

**Time allocated: 3h - No documents allowed.**

**TAKE CARE:** any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

## 1 Problem modeling and solving in PL ( $\simeq$ 4 points)

Here are some informations about a simple world:

*If James eats a lot, then he becomes anxious and he wants to program in Java. If James drinks a lot then he eats a lot and he becomes fat. If James and Emily want to program in Java then Emma is jealous. When Emily wants to practice Software Development, she wants to program in Java and in Python. When Emily dreams and eats chocolate cake, she wants to practice Software Development. When Emily is hungry in the afternoon, she eats chocolate cake. If Emily did not have breakfast, then she is hungry in the afternoon. When Emma is jealous, she fails her exams and doesn't want to leave her house. When Emma doesn't want to leave her house, the police come to take her out. When Emma fails her exams, she says she has passed. Emily did not have breakfast. Emily dreams. James drinks a lot.*

You are asked to:

1. Choose proposition constants wisely and convert these English sentences into propositional logic sentences.
2. Convert these proposition logic sentences into CNF.
3. Use resolution reasoning in proposition logic to prove that: *Emma says she has passed.*

## 2 Problem solving using a truth table ( $\simeq$ 3 points)

Three boxes are presented to you. Each box has imprinted on it a clue as to its contents; the clues are:

- (Box 1) The gold is not here
- (Box 2) The gold is not here
- (Box 3) The gold is in Box 2

Two important facts are that:

- One box contains gold, the other two are empty.
- Only one of the above messages is true; the other two are false.

Which box contains the gold? To answer this question, formalize the problem in Propositional Logic and find the solution using a truth table.

## 3 Validity, unsatisfiability, contingency ( $\simeq$ 3 points)

Using the method studied during the course, say whether the following sentences are unsatisfiable, valid or contingent

1.  $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
2.  $\forall X.\forall Y.((p(X) \wedge p(Y)) \Rightarrow q(X, Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X, Y))$
3.  $(\exists X.q(X) \wedge (\forall X.(p(X) \Rightarrow \neg q(X)))) \Rightarrow \exists X.\neg p(X)$

Note: you must write down all the details of the conversion of each formula to CNF.

## 4 Resolution principle ( $\simeq$ 3 points)

For each pair of clauses below, say whether the resolution principle can be applied. If yes give the resolvent(s), if no, explain why.

1.  $\{p1(X), p2(X, a, f1(X))\}$  and  $\{\neg p2(b, Y, f1(b)), p3(Y)\}$
2.  $\{\neg p1(X, f1(Y), k), \neg p1(f2(a, b), f1(c), W), \neg p2(W)\}$  and  $\{p1(A, f1(B), C), p3(A, B)\}$
3.  $\{p1(X, a, Y), p2(Y)\}$  and  $\{p3(X), \neg p1(b, X, Z)\}$
4.  $\{p1(a, f1(X, f2(Y), Z), b, T), p2(X, Y), p1(X, f1(a, T, c), Y, f2(Y))\}$  and  $\{\neg p1(A, f1(C, D, E), b, F), p3(A, F)\}$
5.  $\{p1(A), \neg p2(b)\}$  and  $\{\neg p1(a), p2(B)\}$
6.  $\{p1(A), p2(A, B), p3(B), p2(a, C), p4(C), p2(A, b)\}$  and  $\{\neg p2(X, b), \neg p2(a, Y), p1(X), p5(Y)\}$

## 5 Problem modeling and solving in FOL ( $\simeq$ 4 points)

Here are some informations about a simple world:

