1. $f(z) = \log_2(1+z)$ where $z = x^T x$, $x \in \mathbb{R}$

Find the derivative.

Answer

Let

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Therefore $x^T = [x_1 \ x_2 \ \dots \ x_n]$

$$\therefore x^T x = [x_1 \ x_2 \ \dots \ x_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= [x_1^2 + x_2^2 + \dots + x_n^2]$$

$$\frac{df}{dx} = \frac{df}{dz} \frac{dz}{dx}$$

$$= \frac{d}{dz} \log_2(1+z) \ \frac{d}{dx}[x_1^2 + x_2^2 + \dots + x_n^2]$$

$$= \frac{1}{1+z} \frac{dz}{dz} (2x_1 + 2x_2 + \dots + 2x_n)$$

$$= \frac{1}{1+z} 2(x_1 + x_2 + \dots + x_n)$$

$$= \frac{2}{1+z} \sum_{i=1}^{n} x_i \quad \text{(Ans.)}$$

2. $f(z) = z^{-z/2}$ where $z = g(y)$

$$g(y) = y^T S^{-1} y$$

$$y = h(x)$$

$$h(x) = x - u$$

Find the ~~derit~~ derivative of the function.

Answer

$$\frac{df}{dx} = \frac{df}{dz} \frac{dz}{dy} \frac{dy}{dx}$$

here $\frac{df}{dz} = \frac{d}{dz} e^{-z/2} = -\frac{e^{-z/2}}{2}$

$$\frac{dz}{dy} = \frac{d}{dy}(y^T S^{-1} y)$$

$$= \lim_{h \to 0} \frac{(y^T + h) S^{-1} (y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} y}{h}$$

P.T.O.

$$= \lim_{h \to 0} \frac{y^T S^{-1} y + y^T S^{-1} h + h S^{-1} y + h^2 S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{h\,(y^T S^{-1} + S^{-1} y + h S^{-1})}{h}$$

$$= \lim_{h \to 0} (y^T S^{-1} + S^{-1} y + h S^{-1})$$

$$= y^T S^{-1} + S^{-1} y$$

$$\frac{dy}{dx} = \frac{d(x - \mu)}{dx} = 1$$

Therefore $\dfrac{df}{dx} = \dfrac{df}{dz} \cdot \dfrac{dz}{dy} \cdot \dfrac{dy}{dx}$

$$= -\frac{e^{-z/2}}{2} (y^T S^{-1} + S^{-1} y) \cdot 1$$

$$= -\frac{e^{-z/2}}{2} \frac{1}{S} (y^T + y) \quad \text{(Ans.)}$$