

Exponentiation

$$[a, b, M]$$

$$a^b \text{ MOD } M$$

$$x \% M \rightarrow [0, M-1]$$

$$M = 10^9 + 7$$

$$5, 3, 7$$

$$5^3 \text{ MOD } 7$$

$$\text{res} = 1$$

$$\text{while } (b > 0)$$

$$\}$$

$$\text{res} = \text{res} * a;$$

$$b--;$$

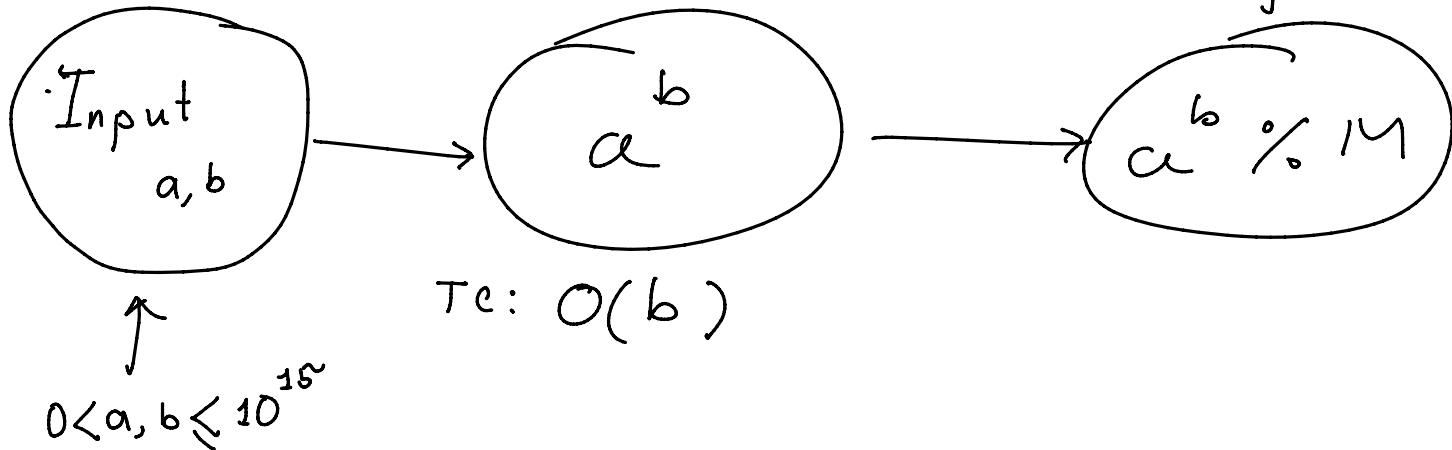
$$y$$

$$a = 5$$

$$b = 3$$

$$5 \xrightarrow{3} 5 \times 5 \times 5$$

$$= 125 \text{ MOD } 7$$

$$= 6$$
How to optimize?

$$a^{10} = \underbrace{a \times a \times a \times a \times a}_{{a^5} = x} \times \underbrace{a \times a \times a \times a \times a}_{{a^5}}$$

$$a^{100} = a^{50} \times a^{50} = x \times x = x^2$$

$$a^{50} = a^{25} \times a^{25}$$

$$a^{25} = a \times a^{24}$$

$$a^{12} = a^6 \times a^6$$

$$a^6 = a^3 \times a^3$$

$$a^0 = 1$$

$$b \downarrow 1$$

$$1 \downarrow K$$

$$a^{25} = \underline{a} \times a^{24}$$

\cancel{a}

$$a^{24} = \underline{a} \times a^{12}$$

$$a^{12} = \underline{a} \times a^6$$

$$a^b = \underline{a} \times a^{b-1}$$

$$a^3 = \underline{a} \times a^2$$

$$a^2 = \underline{a} \times a^1$$

$$a^1 = \underline{a} \times a^0$$

$$\frac{b}{2}$$

$$\frac{b}{4}$$

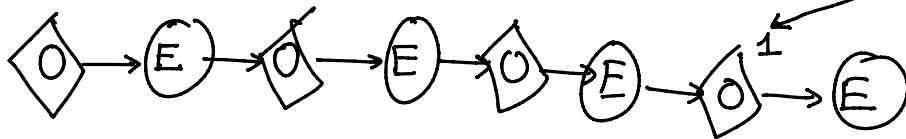
$$\frac{b}{8}$$

$$\dots$$

$$\frac{b}{1b}$$

$$k = \log_2(b)$$

$$O(2k) \asymp O(k) \asymp O(\log_2 b)$$



$$b \rightarrow 100 \rightarrow 50 \rightarrow 25 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 1 \rightarrow 0$$

