

Introduction to Machine Learning and Artificial Neural Networks

Single and Multi-Layer Perceptron

5 December 2021

Part 1

- a) Implement an SLP where the inputs are 100-dimensional flattened images and the outputs are 10-dimensional class probabilities.
- b) Train the perceptron for a number of epochs (e.g., 50) using the training dataset and plot the accuracy values for each epoch on both training and test datasets.
- c) Report the confusion matrices for both training and test datasets using the predictions with the highest accuracy obtained during training.

Confusion train :

```
[[100.  0.  1.  0.  0.  0.  0.  0.  0.  2.]  
 [  0. 94.  0.  0.  0.  0.  0.  0.  2.  0.]  
 [  0.  1. 82.  2.  0.  0.  0.  0.  3.  0.]  
 [  0.  1.  8. 93.  1.  6.  0.  0.  7.  2.]  
 [  0.  0.  1.  0. 97.  1.  0.  0.  0.  2.]  
 [  0.  0.  0.  1.  0. 91.  0.  0.  8.  1.]  
 [  0.  0.  2.  0.  2.  1.100.  0.  1.  0.]  
 [  0.  2.  4.  1.  0.  0.  0.100.  1.  7.]  
 [  0.  2.  2.  2.  0.  0.  0.  0. 75.  0.]  
 [  0.  0.  0.  1.  0.  1.  0.  0.  3. 86.]]
```

Confusion test :

[[91. 0. 0. 0. 0. 0. 2. 0. 0. 0.]

[0.89. 0. 0. 1. 1. 0. 0. 1. 0.]

[0. 0.80. 0. 0. 2. 2. 2. 1. 0.]

[2. 9. 4.91. 6. 6. 1. 6.12. 8.]

[0. 0. 0. 1.84. 5. 1. 2. 3. 3.]

[2. 0. 0. 6. 0.72. 5. 1. 8. 1.]

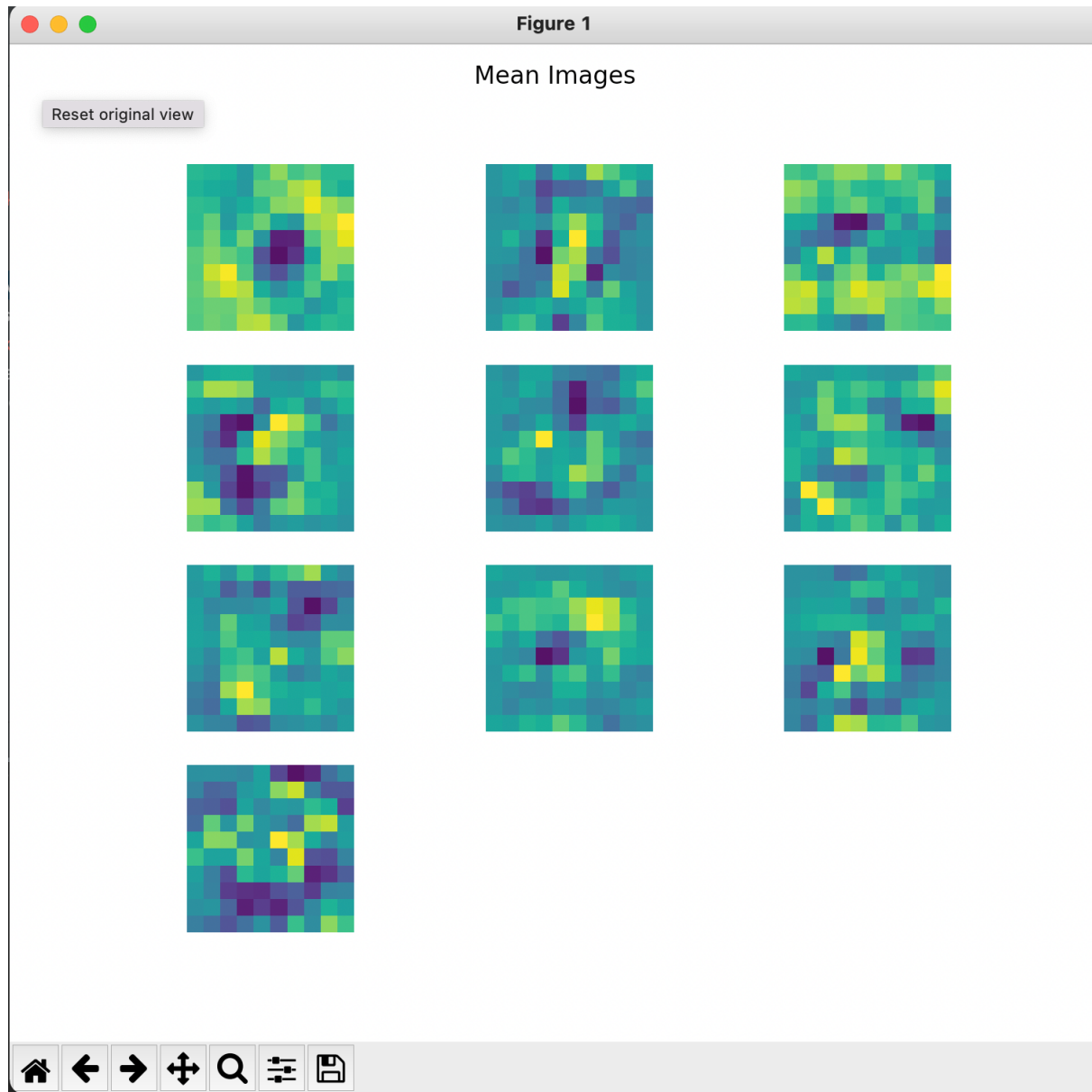
[2. 0. 3. 1. 2. 0.88. 0. 3. 0.]

