**Paper Title**: Real-Time Simulation Method based on Backward Differential Formula for Stiff Systems
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## 1 Summary

This paper introduces a real-time simulation method based on the backward differential formula (BDF) to solve stiff systems of ordinary differential equations (ODEs). Stiff systems, prevalent in engineering applications, pose challenges for real-time simulation due to root finding during iterations and discontinuous states. The proposed method addresses these challenges by dynamically adjusting the order and step size, balancing computing cost and accuracy demands. Through analysis and testing, the method demonstrates efficiency, particularly in systems with stiff equations, offering increased flexibility in simulation applications.

### 1.1 Motivation

The motivation behind this research is the critical need for real-time simulation in testing algorithms and designs, especially in fields like aerospace and applications like virtual reality games. Simulating systems with ODEs involves solving initial value problems using numerical methods, and the choice of a method impacts both accuracy and computational efficiency. The motivation is to develop a method that handles stiff systems efficiently without compromising real-time performance.

### 1.2 Contribution

The primary contribution of this paper is the introduction of a real-time simulation method based on BDF, dynamically adjusting order and step size. This approach mitigates issues related to state discontinuity and excessive iterations for root finding. The method exhibits 4-stability, making it suitable for stiff systems, and its stability is confirmed through numerical experiments. Truncation error analysis guides the adaptive control of minor step size, balancing precision and computational efficiency. The proposed method is tested on various examples, showcasing its accuracy and efficiency in handling real-time simulations, outperforming traditional methods in certain scenarios.

### 1.3 Methodology

The methodology involves utilizing the well-established implicit multistep method, BDF, and adapting it to real-time simulation by dynamically adjusting the order and minor steps. Stability analysis demonstrates 4-stability, ensuring stability even with large step sizes. Truncation error analysis guides the adaptive control of minor step size, maintaining stability and accuracy in simulation. The method is tested on various examples, including systems with changing parameters and a stiff electrical circuit.

### 1.4 Conclusion

In conclusion, the real-time simulation method based on BDF provides a practical solution for handling stiff systems in engineering applications. The dynamic adjustment of order and step size proves advantageous, offering flexibility and efficiency in real-time simulations. The method's stability, accuracy, and adaptability make it a promising choice for a wide range of simulation scenarios, especially those involving stiff ODEs.

## 2 Limitations

### 2.1 First Limitation

One limitation of the proposed method is that its effectiveness might be influenced by specific characteristics of the system under consideration. Further research and testing are needed to identify scenarios where the method might be less suitable or require additional adjustments.

### 2.2 Second Limitation

Another limitation is that the proposed method's computational efficiency may be impacted by the complexity of the system. In highly intricate simulations, the trade-off between accuracy and real-time performance might become more pronounced, potentially affecting the method's applicability.

## 3 Synthesis

In synthesis, the paper presents a novel approach to real-time simulation, addressing the challenges posed by stiff ODEs. While demonstrating notable strengths in stability, accuracy, and adaptability, the method is not without limitations. These limitations highlight areas for future research and refinement. Overall, the proposed method offers a valuable contribution to the field of real-time simulation, particularly in scenarios involving stiff differential equations.