Matrix:

ORDER:

$A = m \times n = 2 \times 3 \qquad B = n \times q = 3 \times 4$

$A \times B = m \times q = 2 \times 4$

Ex. 1) Let, $\mathbf{A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}}$ **and** $\mathbf{B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 2 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}}$**, Calculate the product of AB**

Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & 3 & 2 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+6 & 2+6+3 & -1+4-3 & 3+10+9 \\ 2+3+10 & 4+9+5 & -2+6-5 & 6+15+15 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 11 & 0 & 22 \\ 15 & 18 & -1 & 36 \end{bmatrix}$$

Ex. 2) If $\mathbf{A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}}$**, Prove that,** $\mathbf{A^3 + A^2 - 21A - 45I = 0}$**,**

where I is the identity matrix of order 3.

Solution: $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix} \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} -8 & 38 & -57 \\ 38 & 49 & -114 \\ -19 & -38 & 30 \end{bmatrix}$$

$\therefore A^3 + A^2 - 21A - 54I$

$$= \begin{bmatrix} -8 & 38 & -57 \\ 38 & 49 & -114 \\ -19 & -38 & 30 \end{bmatrix} + \begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix} - 21 \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - 54 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 38 & -57 \\ 38 & 49 & -114 \\ -19 & -38 & 30 \end{bmatrix} + \begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix} - \begin{bmatrix} -42 & 42 & -63 \\ 42 & 21 & -126 \\ -21 & -42 & 0 \end{bmatrix} - \begin{bmatrix} 45 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 45 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Ex. 3) If $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-2} & \mathbf{5} & \mathbf{-1} \\ \mathbf{2} & \mathbf{3} & \mathbf{4} \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} \mathbf{-1} & \mathbf{5} & \mathbf{3} \\ \mathbf{7} & \mathbf{-2} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{-3} \end{bmatrix}$**, then prove that,** $(\mathbf{AB})^{\mathbf{T}} = \mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}$

Solution: Given, $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 5 & -1 \\ 2 & 3 & 4 \end{bmatrix}$ $\quad \therefore A^T = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 5 & 3 \\ 3 & -1 & 4 \end{bmatrix}$

and, $B = \begin{bmatrix} -1 & 5 & 3 \\ 7 & -2 & 1 \\ 2 & 0 & -3 \end{bmatrix}$ $\quad \therefore B^T = \begin{bmatrix} -1 & 7 & 2 \\ 5 & -2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} -1 & 7 & 2 \\ 5 & -2 & 0 \\ 3 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ 2 & 5 & 3 \\ 3 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 35 & 27 \\ 1 & -20 & 4 \\ -4 & 2 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 5 & -1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 5 & 3 \\ 7 & -2 & 1 \\ 2 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 19 & 1 & -4 \\ 35 & -20 & 2 \\ 27 & 4 & -3 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 19 & 35 & 27 \\ 1 & -20 & 4 \\ -4 & 2 & -3 \end{bmatrix}$$

$\therefore (AB)^T = B^T A^T$ (PROVED)