Vanilla QAOA:

- Discretized (Trotterized) version of Quantum Adiabatic Approximation
- However, γ_k (counterpart of f) and βp (counterpart of g) are tuned variationally (Variational Quantum Eigensolver feature)
- Repeated layers of U=U_c U_m, similar to the following equation for QAA. (r is the number of layers)

$$\hat{U}(t) \approx \prod_{k=0}^{r-1} \exp\left[-i\hat{H}(k\Delta\tau)\Delta\tau\right] = \prod_{k=0}^{r-1} \exp\left[-if(k\Delta\tau)\hat{H}_C\Delta\tau\right] \exp\left[-ig(k\Delta\tau)\hat{H}_M\Delta\tau\right] \qquad (9)$$

Process:

1) Defining Hc and Hm

Define Hc according to the problem

Define a Hm that does not commute with Hc

Example Hc and H for MaxCut problem (I and j defining the ith and jth qubit):

$$\hat{H}_C = \frac{1}{2}\sum_{(i,j)\in\mathcal{E}} w_{ij}(I - Z_i Z_j),$$

$$\hat{H}_M = \sum_{j\in\mathcal{V}} X_j,$$

Where each binary variable $x_i = 0.5(I - Z_i)$ i.e., when state= (0 1), x_i= 0.5(1-(-1))=1. And when state=(1 0), x_i= (1-1)=0.

Making Hc the same as the objective function of MaxCut:

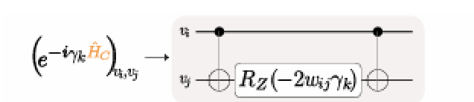
$$C(\mathbf{x}) = \sum_{i,j=1}^{|\mathcal{V}|} w_{ij} x_i (1 - x_j),$$

2) **Creating the circuit ansatz** containing Uc and Um unitaries.

$$\hat{U}_C(\gamma) = e^{-i\gamma\hat{H}_C} = \prod_{i=1,j<i}^{n} R_{Z_iZ_j}(-2w_{ij}\gamma),$$

Cost interaction can be implemented using two CNOTs gates with one Rz gate (acting on the target qubit of CNOTs) in between.

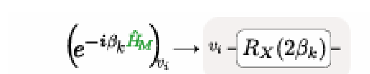
Circuit ansatz Uc part:



$$\hat{U}_M(\beta) = e^{-i\beta\hat{H}_M} = \prod_{i=1}^{n} R_{X_i}(2\beta),$$

Mixer interaction can be implemented using rotation gate Rx.

Circuit ansatz Um part:



3) **Defining the initial state**

The initial state is *typically* defined as tensor products of |+> states for problems like MaxCut. It corresponds to the highest energy state of the Pauli-X basis (i.e., Hm).

$$|s\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{\mathbf{x}\in\{0,1\}^n} |\mathbf{x}\rangle\,,$$

sqrt(2^n) denominator is used to normalize the state i.e. to make the total probability 1. **x** is the binary string of each combination of the states.

(Note: The initial state being highest energy state instead of the ground state still satisfies the adiabatic theorem, as *initial nth state = final nth state* if there is no overlap between states)

4) Layers

- Total number of layers, p, should be at least 1.

- For each layer, define the variational parameters (p no. of parameters for Uc and p no. of parameters for Um. Total =2p) γ and β, such that γ_k ∈ [0,2π) and β_k ∈ [0,π), where k is the kth layer.
  (Note: Due to the symmetry in the full QAOA circuit and the way the optimization landscape repeats, choosing β_k ∈ [0,π) is often enough to cover all unique cases. This keeps the search space smaller.)

- The ansatz state after it goes through the repeated unitaries:

$$|\psi_p(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle = e^{-i\beta_p \hat{H}_M} e^{-i\gamma_p \hat{H}_C} \dots e^{-i\beta_1 \hat{H}_M} e^{-i\gamma_1 \hat{H}_C} |s\rangle$$

5) Classical optimization

The variational parameters are updated iteratively using a classical optimizer for the expectation value of the cost Hamiltonian w.r.t. the ansatz state (F_p) to reach maximum.

$$(\boldsymbol{\gamma}^*, \boldsymbol{\beta}^*) = \arg\max_{\boldsymbol{\gamma}, \boldsymbol{\beta}} F_p(\boldsymbol{\gamma}, \boldsymbol{\beta})$$

6) Repeated measurements of the ansatz state

After each update of variational parameters, the final state is measured and the expectation value of Hc w.r.t. the ansatz state is calculated.

$$F_p(\boldsymbol{\gamma}, \boldsymbol{\beta}) = \langle \psi_p(\boldsymbol{\gamma}, \boldsymbol{\beta}) | \hat{H}_C | \psi_p(\boldsymbol{\gamma}, \boldsymbol{\beta}) \rangle$$

An analog version of QAOA was recently proposed.

Quantum Alternating Operator Ansatzes:

Also known as QAOAnsatz. In this type of variation, the method can vary whether in terms of:

- number of states, or
- alternating unitaries operators in each circuit layer. The unitaries are from a general set of parameters, instead of fixed Hamiltonian.

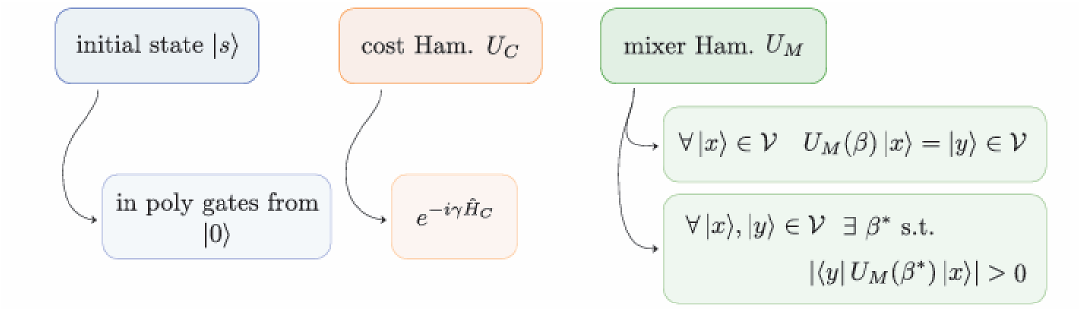


Figure 5: Representation of the QAOAnsatz.

Notable QAOAnsatz Variants:

Grover Mixer QAOA:

Threshold QAOA:

Constraint Preserving Mixers: