Master MLDM/DSC/CPS2 - First year
Introduction to Artificial Intelligence
Exam on Propositional and First Order Logics

Time allocated: 3h - No documents allowed. Scoring will depend on the cleanliness of the copy and the clarity of the explanations. TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Propositional Logic

1.1 Truth table (2 points)

The Sheffer stroke, written "$\uparrow$", denotes a logical operation that is equivalent to the negation of the conjunction operation. Hence, $\Phi \uparrow \Psi$ is logically equivalent to $\neg(\Phi \wedge \Psi)$. Using the truth table method, show that:

1. $\neg p$ is logically equivalent to $p \uparrow p$
2. $p \Rightarrow q$ is logically equivalent to $p \uparrow (q \uparrow q)$
3. $p \Rightarrow q$ is logically equivalent to $p \uparrow (p \uparrow q)$
4. $p \wedge q$ is logically equivalent to $(p \uparrow q) \uparrow (p \uparrow q)$
5. $p \vee q$ is logically equivalent to $(p \uparrow p) \uparrow (q \uparrow q)$

1.2 Truth table (2 points)

1. Using the truth table method, prove that the premises: $p \Rightarrow q$ and $m \Rightarrow p \vee q$, logically entail $m \Rightarrow q$
2. Using the truth table method, prove that $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ is valid

1.3 Validity, unsatisfiability, contingency (2 points)

Using resolution reasoning, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(a \Rightarrow b) \wedge (\neg a \Rightarrow (b \vee c)) \wedge (\neg c \Rightarrow \neg b) \wedge ((b \wedge c) \Rightarrow \neg a) \wedge (c \Rightarrow a)$
2. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \wedge q) \Rightarrow r)$
3. $(p \Rightarrow q) \wedge (p \Rightarrow \neg q)$

1.4 Problem modeling and solving (5 points)

Here are some informations about a simple world: *If Paul can not sleep then he will not pass his exam. When Jack wins to poker, he invites John and Mary for dinner. When Jack invites Mary for dinner, Paul is jealous and angry. If Jack invites John for dinner, Lucy cries or invites Mary for dinner. If Jack or Lucy invites Mary for dinner then Mary is happy. When Paul is angry he can't sleep. Jack wins to poker. Lucy doesn't cry.*

1. Model this universe using Propositional Logic, that is, provide judicious proposition constants and convert each sentence to proposition sentences.

2. Provide a resolution proof of: *Paul doesn't pass his exam and Mary is happy.*

2 First Order Logic

2.1 Unification (2 points)

For each pair of logical sentences below, say whether they are unifiable or not. In case they are unifiable give their most general unifier, in case they are not unifiable, explain why.

1. p(X,a) and p(b,X)
2. p(Y,Y,b) and p(X, f(X), b)
3. p(a,X,c) and p(Y,d)
4. p(a,f(X,Y),b) and p(Z,f(a),W)
5. p(X,X,b) and q(a,a,Y)
6. p(A,a) and p(b,B)
7. p(a,f(X,g(Y),Z),b,T) and p(X,f(a,T,c),Y,g(Y))
8. p(a,f(X,g(Y),Z),b,T) and p(X,f(a,T,c),Y,g(Z))

2.2 Conjonctive Normal Form (2 points)

Convert to CNF the sentences below:

1. $\forall X.\exists Y.\forall Z.(r(X,Y,Z) \Rightarrow \forall Y.\exists Z.p(Y,Z))$
2. $\forall Y.\exists X.p(X,Y) \Leftrightarrow \forall Z.\forall X.p(Z,X)$

2.3 Validity, unsatisfiability, contingency (1.5 points)

Consider a universe that can be modeled using two object constants a and b, and two predicate symbols p and q of arity 1. Using the truth table method, say whether each sentence below is valid, unsatisfiable or contingent (you have to build the table and then explain what you observe and what you conclude from this observation):

1. $\forall X.p(X) \Rightarrow \exists X.p(X)$
2. $\forall X.(p(X) \Rightarrow q(X)) \Rightarrow \exists X.(p(X) \wedge q(X))$
3. $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$

2.4 Problem modeling (2 points)

We are considering the world of the 10 tallest buildings in the U.S. The table below provides some data about them:

Building Name	Location	Height (ft.)	Number of Stories	Year Built
One World Trade Center	New York	1776	104	2014
Willis Tower	Chicago	1451	108	1974
432 Park Avenue	New York	1396	88	2014
Trump International Hotel and Tower	Chicago	1389	98	2009
Empire State Building	New York	1250	102	1931
Bank of America Tower	New York	1200	55	2009
Aon Center	Chicago	1136	83	1973
John Hancock Center	Chicago	1127	100	1969
Chrysler Building	New York	1046	77	1930
New York Times Building	New York	1046	52	2007

Consider four predicates inC, inN, inT and inT1046 such as inC(B) is true if B is a building located in Chicago, inN(B) is true if B is a building located in New York, inT(B) is true if B is a building of the table above and inT1046(B) is true if B is a building of the table above that is exactly 1046ft. tall.

Convert to first order logic sentences the four english sentences below (you will define any additional predicate you need for that):

1. There is at least one building in New York that is taller than every building in Chicago.
2. If we consider every building in Chicago, there is always at least one building in New-York that has less stories.
3. If we consider every building of the table above, if it is located in Chicago and has at least 100 stories, we can be sure the building has been built before 1980.
4. Every building in Chicago has more stories than any building of the table above that is exactly 1046ft. tall.

