

**Problem 1:**

Info(A)

$$-(7/16) \cdot \log_2(7/16) - (9/16) \cdot \log_2(9/16) = 0.989 \text{ bits}$$

RAIN:

$$\begin{aligned} & (4/16) \cdot (-\frac{3}{4}) \cdot \log_2(\frac{3}{4}) - (\frac{1}{4}) \cdot \log_2(\frac{1}{4}) + (12/16) \cdot (-\frac{4}{12}) \cdot \log_2(\frac{4}{12}) - (8/12) \cdot \log_2(8/12) \\ & = (0.25) \cdot (-0.75) \cdot (-0.415) - (0.25) \cdot (-2) + (0.75) \cdot (-0.333) \cdot (-1.586) - (0.667) \cdot (-0.584) = 0.057 \text{ bits} \\ \text{Gain(Rain): } & = 0.989 - 0.057 = 0.932 \text{ bits} \end{aligned}$$

SPRINKLER:

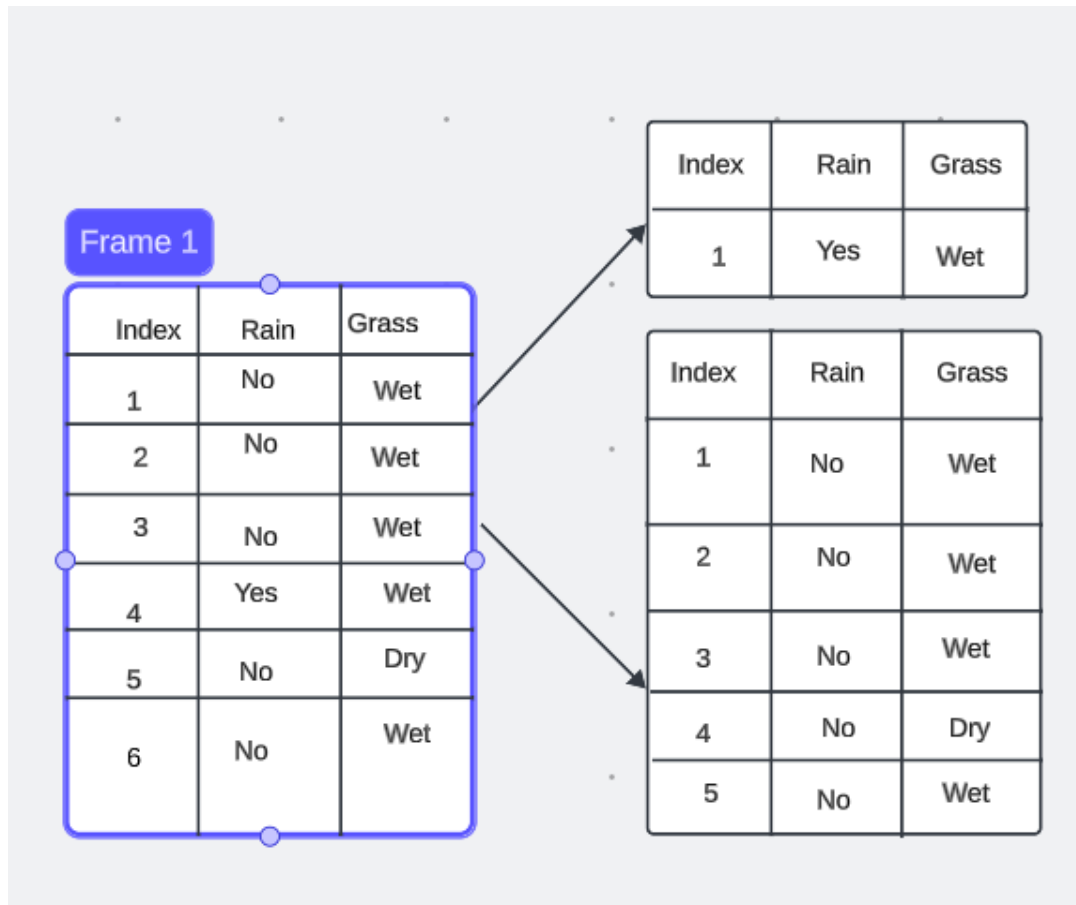
$$\begin{aligned} & (6/16) \cdot (-\frac{5}{6}) \cdot \log_2(\frac{5}{6}) - (\frac{1}{6}) \cdot \log_2(\frac{1}{6}) + (10/16) \cdot (-\frac{2}{10}) \cdot \log_2(\frac{2}{10}) - (8/10) \cdot \log_2(8/10) \\ & = (0.375) \cdot (-0.833) \cdot (-0.264) - (0.167) \cdot (-2.582) + (0.625) \cdot (-0.2) \cdot (0.322) - (0.8) \cdot (-0.322) \\ & = 0.05 \text{ bits} \\ \text{Gain(Sprinkler): } & 0.989 - 0.05 = 0.939 \text{ bits} \end{aligned}$$

