Oscillator

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1. For the circuit in Fig. 1, break the loop at node X and find the loop gain (working backward for simplicity to find V_X in terms of V_O). For R = 10 kΩ, find C and R_f to obtain sinusoidal oscillations at 10 kHz.

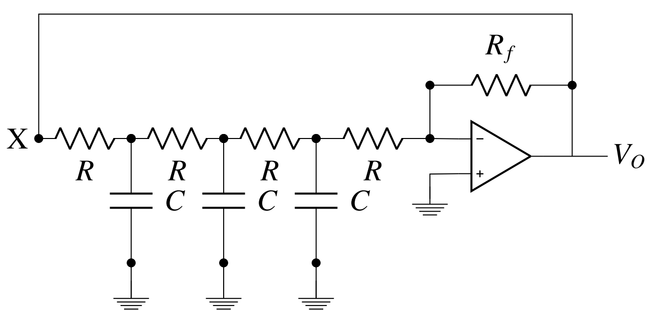

Fig. 1

2. **Solution:** We first calculate the relation between V_3 and V_X in fig 2 by using the relation between the currents as follows:

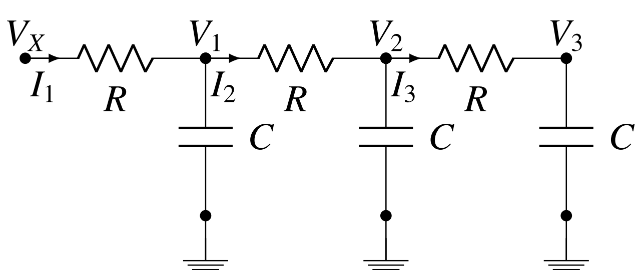

Fig. 2

Applying KVL between V_2 and V_3, we get

$$V_2 = V_3 + I_3 R \quad (2.1)$$

$$V_3 = \frac{I_3}{sC} \quad (2.2)$$

$$\implies V_2 = V_3(1 + sRC) \quad (2.3)$$

Then, KCL between V_1 and V_2 gives

$$V_1 = V_2 + I_2 R \quad (2.4)$$

$$I_2 = I_3 + V_2 sC \implies I_2 = V_3 sC + V_2 sC \quad (2.5)$$

$$\implies V_1 = V_2(1 + sRC) + V_3 sRC \quad (2.6)$$

Now, using the equations eq.2.3 and eq.2.6, we get

$$V_1 = V_3((1 + sRC)^2 + sRC) \quad (2.7)$$

$$\implies V_1 = V_3(1 + 3sRC + (sRC)^2) \quad (2.8)$$

KCL and KVL between V_X and V_1 gives

$$V_X = V_1 + I_1 R \quad (2.9)$$

$$I_1 = I_2 + V_1 sC \quad (2.10)$$

$$\implies I_1 = V_3 sC + V_2 sC + V_1 sC \quad (2.11)$$

$$\implies V_X = V_1(1 + sRC) + V_2 sRC + V_3 sRC \quad (2.12)$$

Substituting eq.2.3 and eq.2.8 in eq.2.12:

$$V_X = V_3((1 + sRC)(1 + 3sRC + (sRC)^2) + (1 + sRC)(sRC) + sRC) \quad (2.13)$$

$$\implies V_X = V_3(1 + 6sRC + 5(sRC)^2 + (sRC)^3) \quad (2.14)$$

Substituting $s = j\omega$ in the eq.2.14 gives:

$$V_X = V_3((1 + 5(\omega RC)^2) + j(6\omega RC - (\omega RC)^3)) \quad (2.15)$$

The relation between V_3 and V_O is:

$$\frac{V_3}{R} = -\frac{V_O}{R_f} \quad (2.16)$$

$$V_3 = -V_O(\frac{R}{R_f}) \quad (2.17)$$

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$$V_X = -V_O\frac{((1-5(\omega RC)^2)+j(6\omega RC-(\omega RC)^3))R}{R_f} \tag{2.18}$$

To make the circuit oscillate at a particular frequency, we equate the imaginary part of the eq.2.15 to zero.

$$6\omega RC-(\omega RC)^3 = 0 \implies (\omega RC)^2 = 6 \tag{2.19}$$

$$V_X = -V_O\frac{(1-5(6))R}{R_f} \tag{2.20}$$

$$\implies V_X = V_O\frac{29R}{R_f} \tag{2.21}$$

So, we get the final expression for the open loop gain and feedback gain as:

$$\alpha = \frac{R_f}{29R}, \beta = 1 \tag{2.22}$$

To make the oscillator stable, we need to have $\alpha\beta = 1$. So, we get

$$\frac{R_f}{29R} = 1 \implies R_f = 29R = 290k\Omega \tag{2.23}$$

Also, as we already know from eq.2.19 that $(\omega RC)^2 = 6$, the oscillator frequency $(2\pi f = \omega)$ can be set to 10kHz by varying the capacitor value.

$$\omega RC = \sqrt{6} \implies C = \frac{\sqrt{6}}{2\pi Rf} \tag{2.24}$$

$$C = 3.89nF \tag{2.25}$$

Therefore, the values of C and R_f for the desired oscillator are 3.89nF and 290kΩ.

Parameter	**Value**
C	3.89nF
R_f	290kΩ

TABLE 2: Final Values

3. Verification: