Pattern Recognition - Coursework 3

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1 Question 1

My King's ID is

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2 Question 2

$$\frac{1+7+7+3+8+6+9}{4} = \frac{41}{4}$$

The remainder for this equation is 1, hence the "one against all" method will be used.

3 Question 3

Sample Class 1	Sample Class 2	Sample Class 3
(4,10)	(9.5,6)	(2.5,4.5)
(4,11.5)	(11.5,4.5)	(2.5,5.5)
(4.5,11)	(11,7)	(3.5,6)
(5.5,10)	(11.5,5.5)	(3.5,6)
(5.5,12)	(12.5,4.5)	(3.5,3)
(6,8)	(12.5,5.5)	(4,2.5)
(6,9)	(13.5,6)	(4,3.5)
(6.5,11.5)	(12.5,5)	(4,5)
(7,10)	(14,5)	(4.5,4.5)
(7,12)	(13.5,7)	(4.5,6)
(7.5,8.5)	(13.5,8)	(5.5,3)
(8,11)	(14.5,6.5)	(5.5,5.5)
(8.5,8)	(14.5,7)	(6,2)
(8.5,12)	(15,7.5)	(6.5,2.5)
(9,11)	(15,6)	(6.5,3.5)
(9.5,9)	(15,8.5)	(6.5,5.5)
(9.5,10.5)	(16.5,8)	(7,4.5)

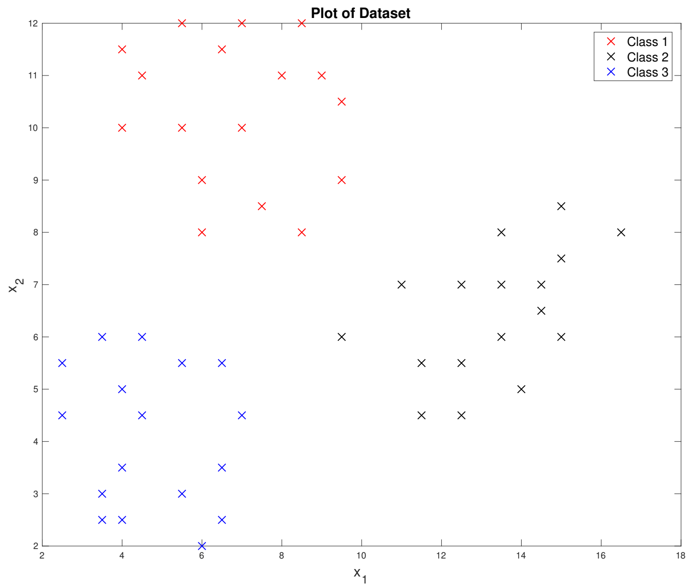
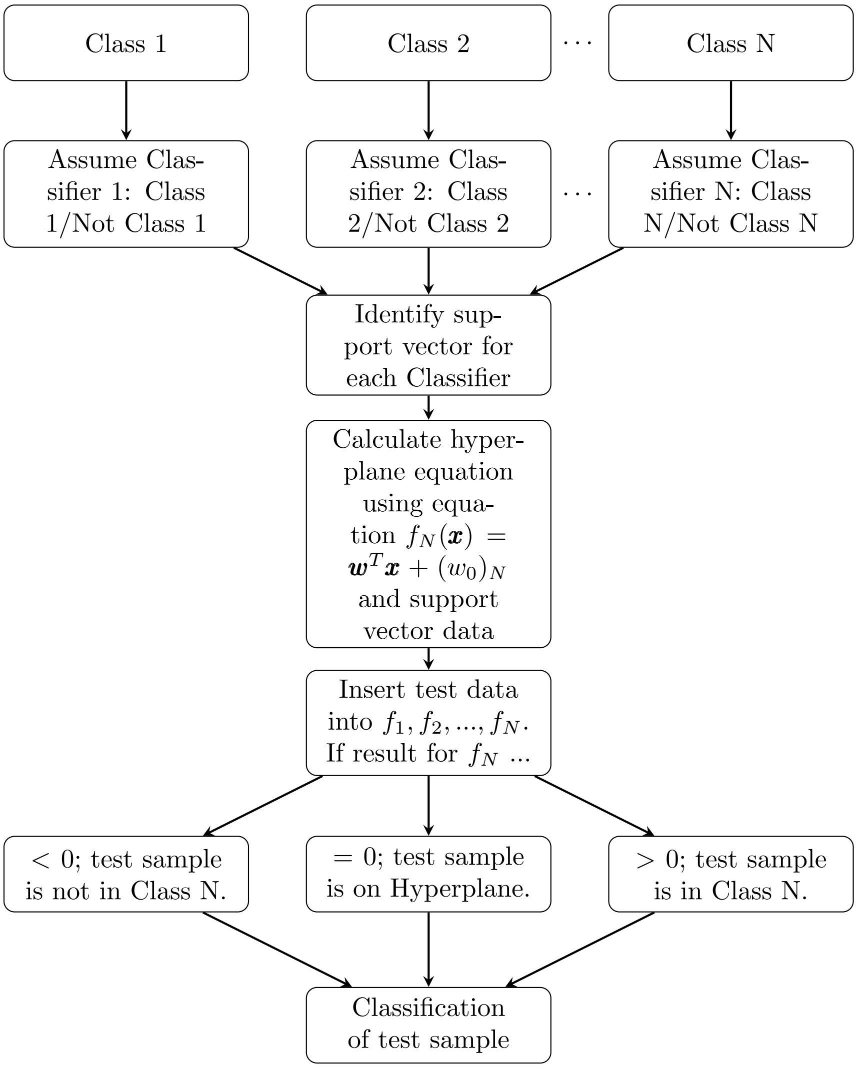


