CSI 5325 Assignment 0

Sadia Nasrin Tisha

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1. **Find the value x that maximizes** $f(x) = 3x^2 + 24x - 30$

   Given that, $f(x) = -3x^2 + 24x - 30$
   So the first derivation of $f(x)$ is
   $f'(x) = -6x + 24$
   Here,
   To find the value x that maximizes $f(x)$, we need to assign $f'(x) = 0$.
   $f'(x) = 0$
   $\Rightarrow -6x + 24 = 0$
   $\Rightarrow 6x = 24$
   $\Rightarrow x = 4$

   So value for $x = 4$ maximizes $f(x)$.

2. **Find the partial derivatives of g(x) with respect to** x_0 **and** x_1**:**
   $g(x) = 3x_0^3 - 2x_0x_1^2 + 4x_1 - 8$

   Given that,$g(x) = 3x_0^3 - 2x_0x_1^2 + 4x_1 - 8$
   So,the partial derivative of $g(x)$ in respect to x_0: $f(x_0) = 9x_0^2 - 2x_1^2$
   And the partial derivative of $g(x)$ in respect to x_1: $f(x_1) = -4x_0x_1 + 4$

3. **What is the value of** $AB^T + C^{-1}$**, if the following define A, B, and C?**
   Here,
   A= $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ C= $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
   So,
   B^T= $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$
   AB^T= $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 26 & 62 \\ 44 & 107 \end{bmatrix}$
   $C^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \frac{1}{(2\times1)-0} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$
   $AB^T + C^{-1}$= $\begin{bmatrix} 26 & 62 \\ 44 & 107 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 27 & 62 \\ 44 & 107.5 \end{bmatrix}$
   The matrix Python code can be found here:
   `https://colab.research.google.com/drive/1b5Zs_024BsEUhQr4lPVDaYv6wYBh1oVZ?usp=sharing`

4. **Write down the mathematical definitions of the simple Gaussian, multivariate Gaussian, Bernoulli, binomial and exponential distributions**

Simple Gaussian Distributions

$$N(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}exp[\frac{(x-\mu)^2)}{2\sigma^2}]$$

Here,
μ = mean
σ^2 = variance (Standard daviation σ)

Multivariate Gaussian Distributions

$$N(X;\mu,\Sigma) = \frac{1}{(2\pi)^{d/2}}|\Sigma|^{-1/2}exp\{-\frac{(X-\mu)\Sigma^{-1}(X-\mu)^T)}{2}\}$$

Here,
X= $d \times 1$ random vector
μ = mean vector
Σ= population variance-covariance matrix
$|\Sigma|$= matrix determinant

Bernoulli Distributions

The Bernoulli distribution is a discrete distribution having two possible outcomes labelled by n=0 and n=1 in which n=1 ("success") occurs with probability p and n=0 ("failure") occurs with probability 1-p, where $0 < p < 1$.
So the bernoulli distribution function is given below:-

$$D(n) = \begin{cases} 1-p, & \text{for } n = 0 \\ p, & \text{for } n = 1 \end{cases}$$

Binomial Distributions

Suppose that n independent trials of the same probability experiment are performed, where each trial results in either a "success" (with probability p), or a "failure" (with probability 1-p). If the random variable X denotes the total number of successes in the n trials, then X has a binomial distribution with parameters n and p , which we write X $\sim$ binomial(n,p) .
The probability mass function of X is given by

$$p(x) = P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \text{for } x = 0, 1, \ldots, n.$$

Exponential Distributions

The continuous random variable, X have an exponential distribution, if it has the following probability density function:

$$f_X(x|\lambda) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } x \geq 0 \\ 0, & \text{for } x < 0 \end{cases}$$

Here,
λ= parameter of distribution

5. **What is the relationship between the Bernoulli and binomial distributions?**

A Bernoulli random variable has two possible outcomes: 0 or 1. A binomial distribution is the sum of

independent and identically distributed Bernoulli random variables.
A bernoulli random variable X is a random variable that satisfies $P(X = 1) = p, P(X = 0) = 1 - p$. If $X = X_1 + X_2 + \ldots + X_n$ and each of $X_1, X_2, \ldots, X_n$ has a bernoulli distribution with the same value of p and they are independent,then X has a binomial distribution, and the possible values of X are $\{0, 1, 2, 3, \ldots, n\}$. So a bernoulli distribution is a special case of binomial distribution. Specifically, when $n = 1$ the binomial distribution becomes bernoulli distribution. Another way to say all bernoulli distributions are binomial distributions, but most binomial distributions are not bernoulli distributions.

6. **Suppose that random variable $X \sim N(1, 3)$. What is its expected value?**
From Theorem we know,
$X \sim N(\mu, \sigma^2)$ for some $\mu \in R, \sigma \in R_{>0}$, where N is the Gaussian distribution. Then expected value, $E(X) = \mu$.
Here, $X \sim N(1, 3)$, so $\mu = 1$.
Hence, expected value, $E(X) = \mu = 1$

7. Given that, random variable Y has distribution

$$P(Y = y) = \begin{cases} \text{exp(-y)}, & \text{if } y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- $\int_{y=-\infty}^{\infty} p(Y = y)\, dy$
$=\int_{y=-\infty}^{0} \text{p(Y=y)}\, dy + \int_{y=0}^{\infty} \text{p(Y=y)}\, dy$
$= \int_{y=0}^{\infty} \text{e}^{-y}\ \text{dy}$
$= [\ \dfrac{-1}{e^y}]_0^\infty$
$=$ -0+1
$= 1$
Hence, $\int_{y=-\infty}^{\infty} p(Y = y) = 1$
- $\mu_Y = \text{E[Y]} = \int\limits_{y=-\infty}^{\infty} p(Y = y) y\, dy$
$=\int\limits_0^\infty y\text{e}^{-y}\ dy = [-y\text{e}^{-y}\ ]_0^\infty + \int\limits_0^\infty \text{e}^{-y}\ dy$
$= [-y\text{e}^{-y}\ ]_0^\infty + [-\text{e}^{-y}\ ]_0^\infty$
$= [-y\text{e}^{-y}\ - \text{e}^{-y}\ ]_0^\infty$
$= \lim\limits_{y\to\infty} [-y\text{e}^{-y}\ - \text{e}^{-y}\ ]$ - $\left[0\text{e}^{-0}\ - \text{e}^{-0}\ \right]$
$= [0 - 0]$ - $[0 - 1]$
$= 1$
- $\sigma^2 = \text{Var[Y]} = \int_{y=-\infty}^{\infty} p(Y = y)(y - \mu_Y)^2\, dy$
$=\int_{y=-\infty}^{\infty} \text{y}^2\text{p(Y=y)}\ \text{dy} - \mu^2$
$=\int_0^\infty \text{y}^2 e^{-y}\ \text{dy} - \mu^2$
$= [-2e^{-y} - 2ye^{-y} - y^2e^{-y}]_0^\infty - \mu^2$
$=2 - 1^2$
$=1$
- $E[Y|Y \geq 10]$
$=\int_{y=10}^{\infty} ye^{-y}\, dy$
$=[-ye^{-y}]_{10}^\infty + [-e^{-y}]_{10}^\infty$
$=[-ye^{-y} - e^{-y}]_{10}^\infty$
$=[\frac{-y}{e^y} - \frac{1}{e^y}]_{10}^\infty$
$=0 + 0 + \frac{10}{e^{10}} + \frac{1}{e^{10}}$
$=\frac{11}{e^{10}}$

$=4.99 \times 10^{-4}$
$=5 \times 10^{-4}$