The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

SEMESTER SEMESTER 2024-2025

MATH3027 - OPTIMIZATION

Coursework 2

Deadline: 1 pm, Friday 13/12/2024

Your neat, clearly-legible solutions should be submitted electronically as a pdf file via the MATH3027 Moodle page by the deadline indicated there. A scan of a handwritten solution is acceptable. As this work is assessed, your submission must be entirely your own work (see the University's policy on Academic Misconduct).

Submissions up to five working days late will be subject to a penalty of 5% of the maximum mark per working day.

Deadline extensions due to Support Plans and Extenuating Circumstances can be requested according to School and University policies, as applicable to this module. Because of these policies, solutions (where appropriate) and feedback cannot normally be released earlier than 10 working days after the main cohort submission deadline.

You should submit a report electronically as a pdf file with the computational code in the appendix. You may be requested to submit your code (read instructions on Moodle). Comment on all your results.

1. Four factories are located at the positions A(1,1), B(1,3), C(2,5), and D(3,1) in a town. An assembly line, which will use parts produced by these four factories, needs to be built in such a way that the total transportation cost is minimized.

   (a) Assuming that the transportation cost is directly proportional to the Euclidean distance between the factories and the assembly line, the goal is to determine the optimal location for the assembly factory by solving a Fermat-Weber problem.

      i) Explain your choice of weights in the Fermat-Weber problem.

      ii) Formulate the minimization problem with objective function f clearly, and specify the iterative steps involved. Ensure that any notations not mentioned in the question are clearly defined.

      iii) Show that the minimization problem in part ii) has a solution. Is the solution unique?

      iv) Starting with $\mathbf{x}^0 = (0,0)^\top$ and a stopping criterion of $\|\nabla f(\mathbf{x}^k)\| < 10^{-5}$, implement the algorithm. Print the number of iterations and the last iteration point.

      v) Can we choose any point in $\mathbb{R}^2$ as starting point? Why?

      vi) How would you choose a starting point to increase the likelihood of achieving the same accuracy with fewer iterations than that in part iv)?

   (b) If the transportation cost from location A is 50% higher than to the other locations per unit distance. Denote the total transportation cost by $h(\mathbf{x})$. Starting with $\mathbf{x}^0 = (0,0)^\top$ and a stopping criterion of $\|\nabla h(\mathbf{x})\| < 10^{-5}$, implement an algorithm to determine the new optimal location of the assembly factory to minimize h. Print the number of iterations and the last iteration point.

2. Apply Newton's method to find the minimizer of the four variables $\mathbf{x} := (x, y, z, w)$ function g defined as

$$100(w^2 - x)^2 + (w-1)^2 + (y-1)^2 + 90(y^2 - z)^2 + 10.1[(x-1)^2 + (z-1)^2] + 19.8(x-1)(z-1).$$

Run the Pure Newton's method with initial point $\mathbf{x}^0 := (0, 1, 2, 3)^T$ and tolerance 10^{-5}. In particular,

(a) Count the number of iterations.

(b) Write down the last iteration point.