Control Systems

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Contents

Abstract—**The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Compensators

1.1 Phase Lead

1.1. For a control system in unity feedback with a transfer function

$$G(s) = \frac{10K}{s(s+1)(s+5)} \tag{1.1.1}$$

Design a lead compensator with a 60° phase margin and an appropriate error constant of 5

1.2. **Solution:** Before adding a compensator, we first find a value of gain for an error constant of 5. As the system has one pole at the origin, the appropriate error constant would be the velocity constant K_v. For an error constant of 5, we solve the following equation:

$$\lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{10K}{(s+1)(s+5)} = 2K \tag{1.2.1}$$

$$\implies K = \frac{K_v}{2} = 2.5 \tag{1.2.2}$$

So, the new transfer function becomes

$$G(s) = \frac{25}{s(s+1)(s+5)} \tag{1.2.3}$$

The gain crossover frequency ω_{gc} and phase margin ϕ_M is calculated from the plots using the following code

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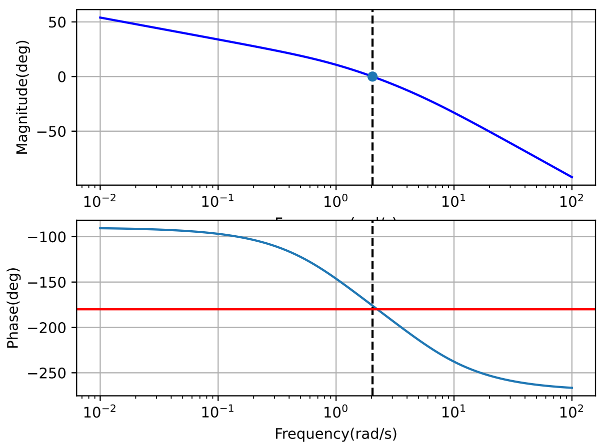
```
codes/ee18btech11051/
    ee18btech11051_code1.py
```



This can also be calculated from the equations

$$\omega\sqrt{\omega^2+1}\sqrt{\omega^2+25} = 25 \tag{1.2.4}$$

$$\phi_M = -90^\circ - \tan^{-1}\omega_{gc} - \tan^{-1}(\frac{\omega_{gc}}{10}) \tag{1.2.5}$$

Solving which, we get $\phi_M = 3.96^\circ, \omega_{gc} = 2.03$ The following code computes the margins and frequencies:

```
codes/ee18btech11051/
    ee18btech11051_code2.py
```

The maximum phase of the compensator is given by-

$$\phi_M = 60^\circ - phasemargin + anglecorrection \tag{1.2.6}$$

$$\phi_M = 56^\circ + anglecorrection \tag{1.2.7}$$

Here, the angle correction is added to compensate the early zero added due to the lead compensator. The highest phase margin is achieved when ϕ_M is close to 90°. To achieve it, we take angle correction = 33°. The phase lead compensator will have a transfer function of the form:

$$G_C(s) = \left(\frac{1+\alpha T s}{1+T s}\right), \alpha > 1 \tag{1.2.8}$$

Note that this transfer function doesn't change the error constant. Now we solve for α using the equation

$$\alpha = \frac{1+\sin\phi_M}{1-\sin\phi_M} = \frac{1+\sin 89^{\circ}}{1-\sin 89^{\circ}} = 13,130.56 \tag{1.2.9}$$

Now to get the phase gain, we set the frequency for maximum phase of compensator ω_m to the previous gain crossover frequency. And using it, the value of T is calculated as follows

$$\omega_m = 2.03 rad/sec \tag{1.2.10}$$

$$T = \frac{1}{\omega_m \alpha} = 0.0000375 \tag{1.2.11}$$

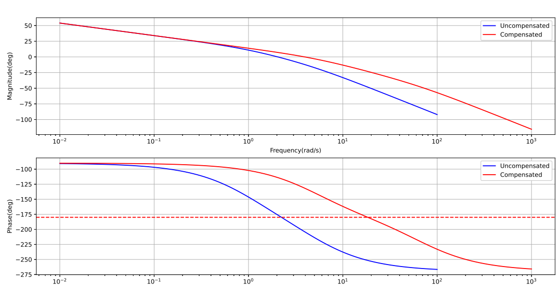
The Compensator will have the transfer function as follows :

$$G_c(s) = \left(\frac{1+0.5s}{1+0.0000375s}\right) \tag{1.2.12}$$

The open loop T.F for the compensated system is :

$$G(s)G_c(s) = 25\left(\frac{(1+0.5s)}{s(s+1)(s+5)(1+0.0000375s)}\right) \tag{1.2.13}$$

To observe the changes, we plot the compensated system



Using the code previously used to calculate the phase margin, the phase margin of the compensated system turns out to not go beyond 46°. So, we use a two stage lead compensator to get the required phase margin. The new compensator can be thought of as the square of the previous compensator

$$G_C^2(s) = \left(\frac{1+\alpha T s}{1+T s}\right)^2, \alpha > 1 \tag{1.2.14}$$

The values of α and T can still be calculated as done previously, but for half of the required increase in phase, as the compensator gives double of the configured phase. So, for

$$\phi_M = \frac{(56^{\circ}+26^{\circ})}{2} = 41^{\circ} \tag{1.2.15}$$

$$\alpha = \frac{1+\sin(38^{\circ})}{1-\sin 38^{\circ}} = 5 \tag{1.2.16}$$

$$\omega_m = 2.6 \tag{1.2.17}$$

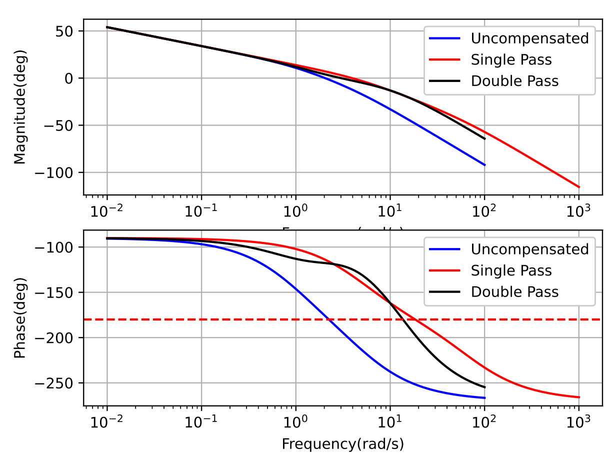
$$T = \frac{1}{\omega_m \alpha} = 0.0769 \tag{1.2.18}$$

Now, the new compensator and the compensated transfer functions are

$$G_c^2(s) = \frac{(1+0.38s)^2}{(1+0.0769s)^2} \tag{1.2.19}$$

$$G(s)G_c^2(s) = 25\left(\frac{(1+0.38s)^2}{s(s+1)(s+5)(1+0.0769s)^2}\right) \tag{1.2.20}$$

1.3. **Verification :** Now we plot the newly compensated transfer function.



The phase margin of the compensated system is found to be 59.64°, and the error constant is 5. Hence, the lead compensator design is done.