Lab Course Kinect Programming for Computer Vision

Transformations and Camera Calibration

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Lecture Outline

Part I: Basics of Transformations

Part II: Basics of Cameras

Outline Part I

- 2D Transformations
- Homogeneous Coordinates in 2D
- Hierarchy of Transformations in 2D
- Hierarchy of Transformations in 3D

2D Transformations

The basic building blocks of transformations are:

- Scaling (isotropic, non-isotropic)
- Rotation (around one rotation axis)
- Translation (motion along the two axes)

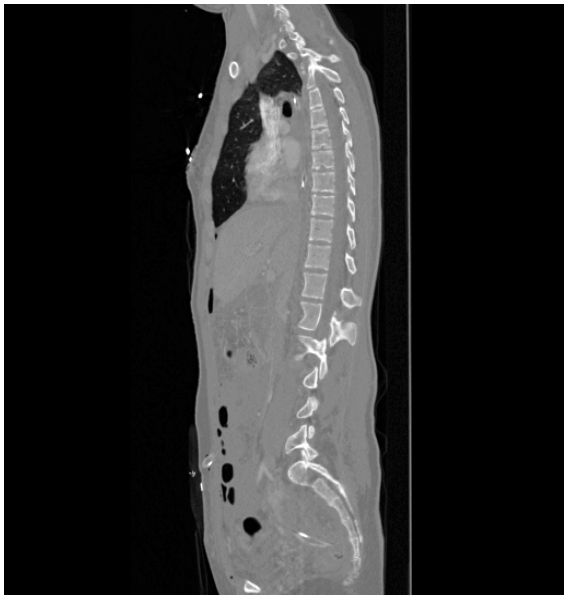
Transformations are defined by their action on point coordinates.

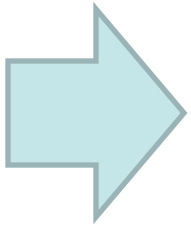
Notation:

$$\mathbf{p} = (x, y)^\top = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \mathbf{p}' = (x', y')^\top = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

original point transformed point

2D Transformations – Scaling



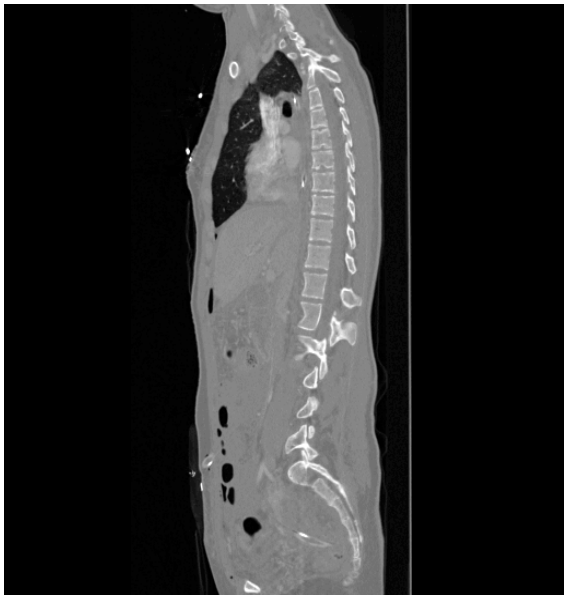


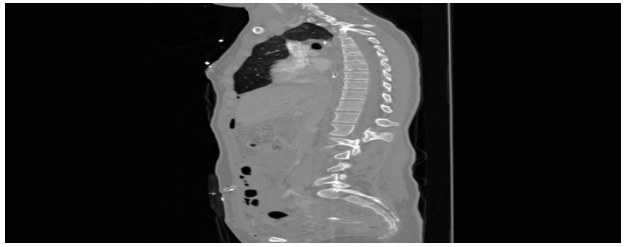
Isotropic scaling: same scaling factor in both spatial dimensions

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Transformations – Scaling

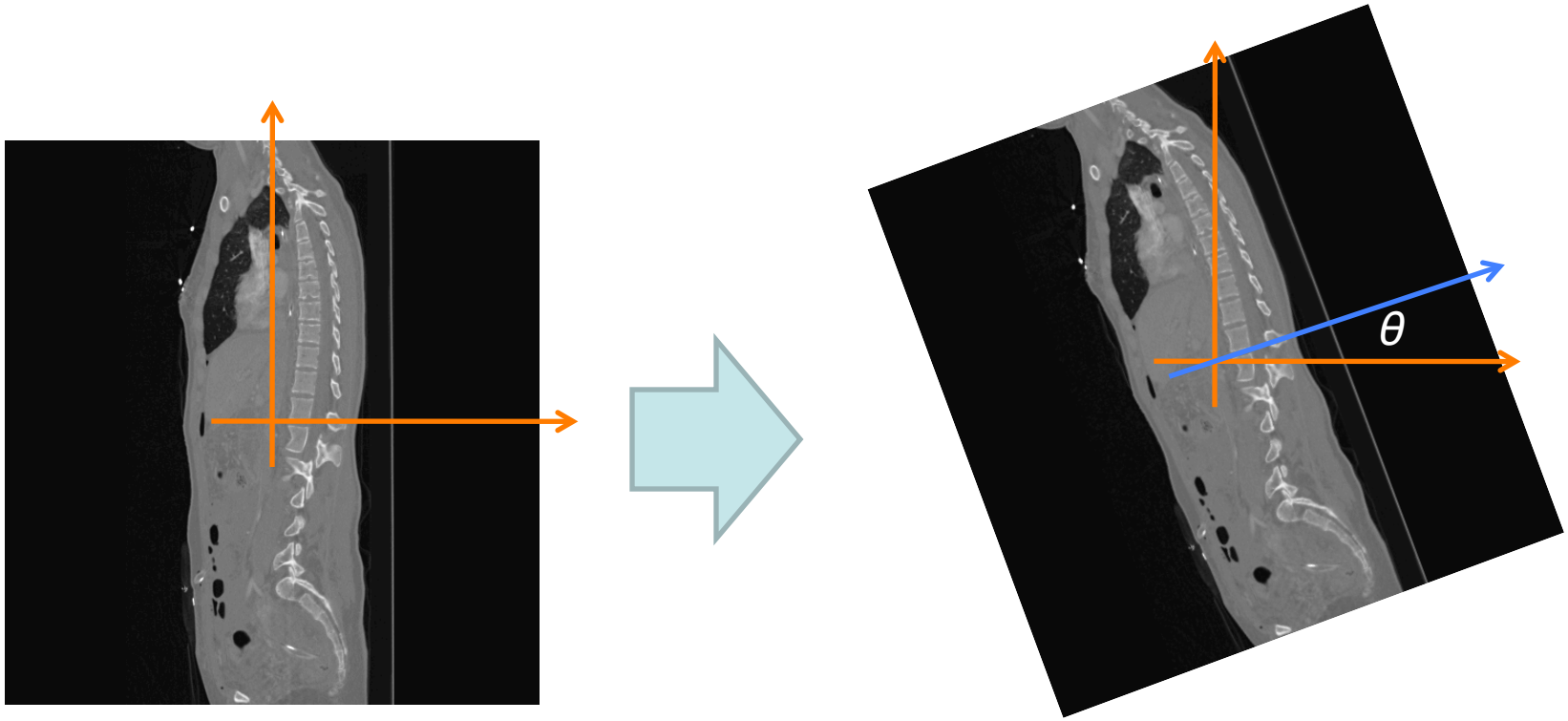
Non-isotropic scaling: different scaling factors in the two spatial dimensions





$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

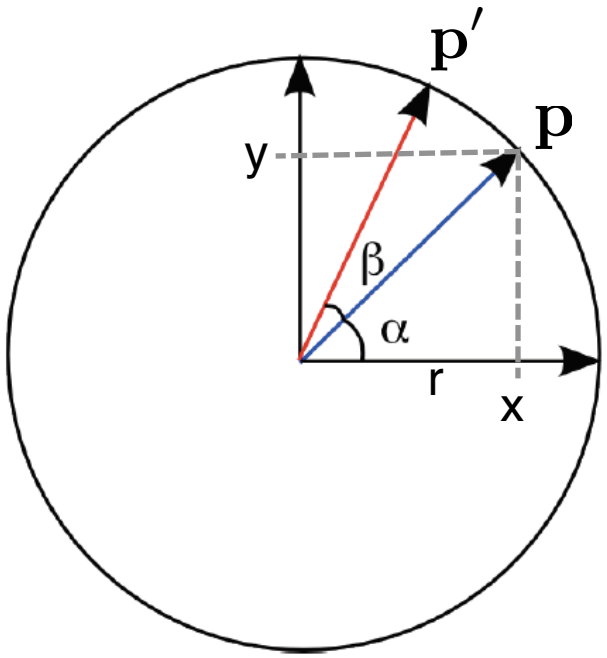
2D Transformations – Rotation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

