Oscillator

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1. For the circuit in Fig. 1, break the loop at node X and find the loop gain (working backward for simplicity to find V_X in terms of V_O). For R = 10 kΩ, find C and R_f to obtain sinusoidal oscillations at 10 kHz.

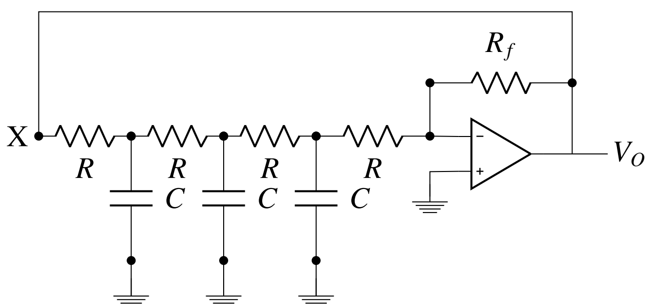

Fig. 1

2. **Solution:** We first calculate the relation between I_4 and V_X in fig 2 by using the relation between the currents and the fact that the inverting terminal of the Op-Amp is virtually grounded as follows:

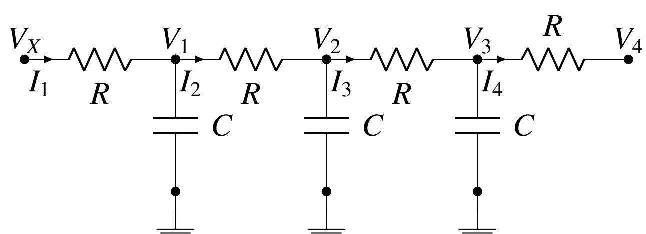

Fig. 2

Here, V_4 has zero voltage. Applying KVL between V_3 and V_4, we get

$$V_3 = I_4 R \tag{2.1}$$

Applying KCL and KVL between V_2 and V_3, and substituting eq.2.1 gives

$$I_3 = I_4 + V_3 sC \implies I_3 = I_4(1 + sRC) \tag{2.2}$$
$$V_2 = V_3 + I_3 R \implies V_2 = I_4 R(2 + sRC) \tag{2.3}$$

Using eq.2.2 and eq.2.3 in KCL at node V_2

$$I_2 = I_3 + V_2 sC \tag{2.4}$$
$$I_2 = I_4((1 + sRC) + sRC(2 + sRC)) \tag{2.5}$$
$$\implies = I_2 = I_4(1 + 3sRC + (sRC)^2) \tag{2.6}$$

Applying KVL between nodes V_1 and V_2, and substituting eq.2.3 and eq.2.6 gives

$$V_1 = V_2 + I_2 R \tag{2.7}$$
$$\implies V_1 = I_4 R(3 + 4sRC + (sRC)^2) \tag{2.8}$$

Substituting eq.2.6 and eq.2.8 in KCL at node V_1 gives

$$I_1 = I_2 + V_1 sC \tag{2.9}$$
$$\implies I_1 = I_4(1 + 6sRC + 5(sRC)^2 + (sRC)^3) \tag{2.10}$$

KVL between V_X and V_1, and using eq.2.8 and eq.2.10 gives

$$V_X = V_1 + I_1 R \tag{2.11}$$
$$\implies V_X = I_4 R(4 + 10sRC + 6(sRC)^2 + (sRC)^3) \tag{2.12}$$

Substituting $s = j\omega$ in the eq.2.12:

$$V_X = I_4 R((4 - 6(\omega RC)^2) + j(10\omega RC - (\omega RC)^3)) \tag{2.13}$$

The relation between I_4 and V_O is:

$$V_O = -I_4 R_f \tag{2.14}$$

So, the transfer function GH from eq.2.13 is

$$GH = \frac{V_O}{V_X} = -\frac{R_f}{R((4 - 6(\omega RC)^2) + j(10\omega RC - (\omega RC)^3))} \tag{2.15}$$

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To make the system oscillate at a frequency f_O,

$$|GH| = 1 \tag{2.16}$$

$$\angle GH = 0^{\circ} \tag{2.17}$$

$$\implies (\omega RC)^2 = 10 \tag{2.18}$$

$$\implies C = \frac{\sqrt{10}}{2\pi f_O R} = 5.03nF \tag{2.19}$$

Putting GH = 1 in eq.2.15 gives:

$$-\frac{R_f}{R(4-6(10))} = 1 \implies R_f = 56R = 560k\Omega \tag{2.20}$$

Therefore, the values of C and R_f for the desired oscillator are 5.03nF and 560kΩ.

Parameter	**Value**
C	5.03nF
R_f	560kΩ

TABLE 2: Final Values

3. Verification:
   The value of R_f is kept slightly above the calculated value for the system to start oscillating