*If a student  $S$  has some knowledge about a problem  $P$  and this student  $S$  has self-confidence, then this problem  $P$  is solvable. If a problem  $P$  is a known problem of a scientific domain  $D$  and a student  $S$  owns a magic wand, then the problem  $P$  is solvable. If a student  $S$  works on a course  $C$  and a problem  $P$  is studied in this course  $C$ , then that student  $S$  has some knowledge about this problem  $P$ . If a teacher  $T$  does a course  $C$  and a student  $S$  takes notes on this course  $C$  and appreciates the teacher  $T$  and learns the course  $C$ , then we can say this student  $S$  works on this course  $C$ . If a teacher  $T$  does a course  $C$  and a student  $S$  owns a smartphone and the course  $C$  is publicly available and the student  $S$  watches it, then we can say this student  $S$  works on this course  $C$ . If a student  $S$  captures a course  $C$  with his camera and converts it to a mp4 file  $F$  and put it on youtube, then this course  $C$  is publicly available. If a student  $S$  owns a paper and owns a pen, and attends a course  $C$  and publishes it on facebook, then this course  $C$  is publicly available. If a teacher  $T$  does a course  $C$  and this teacher  $T$  is in a room  $R$  and a student  $S$  is also in this room  $R$ , then the student  $S$  attends this course  $C$ . Smith does an AI course. Johnson does a ML course. Paul owns a paper, Paul owns a pen, Mary owns a smartphone, Harry owns a magic wand. Overfitting is a known problem of ML. Bias is a known problem of ML. Ethics is a known problem of AI. John has self-confidence. Mary captures the ML course with her camera. Mary converts the ML course to a mp4 file  $f1$ . Mary publishes the file  $f1$  on youtube, Paul publishes the AI course on facebook. Mary watches the ML course. John takes notes on the AI course. John appreciates Smith. John learns the AI course. The problem of overfitting is studied in the ML course. The problem of ethics is studied in the AI course. Smith is in room L125. Paul is in room L125.*

You are asked to:

1. Convert these English sentences into first-order logic sentences.
2. Convert these first-order logic sentences into CNF.
3. Use resolution reasoning in first-order logic, to get **one** answer to the question: "Which problem is solvable?"

## 6 First Order Logic ( $\simeq$ 3 points)

We are considering the world of the 10 tallest buildings in the U.S. The table below provides some data about them:

Building Name	Location	Height (ft.)	Number of Stories	Year Built
One World Trade Center	New York	1776	104	2014
Willis Tower	Chicago	1451	108	1974
432 Park Avenue	New York	1396	88	2014
Trump International Hotel and Tower	Chicago	1389	98	2009
Empire State Building	New York	1250	102	1931
Bank of America Tower	New York	1200	55	2009
Aon Center	Chicago	1136	83	1973
John Hancock Center	Chicago	1127	100	1969
Chrysler Building	New York	1046	77	1930
New York Times Building	New York	1046	52	2007

Consider seven predicates  $inC/1$ ,  $inN/1$ ,  $inT1046/1$ ,  $isTaller/2$ ,  $has100/1$ ,  $before1980/1$  and  $moreStories/2$  such as  $inC(B)$  is true if  $B$  is a building located in Chicago,  $inN(B)$  is true if  $B$  is a building located in New York,  $inT1046(B)$  is true if  $B$  is a building that is exactly 1046ft. tall,  $isTaller(B1,B2)$  is true if the building  $B1$  is taller than the building  $B2$ ,  $has100(B)$  is true if  $B$  is a building that has at least 100 stories,  $before1980(B)$  is true if  $B$  is a building built before 1980 and  $moreStories(B1,B2)$  is true if the building  $B1$  has more stories than the building  $B2$ .

Given the above information, say whether the following first order logic sentences are true or false. Explain each value with english sentences (you may use reasonable abbreviations for building names).

1.  $\exists X.\forall Y.(inN(X) \wedge inC(Y) \wedge isTaller(X, Y))$
2.  $\forall X.(inC(X) \wedge has100(X) \Rightarrow before1980(X))$
3.  $\forall X.\forall Y.(inC(X) \wedge inT1046(Y) \wedge moreStories(X, Y))$
4.  $\forall X.\exists Y.(inC(X) \wedge inN(Y) \wedge \neg moreStories(Y, X))$