Develop Quantum Mechanics Intuition through a Different Quantum Tic Tac Toe

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Abstract

How does one introduce the strangeness of quantum mechanics and quantum information in an interactive and intuitive ways? Quantizing an existing game shows promising potential in performing the task. The paper introduces a different quantization on tic tac toe. We quantize the marking in classical tic tac toe by replacing it with qubit, and introduce unitary operations into the game. The quantization allows players to gain intuition behind the difference between classical and quantum information. The game allows players to perform quantum strategy by exploiting the properties of quantum teleportation and universal quantum gate set.

Keywords— Quantum Tic Tac Toe, Quantum Mechanics, Quantum Game, Quantum Circuit, Quantum Teleportation, Physics Education

1 Introduction

Quantum mechanics is a difficult subject, often counter-intuitive even to a well-trained physicist. This leads to many misconceptions about quantum mechanics in general public, even in mainstream media. However, the future lies in quantum mechanics, with many promising technologies including quantum computer,quantum material[1] and quantum sensing[2], just to name a few. How does one introduce the strangeness of quantum mechanics and quantum information in an interactive and intuitive ways? Quantizing an existing game shows promising potential in performing the task. One of the first game quantization is done by Meyer and Eisert et al on quantum penny game.[3] There have been several games introduced to introduce quantum mechanics at the entry level due to its effectiveness in engaging students. [4][5] A successful gamification can even aid scientist to solve complicated quantum problems by studying players' strategy[6][7].

The goal is to demonstrate as many quantum phenomena as possible with a simple game. The game we chose here is quantum tic tac toe [8], which was introduced as an educational tool to introduce quantum mechanics by Allan Goff. Goff quantizes tic tac toe by quantizing the move - players place the marking at 2 different boxes every turn. His quantization allows the player to create entanglement between moves, and other emergent metaphor follows including the measurement postulates, backward in time causality and many world interpretation.[8] Goff's gameplay is simple, effectively introduces the concept of quantum mechanics which allows educators to introduce quantum mechanics in an interactive way [9][10]. Another version of quantum tic tac toe is introduced by Leon and Cheong [11][12]. Leon and Cheong modified Goff's quantum tic tac toe, forcing the 2 quantum moves to be strictly orthogonal, which allows the quantum game to reduce to classical game easily.

Here, we propose a different quantization - instead of quanziting the move, we quantize the marking of the game by introducing the concept of qubit. The introduction of qubits provides an advantage as qubit is the simplest quantum mechanical system but is capable of capturing most of the central concepts in quantum mechanics.[13][14][15]. Players can visualize a qubit through Bloch sphere, which allows players to gain intuition on what a unitary operation and measurement is. We show that not only does the quantization preserve the emerging quantum mechanics metaphor shown in Goff's Quantum Tic Tac Toe, allow simple reduction to classical tic tac toe, the introduction of unitary moves also allows player to perform quantum strategy. We will also demonstrate the infamous quantum teleportation protocol through the game.[18]

In the next section, we will introduce how the game is quantized and the new moves. Section 3 discusses how quantum entanglement is introduced into the game, while we will talk about some quantum strategy in section 4. We will close off our discussion in section 5.

2 Quantization of the game

A classical tic tac toe can be thought of as a game of classical bits, the marking O and X is the classical bit 0 and 1, each 9 boxes can only store 1 classical bit - they are the memory storage. Our quantum tic tac toe substitute the classical bit to qubit - players take turn to place a pure state qubit in a 3x3 board. 'O' mark and 'X' mark are the spins of the qubit along z axis. We use the braket notation to represent the marking on each box i, where $i \in \{1,2,3,4,5,6,7,8,9\}$, representing the box in the 3x3 board. $|O_i\rangle$ and $|X_i\rangle$, which are defined to be spin up and down along z axis, represent the pure quantum state in box i

$$|O_i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |X_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