[2. 1. 4. 0. 0. 7. 0.82. 2.14.]

[1. 1. 6. 1. 0. 3. 1. 2.61. 0.]

[0. 0. 3. 0. 7. 4. 0. 5. 9.74.]]

d) Each output unit has 100 weights coming to them. Visualize those weights as an image for each output unit.

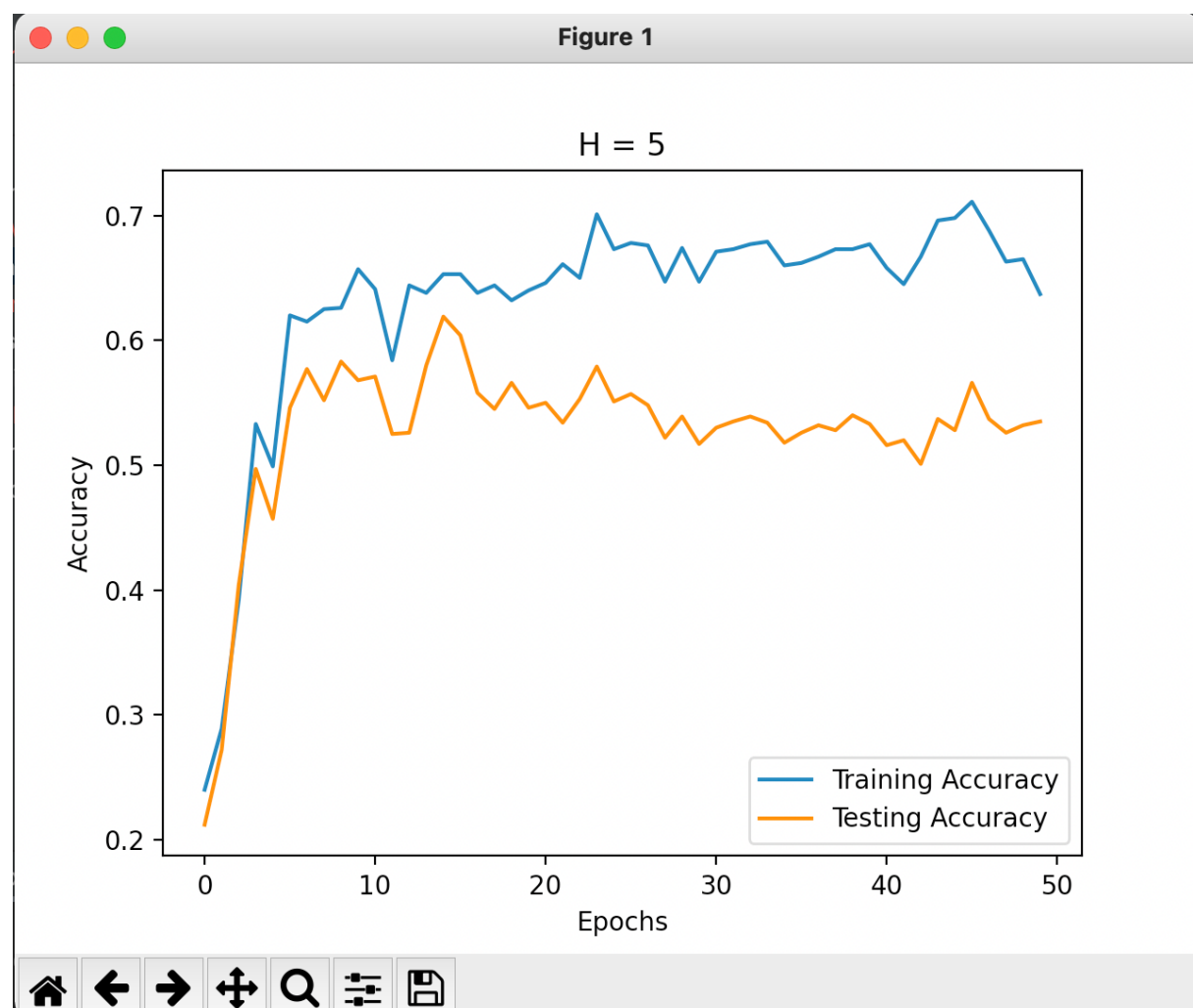


Part 2

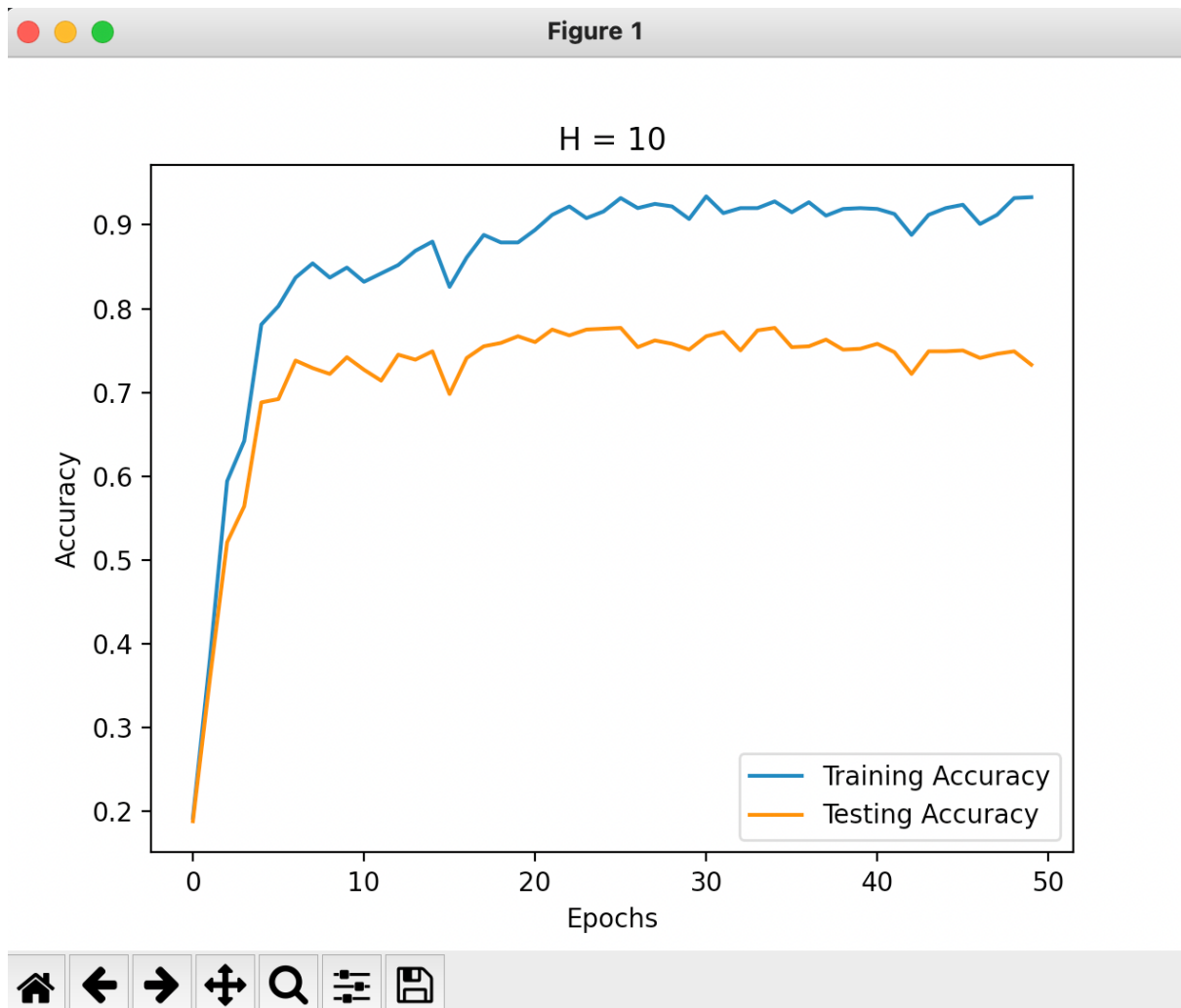
a) Implement an MLP where the input is a 100-dimensional flattened image, the hidden layer has H units, and the outputs are 10-dimensional class probabilities.

b) Train the perceptron for a number of epochs using the training dataset and plot the accuracy values for each epoch on both training and test datasets.

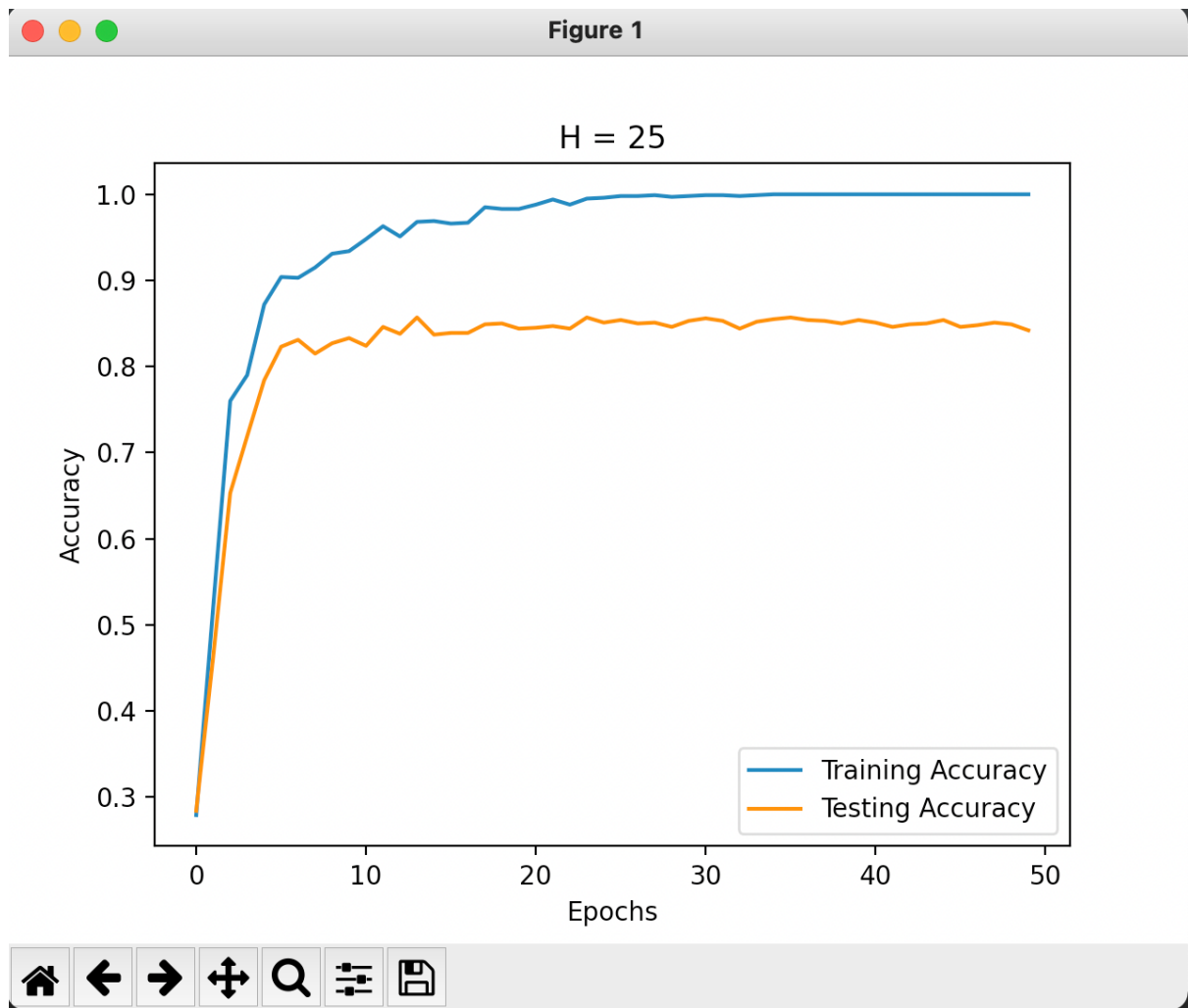
Plot for $H = 5$



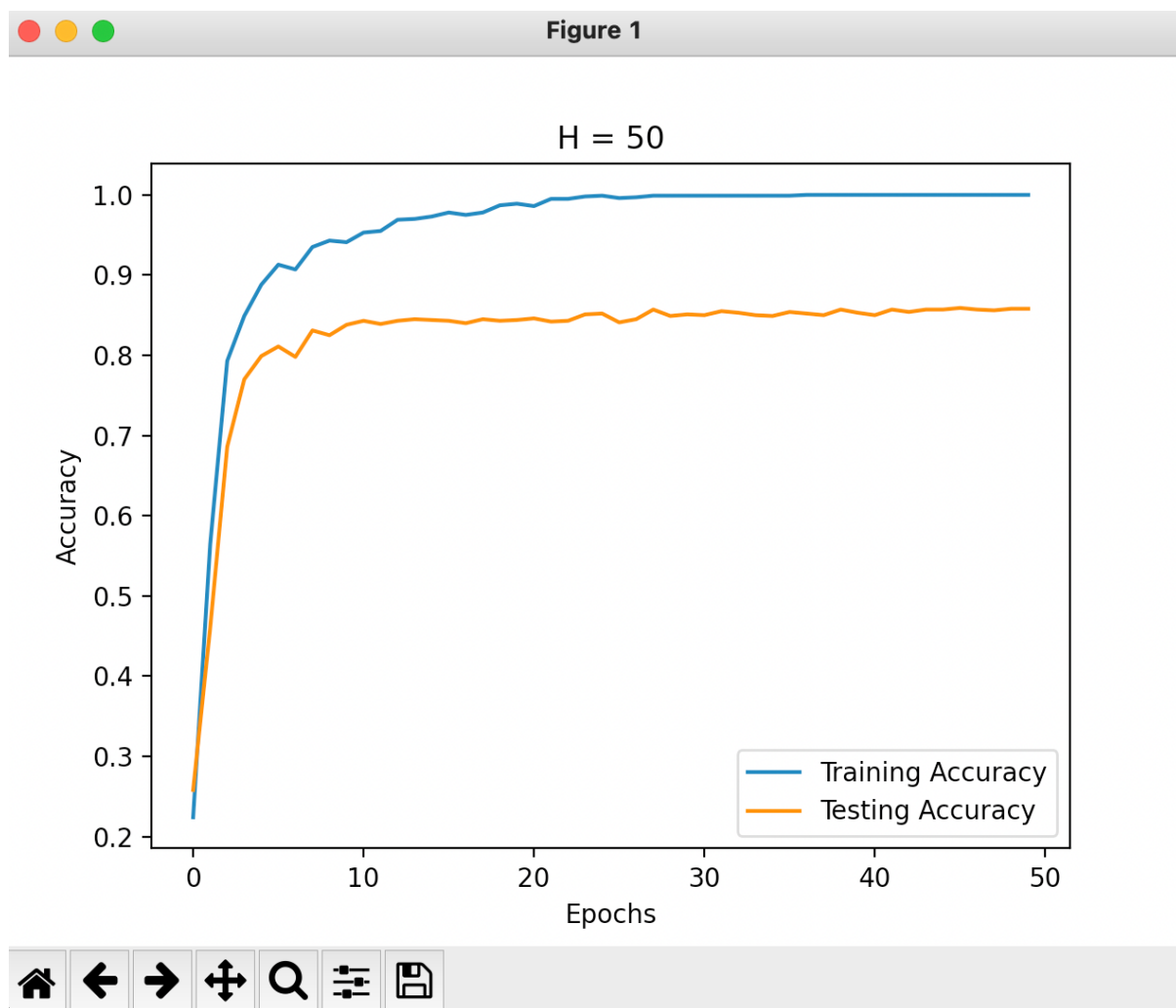
Plot for $H = 10$



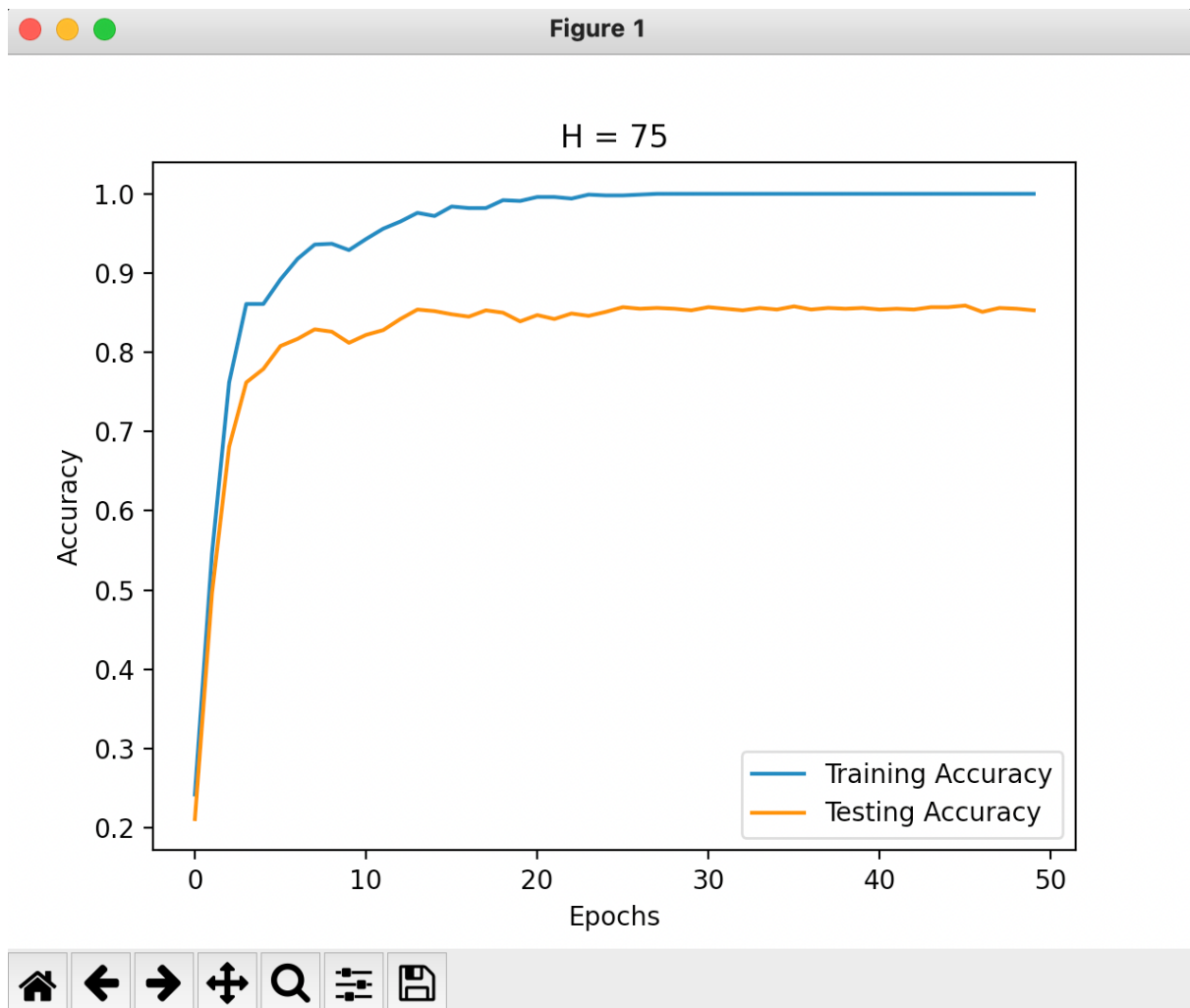
Plot for $H = 25$



Plot for $H = 50$



Plot for $H = 75$



c) Report the confusion matrices for both training and test datasets using the predictions with the highest accuracy obtained during training.

Confusion Matrixes for $H = 5$

Confusion train:

```
[[49. 0. 11. 5. 0. 13. 10. 0. 2. 0.]  
 [0. 92. 4. 0. 0. 0. 0. 1. 0. 0.]  
 [35. 1. 59. 18. 0. 14. 9. 0. 5. 1.]  
 [0. 0. 0. 23. 0. 10. 0. 0. 8. 0.]  
 [1. 0. 0. 0. 94. 8. 5. 1. 0. 13.]  
 [3. 0. 2. 10. 0. 37. 3. 0. 2. 1.]  
 [5. 0. 5. 0. 4. 1. 70. 0. 0. 0.]  
 [0. 3. 3. 2. 0. 2. 0. 87. 5. 5.]  
 [7. 4. 16. 13. 1. 10. 3. 1. 66. 4.]  
 [0. 0. 0. 29. 1. 5. 0. 10. 12. 76.]]
```

Confusion test:

```
[[75. 0. 16. 9. 0. 15. 15. 0. 2. 0.]  
 [0. 88. 1. 0. 0. 1. 0. 0. 2. 0.]  
 [7. 0. 48. 3. 0. 6. 4. 2. 5. 0.]  
 [0. 0. 0. 36. 0. 18. 0. 0. 16. 10.]  
 [3. 0. 2. 1. 86. 3. 6. 7. 2. 17.]  
 [4. 0. 2. 21. 3. 27. 3. 0. 7. 0.]  
 [4. 0. 11. 0. 4. 0. 72. 0. 1. 1.]  
 [0. 8. 4. 3. 1. 4. 0. 75. 5. 9.]  
 [6. 4. 15. 24. 3. 14. 0. 5. 54. 5.]  
 [1. 0. 1. 3. 3. 12. 0. 11. 6. 58.]]
```

Confusion Matrixes for H = 10

Confusion train:

```
[[98. 0. 0. 1. 0. 1. 0. 0. 0. 0.]  
 [0. 99. 0. 0. 0. 1. 1. 2. 0. 0.]  
 [0. 1. 97. 2. 0. 0. 3. 0. 1. 0.]  
 [0. 0. 0. 89. 0. 5. 0. 0. 1. 0.]  
 [0. 0. 0. 0. 90. 1. 0. 0. 0. 3.]  
 [2. 0. 0. 3. 2. 82. 0. 1. 3. 2.]  
 [0. 0. 1. 0. 5. 4. 96. 0. 0. 0.]  
 [0. 0. 2. 2. 1. 0. 0. 95. 0. 3.]  
 [0. 0. 0. 0. 1. 2. 0. 0. 95. 1.]  
 [0. 0. 0. 3. 1. 4. 0. 2. 0. 91.]]
```

Confusion test:

```
[[86. 0. 1. 1. 1. 1. 3. 0. 3. 3.]  
 [0. 99. 1. 0. 0. 1. 0. 2. 0. 0.]  
 [6. 1. 88. 8. 0. 2. 3. 4. 1. 0.]  
 [1. 0. 0. 70. 0. 8. 0. 3. 2. 5.]  
 [3. 0. 1. 0. 70. 3. 3. 2. 2. 5.]  
 [4. 0. 1. 14. 2. 53. 4. 0. 11. 2.]  
 [0. 0. 5. 1. 5. 6. 84. 0. 4. 0.]  
 [0. 0. 1. 1. 0. 4. 0. 83. 0. 9.]  
 [0. 0. 2. 3. 2. 11. 3. 1. 72. 4.]  
 [0. 0. 0. 2. 20. 11. 0. 5. 5. 72.]]
```

Confusion Matrixes for H = 25

Confusion train:

```
[[100. 0. 0. 0. 0. 0. 1. 0. 0. 1.]  
 [0. 98. 1. 0. 0. 0. 0. 0. 0. 0.]
```

```
[0. 1. 97. 3. 0. 1. 1. 0. 0. 0.]
[0. 0. 0. 92. 0. 0. 0. 0. 0. 0.]
[0. 0. 0. 0. 96. 1. 0. 0. 0. 0.]
[0. 0. 0. 3. 0. 97. 0. 0. 1. 1.]
[0. 0. 0. 0. 3. 1. 98. 0. 1. 0.]
[0. 0. 0. 2. 0. 0. 0. 99. 0. 4.]
[0. 1. 1. 0. 0. 0. 0. 0. 98. 1.]
[0. 0. 1. 0. 1. 0. 0. 1. 0. 93.]]
```

Confusion test

```
[[95. 0. 0. 0. 0. 0. 2. 0. 2. 0.]
 [0. 99. 0. 1. 1. 1. 0. 0. 0. 0.]
 [1. 0. 86. 2. 0. 0. 2. 8. 3. 0.]
 [0. 0. 0. 73. 0. 3. 0. 0. 2. 2.]
 [0. 0. 2. 0. 80. 0. 0. 3. 1. 0.]
 [1. 0. 0. 18. 1. 78. 5. 1. 5. 0.]
 [2. 0. 4. 1. 4. 7. 91. 0. 3. 0.]
 [0. 0. 3. 1. 0. 5. 0. 84. 1. 6.]
 [0. 1. 5. 3. 0. 1. 0. 0. 80. 1.]
 [1. 0. 0. 1. 14. 5. 0. 4. 3. 91.]]
```

Confusion Matrixes for $H = 50$

Confusion train:

```
[[100. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 100. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 100. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 100. 0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 100. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 100. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 100. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 100. 0. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 100. 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 0. 100.]]
```

Confusion test:

```
[[ 96.  0.  0.  0.  0.  2.  2.  0.  3.  0.]
 [ 0. 100.  0.  0.  0.  0.  0.  0.  0.  0.]
 [  1.  0. 85.  1.  0.  1.  0.  5.  2.  0.]
 [  0.  0.  2. 80.  1.  1.  0.  9.  3.  4.]
 [  0.  0.  2.  0. 83.  0.  0.  2.  3.  1.]
 [  0.  0.  0. 14.  0. 78.  6.  1.  4.  2.]
 [  3.  0.  3.  2.  2.  5. 91.  0.  1.  0.]
 [  0.  0.  4.  0.  0.  7.  0. 80.  1.  6.]
 [  0.  0.  3.  2.  2.  2.  1.  0. 81.  2.]
 [  0.  0.  1.  1. 12.  4.  0.  3.  2. 85.]]
```

Confusion Matrixes for H =7 5

Confusion train:

```
[[100.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0. 100.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0. 100.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0. 100.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. 100.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0. 100.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 100.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0. 100.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. 100.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0. 100.]]
```

Confusion test:

```
[[ 97.  0.  1.  0.  0.  1.  2.  1.  3.  0.]
 [ 0. 100.  0.  0.  0.  0.  0.  0.  0.  0.]
 [  0.  0. 86.  2.  0.  2.  3.  5.  3.  0.]
 [  0.  0.  1. 79.  1.  2.  0.  8.  3.  4.]
 [  0.  0.  1.  0. 84.  0.  0.  1.  3.  0.]
 [  0.  0.  0. 14.  0. 80.  5.  1.  3.  1.]
 [  1.  0.  3.  2.  4.  5. 89.  0.  2.  0.]
 [  0.  0.  3.  0.  0.  6.  0. 80.  2.  7.]
 [  0.  0.  4.  3.  1.  2.  1.  0. 79.  3.]
 [  2.  0.  1.  0. 10.  2.  0.  4.  2. 85.]]
```

GitHub Link for the code: <https://github.com/SadikhovEmin/CS454>

```

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

LEARNING_RATE = 0.1
CLASS_SIZE = 10
FEATURES_SIZE = 100
training_matrix = pd.read_csv('training.csv', header=None, skiprows=1)
testing_matrix = pd.read_csv('testing.csv', header=None, skiprows=1)

weight_matrix = np.random.uniform(low=-0.01, high=0.01,
                                   size=(FEATURES_SIZE + 1, CLASS_SIZE)) # Random matrix for weights
weight_matrix_best = np.zeros((FEATURES_SIZE + 1, CLASS_SIZE)) # Best possible values for matrix for weights

def softmax(x):
    value = np.exp(x - np.max(x))
    return value / value.sum()

def encode(y_value):
    return np.transpose(np.eye(10)[y_value])

def accuracy_confusion_matrix(x):
    correct = 0
    total = 0
    for i in range(len(x)):
        for j in range(len(x[i])):
            total += x[i][j]
            if i == j:
                correct += x[i][j]
    return correct / total

def forward(slp_object):
    for dataset_row_count in range(slp_object.number_of_rows): # might need to make slp_object SLP_TRAIN
        row = slp_object.x.iloc[dataset_row_count, :]
        row = np.append(row, 1)
        row = np.transpose(row)

        o = np.zeros(CLASS_SIZE)

        for i in range(CLASS_SIZE):
            o[i] = np.dot(weight_matrix[:, i], row)

        slp_object.y_matrix[dataset_row_count] = softmax(o)

class SLP(object):
    def __init__(self, matrix):
        self.labels = matrix.iloc[:, 0] # Gets only the first column (Just class name)
        self.x = matrix.iloc[:, 1:] / 255 # Gets all rows and columns except the first one
        self.number_of_rows = matrix.shape[0]
        self.y_matrix = np.zeros((self.number_of_rows, CLASS_SIZE))

def train(SLP_Train, SLP_Test):
    global weight_matrix_best
    accuracy_best = 0
    best_label_train = []
    best_label_test = []
    confusion_matrix_train = np.zeros((CLASS_SIZE, CLASS_SIZE)) # Confusion matrix for training set with zeroes
    confusion_matrix_test = np.zeros((CLASS_SIZE, CLASS_SIZE)) # Confusion matrix for testing set with zeroes

    for epoch in range(50):

```

```

temp_weight = np.zeros(weight_matrix.shape) # Copy w_matrix to the temp mat
rix

for dataset_row_count in range(SLP_Train.number_of_rows):
    row = SLP_Train.x.iloc[dataset_row_count, :]
    row = np.append(row, 1)
    row = np.transpose(row)

    o = np.zeros(CLASS_SIZE)

    for i in range(CLASS_SIZE):
        o[i] = np.dot(weight_matrix[:, i], row)

    SLP_Train.y_matrix[dataset_row_count] = softmax(o)

    encode_matrix = encode(SLP_Train.labels.iloc[dataset_row_count])

    for i in range(CLASS_SIZE):
        for j in range(FEATURES_SIZE + 1):
            temp_weight[j, i] += (encode_matrix[i] - SLP_Train.y_matrix[dataset_row_count][i]) * row[j]

    for i in range(CLASS_SIZE):
        for j in range(FEATURES_SIZE + 1):
            weight_matrix[j, i] += temp_weight[j, i] * LEARNING_RATE

    forward(SLP_Train)
    forward(SLP_Test)

    label_train = []
    label_test = []

    correct_train = 0
    correct_test = 0
    for dataset_row_count in range(SLP_Train.number_of_rows):
        if SLP_Train.labels.iloc[dataset_row_count] == np.argmax(SLP_Train.y_matrix[dataset_row_count]):
            correct_train += 1
        if SLP_Test.labels.iloc[dataset_row_count] == np.argmax(SLP_Test.y_matrix[dataset_row_count]):
            correct_test += 1

    label_train.append(np.argmax(SLP_Train.y_matrix[dataset_row_count]))
    label_test.append(np.argmax(SLP_Test.y_matrix[dataset_row_count]))

    accuracy_training = correct_train / SLP_Train.number_of_rows
    accuracy_test = correct_test / SLP_Test.number_of_rows

    print("Epoch: ", epoch + 1, "Training Accuracy: ", accuracy_training, "Testing Accuracy: ", accuracy_test)

    if accuracy_test > accuracy_best:
        accuracy_best = accuracy_test
        weight_matrix_best = weight_matrix
        best_label_train = label_train
        best_label_test = label_test

    print('weight matrix best : ', weight_matrix_best.shape)

    for i in range(SLP_Train.number_of_rows):
        confusion_matrix_train[best_label_train[i], SLP_Train.labels[i]] += 1

    for i in range(SLP_Test.number_of_rows):
        confusion_matrix_test[best_label_test[i], SLP_Test.labels[i]] += 1

    print('Confusion train : ', confusion_matrix_train)
    print('Confusion test : ', confusion_matrix_test)

def plot():
    fig, axes = plt.subplots(4, 3)
    fig.set_figheight(8)

