FACULTÉ DES SCIENCES ET TECHNIQUES
SAINT-ÉTIENNE

Master MLDM and 3DMT - Computer Vision course

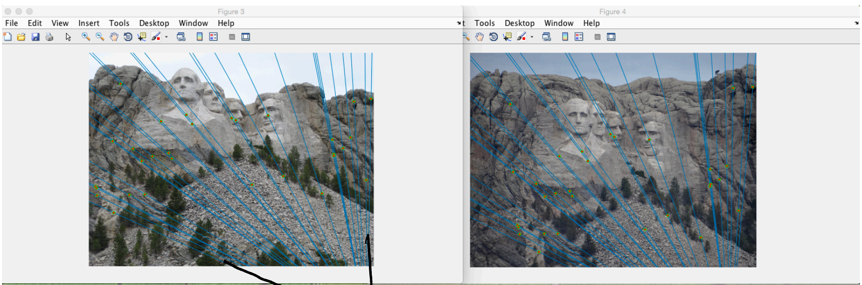
Exam March 2018 - 2h without documents

(6 parts with a total of 14 questions accounting for 25.5 points, the exam will be scored for 20 points)

First Name : ______________________________

Family Name : ______________________________

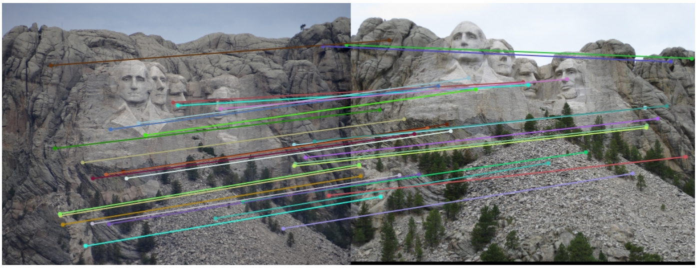
Part 1 *(5 points – 15 mn)*: **Stereo correspondence**



(a) left view

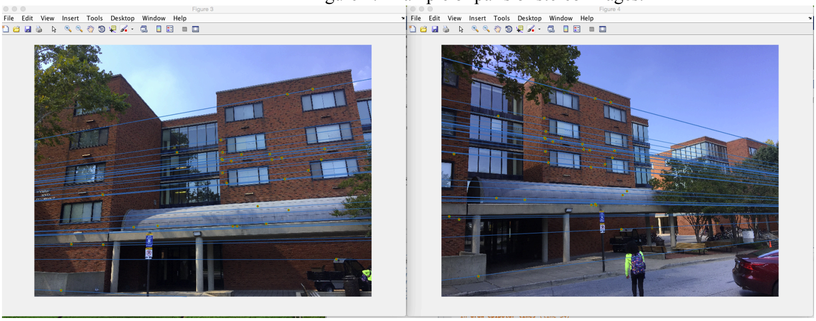
(b) right view

Correspondence pt. rotation, translation & disparity / depth estimation



(c) Examples of corresponding points

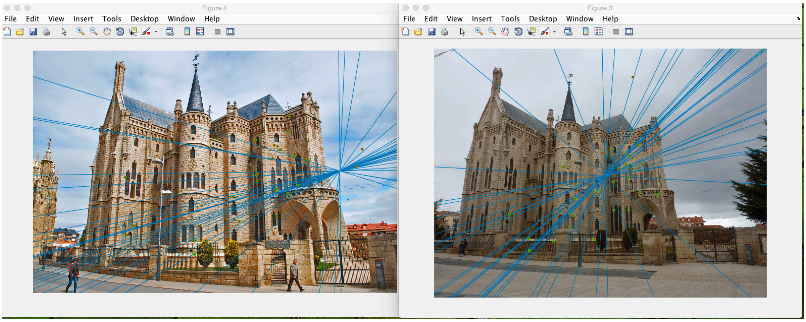
Figure 1: Example of pairs of stereo images.