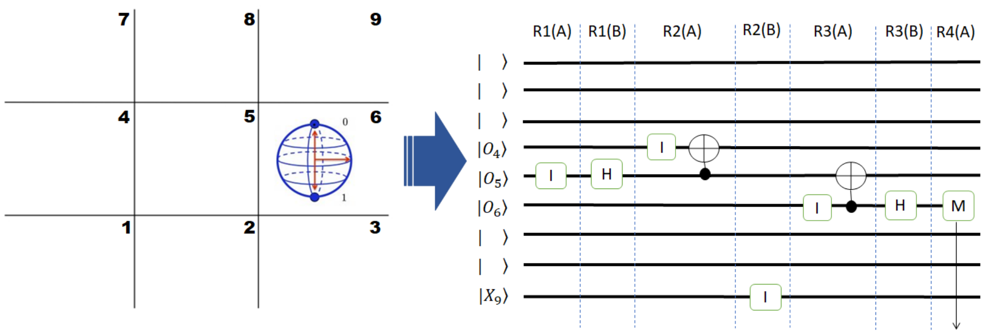


Figure 1: A tic tac toe played with qubit is similar to a 9 qubits quantum computer.The goal is to collapse the qubits into classical bits to form a straight line. Quantum computing and measurement postulate become a direct metaphor through this quantization.

We can treat our quantum tic tac toe as a *9-qubits quantum computer*.(refer to figure 1) The 9 registers represent the 9 boxes in the tic tac toe box. In classical tic tac toe, players win by forming straight lines with 3 classical bit. The winning condition is preserved in quantum tic tac toe. To win the game, players need to collapse the qubit into classical bit to form a straight line from 3 boxes. This is inspired by the *measurement postulate* in quantum mechanics. Translating the winning condition into the 9-qubits quantum computer, the 3 collapsed qubits must come from the following set of register,which shall be called the winning set, W:

$$W = \{(1,2,3)\,,\ (4,5,6)\,,\ (7,8,9)\,,\ (1,4,7),\ (2,5,8),\ (3,6,9),\ (1,5,9),(3,5,7)\} \tag{2}$$

Thus, we formulate the first move for our quantum tic tac toe:

$\boxed{\text{Player may choose to perform measurement on any qubits on the board.}}$

What is the initial state of the qubit? In the classical tic tac toe, the box is empty unless player chooses to put a classical bit onto the box. Analogous to the classical tic tac toe, a register is empty until a player prepares a pure qubit state on the desired register. From now on, we refer Alice as the first player and her pure qubit state is $|O\rangle$, while Bob is the second player and his pure qubit state is $|X\rangle$. This gives rise to the second move in quantum tic tac toe.

$\boxed{\text{Player may choose an empty register to prepare their own pure state qubit}}$

One interesting, and perhaps the most bizarre phenomena in quantum mechanics is *quantum superposition* - which says the physical system with more than 1 state may be in the combination of states. A qubit is a 2-state system, hence our game should allow the superposition of these 2 states in the game. This can be achieved by introducing the Hadamard gate, a single qubit *quantum logic gate* which creates quantum superposition with equal probability, into the game. The 3rd move in quantum tic tac toe is therefore:

$\boxed{\text{Player may choose to create quantum superposition on any desired qubit}}$

We have introduced quite a number of quantum phenomenon in the game, but the game is still not ready to be played. If Hadamard gates are added to every register, the game immediately becomes a game of luck, as the qubit may collapse into either of the pure state with equal chance. This is a form of *quantum randomness*. We desire a game where players are allowed to perform quantum strategy, thus we introduce the following quantum logic gates: Pauli X gate, Pauli Z gate, and CNOT gate. These gates are unitary transformation that preserve the probability amplitude of the quantum state - a consequence of *unitarity principle*. The introduction of unitary transformation is essential - it draws a clear line between a measurement and a unitary transformation, an important concept in understanding the *measurement problem in quantum mechanics*.

