Class 08

Monday, November 1, 2021 8:27 PM

Discrete Math (বিচ্ছিন্ন গণিত)

↳ main base → Set → Number Theory (Crypto, security, probabilistic model, ML/AL etc.)

→ Graph Theory (Graph networks, google map etc.)

What is Set?

↳ an unordered collection of elements

* $S_1 = \{2, 4, 6, 10, 8\}$, $|S_1| = 5$ ← cardinality of set

* $S_{vowel} = \{a, e, i, o, u\}$ ← Roster/Tabular form

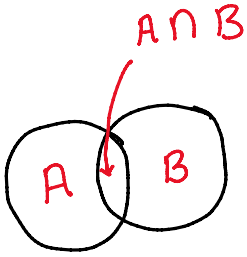
$= \{x : x \text{ is a vowel in English alphabet}\}$ ← Set Builder Notation

Example 1

$S_{name} = \{s, a, k, i, b\}$; $S_{vowel} = \{a, e, i, o, u\}$

$\therefore S_{name} \cap S_{vowel} = \{a, i\}$

$|S_{name} \cap S_{vowel}| = 2$



Types of Set

* Finite Set → size finite. → $S = \{x : 0 < x < 100 \text{ and } x\%2 == 0\}$

* Infinite Set → size infinite!

↳ $S = \{x : x > 0 \text{ and } x\%2 == 0\}$

* Subset : $Y = \{1,2,3,4,5,6,7,8,9,10\}$

$X = \{1,4,7,9\}$

$X \subseteq Y$

* Proper Subset : $X \subset Y$

$X \subseteq Y = X \subset Y$ OR $X == Y$

$X \subset Y = \ldots\ldots$ AND $|X| < |Y|$

Power Set

$S = \{1,2,3\}$

$\emptyset$ $\{1\}$ $\{2\}$ $\{3\}$ $\{1,2\}$ $\{1,3\}$ $\{2,3\}$ $\{1,2,3\}$

$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|P(S)| = 2^{|S|}$

${}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$

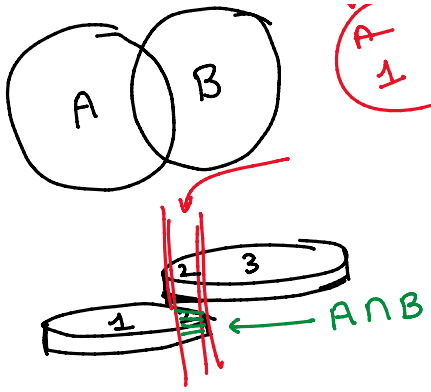
$1 + 3 + 3 + 1 = 2^3 = 8$

${}^nC_r = \frac{n!}{r!(n-r)!}$

Sum of pascal's n^{th} row ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

IEP → Inclusion - Exclusion Principle

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

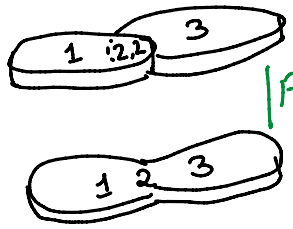


$A = \{1,2,3\}$

$B = \{2,3,6\}$

$A \cup B = \{1,2,3,2,3,6\} - \{2,3\}$

$= \{1,2,3,6\}$



$|A \cup B| = |A| + |B| - |A \cap B|$

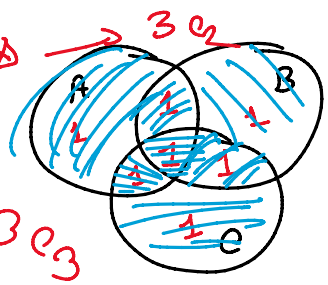
$= 3 + 3 - 2$

$= 4$

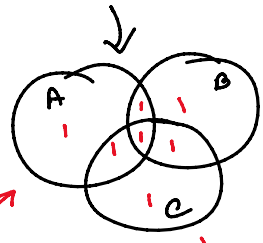
$n(A \cup B \cup C) = n(A) + n(B) + n(C) \rightarrow 3 \rightarrow {}^3C_1$

$- n(A \cap B) - n(A \cap C) - n(B \cap C) \rightarrow 3 \rightarrow {}^3C_2$

$+ n(A \cap B \cap C) \rightarrow 1 \rightarrow {}^3C_3$



$2^n - {}^nC_0 = \boxed{2^n - 1}$



This is our target!

$\left\lfloor \frac{15}{2} \right\rfloor = 7 \qquad \left\lfloor \frac{15}{3} \right\rfloor = 5 \qquad \left\lfloor \frac{15}{6} \right\rfloor = 2$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$n(P_1 \cup P_2) = n(P_1) + n(P_2) - n(P_1 \cap P_2)$

$P_1 = \{2,4,6,8,10,12,14\}$

$P_2 = \{3,6,9,12,15\}$

$$n(P_1 \cup P_2) = n(P_1) + n(P_2) - n(P_1 \cap P_2)$$

$$= \left\lfloor \frac{N}{P_1} \right\rfloor + \left\lfloor \frac{N}{P_2} \right\rfloor - \left\lfloor \frac{N}{lcm(P_1, P_2)} \right\rfloor$$

$$= \left\lfloor \frac{15}{2} \right\rfloor + \left\lfloor \frac{15}{3} \right\rfloor - \left\lfloor \frac{15}{6} \right\rfloor = 10$$

