Function

Type I: $\mathbf{f(x) = \frac{1}{ax + b}}$

For Domain: f(x) is not defined for

$ax + b = 0 \Rightarrow x = -\frac{b}{a}$

$\therefore D_f = \mathbb{R} - \left\{-\frac{b}{a}\right\}$

For Range: Let,

$y = \frac{1}{ax + b}$

$\Rightarrow ax + b = \frac{1}{y} \Rightarrow\Rightarrow x = \frac{1}{a}\left(\frac{1}{y} - b\right); y \neq 0$

$\therefore R_f = \mathbb{R} - \{0\}$

Ex. 1) $\mathbf{f(x) = \frac{2}{x + 3}}$

Solution: For Domain: f(x) is not defined for

$x + 3 = 0 \Rightarrow x = -3$

$\therefore D_f = \mathbb{R} - \{-3\}$

For Range: Let,

$y = \frac{2}{x + 3}$

$\Rightarrow x + 3 = \frac{2}{y} \Rightarrow x = \left(\frac{2}{y} - 3\right); y \neq 0$

$\therefore R_f = \mathbb{R} - \{0\}$

$* \mathbf{f(x) = \frac{1}{x - 2}}$

$* \mathbf{f(x) = \frac{1}{2x + 1}}$

$* \mathbf{f(x) = \frac{x}{x + 1}}$

Type II: $\mathbf{f(x) = \frac{x^2 - a^2}{x - a}}$

For Domain: f(x) is not defined for

$x - a = 0 \Rightarrow x = a$

$\therefore D_f = \mathbb{R} - \{a\}$

For Range: Let,

$y = \frac{x^2 - a^2}{x - a}$

$\Rightarrow y = x + a$

$\Rightarrow x = y - a$

Here, x is defined for all values of y except $y = 2a$ since $x \neq a$

$\therefore R_f = \mathbb{R} - \{2a\}$

Ex. 2) $\mathbf{f(x) = \frac{x^2 - 4}{x - 2}}$

Solution: For Domain: f(x) is not defined for

$x - 2 = 0 \Rightarrow x = 2$

$\therefore D_f = \mathbb{R} - \{2\}$

For Range: Let,

$y = \frac{x^2 - 4}{x - 2}$

$\Rightarrow y = x + 2$

$\Rightarrow x = y - 2$

Here, x is defined for all values of y except $y = 4$ since $x \neq 2$

$\therefore R_f = \mathbb{R} - \{4\}$

Type III: $\mathbf{f(x) = \frac{x - a}{x^2 - a^2}}$

For Domain: f(x) is not defined for

$x^2 - a^2 = 0 \Rightarrow x^2 = a^2 \Rightarrow x = \pm a$

$\therefore D_f = \mathbb{R} - \{-a, a\}$

For Range: Let,

$$y = \frac{x - a}{x^2 - a^2}$$

$$\Rightarrow y = \frac{1}{x + a}$$

$$\Rightarrow x = \frac{1}{y} - a$$

Here, x is defined for all values of y except $y = 0$ and $y = \frac{1}{2a}$ since $x \neq a$

$$\therefore R_f = \mathbb{R} - \left\{0, \frac{1}{2a}\right\}$$

Ex. 3) $\mathbf{f(x) = \frac{x - 3}{x^2 - 9}}$

Solution: For Domain: f(x) is not defined for

$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

$\therefore D_f = \mathbb{R} - \{-3,3\}$

For Range: Let,

$$y = \frac{x - 3}{x^2 - 9}$$

$$\Rightarrow y = \frac{1}{x + 3}$$

$$\Rightarrow x = \frac{1}{y} - 3$$

Here, x is defined for all values of y except $y = 0$ and $y = \frac{1}{6}$ since $x \neq 3$

$$\therefore R_f = \mathbb{R} - \left\{0, \frac{1}{6}\right\}$$

* $\mathbf{f(x) = \frac{x^2 - 36}{x - 6}}$

* $\mathbf{f(x) = \frac{x - 5}{x^2 - 25}}$

Type IV: $\mathbf{f(x) = \sqrt{ax + b}}$

For Domain: f(x) is defined for

$$ax + b \geq 0 \Rightarrow x \geq -\frac{b}{a}$$

$$\therefore D_f = \left\{x :\ x \geq -\frac{b}{a}\right\} = \left[-\frac{b}{a}, \infty\right)$$

For Range: Let,

$$y = \sqrt{ax + b}\ ;\ y \geq 0$$

$$\Rightarrow y^2 = ax + b$$

$$\Rightarrow x = \frac{1}{a}(y^2 - b)$$

$\text{Here, x is defined for } y \geq 0$

$$\therefore R_f = \{y :\ y \geq 0\} = [0, \infty)$$

Ex. 4) $\mathbf{f(x) = \sqrt{3x - 1}}$

Solution: For Domain: f(x) is defined for

$$3x - 1 \geq 0 \Rightarrow x \geq \frac{1}{3}$$

$$\therefore D_f = \left\{x :\ x \geq \frac{1}{3}\right\} = \left[\frac{1}{3}, \infty\right)$$

For Range: Let,

$$y = \sqrt{3x - 1}\ ;\ y \geq 0$$

$$\Rightarrow y^2 = 3x - 1$$

$$\Rightarrow x = \frac{1}{3}(y^2 + 1)$$

$\text{Here, x is defined for } y \geq 0$

$$\therefore R_f = \{y :\ y \geq 0\} = [0, \infty)$$

Ex. 5) $\mathbf{f(x) = \sqrt{1 - 2x}}$

Solution: For Domain: f(x) is defined for

$$1 - 2x \geq 0 \Rightarrow -2x \geq -1 \ \ \Rightarrow 2x \leq 1 \ \Rightarrow x \leq \frac{1}{2}$$

$\therefore D_f = \left\{x : x \leq \frac{1}{2}\right\} = \left(-\infty, \frac{1}{2}\right]$

For Range: Let,

$y = \sqrt{1-2x}\,;\ y \geq 0$

$\Rightarrow y^2 = 1 - 2x$

$\Rightarrow x = \frac{1}{2}(1 - y^2)$

$\text{Here, x is defined for } y \geq 0$

$\therefore R_f = \{y : y \geq 0\} = [0, \infty)$

$*\ \mathbf{f(x) = \sqrt{x+2}}$

$*\ \mathbf{f(x) = -\sqrt{5x-1}}$

$*\ \mathbf{f(x) = -\sqrt{1-x}}$