Player may choose to perform unitary transformation on the desired qubit

We group the first move(measurement) as a projection operation, and the others(prepare pure state qubit, Unitary Transformation) as unitary operation. The game flow can then be summarised as followed:

1. Each turn, player has to choose one of the 9 boxes/9 registers. He/she will then choose to perform either projection or unitary operation on the chosen box/register. The 2 operations cannot be executed together.
2. Player can perform up to 3 unitary operations in each turn on the chosen register.
3. Player can only perform projection operation on the chosen register. The player's turn will end after the operation.

The 5 allowed unitary operations are summarized in the table below:

1	Qubit preparation
2	Hadamard Gate
3	CNOT gate
4	Pauli X gate
5	Pauli Z Gate

Table 1: Unitary Operations

***Remark**: Among the 5 operations, CNOT gate is the only 2-qubits logic gate. Thus, if a player chooses to perform CNOT gate on the desired register, the player requires to choose another register, and determine which is the control and target qubit.*

The new rules has transformed the game into assembling different logic gates to obtain a set of collapsed state described by the winning set W. We present a game play example to familiarize ourselves with the new rules.

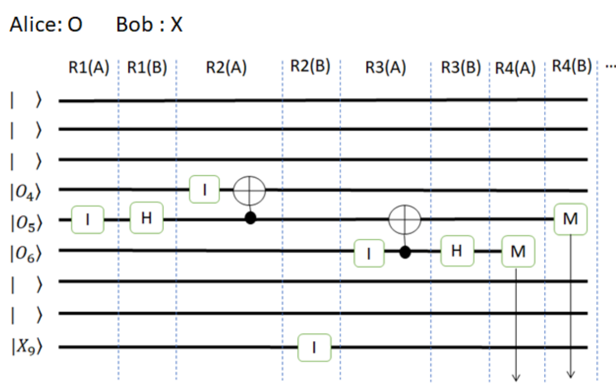


Figure 2: One of the possible game play. The I block means qubit preparation, H block is Hadamard gate and M block means measurement.The circle connected to a dot is the CNOT gate.

Figure 2 shows one of the possible game play for quantum tic tac toe. The game starts with Alice initializing a qubit in box 5. Bob responded by placing a Hadamard gate in box 5, creating superposition. The quantum state of the board is :

$$|\psi_{R1}\rangle = |\psi_5\rangle = \frac{|O_5\rangle + |X_5\rangle}{\sqrt{2}} \tag{3}$$

Round 2 starts with 2 unitary operations from Alice - prepare a qubit in box 4, and perform a CNOT gate with qubit in box 5 as the control qubit, and qubit in box 4 as her target. Bob responded by preparing a qubit in box 9. The quantum state after the 2nd round is :

$$|\psi_{R2}\rangle = (\frac{|O_4\rangle\,|O_5\rangle + |X_4\rangle\,|X_5\rangle}{\sqrt{2}})\,|X_9\rangle \tag{4}$$

An observant reader may notice that the current board contains a pair of Bell state qubits, located in box 4 and box 5. In round 3, Alice prepares a qubit in box 6, and perform CNOT gate operation using box 6 as control qubit, and box 5 as her target qubit. Bob performs Hadamard gate operation and ends his round.

$$|\psi_{R3}\rangle = |\psi_{R_2}\rangle\,(\frac{|O_6\rangle + |X_6\rangle}{\sqrt{2}}) \tag{5}$$

The next round started with Alice performing measurement on box 6. Bob follows and performs measurement on box 5. Note that the probability of the qubit collapsing to any state is described by the probability amplitude of the quantum state. Hence, the board's quantum state and the outcome of the measurement may vary even if both players repeat their steps in another game. Here, there are 4 possible outcomes, listed in the table below:

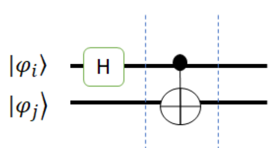
Measurement Outcome	Board's Quantum State, $\lvert\psi_{R4}\rangle$
O_5, O_6	$\lvert O_4\rangle\,\lvert X_9\rangle$
O_5, X_6	$\lvert O_4\rangle\,\lvert X_9\rangle$
X_5, O_6	$\lvert X_4\rangle\,\lvert X_9\rangle$
X_5, X_6	$\lvert X_4\rangle\,\lvert X_9\rangle$

Table 2: Measurement outcome and the board's quantum state after the measurement. We use braket notation to indicate quantum state. A collapsed qubit will be written without braket notation.

Measurement postulate is one of the consequences of *Copenhagen Interpretation of quantum mechanics*. The 4 different measurement outcome comes from the collapse of wavefunction. One may also interpret the 4 different outcomes as the 4 alternate possibilities - *the many world interpretation*[16].

3 Spooky Action in Quantum Tic Tac Toe

Quantum entanglement is one of the bizzare consequence from quantum mechanics which was one widely debated. To quote Einstein, spooky action at a distance. [17] How do we demonstrate quantum entanglement in quantum tic tac toe? We resort to one of the common method used in quantum computation - the CNOT gate.



$\lvert\varphi_i\varphi_j\rangle$	Outcome
$\lvert OO\rangle$	$\frac{1}{\sqrt{2}}(\lvert OO\rangle + \lvert XX\rangle)$
$\lvert OX\rangle$	$\frac{1}{\sqrt{2}}(\lvert OX\rangle + \lvert XO\rangle)$
$\lvert XO\rangle$	$\frac{1}{\sqrt{2}}(\lvert OO\rangle - \lvert XX\rangle)$
$\lvert XX\rangle$	$\frac{1}{\sqrt{2}}(\lvert OX\rangle - \lvert XO\rangle)$

Figure 3: Quantum circuit to prepare Bell state qubit

Figure 3 shows the quantum circuit to create a pair of maximally entangled qubit, also known as *Bell State qubits*. In our game, a Bell state qubit can be prepared in a single turn by using a Hadamard gate and CNOT gate. Depending on the type of Bell states the player desires, he/she has to prepare different pair of qubits before passing the 2 qubits into the quantum circuit shown in figure 3.

4 Quantum Strategy

4.1 Classical vs Quantum Player

Our quantum tic tac toe allows players to play classical tic tac toe by restricting the moves to only pure state qubit preparation and measurement. We can thus define a "classical player" as players who only execute pure state qubit preparation and measurement as their only moves, whereas a "quantum player" uses 5 of the moves in his/her gameplay. Interestingly, a classical player will never win the game if his/her opponent is a quantum player. Suppose Alice is a classical player, while Bob is a quantum player. Since Alice refuses to use any of the quantum logic gates, Bob can win the game by placing a Pauli X gate every time after Alice prepares a pure state qubit.(refer to figure 4) If Alice chooses to measure the qubit, it will collapse to Bob's classical marking. The manipulation of quantum state is important as in this game, players are required to consider both quantum and classical information and use them both in their game strategies.

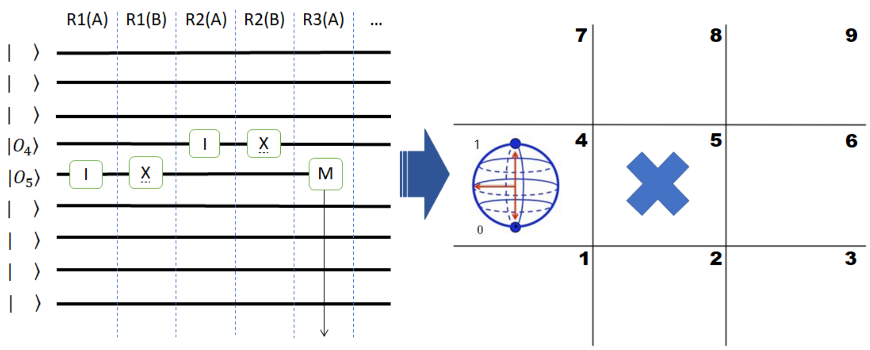
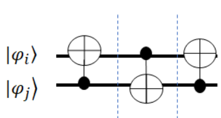


Figure 4: Since the classical player can't unitary operation other than prepare pure state qubit, a quantum player can always reflect the pure state qubit to his/her pure state with CNOT gate or Pauli X gate, and perform measurement in the next turn to collapse the pure quantum state to his/her classical marking.

