

Modular Arithmetic

$$1) (a \pm b) \% m = ((a \% m) \pm (b \% m)) \% m$$

$$(a+b) \% m$$

$$(a \times b) \% m$$

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$$(10^{17} \times 10^{17}) \% 5$$

$$(0 \sim 4) \quad (0 \sim 4)$$

Euler Totient

$\phi(n)$ = count of co-prime numbers with n from 1 to n .

$$\phi(6) = \begin{matrix} 1, 6 \\ 1 \end{matrix} \quad 2, 6 \quad 3, 6 \quad 4, 6 \quad \begin{matrix} 5, 6 \\ 1 \end{matrix} \quad 6, 6$$

$$= 2$$

$$\phi(n) = n \cdot \prod_{i=0}^{k-1} \frac{P_i - 1}{P_i} \quad \left[\text{যৰুণ, } k = \text{number of unique primes in the prime fact. of } n \right]$$

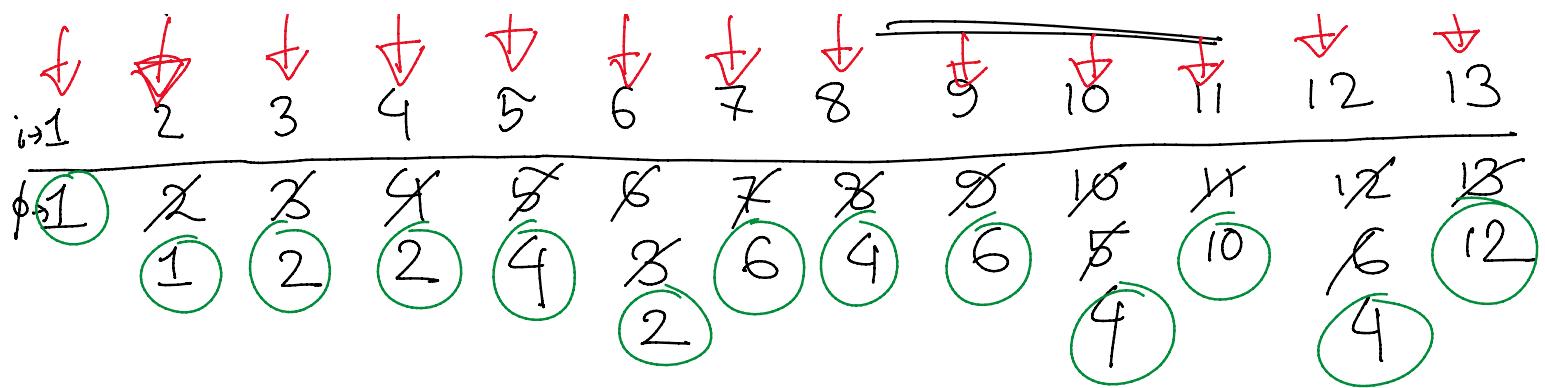
$$\phi(12) = 12 \cdot \frac{2-1}{2} \cdot \frac{3-1}{3}$$

$$12 \rightarrow \begin{matrix} 2 \\ P_0=2 \end{matrix} \times \begin{matrix} 3 \\ P_1=3 \end{matrix}^1$$

$$\phi(4) = 4 \cdot \frac{1}{2} = 2$$

$$\phi(p) = p \cdot \frac{p-1}{p} = p-1$$

$$1 \dots n \quad \begin{matrix} 2/3 \\ 4/5 \end{matrix} \quad \phi(n) = n \cdot (\text{something}) \quad \phi(12) = 12 \cdot \underbrace{\frac{1}{2} \cdot \frac{2}{3}}_{12 \quad 13}$$



$$\begin{aligned}
 x \cdot \frac{p-1}{p} &= \frac{xp}{p} - \frac{x}{p} \\
 &= x - \frac{x}{p} \\
 &= \text{phi} - \frac{\text{phi}}{p}
 \end{aligned}$$

$$\begin{aligned}
 F_n &= \{ \dots \} & F_3 &= \left\{ \frac{1}{3}, \frac{2}{3}, \frac{1}{2} \right\} \\
 F_{n+1} &= \left\{ F_n, \underbrace{\dots} \right\} & F_4 &= \left\{ F_3, \underbrace{\frac{1}{4}, \frac{3}{4}} \right\} \\
 F_n &= F_{n-1} + \underbrace{\phi(n)}_{\phi(4)} & \left\{ \begin{array}{l} 1, \dots, 4 \\ 1, \dots, 3 \end{array} \right\} & \frac{1, 3}{4}
 \end{aligned}$$

Diagram showing a red box around the equation $a \pmod{m} = 1$. Inside the box, $\phi(m)$ is written above a . A red arrow points from the bottom of the box to the left, and another red arrow points from the right side of the box upwards.

$$\left[\because \gcd(a, m) = 1 \right]$$

$$\left(a^{p-1} \% p \right) = 1$$

4% 17

4% 17

43% (7)

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n or

$$7 \% 17 \Rightarrow$$

$$7^1 \quad 7^2 \quad 7^3 \equiv 7^0$$

6
4

$$\% \mathcal{F} = 1$$

10215

% 17

7

7, 11, 1

$$= 11 \%$$

$$a\% m = \frac{1}{m}$$

$$3 \xrightarrow{\hspace{1cm}} 3k$$

Diagram illustrating the mapping of powers of 7 to their digital roots (1, 7, 15, 3, 9, 11) within a red boundary.

- $7^0 \rightarrow 1$
- $7^1 \rightarrow 7$
- $7^2 \rightarrow 15$
- $7^3 \rightarrow 3$
- $7^4 \rightarrow 9$
- $7^5 \rightarrow 11$

$7^6 \rightarrow 9$
 $7^7 \rightarrow 12$
 $7^8 \rightarrow 16$
 $7^9 \rightarrow 10$
 $7^{10} \rightarrow 2$
 $7^{11} \rightarrow 14$

$$\begin{array}{ccc}
 7^{12} & \rightarrow & 13 \\
 7^{13} & \rightarrow & 6 \\
 7^{14} & \rightarrow & 8 \\
 7^{15} & \rightarrow & 5 \\
 7^{16} & \rightarrow & 1 \\
 7^{32} & \rightarrow & 1
 \end{array}$$

$$2^5 \rightarrow \cancel{11} \quad 7 \rightarrow \cancel{14} \quad 7^0 \rightarrow 1$$

$$\left(\frac{7^{16}}{\cancel{11}} \cdot \frac{7^{16}}{\cancel{14}} \right) \% 17$$

$$(1 \cdot 1) \% 17$$

$$7 \% 17$$

$$a^{\phi(m)} \% m = 1$$

$$a^{16} \% 17 = 1$$

$$\frac{\phi(19)}{7 \% 19} = 1$$

$$\frac{18}{7 \% 19} = 1$$

1 3 2 1 3 2 1 3 2

3k

12345612334549999123...

$$53219 \% 12$$

$$a \% 13$$

$$\phi(13) = 12$$

$$a \% 13 \quad [0 \sim 12]$$

$$[0 < a < 10^{18}] \quad [0 < b < 10^{100}]$$

