (6)$

$(6) + (7) \Rightarrow 44 + 16g = 0$

$4g = -11$

$\boxed{g = -\frac{11}{4}}$

$(2) \Rightarrow 10 + 4g + 8f + 2c = 0 \rightarrow (5)$

$25 + 6g + 8f + c = 0 \rightarrow (3)$

$15 + 2g - c = 0 \rightarrow (7)$

$15 - \frac{22}{4} - c = 0$

$\frac{30 - 11}{2} = c$

$\boxed{\frac{19}{2} = c}$

$(1) - (2) \Rightarrow 12 + 6g - 2f = 0$

$12 + 3(-11/4) - 2f = 0$

$\frac{24 - 33}{2} = 2f$

$\boxed{\frac{-9}{4} = f}$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 + 2\left(\frac{-11}{4}\right)x + 2\left(\frac{-9}{4}\right)y + \frac{19}{2} = 0$

$\underline{2x^2 + 2y^2 - 11x - 9y + 19 = 0}$

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Question 4 (Metrix Algebra)

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

1.

(i) $$A+B = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

(ii) $$A-B = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 4 & -2 \end{bmatrix}$$

(iii) $$AB = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} (1\times3+5\times-1) & (1\times2+5\times4) \\ (3\times3+2\times-1) & (3\times2+4\times2) \end{bmatrix} = \begin{bmatrix} -2 & 22 \\ 7 & 14 \end{bmatrix}$$

2.

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ Y \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ -9 \end{bmatrix}$$

$$Ax = b$$

$$x = \bar{A}^1 b$$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\bar{A}^1 = \frac{CM^T}{|A|}$$

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1

C.M $\begin{bmatrix} \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \end{bmatrix}$

$CM = \begin{bmatrix} (-3+4) & -(-6+2) & (4-1) \\ -(6-2) & (-3-1) & -(2+2) \\ (4-1) & -(-2-2) & (1+4) \end{bmatrix}$

$CM = \begin{bmatrix} 1 & 4 & 3 \\ -4 & -4 & -4 \\ 3 & 4 & 5 \end{bmatrix}$

$CM^T = \begin{bmatrix} 1 & -4 & 3 \\ 4 & -4 & 4 \\ 3 & -4 & 5 \end{bmatrix}$

$|A| = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$

$= 1\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} - (-2)\begin{bmatrix} 2 & -2 \\ 1 & -3 \end{bmatrix} + 1\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$= 1(-3+4) - (2)(-6+2) + 1(4-1)$

$= 1(1) + 2(-4) + 1(3)$

$= \underline{\underline{-4}}$

$$x = A^{-1} b = \frac{CM^T}{|A|} \times b$$

$$= \frac{1}{-4} \begin{bmatrix} 1 & -4 & 3 \\ 4 & -4 & 4 \\ 3 & -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -3 \\ -9 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} (7 + 12 - 27) \\ (28 + 12 - 36) \\ (21 + 12 - 45) \end{bmatrix}$$

$$= -\frac{1}{4} \begin{bmatrix} -8 \\ -4 \\ -12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} //$$

Question 5 (Logarithms)

1

(i) $\log_2 2x^2 = \log_2 2 \times x^2$

$= \log_2 2 + \log_2 x^2$

$= \log_2 2 + 2\log_2 x$ //

(ii) $\log_3 (x^2 y^{-3}) = \log_3 x^2 \times y^{-3}$

$= \log_3 x^2 + \log_3 y^{-3}$

$= 2\log_3 x + \log_3 \left(\frac{1}{y}\right)^3$

$= 2\log_3 x + 3\log_3 \left(\frac{1}{y}\right)$ //

(iii) $\ln \left(\frac{a^2\sqrt{b^3}}{5\sqrt{c^2}}\right) = \ln \left(\frac{a^2 \times \sqrt{b^3}}{5\sqrt{c^2}}\right)$

$= \ln a^2 + \ln \sqrt{b^3} - \ln 5\sqrt{c^2}$

$= 2\ln a + \frac{1}{2}\ln b^3 - \frac{1}{5}\ln c^2$

$= 2\ln a + \frac{3}{2}\ln b - \frac{2}{5}\ln c$ //

2.

(i) $\log_7 5 = \frac{\log_{10} 5}{\log_{10} 7}$

$= 0.82718$

(ii) $\log_3 81 \log_5 0.008 \log_8 (0.125) = \log_3 3^4 \cdot \log_5 \frac{8}{1000} \log_8 \frac{125}{1000}$

$= 4\log_3 3 \times \log_5 \frac{1}{125} \times \log_8 \frac{1}{8}$

$= 4\log_3 3 \times \log_5 \frac{1}{5^3} \times \log_8 \frac{1}{8}$

$= 4\log_3 3 \times \log_5 (5^{-3}) \times \log_8 (8^{-1})$

$= 4\log_3 3 \times (-3)\log_5 5 \times (-1)\log_8 8$

$= 4(1) \times (-3)(1) \times (-1)(1)$

$= \underline{12}$

3. $5^{2x} - 12(5^x) + 35 = 0$

If $5^x = t$

$t^2 - 12t + 35 = 0$

$(t-7)(t-5) = 0$

$t = 7$ or $t = 5$

$5^x = 7$ $\quad 5^x = 5$

$\log_5 7 = x$ $\quad \underline{x = 1}$

$x = \frac{\log_{10} 7}{\log_{10} 5}$

$= \underline{1.209}$

Question 6 (Statistics)

1.

Median

The median is the middle value in an orderd array of data that has been ranked from smallest to largest.

$$\text{Median} = \frac{n+1}{2} \text{ ranked value.}$$

Mode

The mode is the value in a set of data that appears most frequently.

2.

Paperbacks	40	$1.00
hardcover	10	$ 3.50

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{n} = \frac{w_1 x_1 + w_2 x_2}{n} = \frac{3.50\,(10) + 1.00\,(40)}{4.50} = \frac{7.50}{4.50}$$

$$= \$1.667$$

3. a) $\text{Mean}(\bar{x}) = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{282}{11} = 25.636$

b) Median (Me)

18, 19, 20, 23, 25, 27, 28, 28, 28, 32, 34

$$\text{Median} = \frac{11+1}{2} = 6^{th} \text{ one.}$$

Median = 27 //

c) Mode (M_o) = 28

$Q_1 = \frac{1}{4}(11+1)$

$= \frac{12}{4}$

$= 3^{rd}$ one

20

$Q_3 = \frac{3}{4}(11+1)$

$= \frac{3}{4} \times 12^3$

$= 9^{th}$ one

28

d. Interquartile range (IQR) : $Q_3 - Q_1$

$= 28 - 20$

$= 8$

e. $s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})}{n}$

$= \frac{\left[\sum_{i=1}^{n} 282 - 25.636\right]}{11}$

$= 23.3058$

$s = \sqrt{23.3058}$

$= 4.8276$

Question 7 (statistics)

1.

a Population

A population is the set of all elements of interest in a particular study.

b. Census.

The process of conducting a survey to collect data of the entire population is called a census.

2.

Profit (Rs. in 000s)	Number of companies (f_i)	F_i	x_i	$u_i = \frac{x_i - A}{C}$ (A = 42.5, C = 5)	$u_i f_i$	$u_i^2 f_i$
20 - 25	02	2	22.5	-4	-8	32
25 - 30	05	7	27.5	-3	-15	45
30 - 35	14	21	32.5	-2	-28	56
35 - 40	20	41	37.5	-1	-20	20
40 - 45	25	66	42.5	0	0	0
45 - 50	17	83	47.5	1	17	17
50 - 55	10	93	52.5	2	20	40
55 - 60	07	100	57.5	3	21	63
	100				-13	273

a) Mean ($\bar{x}$) = $C\bar{u} + A$

$$= 5 \times \left(\frac{\sum_{i=1}^{n} u_i f_i}{\sum_{i=1}^{n} f_i} \right) + A$$

$$= 5 \times \left(\frac{-13}{100} \right) + 42.5$$

$$= 41.85$$

b. Median (Mc) = $L_1 + \left[\frac{N}{2} - (\Sigma F)_1\right] \frac{c}{fm}$

$= 40 + \left[\frac{100}{2} - 41\right] \frac{5}{66}$

$= \underline{40.681}$

c. Mode (Mo) = $L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) c$

$= 40 + \left(\frac{5}{5+8}\right) \times 5$

$= 41.923$

d. $s^2 = c^2 \left[\frac{\Sigma u_i^2 f_i}{\Sigma f_i} - \left(\frac{\Sigma u_i f_i}{\Sigma f_i}\right)^2\right]$

$= 25 \left[\frac{273}{100} - \left(\frac{-13}{100}\right)^2\right]$

$= 67.8275$

$s = \sqrt{67.8275}$

$= \underline{8.2357}$