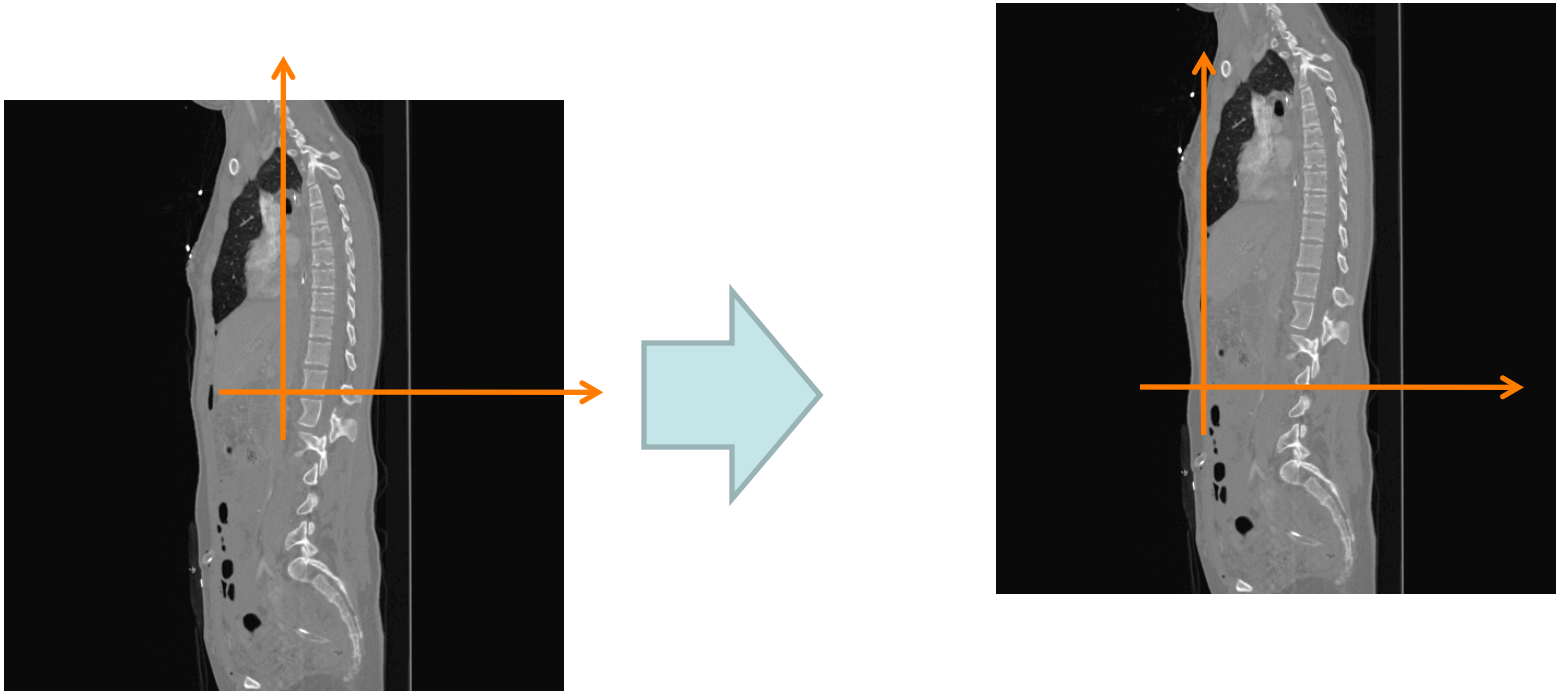
2D Transformations – Rotation

$$\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r\cos(\alpha) \\ r\sin(\alpha) \end{pmatrix}$$



$$\begin{aligned}
\mathbf{p}' &= \begin{pmatrix} r\cos(\alpha+\beta) \\ r\sin(\alpha+\beta) \end{pmatrix} \\
&= \begin{pmatrix} r\cos(\alpha)\cos(\beta) - r\sin(\alpha)\sin(\beta) \\ r\sin(\alpha)\cos(\beta) + r\cos(\alpha)\sin(\beta) \end{pmatrix} \\
&= \begin{pmatrix} x\cos(\beta) - y\sin(\beta) \\ y\cos(\beta) + x\sin(\beta) \end{pmatrix} \\
&= \begin{pmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\
&= \begin{pmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{pmatrix} \mathbf{p}
\end{aligned}$$

2D Transformations – Translation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

2D Transformations

Have you noticed we have two types of representations for the basic transformations seen so far?

Scaling, Rotation: **Matrix-vector product**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Translation: **Sum of vectors**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

How to achieve a matrix-vector product form for all types of transformations?

Outline Part I

- 2D Transformations
- Homogeneous Coordinates in 2D
- Hierarchy of Transformations in 2D
- Hierarchy of Transformations in 3D

Homogeneous Coordinates in 2D

How to achieve a matrix-vector product form for all types of transformations?

- Write a 2D point in **homogeneous coordinates**:

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \in \mathbb{R}^3$$

We will see more properties of homogeneous representation later.

- Then, translation can easily be expressed as a matrix-vector product:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates in 2D

- Rotation in homogeneous representation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Combined transformations (here: rotation and translation):

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta + t_x \\ x\sin\theta + y\cos\theta + t_y \\ 1 \end{bmatrix}$$

2D Projective Space

When dealing with points represented in homogeneous coordinates, we are working in a Projective space.

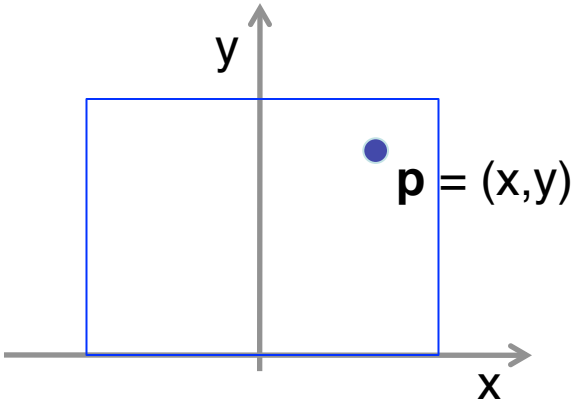
- Points in 2D **Euclidean** space are represented by **Cartesian** coordinates

$$\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$$

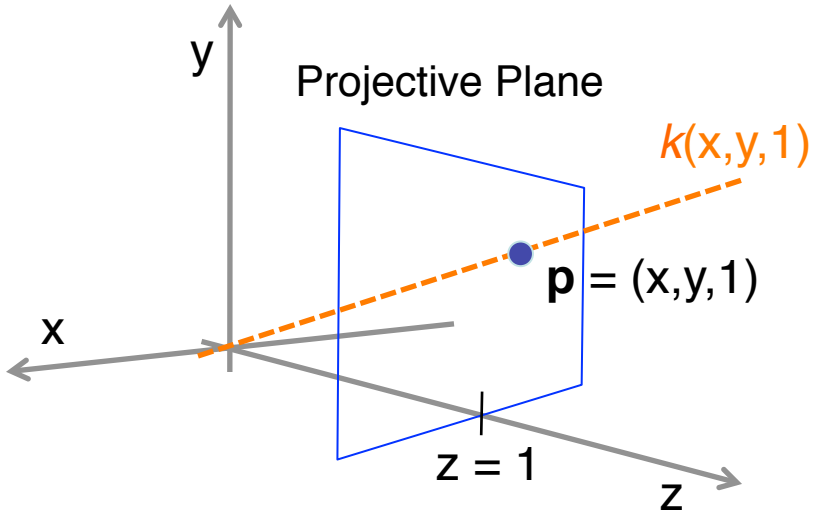
- Points in 2D **Projective** space are represented by **Homogeneous** coordinates:

$$\mathbf{p} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \in \mathbb{P}^2$$

2D Projective Space



Point in **Euclidean** 2D space



Point in **Projective** 2D space.

All points on the orange line represent the **same point**. A homogeneous vector in 2D projective space represents an **equivalence class** of points.

Points and Lines in 2D Projective Space

Properties of points

- A 2D point $(x, y)^\top$ has the **homogeneous** representation $(x, y, 1)^\top$.
- The homogeneous vector $(x, y, w)^\top$ with w≠0 represents the point $(\frac{x}{w}, \frac{y}{w}, 1)^\top$.
- For any non-zero *k*, $(x, y, w)^\top$ and $k(x, y, w)^\top$ represent the same point.

Points and Lines in 2D Projective Space

Properties of lines

- A line $ax + by + c = 0$ has the homogeneous representation $\mathbf{l} = (a, b, c)^\top$.
- A point $\mathbf{x}$ lies on a line $\mathbf{l}$ if and only if $\mathbf{x}^\top \mathbf{l} = 0$.
- Two lines $\mathbf{l}$ and $\mathbf{l}'$ intersect in the point $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$.
- The line joining the two points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$.

Points and Lines in 2D Projective Space

Special points and lines

- Points with 3rd coordinate zero $(x, y, 0)^\top$ are called **points at infinity**.
- All points at infinity lie on the **line at infinity** $\mathbf{l}_\infty = (0, 0, 1)^\top$.

$$\mathbf{l}_\infty^\top (x, y, 0) = (0, 0, 1)^\top (x, y, 0) = 0$$

- Parallel lines intersect at infinity.

In projective space, all lines intersect in a point, even parallel lines.

