Integration- Method of Substitution

Ex. 1) $\int \frac{\sin x}{\sqrt{1+\cos x}} dx$

$= -\int \frac{1}{\sqrt{z}} dz$ Let, $1 + \cos x = z$

$= -2\sqrt{z} + c$ $\Rightarrow -\sin x\, dx = dz$

$= -2\sqrt{1+\cos x} + c$

Ex. 2) $\int e^{\tan^{-1} x} . \frac{1}{1+x^2} dx$

$= \int e^z\, dz$ Let, $\tan^{-1} x = z$

$= e^z + c$ $\Rightarrow \frac{1}{1+x^2} dx = dz$

$= e^{\tan^{-1} x} + c$

Ex. 3) $\int \frac{1}{\sqrt{x}} \cos\sqrt{x}\, dx$

$= \int \cos z\ \ 2\, dz$ Let, $\sqrt{x} = z$

$= 2 \sin z + c$ $\Rightarrow \frac{1}{2\sqrt{x}} dx = dz$

$= 2 \sin\sqrt{x} + c$ $\Rightarrow \frac{1}{\sqrt{x}} dx = 2\, dz$

Ex. 4) $\int \frac{a \cos x - b \sin x}{a \sin x + b \cos x + d} dx$

$= \int \frac{dz}{z}$ Let, $a \sin x + b \cos x + d = z$

$= \ln z + c$ $\Rightarrow (a \cos x - b \sin x)\, dx = dz$

$= \ln(a \sin x + b \cos x + d) + c$

Ex. 5) $\displaystyle\int \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$

$= \displaystyle\int z^2\,dz$ Let, $\sin^{-1}x = z$

$= \dfrac{z^3}{3} + c$ $\Rightarrow \dfrac{1}{\sqrt{1-x^2}}\,dx = dz$

$= \dfrac{(\sin^{-1}x)^3}{3} + c$

Ex. 6) $\displaystyle\int \cos^3 x\,dx$

$= \displaystyle\int \cos^2 x.\cos x\,dx$ Let, $\sin x = z$

$= \displaystyle\int (1-\sin^2 x).\cos x\,dx$ $\Rightarrow \cos x\,dx = dz$

$= \displaystyle\int (1-z^2)\,dz$

$= z - \dfrac{z^3}{3} + c$

$= \sin x - \dfrac{(\sin x)^3}{3} + c$

Ex. 7) $\displaystyle\int \sin^4 x\cos^3 x\,dx$

$= \displaystyle\int \sin^4 x\,.\cos^2 x.\cos x\,dx$ Let, $\sin x = z$

$= \displaystyle\int \sin^4 x\,(1-\sin^2 x).\cos x\,dx$ $\Rightarrow \cos x\,dx = dz$

$= \displaystyle\int z^4\,(1-z^2)\,dz$

$= \displaystyle\int (z^4 - z^6)\,dz$

$= \dfrac{z^5}{5} - \dfrac{z^7}{7} + c = \dfrac{(\sin x)^5}{5} - \dfrac{(\sin x)^7}{7} + c$

Ex. 8) $\int \tan^3 2x \sec 2x\, dx$

$= \int \tan^2 2x\ \tan 2x\ \sec 2x\, dx$ Let, $\sec 2x = z$

$= \int (\sec^2 2x - 1)\tan 2x \sec 2x\, dx$ $\Rightarrow \sec 2x \tan 2x\, dx\,.2 = dz$

$= \int (z^2 - 1)\frac{1}{2}dz$ $\Rightarrow \sec 2x \tan 2x\, dx = \frac{1}{2}dz$

$= \frac{1}{2}\left(\frac{z^3}{3} - z\right) + c$

$= \frac{1}{2}\left(\frac{\sec^3 2x}{3} - \sec 2x\right) + c$

Ex. 9) $\int \frac{\tan x}{\ln \cos x} dx$

$= \int \frac{-dz}{z}$ Let, $\ln \cos x = z$

$= -\ln z + c$ $\Rightarrow -\frac{\sin x}{\cos x} = dz$

$= -\ln(\ln \cos x) + c$ $\Rightarrow \tan x = -dz$

Ex. 10) $\int \frac{\sin 2x}{(a + b\cos x)^2} dx$

$= \int \frac{2\sin x \cos x}{(a + b\cos x)^2}\ dx$ Let, $a + b\cos x = z$

$= 2\int \frac{\frac{z-a}{b}}{z^2}\left(-\frac{1}{b}dz\right)$ $\Rightarrow -b\sin x\, dx = dz$

$= \frac{-2}{b^2}\int \frac{z-a}{z^2}\ dz$ $\Rightarrow \sin x\, dx = -\frac{1}{b}dz$

$= \frac{-2}{b^2}\int \left(\frac{1}{z} - \frac{a}{z^2}\right)\ dz$ and, $\cos x\, dx = \frac{z-a}{b}$

$= \frac{-2}{b^2}\left(\ln z + \frac{a}{z}\right) + c = \frac{-2}{b^2}\left(\ln(a + b\cos x) + \frac{a}{a + b\cos x}\right) + c$

Ex. 11) $\int \frac{\sin^4 x}{\cos^8 x} dx$

$= \int \tan^4 x \ \sec^4 x \, dx$ Let, $\tan x = z$

$= \int \tan^4 x \ \sec^2 x \sec^2 x \, dx$ $\Rightarrow \sec^2 x dx = dz$

$= \int \tan^4 x \ (1 + \tan^2 x) \sec^2 x \, dx$

$= \int z^4 \ (1 + z^2) \, dz$

$= \int (z^4 + z^6)$

$= \frac{z^5}{5} + \frac{z^7}{7} + c$

$= \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c$

H.W::

1) $\int \frac{\sqrt{1 + \ln x}}{x} dx$	6) $\int \frac{\tan x \sec^2 x}{(a^2 + b^2 \tan^2 x)^2} dx$
2) $\int \frac{e^x(1 + x)}{\cos^2(xe^x)} dx$	7) $\int \frac{\sin x}{\sin(x - a)} dx$
3) $\int \frac{1 - \sin x}{x + \cos x} dx$	8) $\int \frac{dx}{x \ln x \ [\ln(\ln x)]}$
4) $\int \frac{e^{\sec^{-1} x}}{x\sqrt{x^2 - 1}} dx$	9) $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$
5) $\int \frac{\sin 2x}{a \sin^2 x + b \cos^2 x} dx$	10) $\int \sqrt{\sin x} \cos^3 x \, dx$