Ex. 1) Find out the solution of the following system of linear equations:

$$\begin{cases} \mathbf{x - 2y + 3z = 7} \\ \mathbf{2x + y - z = 1} \\ \mathbf{x - y - z = -6} \end{cases}$$

Solution: Reducing it to Echelon Form,

$$\Rightarrow \begin{cases} x - 2y + 3z = 7 \\ 5y - 7z = -13 \\ y - 4z = -13 \end{cases} \qquad R_2' = R_2 - 2R_1, R_3' = R_3 - R_1$$

$$\Rightarrow \begin{cases} x - 2y + 3z = 7 \\ 5y - 7z = -13 \\ 13z = 52 \end{cases} \qquad R_3' = R_2 - 5R_3$$

Thus, the system has unique solution, where

$z = 4,$

$5y - 28 = -13 \Rightarrow y = 3$

$x - 6 + 12 = 7 \Rightarrow x = 1$

Ex. 2) Find out the solution of the following system of linear equations:

$$\begin{cases} \mathbf{x + y + 2z = 0} \\ \mathbf{y + z = 0} \\ \mathbf{-2x + 3y + z = 0} \end{cases}$$

Solution: Reducing it to Echelon Form,

Interchanging 2nd and 3rd row,

$$\begin{cases} x + y + 2z = 0 \\ -2x + 3y + z = 0 \\ y + z = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x + y + 2z = 0 \\ 5y + 5z = 0 \\ y + z = 0 \end{cases} \qquad R_2' = R_2 + 2R_1$$

$$\Rightarrow \begin{cases} x + y + 2z = 0 \\ y + z = 0 \\ y + z = 0 \end{cases} \qquad R_2' = \frac{R_2}{5}$$

$$\Rightarrow \begin{cases} x + y + 2z = 0 \\ y + z = 0 \end{cases}$$

We have two equations with three unknowns.

Let, the free variable $z = a$.

Thus, the general solution is,

$z = a, y = -a$

$x - a + 2a = 0 \Rightarrow x = -a$

H.W:

1) Find out the solution of the following system of linear equations:

(i) $\begin{cases} 5x + 3y - 3z = -1 \\ 3x + 2y - 2z = -1 \\ 2x - y + 2z = 8 \end{cases}$

(ii) $\begin{cases} x + 2y - z = 2 \\ 2x + y + z = 1 \\ x + 5y - 4z = 5 \end{cases}$

2) Prove that the system is inconsistent:

(i) $\begin{cases} x + 2y - 3z = -1 \\ 5x + 3y - 4z = 2 \\ 3x - y + 2z = 7 \end{cases}$

(ii) $\begin{cases} x + y + z = 1 \\ 2x + 2y + 2z = 1 \\ 3x + 3y + 3z = 2 \end{cases}$