```

```
fig.set_figwidth(8)
fig.suptitle('Mean Images')
for label in range(12):
    if label < 10:
        axes[label // 3][label % 3].imshow(weight_matrix_best[:,-1, :][:, label].
reshape(10, 10), )
        axes[label // 3][label % 3].axis('off')
plt.show()

if __name__ == '__main__':
    SLP_Train = SLP(training_matrix)
    SLP_Test = SLP(testing_matrix)

    train(SLP_Train, SLP_Test)
    plot()
```



```

import numpy as np
import pandas as pd
import random
import matplotlib.pyplot as plt

training_matrix = pd.read_csv('training.csv', header=None, skiprows=1)
testing_matrix = pd.read_csv('testing.csv', header=None, skiprows=1)
LEARNING_RATE = 0.1
CLASS_SIZE = 10
FEATURES_SIZE = 100

def accuracy_confusion_matrix(x):
    correct = 0
    total = 0
    for i in range(len(x)):
        for j in range(len(x[i])):
            total += x[i][j]
            if i == j:
                correct += x[i][j]
    return correct / total

def encode(y):
    return np.transpose(np.eye(10)[y])

def sigmoid(x):
    return 1. / (1. + np.exp(-x))

def softmax(x):
    e_x = np.exp(x - np.max(x))
    return e_x / e_x.sum()

class MLP(object):
    def __init__(self, matrix):
        self.labels = matrix.iloc[:, 0] # Gets only the first column (Just class name)
        self.x = matrix.iloc[:, 1:] / 255 # Gets all rows and columns except the first one
        self.number_of_rows = matrix.shape[0]
        self.y_matrix = np.zeros((self.number_of_rows, CLASS_SIZE))

def forward(mlp_object, hidden_layer, weight_matrix, field_matrix):
    for instance in range(mlp_object.number_of_rows):
        row = mlp_object.x.iloc[instance, :]
        row = np.append(1, row)
        row = np.transpose(row)

        logit = np.zeros(hidden_layer)
        for h in range(hidden_layer):
            w_h_T = np.transpose(weight_matrix[:, h])
            logit[h] = np.dot(w_h_T, row)

        logit = sigmoid(logit)
        logit = np.append(1, logit)
        logit = np.transpose(logit)

        activation = np.zeros(CLASS_SIZE)
        for i in range(CLASS_SIZE):
            activation[i] = np.dot(np.transpose(field_matrix[:, i]), logit)

        mlp_object.y_matrix[instance] = softmax(activation)

def train(MLP_Train, MLP_Test):
    for H in [5, 10, 25, 50, 75]:
        accuracy_best = 0
        best_label_train = []

```

```

best_label_test = []

training_accuracies = []
test_accuracies = []

field_matrix = np.random.uniform(low=-0.01, high=0.01, size=(H + 1, CLASS_SIZE))
weight_matrix = np.random.uniform(low=-0.01, high=0.01, size=(FEATURES_SIZE + 1, H))

for epochs in range(50):
    random_list = list(range(MLP_Train.number_of_rows))
    random.shuffle(random_list)

    for element in random_list:
        row = MLP_Train.x.iloc[element, :]
        row = np.append(1, row)
        row = np.transpose(row)

        logit = np.zeros(H)

        for h in range(H):
            w_h_T = np.transpose(weight_matrix[:, h])
            logit[h] = np.dot(w_h_T, row)

        logit = sigmoid(logit)

        logit = np.append(1, logit)
        logit = np.transpose(logit)

        activation = np.zeros(CLASS_SIZE)
        for i in range(CLASS_SIZE):
            activation[i] = np.dot(np.transpose(field_matrix[:, i]), logit)

        MLP_Train.y_matrix[element] = softmax(activation)

        change_field_matrix = np.zeros(field_matrix.shape)
        change_weight_matrix = np.zeros(weight_matrix.shape)

        r_t = encode(MLP_Train.labels.iloc[element])

        for i in range(CLASS_SIZE):
            change_field_matrix[:, i] = LEARNING_RATE * (r_t[i] - MLP_Train.
y_matrix[element][i]) * logit

        for h in range(H):
            total = 0.0
            for i in range(CLASS_SIZE):
                total += (r_t[i] - MLP_Train.y_matrix[element][i]) * field_m
atrix[h, i]
            change_weight_matrix[:, h] = LEARNING_RATE * total * logit[h + 1
] * (1 - logit[h + 1]) * row

        for i in range(CLASS_SIZE):
            field_matrix[:, i] += change_field_matrix[:, i]

        for h in range(H):
            weight_matrix[:, h] += change_weight_matrix[:, h]

    forward(MLP_Train, H, weight_matrix, field_matrix)
    forward(MLP_Test, H, weight_matrix, field_matrix)

label_train = []
label_test = []

correct_train = 0
correct_test = 0
for t in range(MLP_Train.number_of_rows):
    if MLP_Train.labels.iloc[t] == np.argmax(MLP_Train.y_matrix[t]):
        correct_train += 1
    if MLP_Test.labels.iloc[t] == np.argmax(MLP_Test.y_matrix[t]):
        correct_test += 1

label_train.append(np.argmax(MLP_Train.y_matrix[t]))

```

```
        label_test.append(np.argmax(MLP_Test.y_matrix[t]))

    accuracy_training = correct_train / MLP_Train.number_of_rows
    accuracy_test = correct_test / MLP_Test.number_of_rows

    training_accuracies.append(accuracy_training)
    test_accuracies.append(accuracy_test)

    print("H: ", H, " Epoch: ", epochs + 1, "Training Accuracy: ", accuracy_
training, "Testing Accuracy: ",
        accuracy_test)

    if accuracy_test > accuracy_best:
        accuracy_best = accuracy_test
        best_label_train = label_train
        best_label_test = label_test

    confusion_matrix_train = np.zeros((CLASS_SIZE, CLASS_SIZE))
    confusion_matrix_test = np.zeros((CLASS_SIZE, CLASS_SIZE))

    for i in range(MLP_Train.number_of_rows):
        confusion_matrix_train[best_label_train[i], MLP_Train.labels[i]] += 1

    for i in range(MLP_Test.number_of_rows):
        confusion_matrix_test[best_label_test[i], MLP_Test.labels[i]] += 1

    print('H = ', H)
    print('Confusion train : ', confusion_matrix_train)
    print('Confusion test : ', confusion_matrix_test)

    plt.title("H = " + str(H))
    plt.plot(training_accuracies, label='Training Accuracy')
    plt.plot(test_accuracies, label='Testing Accuracy')
    plt.xlabel("Epochs")
    plt.ylabel("Accuracy")
    plt.legend(loc='lower right')
    plt.show()

if __name__ == '__main__':
    MLP_Train = MLP(training_matrix)
    MLP_Test = MLP(testing_matrix)

    train(MLP_Train, MLP_Test)
```