Outline Part I

- 2D Transformations
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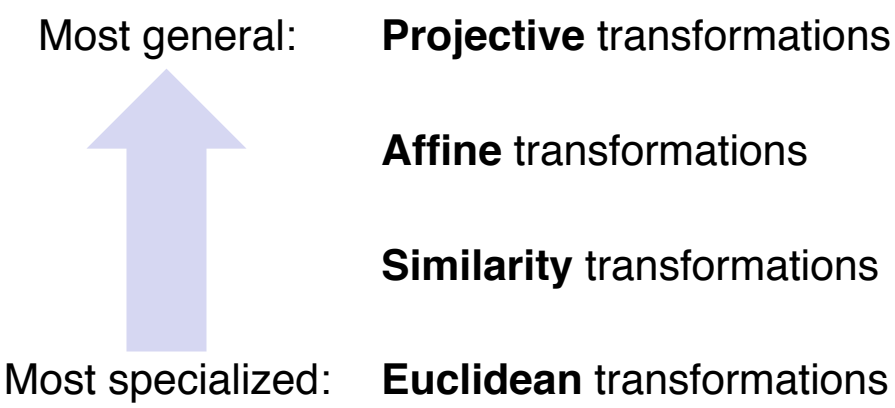
- Hierarchy of Transformations in 2D
- Hierarchy of Transformations in 3D

Hierarchy of Transformations in 2D

- Classification of transformations in terms of quantities or properties that are **invariant** (left unchanged) by the transformation
- Hierarchy: Starting from most specialized type of transformation, successively remove invariants to get more general types of transformations



- Containment relation from general to specialized, e.g.: affine transformations have all properties of projective transformations, plus additional ones.

Hierarchy of Transformations in 2D: **Euclidean Transformations**

- Transformation:

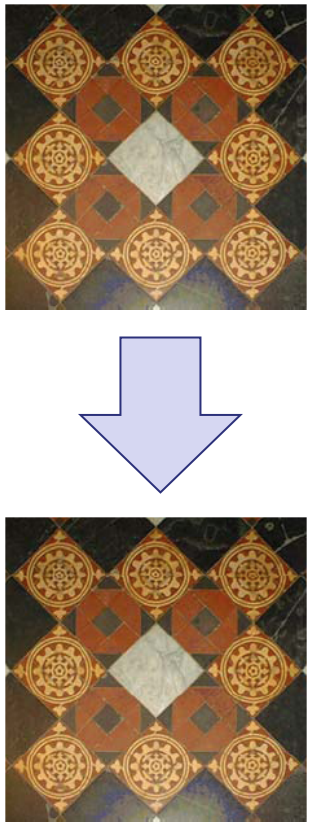
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{euclidean}} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

$$\mathbf{R}^\top \mathbf{R} = \mathbf{I}$$
$$\det(\mathbf{R}) = 1$$

- Degrees of freedom:
  - 3 (1 rotation, 2 translation)
- Invariants: length, area, angles



Hierarchy of Transformations in 2D: **Similarity Transformations**

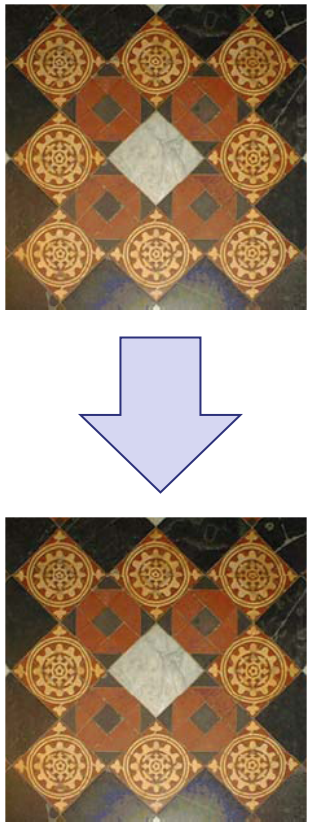
- Transformation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{similarity}} = \begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \qquad \mathbf{R}^\top\mathbf{R} = \mathbf{I} \qquad \det(\mathbf{R}) = 1$$

- Degrees of freedom:
  - 4 (1 rotation, 2 translation, 1 isotropic scaling)
- Invariants: ratio of lengths, angles, parallel lines



Hierarchy of Transformations in 2D: **Affine Transformations**

- Transformation:

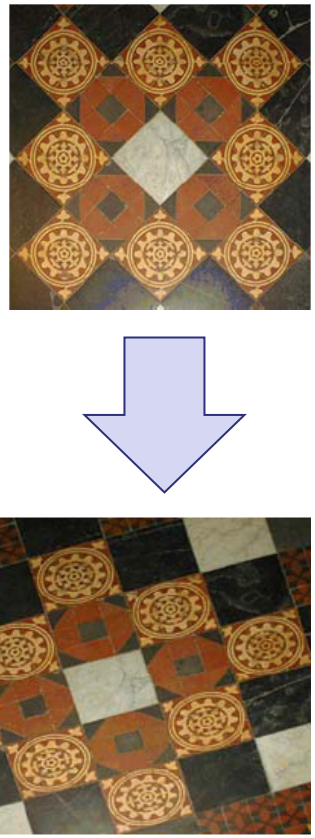
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{affinity}} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

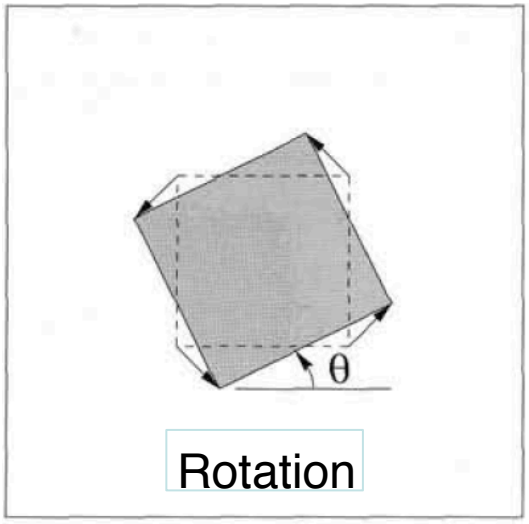
$\mathbf{A}$ non-singular

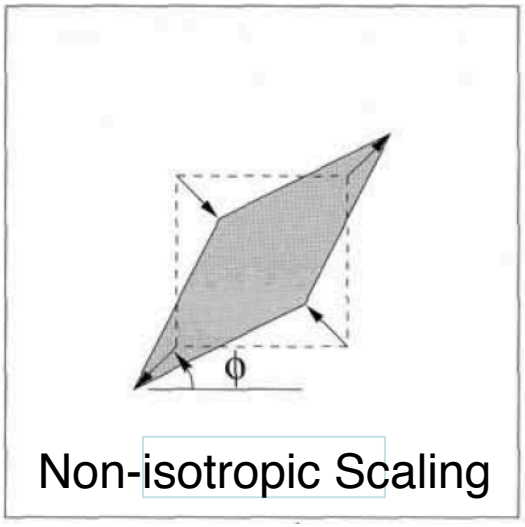
- Degrees of freedom:
  - 6 (2 translation, 4 entries of **A**)
- Invariants: parallel lines, ratios of areas, line at infinity

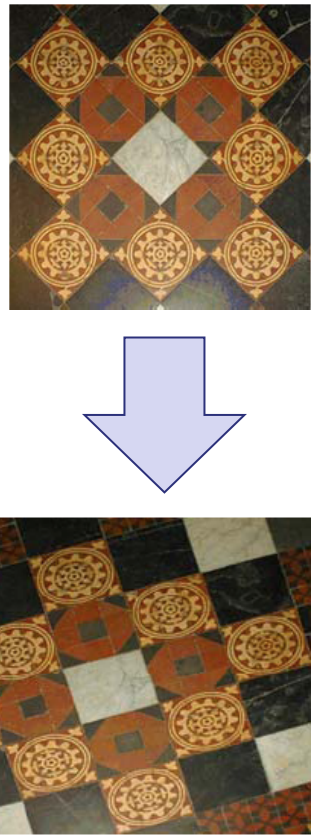


Hierarchy of Transformations in 2D: **Affine Transformations**

Affine transformations geometrically consist of two components:







$$\mathbf{A} = \mathbf{R}(\theta)\mathbf{R}(-\phi)\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\mathbf{R}(\phi)$$

Hierarchy of Transformations in 2D: **Projective Transformations**

- Transformation:

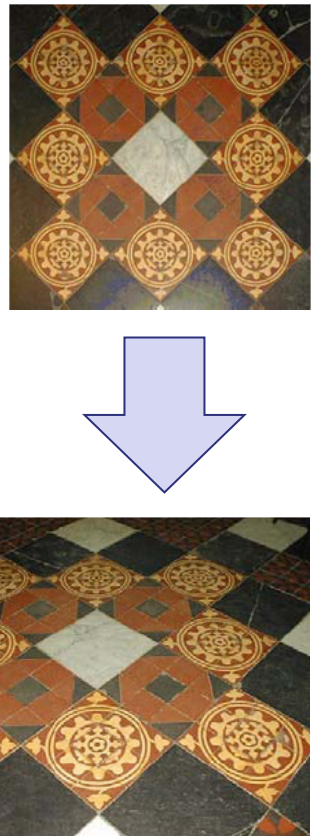
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & v \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

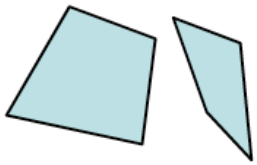
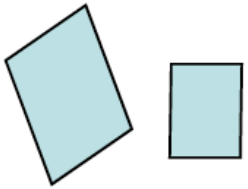
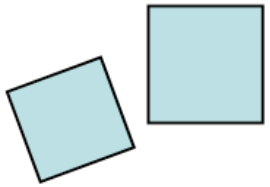
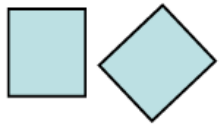
$$\mathbf{T}_{\text{projectivity}} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{v}^\top & v \end{bmatrix}$$

$\mathbf{A}$ non-singular

- Degrees of freedom:
  - 8 (9 arbitrary matrix entries, up to scale)
- Invariants: Intersection points, tangency, cross-ratios



