Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Compensators

1.1 Phase Lead

1.1. For a control system in unity feedback with a transfer function

$$G(s) = \frac{10K}{s(s+1)(s+5)} \tag{1.1.1}$$

Design a lead compensator with a 60° phase margin and an appropriate error constant of 5

1.2. **Solution:** Before adding a compensator, we first find a value of gain for an error constant of 5. As the system has one pole at the origin, the appropriate error constant would be the velocity constant K_v. For an error constant of 5, we solve the following equation:

$$\lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{10K}{(s+1)(s+5)} = 2K \tag{1.2.1}$$

$$\implies K = \frac{K_v}{2} = 2.5 \tag{1.2.2}$$

So, the new transfer function becomes

$$G(s) = \frac{25}{s(s+1)(s+5)} \tag{1.2.3}$$

The gain crossover frequency ω_{gc} and phase margin ϕ_M is calculated from the plots using the following code

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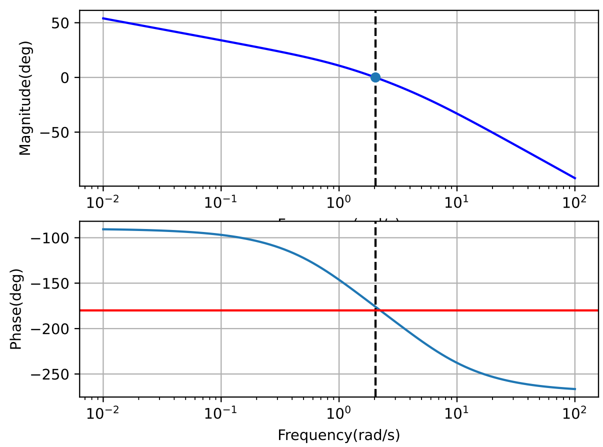
```
codes/ee18btech11051/
    ee18btech11051_code1.py
```



This can also be calculated from the equations

$$\omega\sqrt{\omega^2+1}\sqrt{\omega^2+25} = 25 \tag{1.2.4}$$

$$\phi_M = -90^\circ - \tan^{-1}\omega_{gc} - \tan^{-1}\left(\frac{\omega_{gc}}{10}\right) \tag{1.2.5}$$

Solving which, we get $\phi_M = 3.96^{\circ}, \omega_{gc} = 2.03$ The following code computes the margins and frequencies:

```
codes/ee18btech11051/
    ee18btech11051_code2.py
```

The maximum phase of the compensator is given by-

$$\phi_M = 60^{\circ} - phasemargin + anglecorrection \tag{1.2.6}$$

$$\phi_M = 56^{\circ} + anglecorrection \tag{1.2.7}$$

Here, the angle correction is added to compensate the early zero added due to the lead compensator. We set the angle correction to be

20°. The phase lead compensator will have a transfer function of the form:

$$G_C(s) = \left(\frac{1 + \alpha T s}{1 + T s}\right), \alpha > 1 \tag{1.2.8}$$

Note that this transfer function doesn't change the error constant. Now we solve for α using the equation

$$\alpha = \frac{1 + \sin \phi_M}{1 - \sin \phi_M} = \frac{1 + \sin 76^{\circ}}{1 - \sin 76^{\circ}} = 66.33 \tag{1.2.9}$$

Now to get the phase gain, we set the frequency for maximum phase of compensator ω_m to the previous gain crossover frequency. And using it, the value of T is calculated as follows

$$\omega_m = 2.03 rad/sec \tag{1.2.10}$$

$$T = \frac{1}{\omega_m \alpha} = 0.0074 \tag{1.2.11}$$

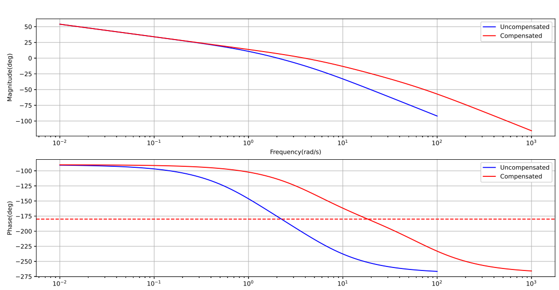
The Compensator will have the transfer function as follows :

$$G_c(s) = \left(\frac{1 + 0.5s}{1 + 0.0074s}\right) \tag{1.2.12}$$

The open loop T.F for the compensated system is :

$$G(s)G_c(s) = 25\left(\frac{(1 + 0.5s)}{s(s + 1)(s + 5)(1 + 0.0074s)}\right) \tag{1.2.13}$$

1.3. **Verification :** To observe the changes, we plot the compensated system



Clearly, the gain crossover frequency is slightly shifted to the right due to addition of the early zero, and an overall increase in phase is observed.