$$(a \times b) \% M = ((a \% M) \times (b \% M)) \% M$$

$$(16 \times 17) \% 5 = ((1 \times 2) \% 5) \\ = 2 \% 5 \\ = 2$$

$$a^5 = a \times a^4$$

$$a^b = \begin{cases} a \times a^{b-1} & ; \quad b \text{ is odd} \\ a^{b/2} \times a^{\cancel{b/2}} & ; \quad b \text{ is even} \\ 1 & ; \quad b = 0 \end{cases}$$

$$f(a, b) = \begin{cases} \cancel{a} \times f(a, b-1) & \rightarrow b \text{ odd} \\ f(a, b/2) \times f(a, b/2) & \rightarrow b \text{ even} \end{cases}$$

$$a^b \% M$$

$$\dots \% M$$

$$a^b \% M \quad f(a, b) \% M$$

$$f(a, b) \% M = \begin{cases} ((a \% M) \times (f(a, b-1) \% M)) \% M \\ ((\underline{f(a, b/2)} \% M) \times \underline{f(a, b/2)} \% M) \% M \end{cases}$$

$$5^{20} \% 7 = ((5^{10} \% 7) \times (5^{10} \% 7)) \% 7$$

$$= (2 \times 2) \% 7$$

$$= 4 \% 7$$

$$= 4$$

$$s[i] =), \{,]$$

$s[i] = = ')$

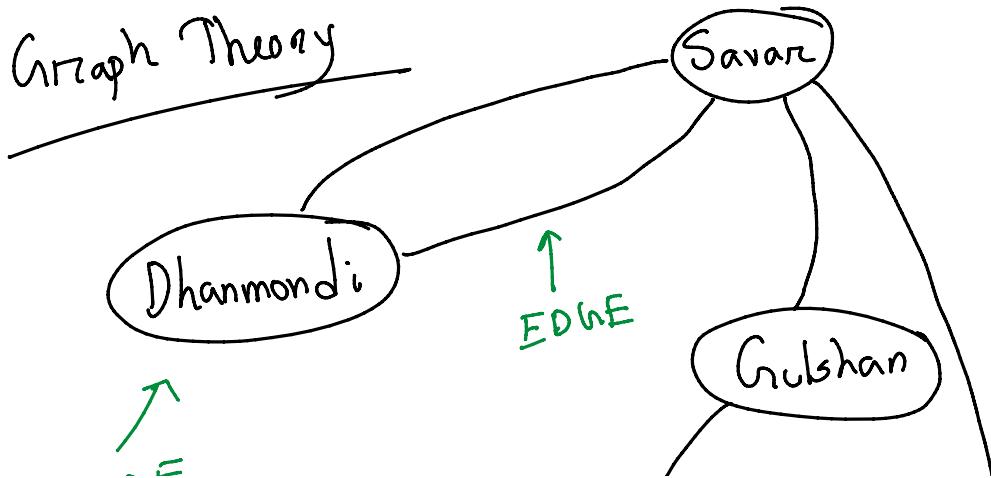
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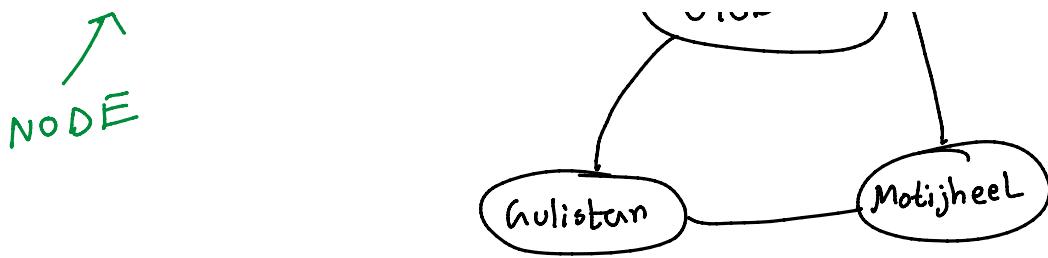
\downarrow

$($

\downarrow

bracketMap[')']