(a) left view (b) right view

(c) Examples of corresponding points

Figure 2: Example of pairs of stereo images.



(a) left view (b) right view

(c) Examples of corresponding points

Figure 3: Example of pairs of stereo images.

Question 1 *(2 points)***:** In Figures 1 to 3, what do represent blue lines in images (a) and (b)? What these lines tell us?

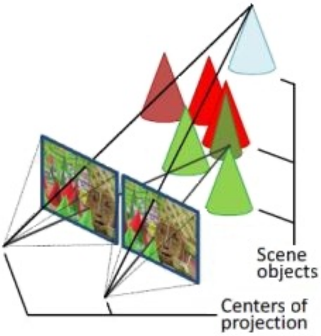
Epipolar lines.
Images need to rectified

Question 2 *(2 points)***:** In Figures 1 to 3, what do represent color lines in images (c)? What these lines tell us?

Question 3 *(1 point)***:** What is the main interest of the epipolar geometry in stereo views matching?

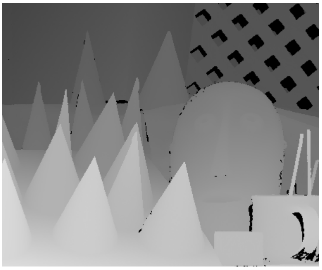
Searching for pt. correspondence in 1D

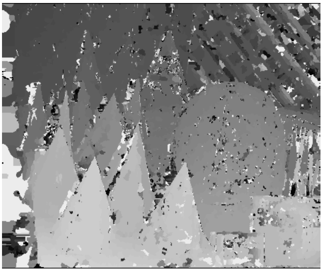
Part 2 *(3 points – 15 mn)***: Stereo correspondence**



(a) Middlebury *s*tereo image “cones”

(b) left view





(c) Disparity map (ground truth)

(d) Disparity map computed from stereo matching

Figure 4: Example of pairs of stereo images.

Question 1 *(2 points)***:** In Figure 4 (c), what do represent the disparity map? Why stereo matching methods based on local spatial correlation technique cannot perform well on such image (see Fig. 4 (d)).

Question 2 *(1 point)***:** It seems in Fig 4 (b) that all cones have the same size, does this assumption is true?

Part 3 *(4 points – 15 mn)***: Projective, Affine, Similarity, and Isometric Transformations**

Question 1 *(2 points)***:** Classify each of the following transformations. Fill-in the circle corresponding to the most specific classification.

$$\left(\frac{9}{16}+1\right)s^2=1$$
$$\Rightarrow \frac{25}{16}s^2=1$$
$$\Rightarrow s=\frac{4}{5}$$

$$\begin{pmatrix} c\theta & -s\theta & \theta_x \\ s\theta & c\theta & \theta_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} sc & -ss & \theta_x \\ ss & sc & \theta_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & \theta_x \\ a_{21} & a_{22} & \theta_y \\ 0 & 0 & 1 \end{pmatrix}$$

i. $H_1 = \begin{bmatrix} 3/4 & -1 & 0 \\ 1 & 3/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ○ Projective ○ Affine ● Similarity ○ Isometric

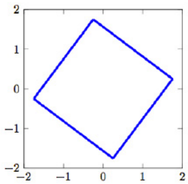
ii. $H_2 = \begin{bmatrix} 3/5 & -4/5 & 1 \\ 4/5 & 3/5 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ ○ Projective ○ Affine ○ Similarity ● Isometric

iii. $H_3 = \begin{bmatrix} 3/16 & -1 & -1/4 \\ 1/4 & 3/4 & 1/2 \\ 1/4 & 1/4 & 1 \end{bmatrix}$ ● Projective ○ Affine ○ Similarity ○ Isometric

iv. $H_4 = \begin{bmatrix} 3/8 & -5/8 & 0 \\ 1/2 & 5/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ○ Projective ○ Affine ○ Similarity ○ Isometric

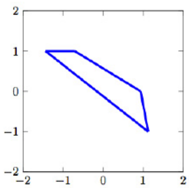
Question 2 *(1.5 point)*: The figures below show the outputs of applying one of transformations (projective, affine, similarity, and isometric) to a square with vertices at (1,1), (1,-1), (-1,-1), (-1,1). Fill in the circle corresponding to the most specific transformation used to generate each output.

i.



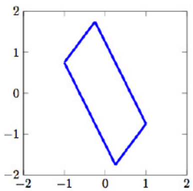
○ Projective ○ Affine ○ Similarity ○ Isometric

ii.



○ Projective ○ Affine ○ Similarity ○ Isometric

iii.



○ Projective ○ Affine ○ Similarity ○ Isometric

Part 4 *(4 points – 20 mn)*: **Camera model and calibration**

Suppose we want to solve for the camera matrix K and that we know that the world coordinate system is the same as the camera coordinate system. Assume the matrix K has the structure outlined below.

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$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ 0 & K_{22} & K_{23} \\ 0 & 0 & K_{33} \end{pmatrix}$$

Note that, in general K_{33} is set to 1, but here K_{33} is an unknown. Assume that we are given n correspondences. Each correspondence consists of a world point (X_i; Y_i; Z_i) and its projection (u_i; v_i) for i = 1;   ; n.

Question 1 *(1 point)*: What is the minimum number of correspondences needed to solve for the unknowns in the matrix K?

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Question 2 *(2 points)*: Set up an equation of the form Ax = 0 to solve for the unknowns in K (where A is a matrix, and x and 0 are vectors). Be specific about what the matrix A and vector x are.

Question 3 *(Bonus - 1 point)*: Explain how to solve for the unknowns in the camera matrix K. Make sure K33 = 1.

Calibration method

Part 5 *(5 points – 25 mn)*: **Epipolar geometry**

The camera projection matrices of two cameras (given in the coordinate system attached to the first camera) are

C = [I 0]   and   C' = [R t]

where R is a rotation matrix and t = (t_x; t_y; t_z) describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).

The epipolar constraint is illustrated in the Figure 5 below and it implies that if *p* and *p'* are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{O'O}$ are coplanar.

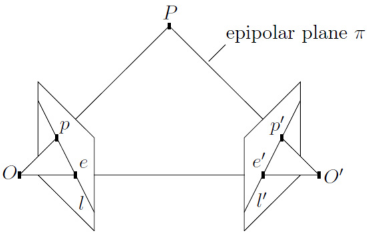

Figure 5: Epipolar geometry. Given a point *p* in the first image its corresponding point in the second image is constrained to lie on the line *l'* which is the epipolar line of *p*. Correspondingly, the line *l* is the epipolar line of *p*'. Points *e* and *e'* are the epipoles.

$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) = 0$ Consequently: (1)

with x represents the cross product between vectors $\overrightarrow{O'O}$ and $\overrightarrow{Op}$

and . represents the inner product between vectors $\overrightarrow{O'p'}$ and $\overrightarrow{O'O} \times \overrightarrow{Op}$

Let $p = (x; y; 1)^T$ and $p' = (x'; y'; 1)^T$ denote the homogeneous image coordinate vectors of p and p'.

Questions 1 *(2 points)***:** Show that the equation (1) can be written in the form:

$$p'^T E\, p = 0 \qquad (2)$$

where matrix E is the essential matrix defined by: $E = [t]\, R$

Hints:

In the coordinate system of the second camera we have: $\overrightarrow{O'p'} = \mathbf{p}', \quad \overrightarrow{O'O} = \mathbf{t}, \quad \overrightarrow{Op} = \mathbf{R}\mathbf{p}$

The cross product a x b, of two arbitrary 3D vectors $a = (i; j; k)^T$ and $b = (l; m; n)^T$, can be written as:

$$a \times b = \begin{pmatrix} 0 & -k & j \\ k & 0 & -i \\ -j & i & 0 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = [a] \,.\, b$$

where [a] is a 3x3 skew matrix associated to the vector a.

