Assignment 1

a) The assignments have to be done individually.

b) The assignments have to be answered in English.

c) The answers have to be uploaded to Toledo trough "Assignments", not using any other Toledo tool and not by email.

d) Clearly indicate your name and student number in the uploaded answer.

e) Indicate the time that you have (approximately) spend on the assignment. This will not be taken into account in any way for the quotation but will give us an idea of the load of the assignments. We estimate a load of maximally two hours and half per assignment.

f) You will be rewarded for correct answers, not for the format of your answer.

g) The name of the uploaded document should be **rXXXXXXX.pdf** where you replace XXXXXXX with your student number.

h) **Use the template of the solution sheet to submit your solution.** In case no printer is available, feel free to mimic this template by hand.

Exercise 1

Consider the network shown in Figure 1. We model the influences on someone's test of high cholesterol being positive or negative. The meaning of the variables is as follows.

FT	has free time
E	exercises
M	has muscles
WN	has normal weight
GD	follows a good diet
HC	has high cholesterol
TC	tested positive for high cholesterol

a) How many values need to be stored to represent the joint probability distribution for the seven boolean nodes in the network, assuming that no conditional independence relations are known to hold among them?

b) How many probability values need to be stored, according to the factorization of the network? Justify your answer.

c) Add the boolean variable HA for *health awareness* to the network.

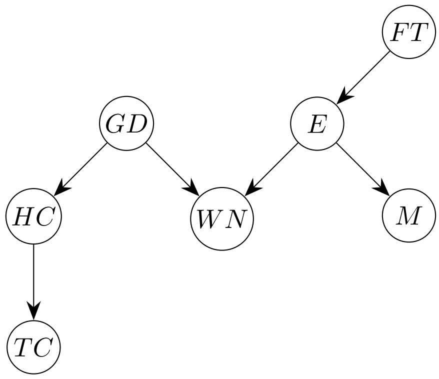
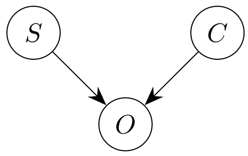


Figure 1: A belief network modelling the influences on the probability of testing positive for high cholesterol.

Exercise 2

Many of you will be familiar with the *Monty Hall problem*. The classical setup is as follows: *there are three doors, behind one of them is a car and the other two host goats. The contestant is asked to pick one of the doors, after which the host opens a different door that certainly contains a goat. The contestant is then allowed to switch doors, but should they?* In this exercise, we will answer this question for the case of **four doors** and the host still only opens one door. In this case, there is one door that contains a car and three doors that contain a goat.

We can model the situation in a simple belief network using three variables having domain $\{1, 2, 3, 4\}$ as follows



with S the door that was selected by the contestant, C the door that contains a car and O the door that is opened by the host. *Think about why we model it like this!*

a) Define the probability distributions $P(S)$, $P(C)$ and $P(O \mid S = 1, C)$.

b) Compute, using these probability distributions, whether it's better to switch doors or keep your initial pick. What is the probability of winning by switching? *Hint: fix some variables first to consider just one case.*

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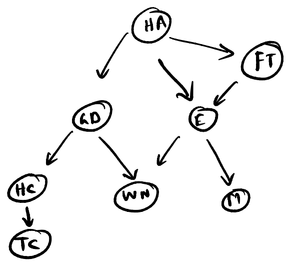
Solution

Fill in each answer in the respective box. Intermediate computations can be done on a separate page.

1a. $2^7 - 1 = 127$

1b. 11 (Reason is stated below)

1c.



Health awareness has positive impact on FT, GD, E

2a. Let $X = \{1,2,3,4\}$

$P(S = x_i) = \frac{1}{4} \quad \forall x_i \in X$

$P(C = x_i) = \frac{1}{4} \quad \forall x_i \in X$

$$P(O \mid S=1, c) = \begin{cases} \frac{1}{3} & \text{if } c = 1 \\ \frac{1}{2} & \text{if } c \neq 1 \\ 0 & \text{if } O = 1 \ \forall c \end{cases}$$

2b. $P(\quad = \quad \mid \quad = \quad , \quad = \quad) =$ and therefore Switching is better.

(Reason stated in next pages).

1.b $P(TC, HC, GD, WN, E, M, FT) = P(TC \mid HC)\, P(HC \mid GD)\, P(WN \mid GD, E)\, P(M \mid E)\, P(FT)$

No. of probability values needed for each term.

$P(TC \mid HC) \rightarrow 2$

$P(HC \mid GD) \rightarrow 2$

$P(WN \mid GD, E) \rightarrow 2^2$

$P(M \mid E) \rightarrow 2$

$P(FT) \rightarrow 1$

Total number $= 2+2+2^2+2+1 = 11$

2.b ∴ Without loss of generality, let $s=2, 0=1$

Let SW = switching to another door.

There are total two events. Each event has two possible outcomes (winning or losing).

① Event 1 (No switching)

→ $P(\text{Winning}) = P(s=2 \mid c=2) = P(s=2) = \frac{1}{4}$

→ $P(\text{Losing}) = P(s \neq 2 \mid c=2) = \frac{3}{4}$

② Event 2 (Switching)

→ $P(\text{Winning}) = \frac{3}{8}$

→ $P(\text{Losing}) = \frac{5}{8}$

} → calculations below

Since $P(\text{winning by Switching}) = \frac{3}{8} > \frac{1}{4}$, $P(\text{Winning by no switching})$

Switching is better.

New

$$P(C=2 \mid S=2, O=1) = \frac{P(S=2, O=1, C=2)}{P(S=2, O=1)}$$

$$= \frac{P(O=1 \mid S=2, C=2)\, P(S=2, C=2)}{\sum_{i=1}^{4} P(S=2, O=1, C=i)}$$

$$= \frac{\frac{1}{3} \times \cancel{\frac{1}{4}}\frac{1}{2}}{\cancel{\frac{1}{4}}2\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{2}\right)}$$

$$= \frac{\frac{1}{3}}{4/3} = 1/4$$

$$P(C=3 \mid S=2, O=1)$$

$$= \frac{P(S=2, O=1, C=3)}{P(S=2, O=1)}$$

$$= \frac{P(O=1 / S=2, C=3)}{\sum_{i=1}^{3} P(O=1 / S=2, C=i)} = \frac{\frac{1}{2}}{\frac{1}{3}+1}$$

$$= \frac{3}{8}$$

$$P(C=4 / S=2, O=1) = \frac{3}{8}$$

$$P(C=j / S=K, O=i) = \frac{3}{8} \quad \text{if } j \neq K,\ i \neq j,\ i \neq K$$

$$= \frac{1}{4} \quad \text{if } j = K,\ i \neq j,\ i \neq K$$

$$= 0 \quad \text{oth}$$

Now given that $s=2$ & $o=1$, the contestant has 2 choices among $\{3,4\}$ to win.

$P(SW=j) = \frac{1}{2} \quad \forall j = 3,4$

$P(\text{winning by switching})$

$= P(SW=3) \cdot P(C=3 \mid S=1, O=2)$

$+ P(SW=4)\, P(C=3 \mid S=1, O=2)$

$= \frac{1}{2} \cdot \left(\frac{3}{8} + \frac{3}{8}\right) = \frac{3}{8}$

$P(\text{losing by switching}) = 1 - \frac{3}{8} = \frac{5}{8}$