

## SOLUTION

$$1) \int \frac{\sqrt{1 + \ln x}}{x} dx$$

$$= \int \sqrt{z} dz$$

$$\text{Let, } 1 + \ln x = z$$

$$= \frac{z^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow \frac{1}{x} dx = dz$$

$$= \frac{2}{3} (1 + \ln x)^{\frac{3}{2}} + c$$

$$2) \int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$\text{Let, } xe^x = z$$

$$= \int \sec^2 z dz$$

$$\Rightarrow (xe^x + e^x) dx = dz$$

$$= \tan z + c$$

$$\Rightarrow e^x(x+1) dx = dz$$

$$= \tan(xe^x) + c$$

$$3) \int \frac{1 - \sin x}{x + \cos x} dx$$

$$= \int \frac{dz}{z}$$

$$\text{Let, } x + \cos x = z$$

$$= \ln z + c$$

$$\Rightarrow (1 - \sin x) dx = dz$$

$$= \ln(x + \cos x) + c$$

$$4) \int \frac{e^{\sec^{-1} x}}{x\sqrt{x^2-1}} dx$$

$$= \int e^z dz$$

$$= e^z + c$$

$$= e^{\sec^{-1} x} + c$$

$$\text{Let, } \sec^{-1} x = z$$

$$\Rightarrow \frac{1}{x\sqrt{x^2-1}} dx = dz$$

$$5) \int \frac{\sin 2x}{a \sin^2 x + b \cos^2 x} dx$$

$$= \int \frac{dz}{z}$$

$$= \frac{1}{a-b} \int \frac{dz}{z}$$

$$= \frac{1}{a-b} \ln z + c$$

$$= \frac{1}{a-b} \ln(a \sin^2 x + b \cos^2 x) + c$$

$$\text{Let, } a \sin^2 x + b \cos^2 x = z$$

$$\Rightarrow [a 2 \sin x \cos x + b 2 \cos x \cdot (-\sin x)] dx = dz$$

$$\Rightarrow (a \sin 2x - b \sin 2x) dx = dz$$

$$\Rightarrow \sin 2x (a - b) dx = dz$$

$$\Rightarrow \sin 2x dx = \frac{dz}{(a-b)}$$

$$6) \int \frac{\tan x \sec^2 x}{(a^2 + b^2 \tan^2 x)^2} dx$$

$$= \int \frac{dz}{z^2}$$

$$= \frac{1}{2b^2} \int \frac{dz}{z^2}$$

$$= \frac{1}{2b^2} \left( -\frac{1}{z} \right) + c$$

$$= \frac{-1}{2b^2} \cdot \left( \frac{1}{a^2 + b^2 \tan^2 x} \right) + c$$

$$\text{Let, } a^2 + b^2 \tan^2 x = z$$

$$\Rightarrow b^2 \cdot 2 \tan x \sec^2 x dx = dz$$

$$\Rightarrow \tan x \sec^2 x dx = \frac{dz}{2b^2}$$

$$7) \int \frac{\sin x}{\sin(x-a)} dx$$

$$= \int \frac{\sin(z+a)}{\sin z} dz$$

$$\text{Let, } x - a = z$$

$$= \int \frac{\sin z \cos a + \cos z \sin a}{\sin z} dz$$

$$\Rightarrow dx = dz$$

$$= \int \left( \frac{\sin z \cos a}{\sin z} + \frac{\cos z \sin a}{\sin z} \right) dz$$

$$\text{and, } x = z + a$$

$$= \int (\cos a + \sin a \cot z) dz$$

$$= \cos a \int dz + \sin a \int \cot z dz$$

$$= \cos a \cdot z + \sin a \ln|\sin z| + c$$

$$= \cos a (x - a) + \sin a \ln|\sin(x - a)| + c$$

$$8) \int \frac{dx}{x \ln x [\ln(\ln x)]}$$

$$= \int \frac{dz}{z}$$

$$\text{Let, } \ln(\ln x) = z$$

$$= \ln z + c$$

$$\Rightarrow \frac{1}{\ln x} \cdot \frac{1}{x} dx = dz$$

$$= \ln(\ln(\ln x)) + c$$

$$9) \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{\cos 2x}{\sin^2 x + 2 \sin x \cos x + \cos^2 x} dx$$

$$= \int \frac{\cos 2x}{1 + \sin 2x} dx$$

$$\text{Let, } 1 + \sin 2x = z$$

$$= \int \frac{dz}{z}$$

$$\Rightarrow 2 \cos 2x \, dx = dz$$

$$= \frac{1}{2} \ln z + c$$

$$\Rightarrow \cos 2x \, dx = \frac{dz}{2}$$

$$= \frac{1}{2} \ln(1 + \sin 2x) + c$$

$$10) \int \sqrt{\sin x} \cos^3 x \, dx$$

$$= \int \sqrt{\sin x} \cdot \cos^2 x \cdot \cos x \, dx$$

$$\text{Let, } \sin x = z$$

$$= \int \sqrt{\sin x} (1 - \sin^2 x) \cdot \cos x \, dz$$

$$\Rightarrow \cos x \, dx = dz$$

$$= \int \sqrt{z} (1 - z^2) \, dz$$

$$= \int \left( z^{\frac{1}{2}} - z^{\frac{5}{2}} \right) dz$$

$$= \left( \frac{z^{\frac{3}{2}}}{\frac{3}{2}} - \frac{z^{\frac{7}{2}}}{\frac{7}{2}} \right) + c$$

$$= \frac{2}{3} z^{\frac{3}{2}} - \frac{2}{7} z^{\frac{7}{2}} + c$$

$$= \frac{2}{3} (\sin x)^{\frac{3}{2}} - \frac{2}{7} (\sin x)^{\frac{7}{2}} + c$$