Figure 1: Data for all classes.

4 Question 4

Linear seperability means that a class of the data can be seperated from all other classes via a linear graph. As can be seen in figure 1, the classes can be easily seperated into three zones; furthermore the boundaries to these zones can be represented via straight lines, which will be demonstrated further on.

5 Question 5



The number of inputs is detrmined by the number of datapoints as training data. The data then determines the number of unknowns to be calculated for each hyperplane. For d-dimensions, there will be d+1 unknowns, w_i for $i = 0, 1, ..., N$, which need to be calculated. Using the support vectors (values for points closest to proposed hyperplane), equation f will be solved by setting them to +1 or -1, depending on their location. Once the w_i values have been determined, equation f can be used to for any dataset to determine its location to specific classes. In the 'One-against-all' method, classification for each hyperplane equation will only be to determine, whether the data point is in the class or not. A collection of all these classification then determines the class of the data, or whether it it can not be classified.

6 Question 6

We need to design a hyperplane that is determined by the following equation:

$$f = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0 = \begin{cases} 0 & \text{hyperplane} \\ +1 & \text{class A support vector} \\ \text{-1} & \text{not class A support vector} \end{cases}$$

Using this classification method, the data was analysed. Data points from the samples were used to calculate the values for w_1, w_2, and w_0 using $f = +1$ and $f = -1$ for the support vectors, depending on their position in relation to the data and proposed hyperplane.

These values were then used to to determine the equation for the required hyperplane by setting $f = 0$ with the values obtained for w_1, w_2 and w_0.

6.1 Hyperplane 1:

For Class A the support vectors were identified and circled as can be seen in figure 2.

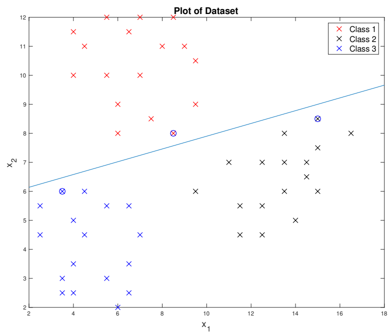


Figure 2: Support vectors for Class 1 hyperplane have been indicated. The hypreplanes, as calculted using SVs, is also plotted in.

The values were then used in the equation f and equated to -1 or +1 as required. The following equation was obtained for the hyperplane.

$$\begin{bmatrix} -0.48 \\ 2.19 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + -12.48 = 0$$

6.2 Hyperplane 2:

For Class B the support vectors were identified and circled as can be seen in figure 3.

The values were then used in the equation f and equated to -1 or +1 as required. The following equation was obtained for the hyperplane.

$$\begin{bmatrix} 1.21 \\ -0.47 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 7.70 = 0$$

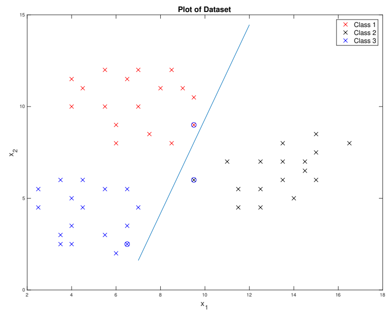


Figure 3: Support vectors for Class 2 hyperplane have been indicated. The hypreplanes, as calculted using SVs, is also plotted in.

6.3 Hyperplane 3:

For Class C the support vectors were identified and circled as can be seen in figure 4.

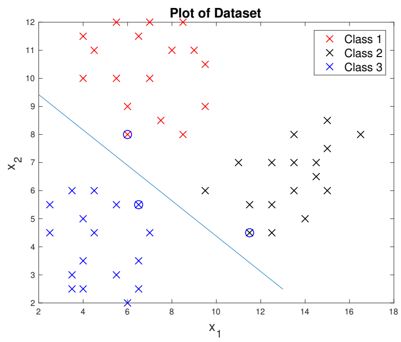


Figure 4: Support vectors for Class 3 hyperplane have been indicated. The hypreplanes, as calculted using SVs, is also plotted in.

The values were then used in the equation f and equated to -1 or +1 as required. The following equation was obtained for the hyperplane.

$$\begin{bmatrix} -0.58 \\ 0.92 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 9.83 = 0$$

7 Question 7

Test Sample	Output of SVM 1	Output of SVM 2	Output of SVM 3	Classification
(5.33, 6.83)	-0.0807	-4.4608	0.4550	Class 3
(6.00,7.17)	0.3423	-3.8099	-0.2464	Class 1
(6.33,6.83)	-0.5607	-3.2508	-0.1250	undefined
(6.83,7.17)	-0.0561	-2.8056	-0.7278	undefined
(7.17,6.50)	-1.6866	-2.0793	-0.3086	undefined
(7.50,5.33)	-4.4073	-1.1301	0.5764	Class 3
(7.83,6.17)	-2.7261	-1.1256	-0.3878	undefined
(7.67,7.83)	0.9861	-2.0994	-1.8222	Class 1
(8.50,6.50)	-2.3250	-0.4700	-1.0800	undefined
(8.33,8.33)	1.7643	-1.5358	-2.6650	Class 1
(8.83,6.50)	-2.4834	-0.0707	-1.2714	undefined
(9.33,7.67)	-0.1611	-0.0156	-2.6378	undefined
(9.67,5.67)	-4.7043	1.3358	-0.9950	Class 2
(10.00,7.33)	-1.2273	0.9549	-2.7136	Class 2
(10.17,6.83)	-2.4183	1.4319	-2.3696	Class 2
(10.33,7.67)	-0.6267	1.1581	-3.2004	Class 2
(11.00,7.67)	-0.9627	2.0051	-3.6064	Class 2

As can be seen in figure 5, seven points are located in the region bounded by all three hyperplanes. These seven results are are inconclusive in the hard SVM classifier, as shown here.

The hyperplanes were created such that it plane created two areas. Datapoints from one area would only yield positive values when inserting them into the hyperplane equation; the results for the same calculation for data points in the other area would create negative values. Using this principle, the hyperplanes were designed to give positive values, when data points were located in the area "Class N", and to be negative in the area allocated to "Not Class N".

To demonstrate the process of classification in the above table, the first datapoint will be used. The output valued for this datapoint are all negative except for SVM3. This means that this datapoint is located in the area allocated to "Class 3"; hence it is classified as belonging to Class 3.

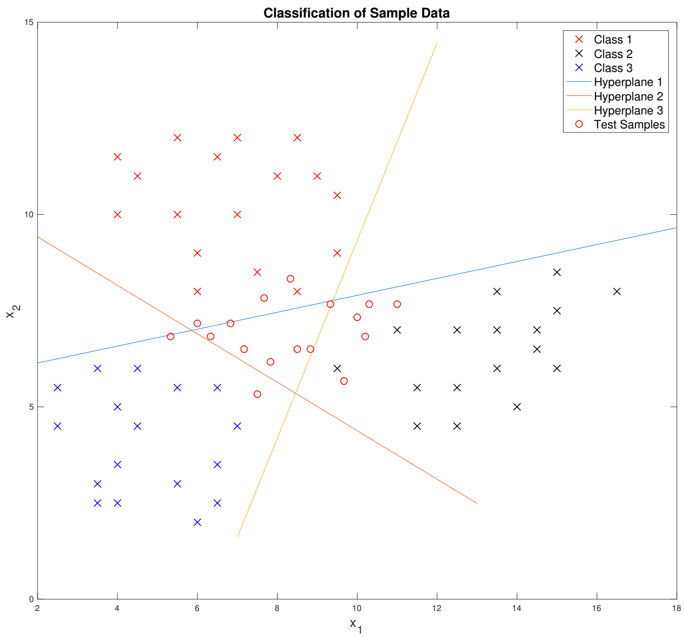


Figure 5: Test samples represented with red circles.