# Integration by Parts

**Formula**: $\int \mathbf{uv\,dx} = \mathbf{u}\int \mathbf{v\,dx} - \int \left\{\frac{\mathbf{d}}{\mathbf{dx}}(\mathbf{u})\int \mathbf{v\,dx}\right\}\mathbf{dx}$

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|---|---|---|---|---|---|
| **L** | **I** | **A** | **T** | **E** | **C** |

L = Logarithm (log x)

I = Inverse ($x^{-1}$, $\sin^{-1} x$)

A = Algeraic Function ($x^2$)

T = Trigonometric Function (sin x, cos x)

E = Exponential ($e^x$)

C = Constant (1,2,3, … . )

**Ex. 1)** $\int \mathbf{x \cos x\,dx}$

$= x\int \cos x\,dx - \int \left\{\frac{d}{dx}(x)\int \cos x\,dx\right\}dx$

$= x \sin x - \int \sin x\,dx$

$= x \sin x + \cos x + c$

**Ex. 2)** $\int \mathbf{\ln x\,dx}$

$= \ln x\int 1\,dx - \int \left\{\frac{d}{dx}(\ln x)\int 1\,dx\right\}dx$

$= \ln x\,.\,x - \int \frac{1}{x}.\,x\,dx$

$= x\ \ln x - \int 1\,dx$

$= x\ \ln x - x + c$

**Ex. 3)** $\int \cos^{-1} x \, dx$

$$= \cos^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} (\cos^{-1} x) \int 1 \, dx \right\} dx$$

$$= \cos^{-1} x . x - \int \frac{-x}{\sqrt{1 - x^2}} dx \qquad \text{Let}, 1 - x^2 = z$$

$$= x \cos^{-1} x - \int \frac{\frac{dz}{2}}{\sqrt{z}} \qquad \Rightarrow -2xdx = dz$$

$$= x \cos^{-1} x - \frac{1}{2} \frac{\sqrt{z}}{\frac{1}{2}} + c \qquad \Rightarrow -xdx = \frac{dz}{2}$$

$$= x \cos^{-1} x - \frac{1}{2} . 2\sqrt{z} + c$$

$$= x \cos^{-1} x - \sqrt{1 - x^2} + c$$

**Ex. 4)** $\int \tan^{-1} x \, dx$

$$= \tan^{-1} x \int 1 \, dx - \int \left\{ \frac{d}{dx} (\tan^{-1} x) \int 1 \, dx \right\} dx$$

$$= \tan^{-1} x . x - \int \frac{1}{1 + x^2} x \, dx \qquad \text{Let}, 1 + x^2 = z$$

$$= x \tan^{-1} x - \int \frac{\frac{dz}{2}}{z} \qquad \Rightarrow 2xdx = dz$$

$$= x \tan^{-1} x - \frac{1}{2} . \ln z + c \qquad \Rightarrow xdx = \frac{dz}{2}$$

$$= x \tan^{-1} x - \frac{1}{2} . \ln(1 + x^2) + c$$

**Ex. 5)** $\int x^2 e^x \, dx$

$$= x^2 \int e^x \, dx - \int \left\{ \frac{d}{dx} (x^2) \int e^x \, dx \right\} dx$$

$$= x^2 e^x - \int 2x \, e^x \, dx$$

$$= x^2e^x - 2\left[x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int e^x\,dx\right]$$

$$= x^2e^x - 2(xe^x - e^x) + c$$

$$= x^2e^x - 2xe^x + 2e^x + c$$

**Ex. 6)** $\int \ln\left(x+\sqrt{x^2+a^2}\right)dx$

$$= \ln\left(x+\sqrt{x^2+a^2}\right)\int 1\,dx - \int\left\{\frac{d}{dx}\ln\left(x+\sqrt{x^2+a^2}\right)\int 1\,dx\right\}dx$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{1}{\left(x+\sqrt{x^2+a^2}\right)}\left(1+\frac{1}{2\sqrt{x^2+a^2}}.2x\right)x\,dx$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{1}{\left(x+\sqrt{x^2+a^2}\right)}\left(1+\frac{x}{\sqrt{x^2+a^2}}.\right)x\,dx$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{1}{\left(x+\sqrt{x^2+a^2}\right)}\left(\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}.\right)x\,dx$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{x}{\sqrt{x^2+a^2}}\,dx$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{x}{\sqrt{x^2+a^2}}\,dx \qquad \text{Let, } x^2+a^2 = z$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \int\frac{\frac{dz}{2}}{\sqrt{z}} \qquad \Rightarrow 2xdx = dz$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \frac{1}{2}\frac{\sqrt{z}}{\frac{1}{2}} + c \qquad \Rightarrow xdx = \frac{dz}{2}$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \frac{1}{2}.2\sqrt{z} + c$$

$$= x\ln\left(x+\sqrt{x^2+a^2}\right) - \sqrt{x^2+a^2} + c$$

**H.W:**

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|---|---|
| 1) $\int x \ln x \, dx$ | 6) $\int \frac{x + \sin x}{1 + \cos x} \, dx$ |
| 2) $\int \left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right) dx$ | 7) $\int \frac{\ln(\ln x)}{x} \, dx$ |
| 3) $\int (\sin^{-1} x)^2 \, dx$ | 8) $\int \frac{\ln(x + 1)}{\sqrt{x + 1}} \, dx$ |
| 4) $\int x^2 \sin^2 x \, dx$ | 9) $\int \frac{x}{\sec x + 1} \, dx$ |
| 5) $\int \frac{x}{1 + \cos x} \, dx$ | 10) $\int (\ln x)^2 \, dx$ |