Vanilla QAOA:

Components:

- QAA inspired: Discretized (Trotterized) version of Quantum Adiabatic Approximation
  Repeated layers of U=U_c U_m, similar to the following equation for QAA (except f and g). (r is the number of layers)

$$\hat{U}(t) \approx \prod_{k=0}^{r-1} \exp\left[-i\hat{H}(k\Delta\tau)\Delta\tau\right] = \prod_{k=0}^{r-1} \exp\left[-if(k\Delta\tau)\hat{H}_C\Delta\tau\right] \exp\left[-ig(k\Delta\tau)\hat{H}_M\Delta\tau\right] \quad (9)$$

- VQE method: Minimum expectation value of cost Hamiltonian is found (Variational Quantum Eigensolver feature i.e., expectation value >= lowest energy value).
- Classical optimization: γ_k (counterpart of f) and β_k (counterpart of g) are tuned classically.

Note: the QAA feature in QAOA help us find a suitable initial trial function when we don't know any eigenstate of cost Hamiltonian (but we do know of mixer Hamiltonian)

Process:

1) **Defining Hc and Hm**

   Define Hc according to the problem
   Define a Hm that does not commute with Hc

   Example Hc and H for MaxCut problem (I and j defining the ith and jth qubit):

$$\hat{H}_C = \frac{1}{2}\sum_{(i,j)\in\mathcal{E}} w_{ij}(I - Z_iZ_j),$$

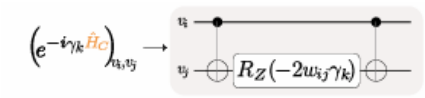
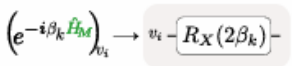
$$\hat{H}_M = \sum_{j\in\mathcal{V}} X_j,$$

Where each binary variable $x_i = 0.5(I - Z_i)$ i.e., when state= (0 1), x_i= 0.5(1-(-1))=1. And when state=(1 0), x_i= (1-1)=0.

Making Hc the same as the objective function of MaxCut:

$$C(\mathbf{x}) = \sum_{i,j=1}^{|\mathcal{V}|} w_{ij}x_i(1 - x_j),$$

2) **Creating the circuit ansatz** containing Uc and Um unitaries.

$\hat{U}_C(\gamma) = e^{-i\gamma\hat{H}_C} = \prod_{i=1,j<i}^{n} R_{Z_iZ_j}(-2w_{ij}\gamma),$	$\hat{U}_M(\beta) = e^{-i\beta\hat{H}_M} = \prod_{i=1}^{n} R_{X_i}(2\beta),$
Cost interaction can be implemented using two CNOTs gates with one Rz gate (acting on the target qubit of CNOTs) in between.	Mixer interaction can be implemented using rotation gate Rx.
Circuit ansatz Uc part: 	**Circuit ansatz Um part:** 

3) Defining the initial state

The initial state is *typically* defined as the highest energy state of the mixer for problems like MaxCut i.e., tensor products of |+> (highest energy state of the Pauli-X basis).

$$|s\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{\mathbf{x}\in\{0,1\}^n} |\mathbf{x}\rangle \,,$$

sqrt(2^n) denominator is used to normalize the state i.e. to make the total probability 1. **x** is the binary string of each combination of the states.

(Note: Here, the initial state being a highest energy eigenstate of the mixer (depending on sign convention) instead of the ground state still satisfies the adiabatic theorem, as simply an eigenstate of mixer is required as initial state- in QAOA it is not our objective to evolve into the corresponding eigenstate of the cost Hamiltonian. Rather, we utilize the variational principle: *the lowest energy state expectation value >= ground state energy)*

4) Layers

- Total number of layers, p, should be at least 1.

- For each layer, define the variational parameters (p no. of parameters for Uc and p no. of parameters for Um. Total =2p) γ and β, such that γ_k ∈ [0,2π) and β_k ∈ [0,π), where k is the kth layer.
  (Note: Due to the symmetry in the full QAOA circuit and the way the optimization landscape repeats, choosing β_k ∈ [0,π) is often enough to cover all unique cases. This keeps the search space smaller.)

- The ansatz state after it goes through the repeated unitaries:

$$|\psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle = e^{-i\beta_p \hat{H}_M} e^{-i\gamma_p \hat{H}_C} \ldots e^{-i\beta_1 \hat{H}_M} e^{-i\gamma_1 \hat{H}_C} |s\rangle$$

5) Repeated measurements of the ansatz state

The final state is measured and the expectation value of Hc w.r.t. the ansatz state is calculated.

$$F_p(\boldsymbol{\gamma},\boldsymbol{\beta}) = \langle\psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})|\hat{H}_C|\psi_p(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle$$

6) Classical optimization

The variational parameters are updated iteratively using a classical optimizer for the expectation value of the cost Hamiltonian w.r.t. the ansatz state to reach maximum.

$$(\boldsymbol{\gamma}^*,\boldsymbol{\beta}^*) = \arg\max_{\boldsymbol{\gamma},\boldsymbol{\beta}} F_p(\boldsymbol{\gamma},\boldsymbol{\beta})$$

An analog version of QAOA was recently proposed.

Quantum Alternating Operator Ansatzes:

Also known as QAOAnsatz. In this type of variation, the method can vary whether in terms of:

- number of states, or
- alternating unitaries operators in each circuit layer. The unitaries are from a general set of parameters, instead of fixed Hamiltonian.

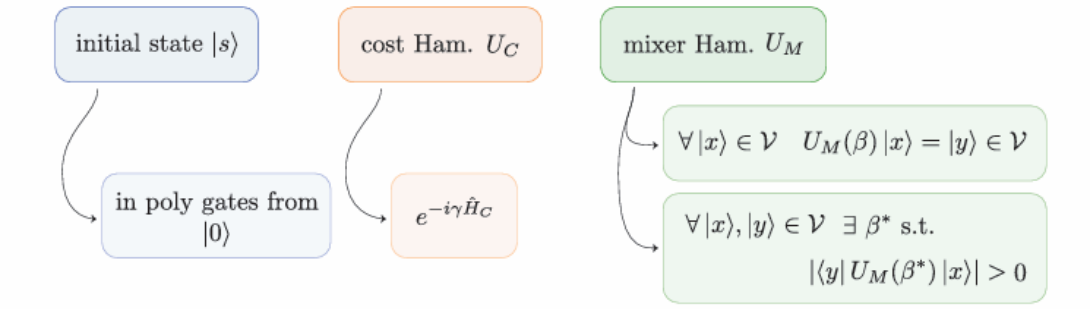


Figure 5: Representation of the QAOAnsatz.

Notable QAOAnsatz Variants:

Grover Mixer QAOA:

Threshold QAOA:

Constraint Preserving Mixers: