CS404: Agent Based Systems Coursework - Extensive Auction Games

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I. Introduction

The coursework for agent based system is on extensive auction games. There are mainly two types of auction games in this coursework. The first game is a second price value game and the second game is a first price collection game. The task is to find out optimal strategies for these two auction games. In this report, we will discuss and evaluate our strategies for the two auction games.

II. Strategy for Value Game

In this game, there will be 200 rounds where in each round, there is only one winner (bot with the highest score). Each painting has their corresponding scores. Picasso is worth 2, Van Gogh worth 11, Rembrandt worth 23 and Da Vinci worth 47. The bot that earns the highest score among every bot will win the game.

A. Second-Price Auction

In a second price auction game, it can be shown that it is always optimal for the bidder to bid the true value [1] for the item. Here, true value means the actual estimated price of the painting considering the current state of the game.

Let's simulate two scenarios for a single round where the true value of the painting is V and there are two players X and Y.

Firstly, if X decides to bid V and Y decides to bid W ($W > V$), then Y will win the current round. But if X wanted to win this round by bidding higher than Y's bid W, then X would actually lose money because of overpaying for the painting. So, even though in this scenario X loses, it is optimal for X to bid the true value because X will actually be saving money for the later rounds.

Secondly, if X decides to bid V and Y decides to bid Z ($Z < V$), then X will win the current round. If X wanted to decrease their value, X will still have to pay the second highest bid (Z) so in this case, changing X's bid has no effect at all. But it is also possible that X might end up with a value lower than Z (because it is a sealed auction), and end up losing this round. So again, in this scenario, it is optimal for X to bid the true value.

B. Main Strategy

For any round, our optimal strategy should be to bid our true value regardless of how others bid. So our main goal is to find the true value of the current painting in each round.

The true value of a painting depends on two factors.

1) The remaining paintings in the auction
2) The remaining total budgets of all the bots.

We first calculate the price for every single point that is still available in the auction. This per point price is basically the sum of remaining budgets divided by the sum of remaining points.

For example, let's say, in an auction with 3 bots, the points of the remaining paintings are: [P1, P2, P3, P4] and the remaining budgets of bots are [R1, R2, R3].

So the per point price, P is,

$$P = (R1 + R2 + R3)/(P1 + P2 + P3 + P4)$$

We know that every painting's weight is not the same, for example, a Picasso is not worth the same as a Da Vinci. So the price should be directly proportional to the weight of the painting that is up for auction. So the true value of the current painting is basically the multiplication of per point price P and the current painting's weight.

C. Evaluation of Value Game Strategy

TABLE I
Value Game Test Scores

Opponent Bots (9 Bots)	Score of our Bot (in 100 games)
Random bots	100
Flat bots (10)	100
Flat bots (10, 20, 50, 170)	80
Random + Flat bots (10, 20, 50, 170)	84
Random + Flats + Intelligent bots(previous versions)	100
Intelligent Bots(previous versions)	100

For evaluating our value bot, we used different levels of bots. At first, we tried to optimize and win against all random bots and flat 10 bots. Then, we tried to optimize against different flat bots, for example, flat 20, flat 50 and so on. After that, we used our own previous versions of intelligent bots to test our best version. We used run_count = 100 in arena.py file to test our best versions. The results are given in the Table I.

From the scores, we can see that our bot performed well against random bots and intelligent bots. It was possible to optimize more against different flat bots (flat 20, flat 50 etc) but that would make our bot perform significantly bad against other rational strategies. So, we focused on making our bot perform better against a mix of different strategies.

D. Time Complexity of Value Game Solution

The time complexity of the solution is as follows -

Let us assume that the total number of rounds in the game is n, then the time complexity of our solution is, $O(n)$.

III. Strategy for Collection Game

In the collection game, the main goal is to achieve a collection set of (3, 2, 1) different paintings. Here, the set (3, 2, 1) indicates 3 paintings of any artist, 2 paintings of another artist and 1 painting of another artist. Whoever completes the collection first, wins the game. Because of this reason, this game will generally end before 200 rounds. On average, if every bot has the same strategy and tie breakers are decided randomly (as specified), in a game with 10 bots, the game should end before or in 60 rounds, given that painting orders are completely random.

Thus, in this game, it will not be optimal to save money in the earlier rounds and then try to win in the later rounds as there will always be a high chance that the game will end before that happens.

