

Integration by Method of Partial Fraction

Ex. 1) $\int \frac{1}{(x+1)(x-5)} dx$

Solution: Let,

$$\frac{1}{(x+1)(x-5)} = \frac{A}{x+1} + \frac{B}{x-5} \dots\dots\dots (i)$$

$$\Rightarrow 1 = A(x-5) + B(x+1) \dots\dots\dots (ii)$$

Putting $x = 5$ in (ii) we get,

$$\Rightarrow 1 = A.0 + B.6$$

$$\Rightarrow 6B = 1 \Rightarrow B = \frac{1}{6}$$

Putting $x = -1$ in (ii) we get,

$$\Rightarrow 1 = A.-6 + B.0$$

$$\Rightarrow -6A = 1 \Rightarrow A = \frac{-1}{6}$$

From (i),

$$\frac{1}{(x+1)(x-5)} = \frac{A}{x+1} + \frac{B}{x-5}$$

$$\Rightarrow \frac{1}{(x+1)(x-5)} = \frac{\frac{-1}{6}}{x+1} + \frac{\frac{1}{6}}{x-5}$$

$$\Rightarrow \int \frac{dx}{(x+1)(x-5)} = \int \frac{\frac{-1}{6}}{x+1} dx + \int \frac{\frac{1}{6}}{x-5} dx$$

$$\Rightarrow \int \frac{dx}{(x+1)(x-5)} = \frac{-1}{6} \int \frac{1}{x+1} dx + \frac{1}{6} \int \frac{1}{x-5} dx$$

$$\Rightarrow \int \frac{dx}{(x+1)(x-5)} = \frac{-1}{6} \ln(x+1) + \frac{1}{6} \ln(x-5) + c$$

Ex. 2) $\int \frac{1}{x^2 + x} dx$

Solution: Let,

$$\frac{1}{x^2 + x} = \frac{1}{x(x + 1)} = \frac{A}{x} + \frac{B}{x + 1} \dots \dots \dots (i)$$

$$\Rightarrow 1 = A(x + 1) + Bx \dots \dots \dots (ii)$$

Putting $x = 0$ in (ii) we get,

$$\Rightarrow 1 = A.1 + B.0$$

$$\Rightarrow A = 1$$

Putting $x = -1$ in (ii) we get,

$$\Rightarrow 1 = A.0 + B.-1$$

$$\Rightarrow -B = 1 \Rightarrow B = -1$$

From (i),

$$\frac{1}{x(x + 1)} = \frac{1}{x} + \frac{-1}{x + 1}$$

$$\Rightarrow \int \frac{1}{x(x + 1)} dx = \int \frac{1}{x} dx - \int \frac{1}{x + 1} dx$$

$$\Rightarrow \int \frac{1}{x(x + 1)} dx = \ln x - \ln(x + 1) + c$$

Ex. 3) $\int \frac{x - 1}{(x - 2)(x - 3)} dx$

Solution: Let,

$$\frac{x - 1}{(x - 2)(x - 3)} = \frac{A}{x - 2} + \frac{B}{x - 3} \dots \dots \dots (i)$$

$$\Rightarrow x - 1 = A(x - 3) + B(x - 2) \dots \dots \dots (ii)$$

Putting $x = 2$ in (ii) we get,

$$\Rightarrow 1 = A \cdot -1 + B \cdot 0$$

$$\Rightarrow A = -1$$

Putting $x = 3$ in (ii) we get,

$$\Rightarrow 2 = A \cdot 0 + B \cdot 1$$

$$\Rightarrow B = 2$$

From (i),

$$\frac{x-1}{(x-2)(x-3)} = \frac{-1}{x-2} + \frac{2}{x-3}$$

$$\Rightarrow \int \frac{x-1}{(x-2)(x-3)} dx = \int \frac{-1}{x-2} dx + \int \frac{2}{x-3} dx$$

$$\Rightarrow \int \frac{x-1}{(x-2)(x-3)} dx = -\ln(x-2) + 2\ln(x-3) + c$$

Ex. 4) $\int \frac{2x-1}{x(x-1)(x-2)} dx$

Solution: Let,

$$\frac{2x-1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2} \dots \dots \dots (i)$$

$$\Rightarrow 2x-1 = A(x-1)(x-2) + Bx(x-2) + Cx(x-1) \dots \dots \dots (ii)$$

Putting $x = 0$ in (ii) we get,

$$\Rightarrow -1 = A \cdot (-1)(-2) + B \cdot 0 + C \cdot 0$$

$$\Rightarrow 2A = -1 \Rightarrow A = \frac{-1}{2}$$

Putting $x = 1$ in (ii) we get,

$$\Rightarrow 1 = A \cdot 0 + B \cdot 1 \cdot (-1) + C \cdot 0$$

$$\Rightarrow B = -1$$

Putting $x = 2$ in (ii) we get,

$$\Rightarrow 3 = A.0 + B.0 + C.2.1$$

$$\Rightarrow 2C = 3 \Rightarrow C = \frac{3}{2}$$

From (i),

$$\frac{2x-1}{x(x-1)(x-2)} = \frac{\frac{-1}{2}}{x} + \frac{-1}{x-1} + \frac{\frac{3}{2}}{x-2}$$

$$\Rightarrow \int \frac{2x-1}{x(x-1)(x-2)} dx = \frac{-1}{2} \int \frac{1}{x} dx - \int \frac{1}{x-1} dx + \frac{3}{2} \int \frac{1}{x-2} dx$$

$$\Rightarrow \int \frac{2x-1}{x(x-1)(x-2)} dx = -\frac{1}{2} \ln x - \ln(x-1) + \frac{3}{2} \ln(x-2) + c$$

$$5) \int \frac{x}{(x-1)^2(x+2)} dx$$

Solution: Let,

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \dots \dots \dots (i)$$

$$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \dots \dots \dots (ii)$$

Putting $x = 1$ in (ii) we get,

$$\Rightarrow 1 = A.0 + 3B + C.0$$

$$\Rightarrow B = \frac{1}{3}$$

Putting $x = -2$ in (ii) we get,

$$\Rightarrow -2 = A.0 + B.0 + C.(-2-1)^2$$

$$\Rightarrow -2 = 9C \Rightarrow C = \frac{-2}{9}$$

Calculating the coefficients of x^2 from both sides,

$$\Rightarrow 0 = A + C \Rightarrow 0 = A - \frac{2}{9} \Rightarrow A = \frac{2}{9}$$

From (i),

$$\frac{x}{(x-1)^2(x+2)} = \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2}$$

$$\Rightarrow \int \frac{2x-1}{x(x-1)(x-2)} dx = \frac{2}{9} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{x+2} dx$$

$$\Rightarrow \int \frac{2x-1}{x(x-1)(x-2)} dx = \frac{2}{9} \ln(x-1) + \frac{1}{3} \left(\frac{1}{x-1} \right) - \frac{2}{9} \ln(x+2) + c$$

H.W:

1) $\int \frac{x+1}{x^2-5x+6} dx$

2) $\int \frac{2x+3}{x^3+x^2-2x} dx$

3) $\int \frac{1}{x^2(x-1)} dx$

4) $\int \frac{x+1}{x^2-7x+10} dx$

5) $\int \frac{1}{x(x+1)^2} dx$