$p' = R(p - t)$

$(p-t)^T (p \times t) = 0$

$\Rightarrow (R^T p')^T [t].p = 0$

$\Rightarrow p'^T R.[t].p = 0$

Question 2 *(1 point)***:** How can the epipolar constraint be utilized when searching point correspondences between two views?

$[t]_x = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

Question 3 *(2 points)***:** Let

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and } t = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

be the rotation and translation between two views. Compute the essential matrix and the epipolar line which corresponds to the principal point of the first image (i.e. the point (0; 0) in the normalized image coordinate system).

Part 6 *(4 points – 15 mn)***: Image Transformations**

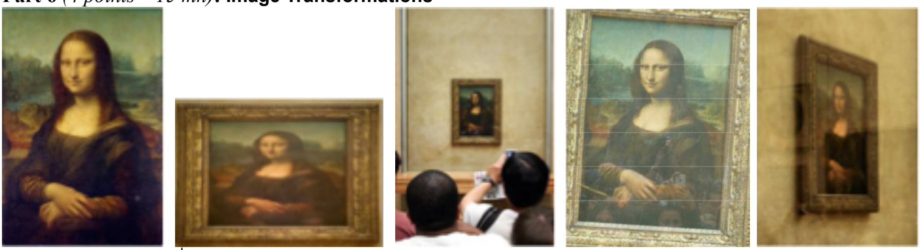

Figure 6 : source ; image 1 image 2 image 3 image 4

Question 1 *(2 points)***:** The figures above show the outputs of applying one of the following transformations to Mona Lisa. Fill in the circle corresponding to the transformation used to generate each output.

Image 1	○ $\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	○ $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Image 2	○ $\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	○ $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Image 3	○ $\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	○ $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Image 4	○ $\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	○ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	○ $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Question 2 *(2 points)***:** For each of images shown above how many degrees of freedom the corresponding transformation has? Fill in the circle corresponding to the right answer.

Image 1	1 DOF ○	2 DOF ○	4 DOF ○	6 DOF ○	8 DOF ○	9 DOF ○
Image 2	1 DOF ○	2 DOF ○	4 DOF ○	6 DOF ○	8 DOF ○	9 DOF ○
Image 3	1 DOF ○	2 DOF ○	4 DOF ○	6 DOF ○	8 DOF ○	9 DOF ○
Image 4	1 DOF ○	2 DOF ○	4 DOF ○	6 DOF ○	8 DOF ○	9 DOF ○

$$\begin{pmatrix} x' \\ y' \\ w \end{pmatrix} = \begin{pmatrix} K11 & K12 & K13 & 0 \\ & K22 & K23 & 0 \\ & & K33 & 0 \end{pmatrix} \begin{pmatrix} X^c \\ Y^c \\ Z^c \\ 1 \end{pmatrix}$$

$$\Rightarrow x' = K_{11}X + K_{12}Y + K_{13}Z$$

$$y' = K_{22}Y + K_{23}Z$$

$$\lambda = K33\,Z$$

$$x = \frac{x'}{\lambda} = \frac{K_{11}X + K_{12}Y + K_{13}Z}{K_{33}Z}$$

$$\Rightarrow K_{11}X + K_{12}Y + K_{13}Z - K_{33}Zx = 0$$

$$0X + K_{22}Y + K_{23}Z - K33Zy = 0$$

$$\begin{pmatrix} X_i & Y_i & Z_i & 0 & 0 & -Z_i x_i \\ 0 & 0 & 0 & Y_i & Z_i & -Z_i y_i \\ & & & & & \end{pmatrix} \begin{pmatrix} K_{11} \\ K_{12} \\ K_{13} \\ K_{22} \\ K_{23} \\ K_{33} \end{pmatrix}$$