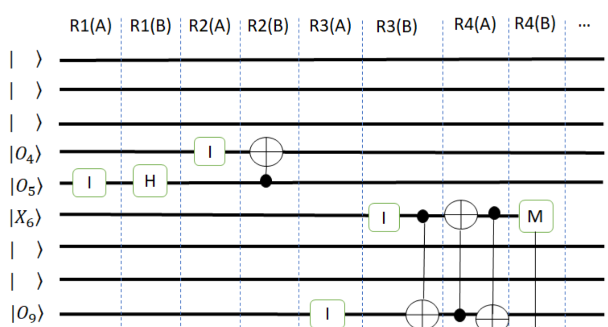
4.2 Universal Gate set and Gate Decomposition as Strategy

A universal gate set is a set of quantum logic gates which can approximate any unitary operation in quantum computing. One universal quantum gate set is the CNOT gate and other single qubit gates. In this game, we didn't introduce the Pauli Y gate, Phase gate and the T gate. Thus, there will be some unitary operations which are not executable in the game. Regardless, players can exploit the universality of quantum gate set in their game play. With the limited quantum gates in the game, players can build a quantum swap gate to swap the qubits between 2 registers. This can be done with 3 CNOT gates as shown in Figure 5a.



$\|\varphi_i\varphi_j\rangle$	Outcome
$\|OO\rangle$	$\|OO\rangle$
$\|OX\rangle$	$\|XO\rangle$
$\|XO\rangle$	$\|OX\rangle$
$\|XX\rangle$	$\|XX\rangle$

(a) Swap gate quantum circuit using 3 CNOT gates and its logic table



(b) Using swap gate as strategy. In the game, Alice swaps the qubits between register 6 and 9.

Figure 5

To use this as a strategy, consider the game play shown in figure 5b. The game starts with Alice and Bob fighting over for box 5 and box 6 for the 1st 2 rounds by creating a pair of Bell State qubits. In Round 3, Alice places a pure state qubit in box 9, Bob places a pure state qubit in box 6 and uses his qubit to flip Alice's qubit. The quantum state after Round 3 can be written as:

$$|\psi_{R3}\rangle = [\frac{|O_5O_6\rangle + |X_5X_6\rangle}{\sqrt{2}}][|X_6X_9\rangle] \tag{6}$$

Alice makes use of Bob's CNOT gate and places 2 more CNOT gate in box6 and 9 in round 4. By doing so, she has successfully swapped the qubit in box 6 and box 9. The quantum state after Alice's turn is:

$$|\psi_{R4(Alice)}\rangle = [\frac{|O_5O_6\rangle + |X_5X_6\rangle}{\sqrt{2}}][|O_6X_9\rangle] \tag{7}$$

Not knowing about quantum swap gate, Bob measures box 6. Box 6 collapses to the classical mark O, giving Alice an advantage.

In quantum computation, the universality of quantum logic gate set allows complicated unitary operation to be decompose to combinations of the element in the universal quantum logic gate set. The quantum swap gate can be decompose into 3 CNOT gates, thus allowing players to execute it as a strategy. The game allows players to explore other strategies by forming complex quantum gates from the 4 gates offered in the game.

4.3 Quantum Teleportation as a Strategy

We will now proceed to show one of the main result of the paper: quantum teleportation as a game strategy.

