Matrix Project

EE1390- Intro to AI and ML

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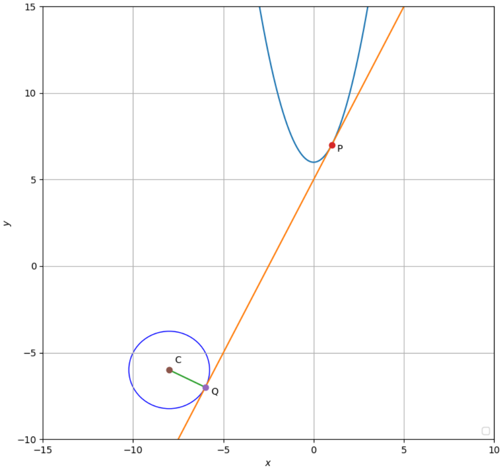
Outline

IIT JEE 2005 Question

Q.Tangent to the curve $y = x^2 + 6$ at a point (1,7) touches the circle $x^2 + y^2 + 16x + 12y + k = 0$. Then the Coordinates of Q are
(a) (-6, -11)
(b) (-9, -13)
(c) (10, -15)
(d) (-6, -7)

Solution



Solution

The given parabola equation is

$$y = x^2 + 6 => x^2 - y + 6 = 0 \tag{1}$$

writing it in matrix form

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \mathbf{x} + 6 = 0 \tag{2}$$

Equation of a standard second order conic is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{U}^T \mathbf{x} + F = 0 \tag{3}$$

On comparing

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad U = \begin{pmatrix} 0 & -1/2 \end{pmatrix} \, and \;\; F = 6 \tag{4}$$

Solution

And for a standard second order conic the equation of tangent at P is

$$(\mathbf{P}^\top \mathbf{V} + \mathbf{U}^\top)\mathbf{x} + \mathbf{P}^\top \mathbf{U} + F = 0 \tag{5}$$

And now substituting all the values the equation of tangent is

$$\begin{pmatrix}-2 & 1\end{pmatrix} \mathbf{x} = 5 \tag{6}$$

Any equation of a circle is given as

$$||x - c||^2 = r^2 \tag{7}$$

$$(\mathbf{x} - \mathbf{c})^\top (\mathbf{x} - \mathbf{c}) = r^2 \tag{8}$$

$$\mathbf{x}^\top \mathbf{x} - 2\mathbf{c}\mathbf{x} = r^2 - \mathbf{c}^\top \mathbf{c} \tag{9}$$

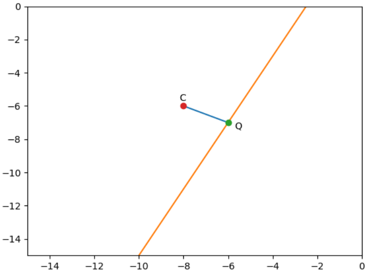
Solution

On comparing it with the given equation

$$x^2 + y^2 + 16x + 12y + k = 0 \tag{10}$$

Centre of the circle c is

$$c = \begin{pmatrix} -8 \\ -6 \end{pmatrix} \tag{11}$$



Solution

Equation of tangent $\mathbf{n_1 x} = p_1$
Equation of a line perpendicular to tangent and passing through c is,

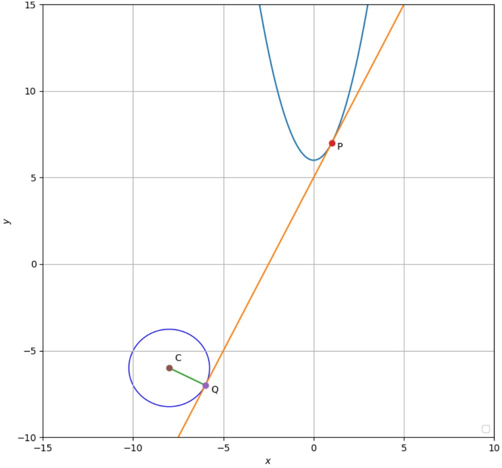
$$\mathbf{n_2}(\mathbf{x} - \mathbf{c}) = 0 \tag{12}$$

Where $\mathbf{n_2}$ can be easily calculated using $\mathbf{n_1}$ and orthogonal matrix Which can be written as

$$\mathbf{n_2 x} = p_2 \ where \ p_2 = \mathbf{n_2 c} \tag{13}$$

And then the point Q is the intersection of both the lines i.e the Tangent and the Normal Which gives $Q = (-6, -7)$

Graphical representation



Final Slide

THANK YOU