INDEX	RAIN	GRASS
1	No	Wet
2	No	Wet
3	No	Wet
4	Yes	Wet
5	No	Dry
6	No	Wet

Index	Rain	Grass
1	No	Dry
2	No	Dry
3	Yes	Wet
4	No	Dry
5	No	Dry
6	Yes	Wet
7	No	Dry

8	No	Dry
9	Yes	Wet
10	No	Dry

Split the tuples



Index	Rain	Grass
1	No	Dry
2	No	Dry
3	Yes	Wet
4	No	Dry
5	No	Dry
6	Yes	Wet
7	No	Dry
8	No	Dry
9	Yes	Wet
10	No	Dry

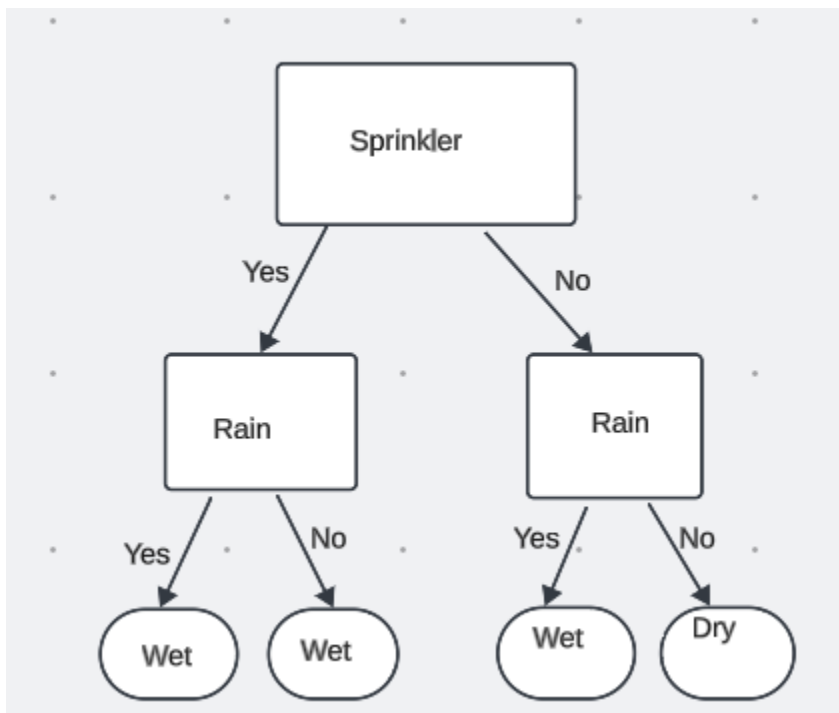
  

Index	Rain	Grass
1	Yes	Wet
2	Yes	Dry
3	Yes	Wet

Index	Rain	Grass
1	No	Dry
2	No	Dry
3	No	Dry
4	No	Dry
5	No	Dry
6	No	Dry
7	No	Dry

Final Decision Tree



**Problem 2:**

Class	Wet	Dry	Total	Recognition
Grass = wet	TP = 4	FN = 1	5	80%
Grass = Dry	FP = 2	TN = 3	5	60%
	Total = 6	Total = 4	10	70%

- Classification Accuracy:  $(TP + TN) / (P + N) = 7/10 = 70\%$
- Error Rate:  $(FP + FN) / (P + N) = 3/10 = 30\%$
- Sensitivity:  $TN / P = \frac{4}{5} = 80\%$
- Specificity:  $TN / N = \frac{3}{5} = 60\%$
- Precision:  $TP / (TP + FP) = 4/6 = 66.67\%$
- Recall:  $TP / (TP + FN) = \frac{4}{5} = 80\%$
- F-score:  $(2 * \frac{4}{6} * \frac{4}{5}) / (\frac{4}{6} + \frac{4}{5}) = 8/11 = 72.73\%$

**Problem 3:**

$$P(\text{Grass} = \text{Wet}) = 7/16 = 0.438$$

$$P(\text{Grass} = \text{Dry}) = 9/16 = 0.563$$

$$P(\text{Rain} = \text{"No"} | \text{Grass} = \text{"Wet"}) = (4/16) * (7/16) = 4/7 = 0.571$$

$$P(\text{Rain} = \text{"No"} | \text{Grass} = \text{"Dry"}) = (8/16) * (9/16) = 8/9 = 0.889$$

$$P(\text{Sprinkler} = \text{"yes"} | \text{Grass} = \text{"Wet"}) = (5/16) / (7/16) = 0.714$$

$$P(\text{Sprinkler} = \text{"yes"} | \text{Grass} = \text{"Dry"}) = (1/16) / (9/16) = 0.111$$

$$P(X | \text{Grass} = \text{"Wet"}) = P(\text{Rain} = \text{"No"} | \text{Grass} = \text{"Wet"}) * P(\text{Sprinkler} = \text{"yes"} | \text{Grass} = \text{"Wet"})$$

$$\rightarrow 0.571 * 0.714 = 0.408$$

$$P(X | \text{Grass} = \text{"Dry"}) = P(\text{Rain} = \text{"No"} | \text{Grass} = \text{"Dry"}) * P(\text{Sprinkler} = \text{"yes"} | \text{Grass} = \text{"Dry"})$$

$$\rightarrow 0.889 * 0.111 = 0.099$$

$$P(X | \text{Grass} = \text{"Wet"}) * P(\text{Grass} = \text{"Wet"}) \rightarrow 0.408 * 0.438 = 0.179$$

$$P(X | \text{Grass} = \text{"Dry"}) * P(\text{Grass} = \text{"Dry"}) \rightarrow 0.099 * 0.563 = 0.056$$

Since  $P(X | \text{Grass} = \text{"Wet"})$ ,  $P(\text{Grass} = \text{"Wet"})$  is maximum, the fourth record (Rain = "No", Sprinkler = "yes" and Grass = Wet) is predicted as Wet by the naïve Bayesian classifier.

**Problem 4:**

RAIN:

$$P(\text{Rain} = \text{Yes}) = 4/16 = 0.25$$

$$P(\text{Rain} = \text{No}) = 12/16 = 0.75$$

#### SPRINKLER:

$$P(\text{Sprinkler} = \text{yes} \mid \text{Rain} = \text{Yes}) = P(\text{Sprinkler} = \text{yes}, \text{Rain} = \text{Yes}) / P(\text{Rain} = \text{Yes})$$

$$\rightarrow (1/16) / (4/16) = \frac{1}{4} = 0.25$$

$$P(\text{Sprinkler} = \text{No} \mid \text{Rain} = \text{Yes}) = P(\text{Sprinkler} = \text{No}, \text{Rain} = \text{Yes}) / P(\text{Rain} = \text{Yes})$$

$$\rightarrow (3/16) / (4/16) = \frac{3}{4} = 0.75$$

$$P(\text{Sprinkler} = \text{yes} \mid \text{Rain} = \text{No}) = P(\text{Sprinkler} = \text{yes}, \text{Rain} = \text{No}) / P(\text{Rain} = \text{No})$$

$$\rightarrow (5/16) / (12/16) = 5/12 = 0.42$$

$$P(\text{Sprinkler} = \text{No} \mid \text{Rain} = \text{No}) = P(\text{Sprinkler} = \text{No}, \text{Rain} = \text{No}) / P(\text{Rain} = \text{No})$$

$$\rightarrow (7/16) / (12/16) = 0.58$$

#### GRASS:

$$P(\text{Grass} = \text{Wet} \mid \text{Sprinkler} = \text{Yes}, \text{Rain} = \text{Yes})$$

$$\rightarrow (1/16) / (1/16) = 1$$

$$P(\text{Grass} = \text{Wet} \mid \text{Sprinkler} = \text{Yes}, \text{Rain} = \text{No})$$

$$\rightarrow (2/16) / (3/16) = \frac{2}{3} = 0.67$$

$$P(\text{Grass} = \text{Wet} \mid \text{Sprinkler} = \text{No}, \text{Rain} = \text{Yes})$$

$$\rightarrow (4/16) / (5/16) = \frac{4}{5} = 0.80$$

$$P(\text{Grass} = \text{Wet} \mid \text{Sprinkler} = \text{No}, \text{Rain} = \text{No})$$

$$\rightarrow (0/16) / (7/16) = 0$$

$$P(\text{Grass} = \text{Dry} \mid \text{Sprinkler} = \text{Yes}, \text{Rain} = \text{Yes})$$

$$\rightarrow (0/16) / (1/16) = 0$$

$$P(\text{Grass} = \text{Dry} \mid \text{Sprinkler} = \text{Yes}, \text{Rain} = \text{No})$$

$$\rightarrow (1/16) / (3/16) = \frac{1}{3} = 0.33$$

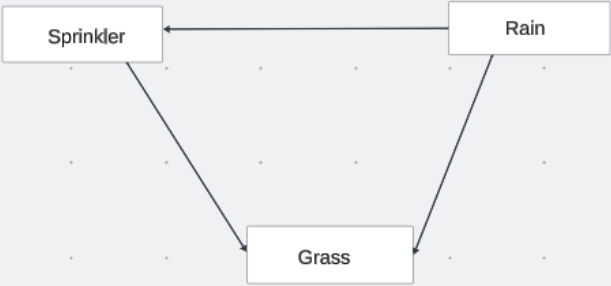
$$P(\text{Grass} = \text{Dry} \mid \text{Sprinkler} = \text{No}, \text{Rain} = \text{Yes})$$

$$\rightarrow (1/16) / (5/16) = \frac{1}{5} = 0.20$$

$$P(\text{Grass} = \text{Dry} \mid \text{Sprinkler} = \text{No}, \text{Rain} = \text{No})$$

$$\rightarrow (7/16) / (7/16) = 1$$

	P(Sprinkler)	
Rain	Yes	No
Yes	0.25	0.75
No	0.42	0.58



	P(Rain)
Yes	0.25
No	0.75

		P(Grass)	
Sprinkler	Rain	Wet	Dry
Yes	Yes	1	0
Yes	No	0.67	0.33
No	Yes	0.80	0.20
No	No	0	1