To recap, quantum teleportation is a protocol to transmit quantum information from one place to another with the help of classical communication channel. The idea is as followed: Alice, the sender, wishes to send a qubit ψ to Bob, which is far away from her location. They shared a Bell state qubit. Since Bob can perform unitary operation on his qubit to change it to the state ψ, Alice can send Bob the information on the state vector $|\psi\rangle$. However, due to the measurement postulate in quantum mechanics, it is impossible to learn everything about the state of the qubit. Lucky for Alice, we can divide the full information encoded in the qubit into classical and quantum part, and send both of them separately. To do that, Alice must entangle her Bell state qubit with qubit $|\psi\rangle$, which allows the quantum information of the state vector to be kept inside her qubit once she performs measurement on ψ. She then measures ψ and her Bell state qubit, and tells Bob the results she obtained via a classical channel. The classical information is sent via the classical channel, while the quantum information is sent to Bob due to entanglement. From the classical information, Bob can perform unitary operation on the qubit and reconstruct ψ from his Bell state qubit [18].

How does one demonstrate this protocol in the game? It can be done easily if we treat the marking as qubits. We perform a little bit of modification on the game play shown in figure 2. Here, Alice ends her turn after preparing a pure state qubit in round 2. Bob doesn't want to give up box 4 without a fight, and perform CNOT gate to create a pair of Bell State qubit at the end of round 2. The board quantum state is thus:

$$|\psi'_{R2}\rangle = \frac{|O_4\rangle|O_5\rangle + |X_4\rangle|X_5\rangle}{\sqrt{2}} \tag{8}$$

Alice however, wishes to secure box 4. She has studied quantum teleportation protocol and decided to exploit the Bell state on the board. She first creates a pure state qubit in box 6 and places a CNOT gate in box 5 and box 6 together. Qubit 6 is the qubit she wishes to send to box 4. Bob creates a quantum superposition on box 6 after Alice's turn. The current board quantum state is now:

$$|\psi'_{R3}\rangle = \frac{|O_4\rangle|O_5\rangle + |X_4\rangle|X_5\rangle}{\sqrt{2}}(\frac{|O_6\rangle + |X_6\rangle}{\sqrt{2}}) \tag{9}$$

$$= |O_4\rangle(\frac{|O_5O_6\rangle + |O_5X_6\rangle}{2}) + |X_4\rangle(\frac{|X_5O_6\rangle + |X_5X_6\rangle}{2}) \tag{10}$$

Alice can now execute her plan to teleport the qubit in box 6 to box 4. She performs measurement on box 6. Oblivious to Alice's plan, Bob performs measurement on box 5. There are 4 possible outcomes after the measurement.

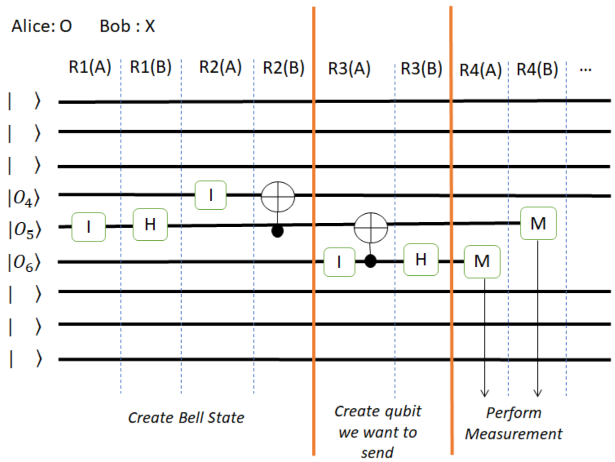
Measurement Outcome	Board's Quantum State,$|\psi_{R4}\rangle$
O_5, O_6	$|O_4\rangle$
O_5, X_6	$|O_4\rangle$
X_5, O_6	$|X_4\rangle$
X_5, X_6	$|X_4\rangle$