$A \rightarrow B$

$$5 \longrightarrow 16$$

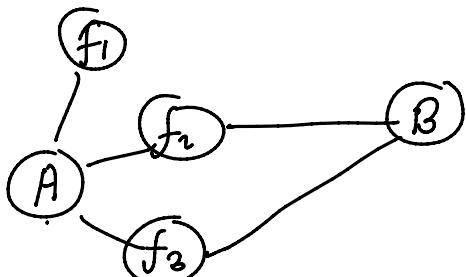
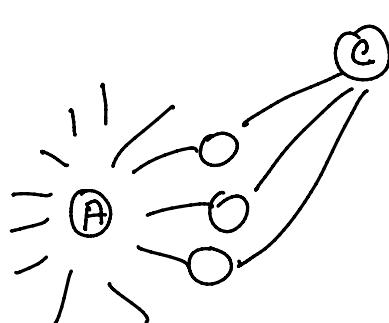
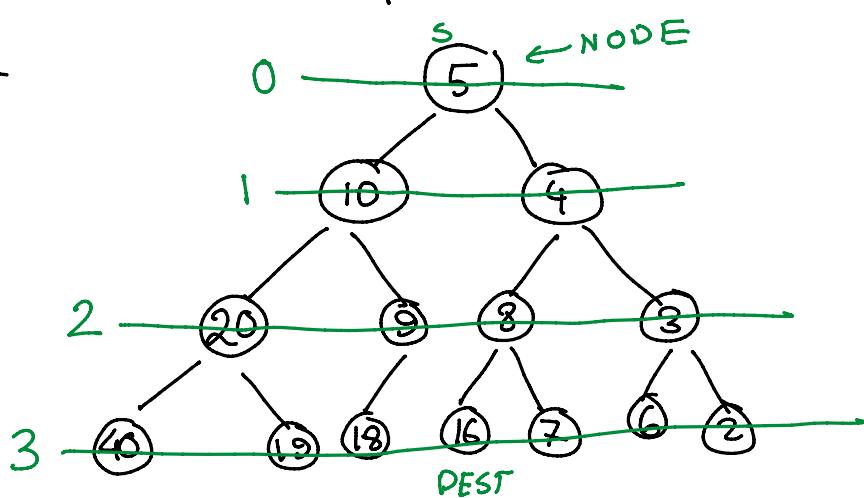
- 1) Multiply A by 2
- 2) Subtract 1 from A

$$\begin{array}{r} 5 \rightarrow 10 \\ \downarrow \\ 20 \\ \downarrow \\ 19 \end{array}$$

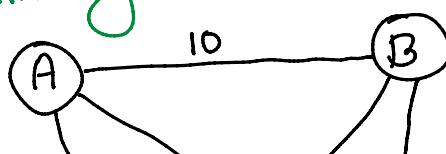
$$5 \rightarrow 4 \rightarrow 8 \rightarrow 16$$

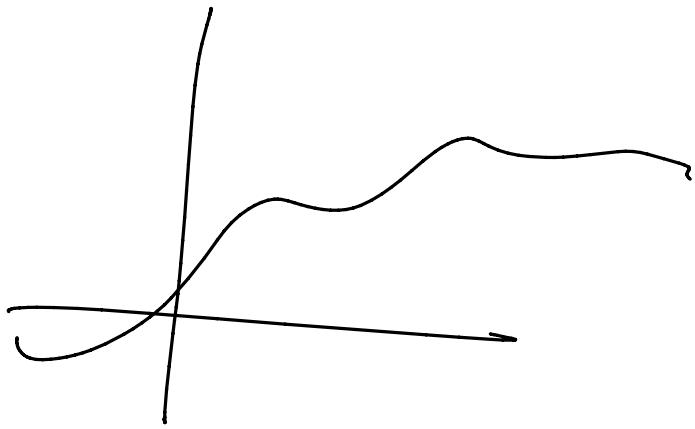
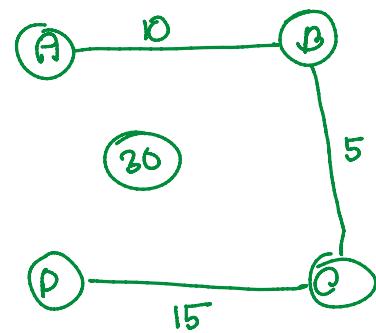
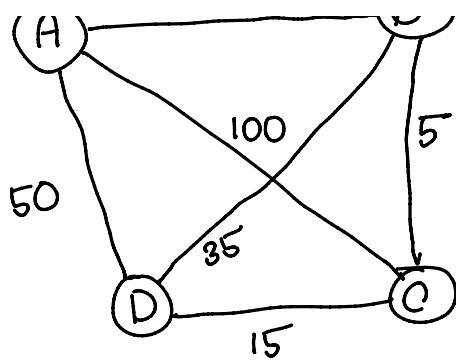
Dijkstra

Neo4j



Minimum Spanning Tree (MST)





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