Hierarchy of Transformations in 2D – Summary

Projective	8 DOF		$\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{v}^\top & v \end{bmatrix}$
Affine	6 DOF		$\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$
Similarity	4 DOF		$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$
Euclidean	3 DOF		$\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$

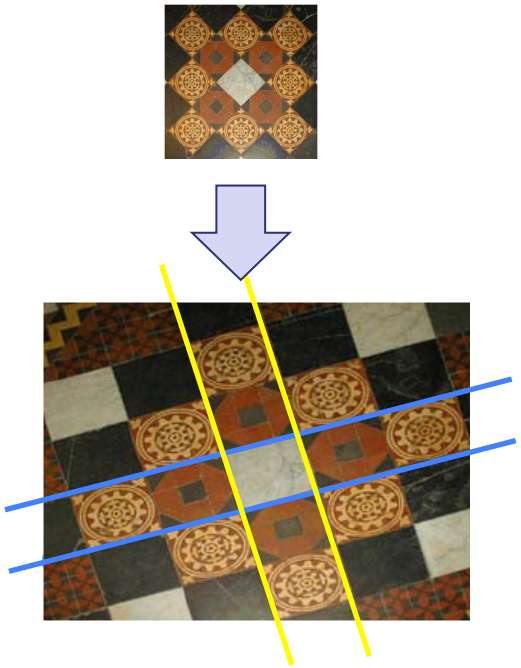
Action of Transformations in 2D on Points at Infinity

- **Affine** Transformations:

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \\ 0 \end{bmatrix}$$

Point at infinity, e.g. intersection of blue lines

Mapped by affine transformation to another point at infinity



Action of Transformations in 2D on Points at Infinity

- **Projective** Transformations:

$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & v \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \\ v_1 x + v_2 y \end{bmatrix}$$

Point at infinity, e.g. intersection of blue lines

Mapped by projective transformation to a finite point (**vanishing point**)

Only projective transformations can make points at infinity finite points.

Outline Part I

- 2D Transformations
- Homogeneous Coordinates in 2D
- Hierarchy of Transformations in 2D
- Hierarchy of Transformations in 3D

Homogeneous Coordinates in 3D

How to achieve a matrix-vector product form for all types of transformations?

- Write a 3D point in **homogeneous coordinates**:

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \in \mathbb{R}^4$$

- 3D projective space: properties analogous to 2D homogeneous representation
- All types of transformations (scaling, translation, rotation, etc.) can be written as a matrix-vector product
- Several transformations can be composed into one transformation matrix

Hierarchy of Transformations in 3D (1): **Euclidean Transformations**

- Transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{euclidean}} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \qquad \begin{aligned} \mathbf{R}^\top \mathbf{R} &= \mathbf{I} \\ \det(\mathbf{R}) &= 1 \end{aligned}$$

- Degrees of freedom:
  - 6 (3 rotation, 3 translation)
- Invariants: volume, angles

Hierarchy of Transformations in 3D (2): **Similarity Transformations**

- Transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} sr_{11} & sr_{12} & sr_{13} & t_x \\ sr_{21} & sr_{22} & sr_{23} & t_y \\ sr_{31} & sr_{32} & sr_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{similarity}} = \begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \qquad \begin{aligned} \mathbf{R}^\top\mathbf{R} &= \mathbf{I} \\ \det(\mathbf{R}) &= 1 \end{aligned}$$

- Degrees of freedom:
  - 7 (3 rotation, 3 translation, 1 scaling)
- Invariants: angles

Hierarchy of Transformations in 3D (3): **Affine Transformations**

- Transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & t_x \\ a_{21} & a_{22} & a_{23} & t_y \\ a_{31} & a_{32} & a_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{affinity}} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

$\mathbf{A}$ non-singular

- Degrees of freedom:
  - 12 (3 translation, 9 entries of **A**)

- Invariants: parallel planes, ratios of volumes, plane at infinity

Hierarchy of Transformations in 3D (4)
Projective Transformations

- Transformation:

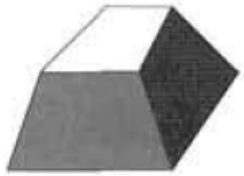
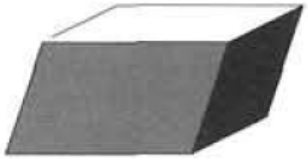
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & t_x \\ a_{21} & a_{22} & a_{23} & t_y \\ a_{31} & a_{32} & a_{33} & t_z \\ v_1 & v_2 & v_3 & v \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Concise notation for transformation matrix:

$$\mathbf{T}_{\text{projectivity}} = \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{v}^\top & v \end{bmatrix} \qquad \mathbf{A} \text{ non-singular}$$

- Degrees of freedom:
  - 15 (16 arbitrary matrix entries, up to scale)
- Invariants: Intersection points, tangency

Hierarchy of Transformations in 3D – Summary

Projective	15 DOF		$\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{v}^\top & v \end{bmatrix}$
Affine	12 DOF		$\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$
Similarity	7 DOF		$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$
Euclidean	6 DOF		$\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix}$

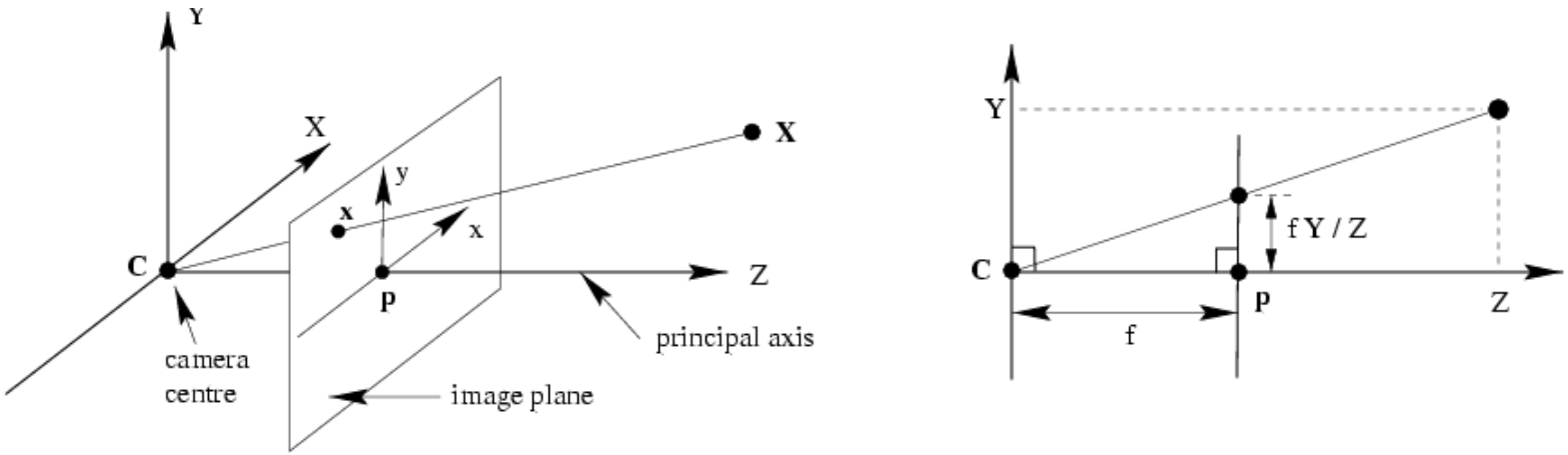
Lecture Outline

Part I: Basics of Transformations

Part II: Basics of Cameras

Pinhole Camera Model

Mapping between 3D world and 2D image by central projection

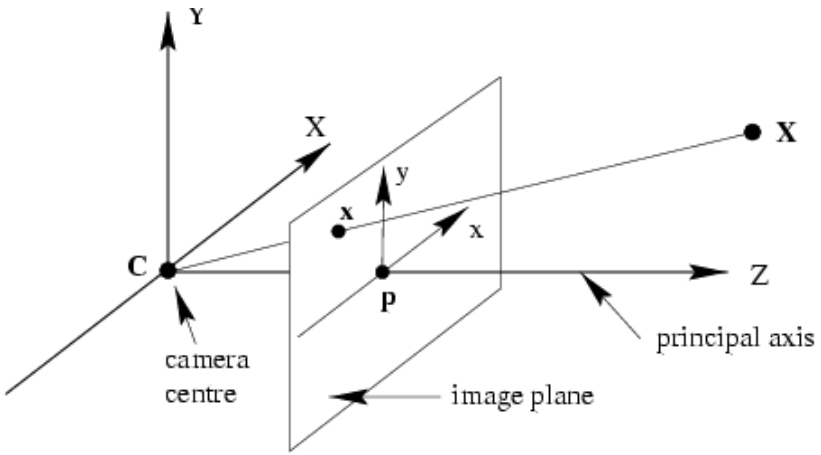


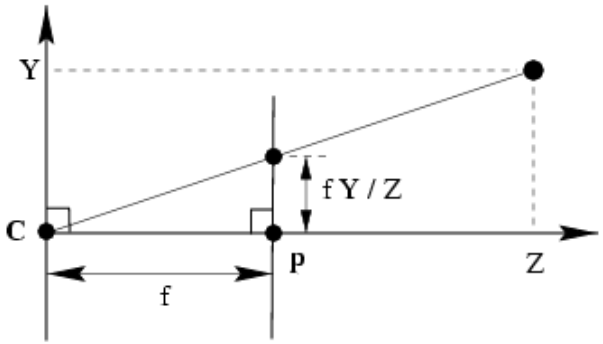
$$(X, Y, Z)^\top \mapsto (fX/Z, fY/Z, f)^\top$$

$$(X, Y, Z)^\top \mapsto (fX/Z, fY/Z)^\top = (x, y)^\top$$

Pinhole Camera Model

In homogeneous coordinates…

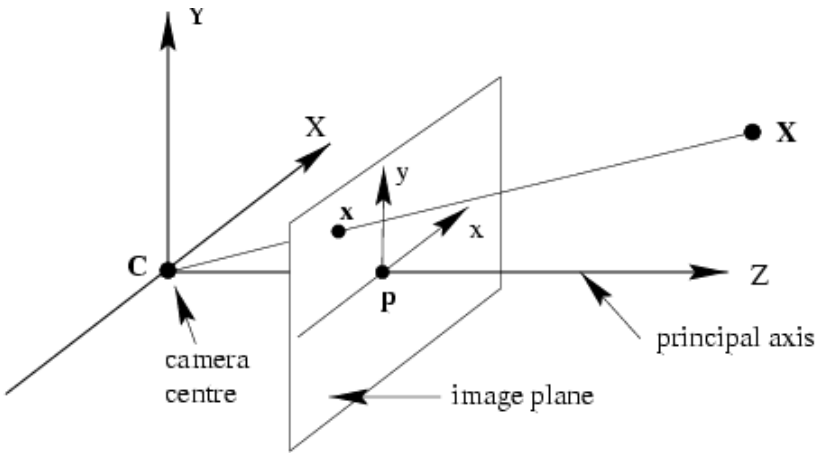


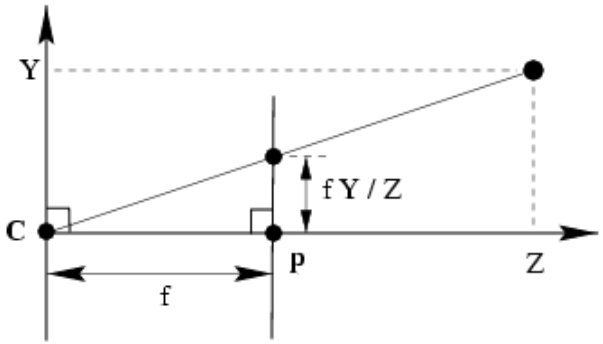


$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Pinhole Camera Model

Mapping between 3D world and 2D image by central projection



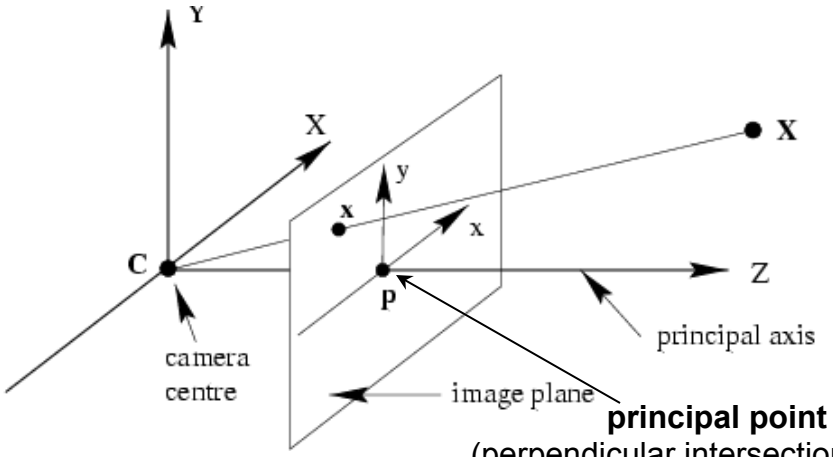


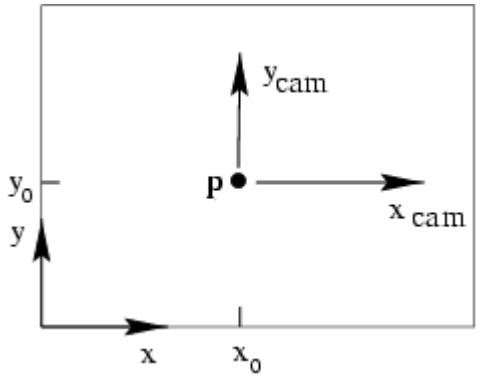
$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \mathrm{diag}(f, f, 1)[\mathbf{I}|\mathbf{0}]\mathbf{X}$$

Pinhole Camera Model

Origin of image plane is not necessarily at the camera center





principal point
(perpendicular intersection point of principal axis and image plane)

$$\mathbf{p} = (p_x, p_y)^\top$$

$$(X, Y, Z) \mapsto (fX/Z + p_x, fY/Z + p_y)$$

Pinhole Camera Model

Origin of image plane is not necessarily at the camera center

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

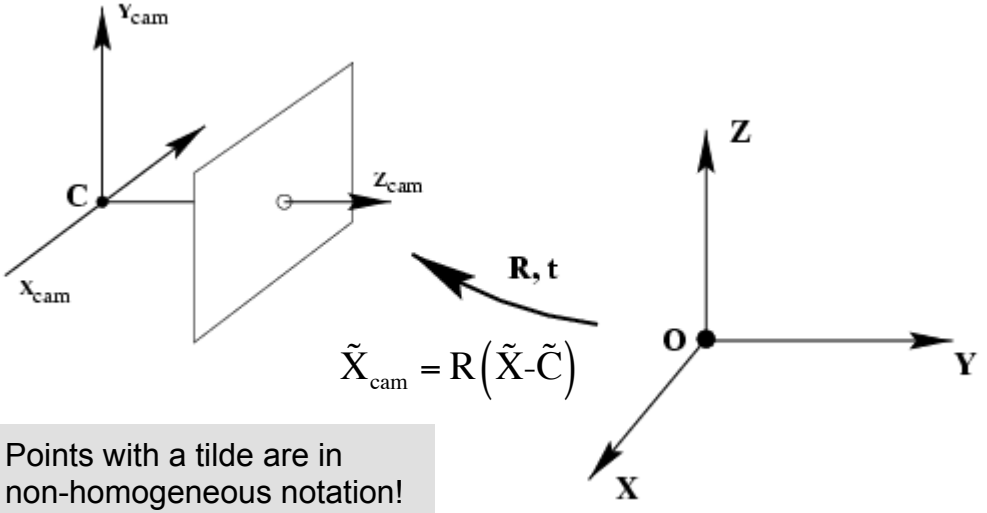
$$\mathbf{x} = \mathbf{K}[\mathbf{I}|\mathbf{0}]\mathbf{X}$$

$$\mathbf{K} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

intrinsic camera parameters matrix

Pinhole Camera Model

Camera center is not necessarily at the world coordinate system origin



$$\tilde{X}_{cam} = R\left(\tilde{X}-\tilde{C}\right)$$

Points with a tilde are in non-homogeneous notation!

$\tilde{\mathbf{C}}$

camera center in world coordinates

$$\mathbf{X}_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \mathbf{X}$$

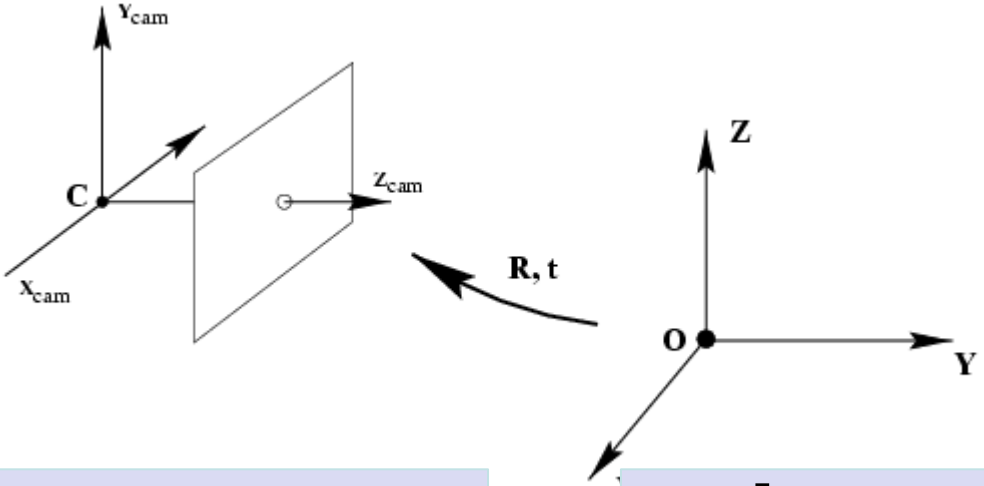
3D point in camera coordinates

$$\mathbf{x} = K\left[I \mid 0\right]\mathbf{X}_{cam}$$ projection to image plane from camera coordinates

$$\mathbf{x} = KR\left[I \mid -\tilde{C}\right]\mathbf{X}$$ projection to image plane from world coordinates

Pinhole Camera Model

Camera center is not necessarily at the world coordinate system origin



$$\mathbf{x} = \mathrm{KR}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]\mathbf{X}$$

$$\mathbf{x} = \mathrm{P}\mathbf{X}$$

$$\mathrm{P} = \mathbf{KR}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]$$

projection matrix, 9 DOF

$$\mathbf{K} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

intrinsic parameters matrix, 3 DOF

$$\mathbf{R}, \tilde{\mathbf{C}}$$

extrinsic parameters, each 3 DOF

CCD Camera Model

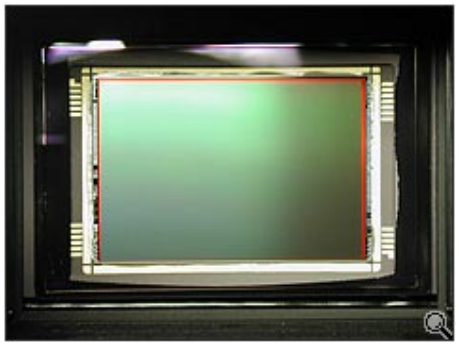
Camera coordinates are mapped to pixels of the CCD chip

m_x, m_y number of pixels per unit distance in camera coordinates

$$\mathbf{K} = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$ 4 DOF

$$\mathrm{P} = \mathbf{K}\mathbf{R}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]$$ 10 DOF



Finite Projective Camera Model

Both axes in camera coordinates need not have identical scaling

s skew parameter

$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix} \qquad \text{5 DOF}$$

$$\mathrm{P} = \mathbf{KR}\left[\mathrm{I} \,|\, -\tilde{\mathrm{C}}\right] \qquad \text{11 DOF}$$

Camera Calibration

- Relating the ideal camera model to the properties of an actual physical device and determining the position and orientation of the camera w.r.t. the world coordinate frame[1].

$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

- Estimating the entries of $\mathbf{K}, \mathbf{R}, \tilde{\mathrm{C}}$

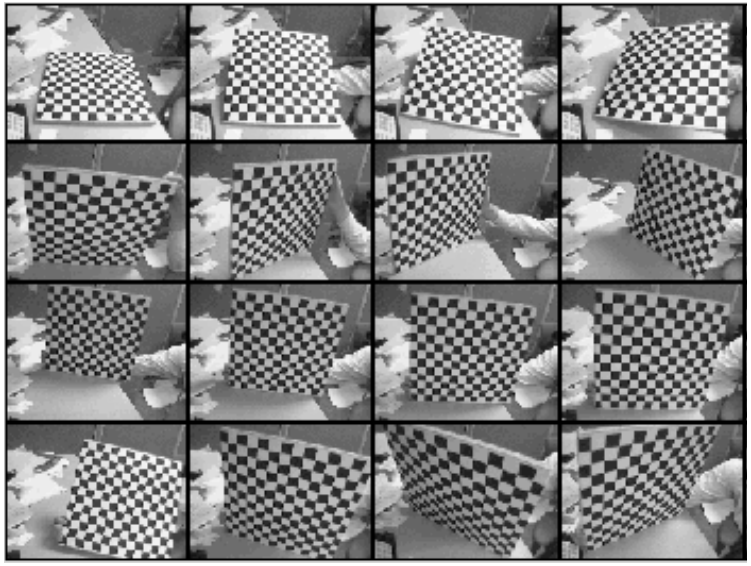
$$\mathrm{P} = \mathbf{K}\mathbf{R}\left[\mathrm{I} \mid -\tilde{\mathrm{C}}\right]$$

- Camera models with increasing generality:
  - pinhole camera 9 DOF
  - CCD camera 10 DOF
  - finite projective camera 11 DOF

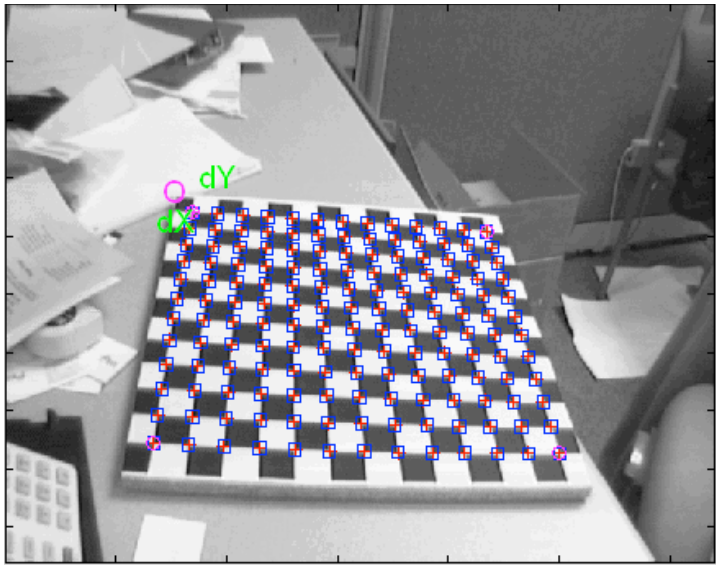
- In addition: lens distortion parameters

[1] http://www.dis.uniroma1.it/~iocchi/stereo/calib.html

Camera Calibration



Checkerboard pattern captured from multiple views (the checker sizes in the real world are known)

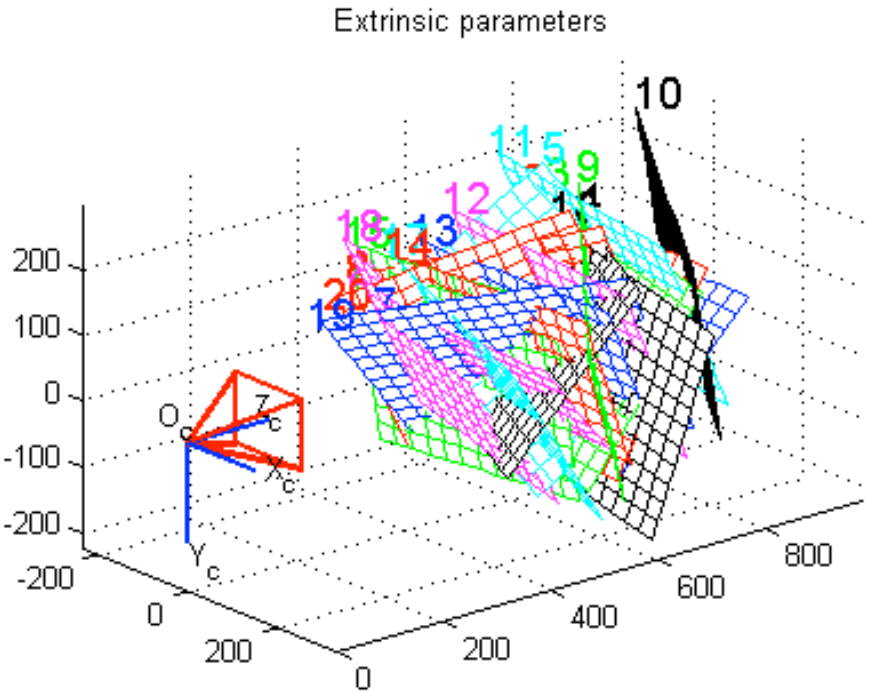


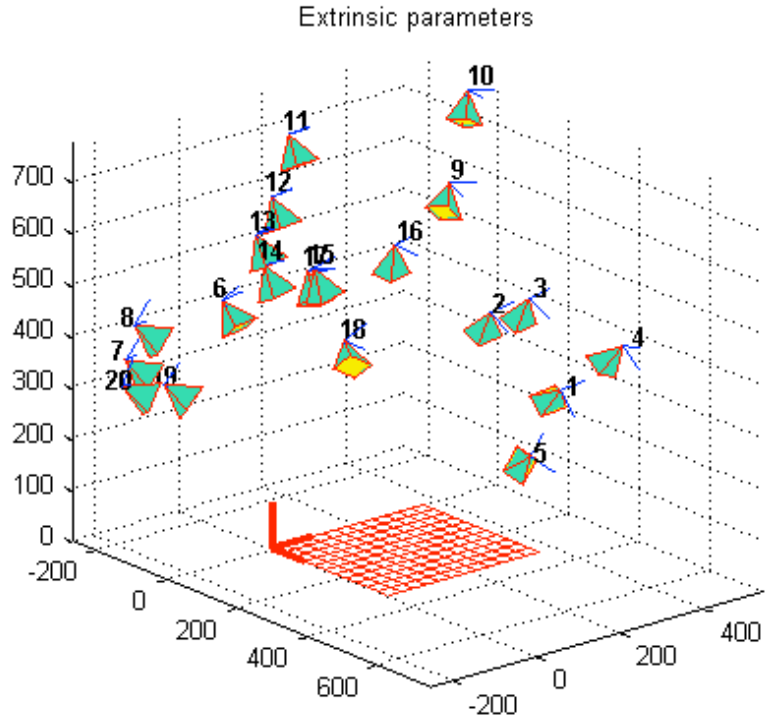
Semi-automatic measurements to find sizes in each of the images

- Linear and non-linear optimization for estimating camera parameters
- MATLAB calibration toolbox: http://www.vision.caltech.edu/bouguetj/calib_doc

Camera Calibration

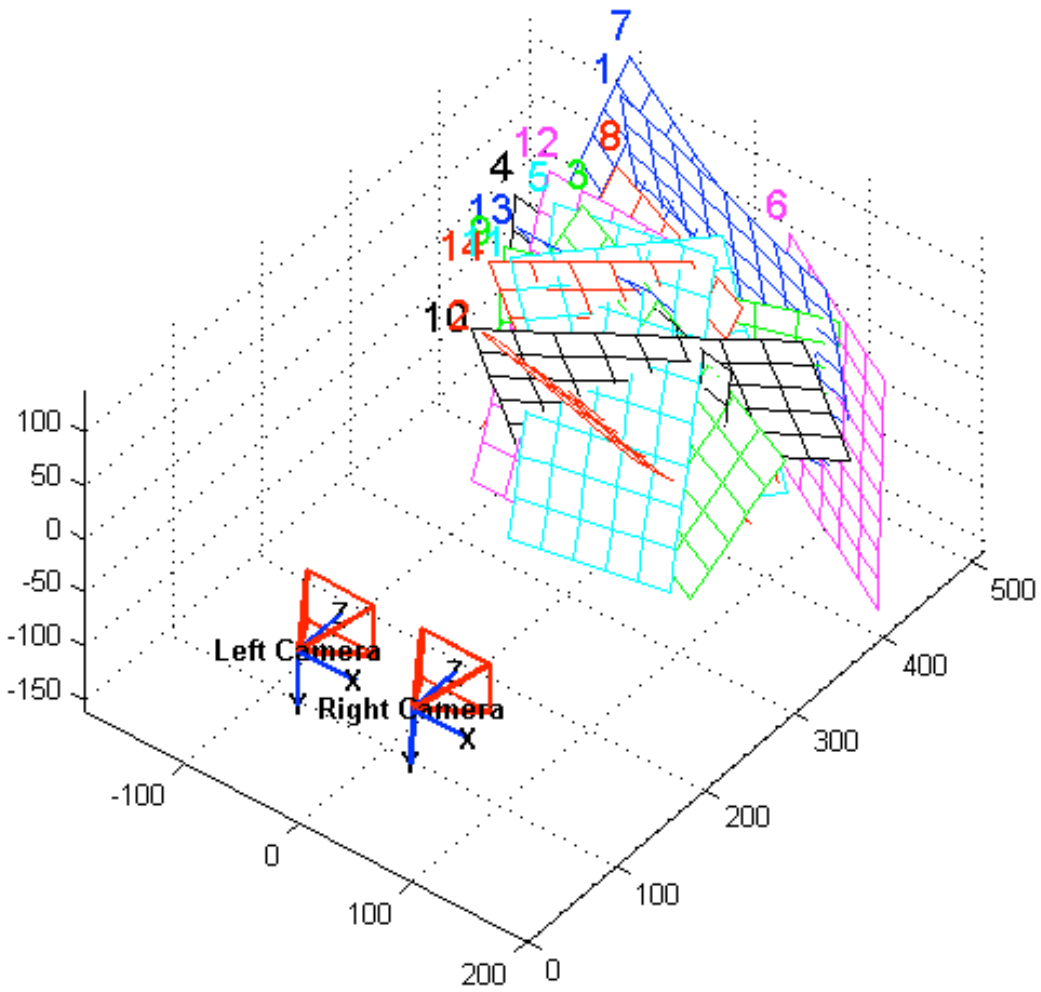
```
Calibration results after optimization (with uncertainties):

Focal Length:          fc = [ 661.67001   662.82858 ] ± [ 1.17913   1.26567 ]
Principal point:       cc = [ 306.09590   240.78987 ] ± [ 2.38443   2.17481 ]
Skew:             alpha_c = [ 0.00000 ] ± [ 0.00000  ]   => angle of pixel axes = 90.00000 ± 0.00000 degrees
Distortion:            kc = [ -0.26425   0.22645   0.00020   0.00023   0.00000 ] ± [ 0.00934   0.03826   0.00052
Pixel error:          err = [ 0.45330   0.38916 ]
```





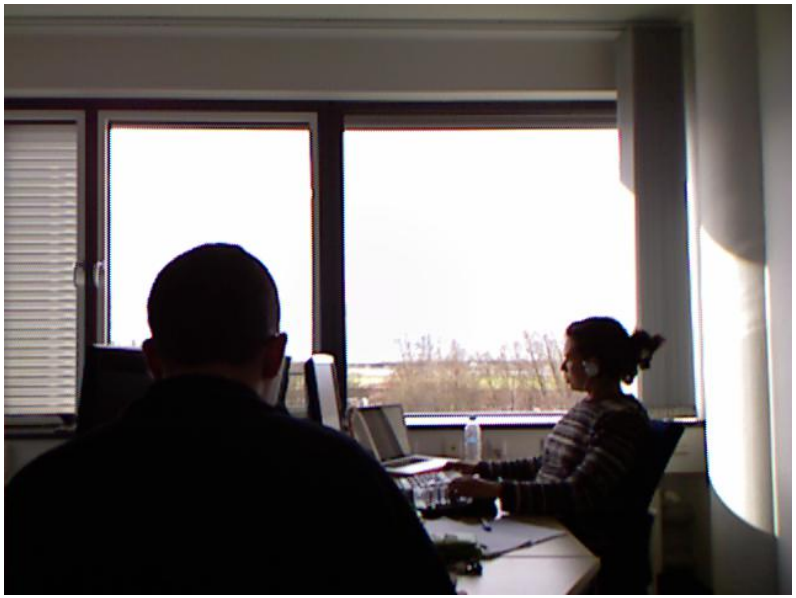
Stereo Calibration

- Find the spatial relation between two cameras observing the same scene (stereo setup)
- For each orientation of the calibration pattern, capture corresponding left and right camera images
- Calibrate each camera separately, as before
- Perform stereo calibration using the result of separate calibration as an input

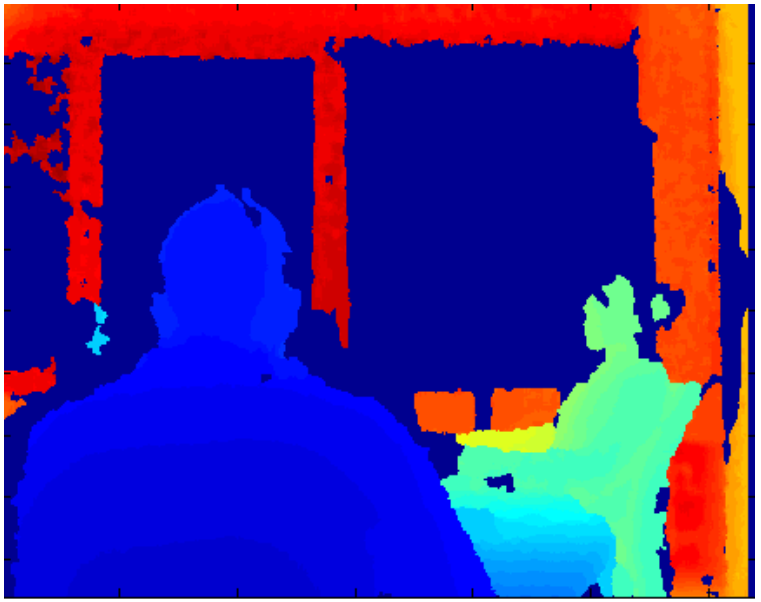


Calibrating the Kinect

- Depth and RGB images do not match exactly, there is an offset
- Stereo calibration for getting both images from the same camera perspective



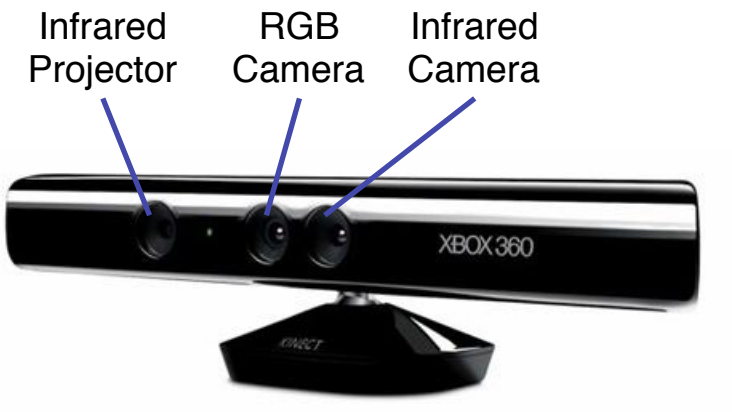
RGB Image

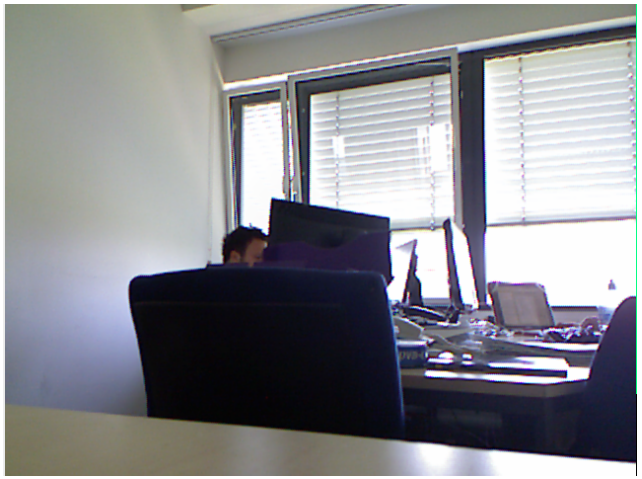


Depth Image

Calibrating the Kinect

- Kinect has two cameras with standard optics
  - RGB camera
  - Infrared (IR) camera
- Infrared image is used internally for computing depth image, but is accessible through the API
- Checkerboard pattern is not visible in depth images, but visible in IR images





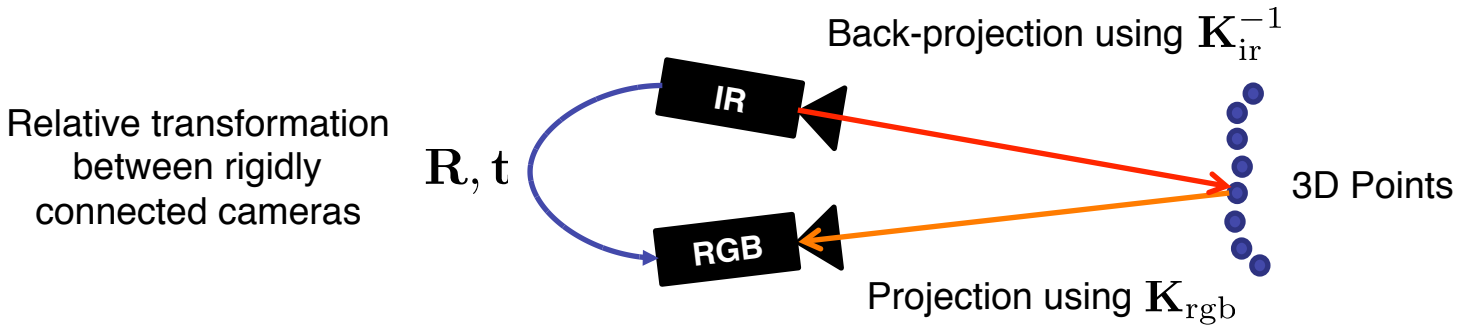
RGB Image

Infrared Image

Relating RGB and Depth Images

Stereo calibration gives $\mathbf{R}, \mathbf{t}$ and $\mathbf{K}_{\text{rgb}}, \mathbf{K}_{\text{ir}}$.