2.5 Resolution reasoning in FOL (1.5 points)

Given the three sentences below:

1. Every human is a primate.
2. Dolphins are not primates.
3. There are dolphins who are intelligent.

prove, by a FOL resolution proof, that *it is possible not to be a human and to be intelligent.*

1

a	b	c	$a \Rightarrow b$	$\neg a \Rightarrow b \lor c$	$\neg c \Rightarrow \neg b$	$b \land c \Rightarrow \neg a$	$c \Rightarrow a$
0	0	0	1	0	1	1	1
0	0	1	1	1	1	1	0
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	0
1	0	0	0	1	1	1	
1	0	1	0	1	1	1	
1	1	0	1	1	0	1	
1	1	1	1	1	1	0	

Unsatisfiable

1

p	$\neg p$	$\neg(p \land p)$
0	1	1
1	0	0

$p \models \neg(p \land p)$

2

p	q	$p \Rightarrow q$	$\neg(p \land \neg(\neg q \land q))$
0	1	1	1
0	0	1	1
1	0	0	0
1	1	1	1

m	p	q	$p \Rightarrow q$	$m \Rightarrow p \lor q$	$m \Rightarrow q$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	1	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

2 P = Paul can sleep

Q = He wouldn't pass.

J = Jack wins Poker

I_M = Jack is Mary

I_J = " " John

J_P = Paul is jealous.

A_P = " " angry.

L_C = Lucy Cr.

L_M = Lucy-May

M_H = May is happy.

1. $\neg P \Rightarrow \neg Q$
2. $J \Rightarrow I_M \vee I_J$
3. $I_M \Rightarrow J_P \wedge A_P$
4. $I_J \Rightarrow L_C \vee L_M$
5. $I_M \vee L_M \Rightarrow M_H$
6. $A_P \Rightarrow \neg P$
7. J
8. $\neg L_C$
9. $\neg(P \wedge M_H)$

1. $\{P, \neg Q\}$
2. $\{\neg J, I_M, I_J\}$ $\{\neg J, I_J, A_P\}$
3. $\{\neg I_M, J_P\}$ $\{\neg J, I_J, \neg P\}$
4. $\{\neg I_M, A_P\}$
5. $\{\neg I_J, L_C\}$ $\{I_J, \neg P\}$
6. $\{\neg I_J, L_M\}$ $\{L_M, \neg P\}$
7. $\{\neg L_M, M_H\}$ 7.2 $\{\neg I_M, M_H\}$
8. $\{\neg A_P, \neg P\}$ $\{M_H, \neg P\}$
9. $\{J\}$
10. $\{\neg L_C\}$
11. $\{P, \neg M_H\}$
12. $\{P, \neg P\}$
13. P

1. $\forall X. \exists Y. \forall Z \left(r(X,Y,Z) \Rightarrow \forall Y. \exists Z\, P(Y,Z) \right)$

$\Rightarrow \forall X. \exists Y. \forall Z \left(r(X,Y,Z) \Rightarrow \forall A. \exists B\, P(A,B) \right)$

$\Rightarrow \forall X\, \exists Y\, \forall Z \left(\neg r(X,Y,Z) \vee \forall A. \exists B\, P(A,B) \right)$

$\Rightarrow \forall X. \exists Y \forall Z \left(\neg r(X,Y,Z) \vee \forall A. P(A, f(X,Z)) \right)$

$\Rightarrow \forall Z. \forall Z \left(\neg r(X,a,Z) \vee \forall A. P(A, f(X,Z)) \right)$

$\Rightarrow \{ \neg r(X,a,Z),\ P(A, f(X,Z)) \}$

(2) $\forall Y . \exists X\ P(X,Y) \Leftrightarrow \forall Z . \forall \cancel{P}\ P(Z,X)$

$\forall Y . \exists X\ P(X,Y) \Leftrightarrow \forall Z . \forall A\ P(Z,A)$

$(\neg \forall Y . \exists X\ P(X,Y) \vee \forall Z . \forall A\ P(Z,A)) \wedge$

$(\neg \forall Z . \forall A\ P(Z,A) \vee \forall Y . \exists X\ P(X,Y))$

$\Rightarrow (\exists Y . \forall X\ \neg P(X,Y) \vee \forall Z . \forall A\ P(Z,A)) \wedge$

$(\exists Z . \exists A \neg P(Z,A) \vee \forall Y . \exists X\ P(X,Y))$

$\Rightarrow (\forall X\ \neg P(X,a) \vee P(Z,A)) \wedge$

$(\neg P(c,b) \vee \forall Y\ P(d,Y))$

$\{\neg P(X,a), P(Z,A)\}, \{\neg P(c,b), P(d,Y)\}$

$X=a$	$x=b$	$P(x) \Rightarrow \exists x\, P(x)$
0	0	1
0	1	1
1	0	1
1	1	1

$P(a)$	$P(b)$	$q(a)$	$q(b)$	$\forall x(P(x) \Rightarrow q(x))$	$\exists X \cdot (P(X) \land q(X))$
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0

2.4

① $\exists X . \forall Y . taller(inN(X), inC(Y))$.

② $\forall Y . \exists X \; stories(inN(X), inC(Y)))$.

③ $\exists X . (inT(X) \wedge inC(X)) \wedge tall(X, 100)$
$\Rightarrow yearBuilt(X, 1980)$

2.5 ① $human(X)$
$\forall X . (human(X) \Rightarrow primate(X))$
$\forall X (Dolphin(X) \Rightarrow \neg primate(X))$
$\exists X . (Dolphin(X) \wedge Intelligent(X))$
$\neg human(X) \wedge Intelligent(X)$.

1. $\{h(x)\}$
2. $\{\neg h(x), p(x)\}$
3. $\{\neg d(x), \neg p(x)\}$
4. $\{d(a), i(a)\}$
5. $\{h(x), \neg i(x)\}$
6. $\{d(a), h(a)\}$
7. $\{\neg p(a), h(a)\}$
8. $\{p(x)\}$
9. $\{\neg p(a)\}$
10. False