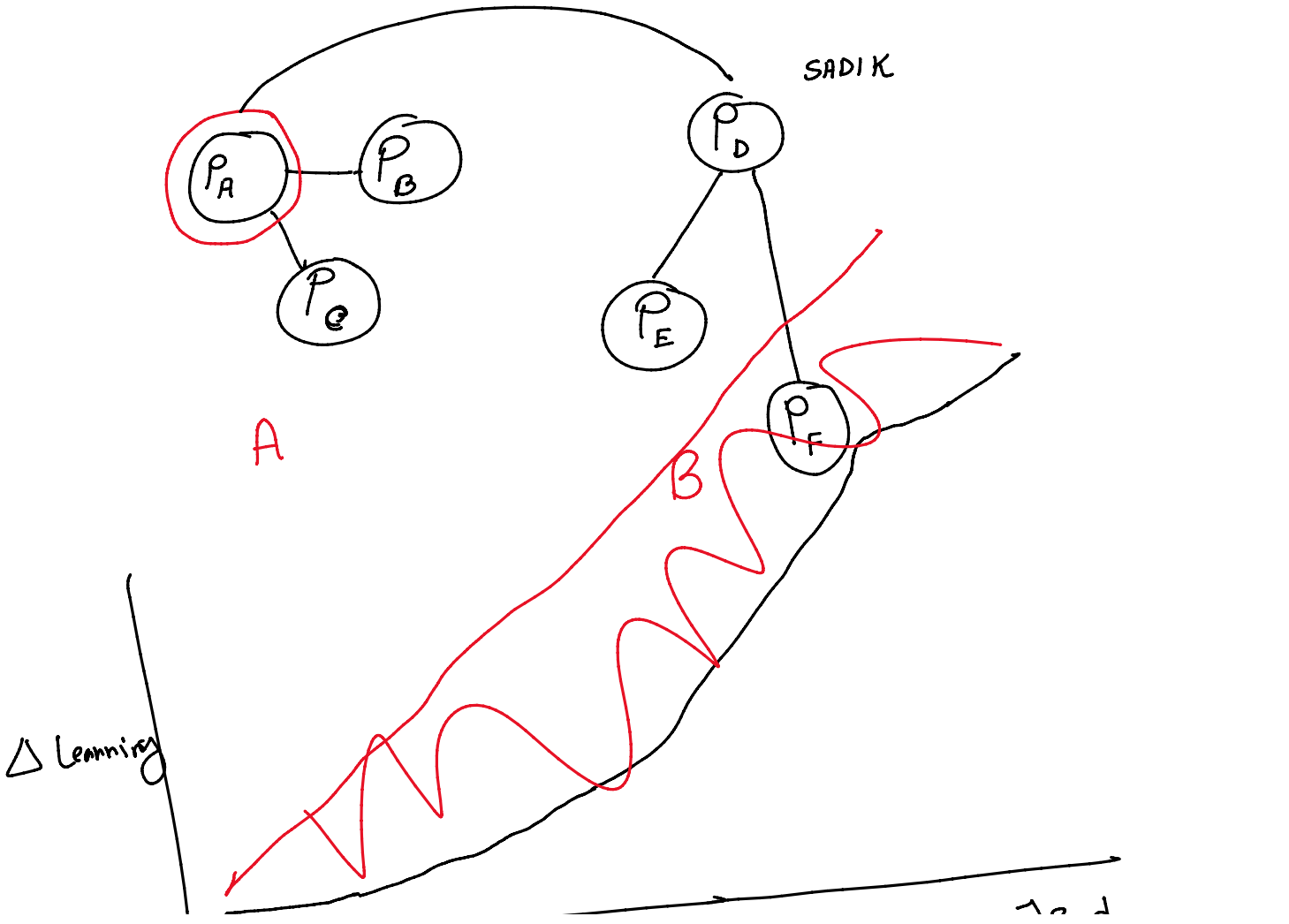
$P_2 = \{3, 6, 9, 12, 15\}$

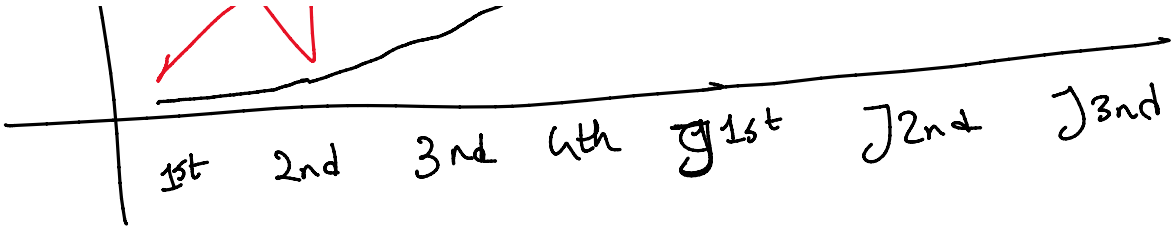
$P_1 \cup P_2 = ?$

$$N - n(P_1 \cup P_2)$$

Disjoint Set Union: (DSU)

$|A \cap B| = 0$ $\quad A \cap B = \emptyset$




1st
2nd
3rd
4th
1st
2nd
3rd