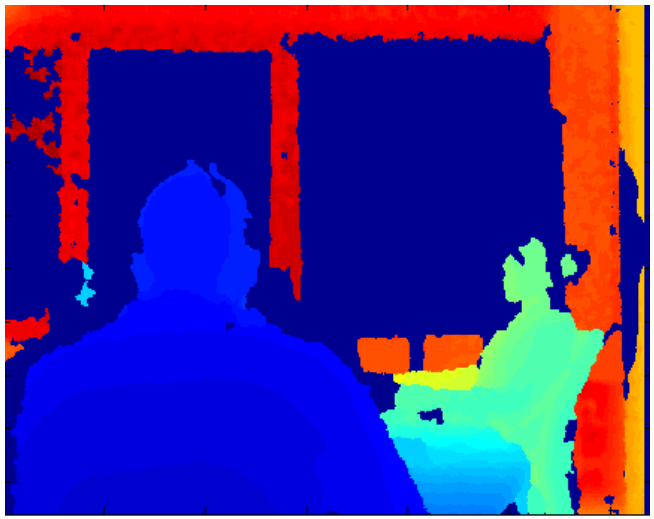
Step 1: Back-project every 2D point from depth image into 3D space
Step 2: Apply rigid transformation between two cameras to 3D points
Step 3: Project every transformed 3D point into the RGB image



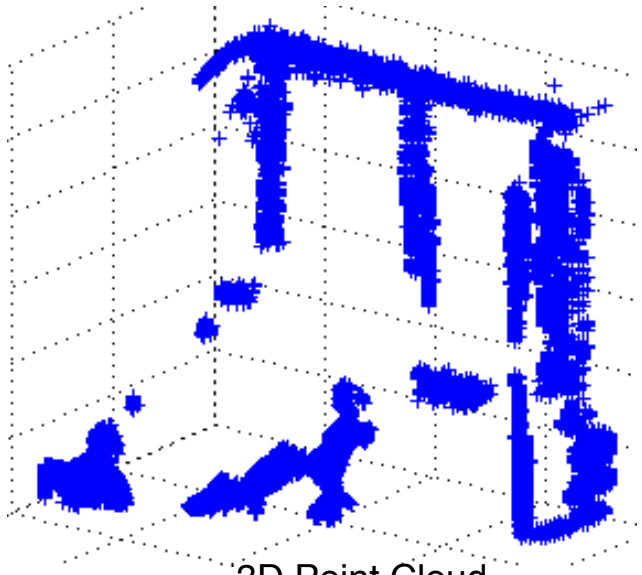
Relating RGB and Depth Images

Step 1: Back-project every 2D point from depth image into 3D space

- ToF data: depth *d* in meters for every pixel location $\mathbf{x} = (x, y)^\top$
- Desired data: 3D coordinates $\mathbf{X} = (X, Y, Z)^\top$ for every pixel



Depth Image



3D Point Cloud

Relating RGB and Depth Images

Step 1: Back-project every 2D point from depth image into 3D space

Apply inverse of IR camera projection to obtain 3D points:

$$\mathbf{x}_{\text{ir}} = \mathbf{P}_{\text{ir}}\mathbf{X} = \mathbf{K}_{\text{ir}}[\mathbf{I}|\mathbf{0}]\mathbf{X}$$

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} = \begin{pmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{pmatrix} f_x X + c_x Z \\ f_y Y + c_y Z \\ Z \end{pmatrix}$$

$$X = \frac{(x - c_x)Z}{f_x} \qquad Y = \frac{(y - c_y)Z}{f_y}$$

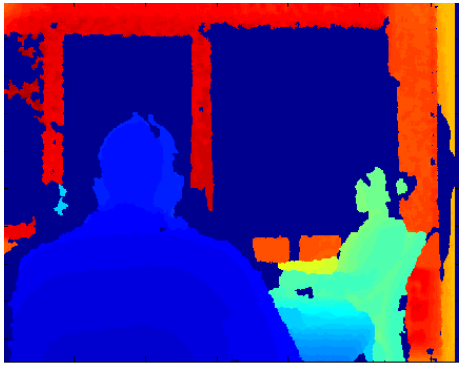
Inverse relation for X and Y

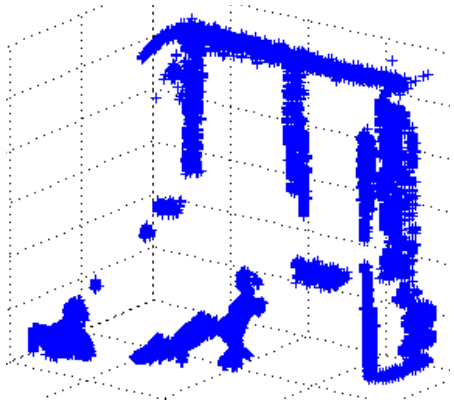
Relating RGB and Depth Images

Step 1: Back-project every 2D point from depth image into 3D space

For every depth pixel at location $\mathbf{x} = (x, y)^\top$ with depth *d*, compute a 3D point $\mathbf{X} = (X, Y, Z)^\top$ with coordinates:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \frac{(x - c_x)d}{f_x} \\ \frac{(y - c_y)d}{f_y} \\ d \end{pmatrix}$$





c_x, c_y, f_x, f_y

Intrinsics of the IR (depth) camera

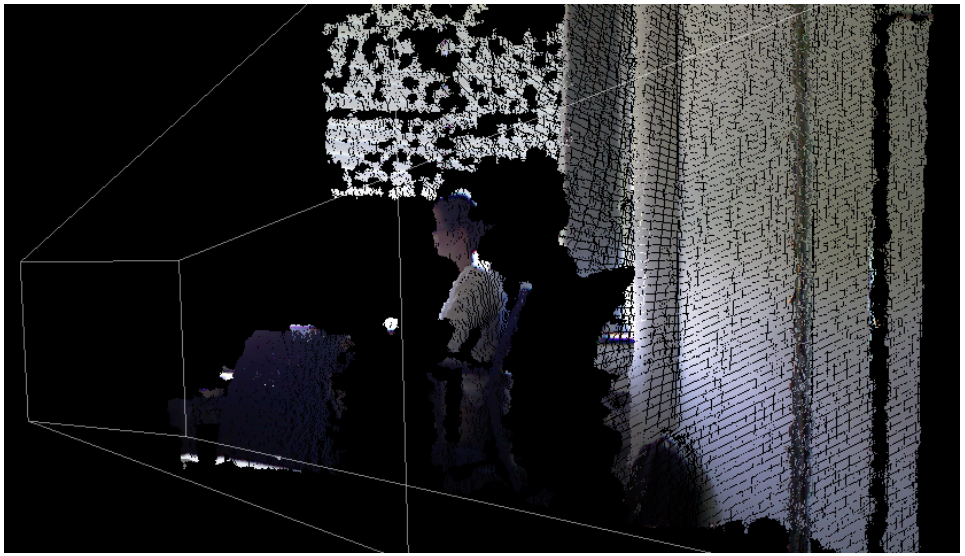
Relating RGB and Depth Images

Step 2: Apply rigid transformation between two cameras to 3D points

Step 3: Project every transformed 3D point into the RGB image

$$\mathbf{x}_{\mathrm{rgb}} = \mathbf{P}_{\mathrm{rgb}}\mathbf{X} = \mathbf{K}_{\mathrm{rgb}}[\mathbf{R}|\mathbf{t}]\mathbf{X}$$

- $\mathbf{x}_{\mathrm{rgb}}$ are the coordinates of the projection of X into the RGB image
- look up color corrsponding to a given 3D point



Assignment 3

- Extend your application such that it can read out Kinect IR images
- Extend your application for saving IR and RGB images to bitmap files
- Familiarize yourself with the Matlab Camera Calibration Toolbox
  http://www.vision.caltech.edu/bouguetj/calib_doc/
  - Read the examples (especially examples 1 and 5)
  - Download the toolbox
  - Try out the examples
- Perform stereo-calibration for your Kinect device
  - Note that you cannot extract IR and RGB images simultaneously…

Outlook Assignment 4:

- Implement a 3D point cloud visualization for kinect depth data
- Add a feature to color the 3D points with the colors from the RGB camera