Thus, our strategy will be to have an aggressive approach in the auction. There are two main prerequisites for winning this game. They are:

1) Achieving the winning set before any other bots
2) Preventing other bots from completing their winning set

If we are only focused on our own set, someone else could make their set before us and win. So, in this game, we have to change our strategy based on others' status. This is why we will have different types of bets in different cases. To determine the different cases, we have used a metric called ***importance*** which will provide information about everyone's status in the game.

There are 3 main importance level in our strategy. The levels are described below:

(a) **Importance 0**: Painting is not required. For example, from set (3, 1, 1) to set (4, 1, 1) is unnecessary.
(b) **Importance 1**: Painting is required for collection/will make collection better. For example, set (1, 1, 1) to set (2, 1, 1) is one step closer towards winning set.
(c) **Importance 2**: Painting is necessary to win (end game). For example, set (3, 1, 1) to set (3, 2, 1) which is the winning set.

In each case, the strategy will be different. At each round, we need to calculate the importance score for all the bots including our own. Based on those importance scores, we can estimate a bidding value.

Let us consider the importance score pair (A, B) where A is our importance score and B is the maximum importance score of all the other bots. Now we will discuss all the possible scenarios below.

Case 1 (A = 2, B = Any)

Here, our own importance score is 2, it means we will win the game if we can get the current painting. So, in this scenario, it is definitely optimal to bid our full budget.

Case 2 (A = Any, B = 2)

Here, there exists at least one bot who can complete their set by getting the painting in the current round. So, we definitely have to stop them from winning by bidding a higher value than their budget. So we'll need to calculate the highest budget of the bots who are in the winning position, and we will bid a higher value than that to stop them from winning.

Case 3 (A = 1, B = 0)

In this case, the current painting improves our collection set but it is not important for any other bots. So, we can bid our average value. This average value is calculated by dividing our total remaining budget by the total paintings we still need to win.

Case 4 (A = 1, B = 1)

In this case, the current painting improves our collection set as well as some other bot's set. So, we should bid a bit higher than that bot's average value to be able to acquire that painting.

TABLE II
STRATEGY SUMMARY FOR COLLECTION GAME

	Own = 0	Own = 1	Own = 2
Opponents = 0	Bid 0	Bid average budget	Bid Full Budget
Opponents = 1	Bid 0	Bid higher than opponents average budgets	Bid Full Budget
Opponents = 2	Bid Higher than Opponents Max Budget	Bid Higher than Opponents Max Budgets	Bid Full Budget

Case 5 (A = 0, B = 0, 1)

In this case, we don't need the current painting at all. Even though it may improve some other's collection set, we should not worry about it as they are not in a winning position yet. So we will bid 0 instead bidding a higher value and losing money unnecessarily.

The summary of our strategy is given in Table II)

TABLE III
COLLECTION GAME TEST SCORES

Opponent Bots (9 Bots)	Score of our Bot (in 100 games)	Score of Runner-up
Random bots	100	0
Flat bots (10s)	100	0
Flat bots (10s, 20s, 50s, 170s)	82	11
Random + Flat bots (10s, 20s, 50s, 170s)	82	18
Random + Flats + Intelligent bots	48	20
Intelligent Bots	51	14

A. *Evaluation of Collection Game Strategy*

For evaluation, we used various different kinds of bots to evaluate our strategy. We used random bots and different kinds of flat bots like flat 10, flat 20, flat 50 and so on. We also played against previous versions of our intelligent bots to check how it performs as well. The overall scores are shown in table III.

From the score table we can see that, our performed well against random and flat bots. In terms of intelligent bots, it did not win every game but it did win the most games among the intelligent bots.

B. *Complexity of Collection Game Solution*

The time complexity of this collection game is as follows -

Let us assume, that number of bots in the game are denoted by n and number of painting collected by each bot is denoted by m. Then the time complexity of the collection game strategy is $O(n * m)$.

IV. CONCLUSION

The main objective of this coursework was to create optimal bots for two different auction games that will perform fairly well against different strategies. It is not possible to have a perfect strategy in these games. So, our strategy was to design solutions based on approximation. We tried to optimize our bots against different types of strategy as much as possible.

REFERENCES

[1] EconPort - Handbook - Auctions - Second Price Sealed-Bid, 2021