Table 3: Measurement outcome and the board's quantum state after the measurement

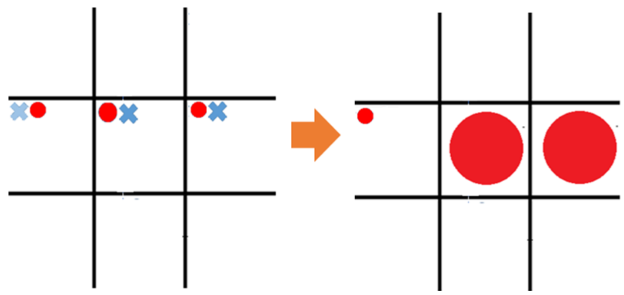
Alice has a $\frac{1}{4}$ chance of winning the game in the next round if qubit 5 and 6 collapse to 'O'. Bob may think he possesses equal chance as Alice to win the game, but he has forgotten that Alice will be performing a move in the next turn. The measurement outcome is visible to both players. If the outcome is undesired, Alice can always place a Pauli X gate such that

$$\hat{P}_x |X_4\rangle = |O_4\rangle \tag{11}$$

Alice has teleported her qubit in box 6 to box 4. Bob lost an initiative in the game, putting himself in a danger position.



(a) Quantum Teleportation protocol.



(b) One of the possible outcome after quantum teleportation protocol. In this scenario, Alice wins the game by performing measurement on box 4 the next round.

Figure 6

The quantum teleportation strategy we demonstrate here differs a little from the original quantum teleportation protocol. In the original protocol, Bob(receiver) doesn't know about the state of the qubit Alice(sender) is sending to him. In the game, Alice is both the sender and the receiver - she is simply exploiting the quantum circuit used in quantum teleportation protocol. Furthermore, the game uses an open classical channel since the measurement result is known to the both parties immediately., Despite the difference, the game is able to highlight the important intuition behind quantum teleportation - information can be divided into classical and quantum information, and quantum information can only be sent via a quantum communication channel - which is quantum entanglement. The demonstration also breaks the common misconception that a quantum teleported qubit will "disappear" after being teleported when in fact it should collapse to a classical bit. For quantum teleportation to be a strategy, the collapse of the transferred qubit is crucial, as it may help the player to win the game if the collapsed state belongs to the player.

5 Discussion and Conclusion

Quantum tic tac toe offers an intuitive way to show that information is physical[19]. The effect of quantum information on the game play is immediate - if one ignores quantum information, one may fall into the quantum teleportation trap like Bob did. Players have to understand the difference between classical information and quantum information, and exploiting those 2 information is crucial in winning the game.

The game encourages players to explore the use of quantum logic gate by allowing 3 unitary moves each round. A classical player in this game will be in severe disadvantage if his/her opponent utilizes the unitary moves. This forces player to think "quantum mechanically". Players are challenged to visualize and manipulate qubit, which is helping in developing intuition on quantum mechanics.

To conclude, we have introduced a different quantum tic tac toe by quantizing the marking. Through this quantization, we demonstrated the emerging metaphor in quantum mechanics with the game, which includes quantum computing, quantum superposition, quantum entanglement, measurement postulate, Copenhagen Interpretation, and Many-World Interpretation. The game also introduced a new application for quantum teleportation - as a game strategy in quantum zero sum game. The game shows potential in demonstrating complex quantum phenomenon such as many body entanglement, and might help in discovering a new quantum information processing protocol.

This game might be helpful in introducing quantum programming, cultivating quantum thinking through game.

The game is available here: `https://github.com/honlin96/Quantum-Game/blob/master/tictactoev4.py`

6 Acknowledgement

We wish to acknowledge the funding support for this project from Nanyang Technological University under the Undergraduate Research Experience on CAmpus (URECA) programme. The author thanks Assoc Prof Chew Lock Yue for the constant support and helpful guidance. The author also thanks Gan Beng Yee for useful discussion.

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