Rank of a Matrix reducing it to Echelon form

- Only row operations.
- Reduce it to upper triangular matrix.
- Number of non − zero rows = Rank of the matrix.
- Nullity of A + Rank of A = Total number of attributes of A (i.e. total number of columns in A)

Ex. 1) Find the Rank of the matrix $\mathbf{A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}}$ **reducing it to echelon form.**

Solution: Given,

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix} \qquad R_2' \rightarrow R_2 + 2R_1, \;\; R_3' \rightarrow R_3 - R_1$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_3' \rightarrow 2R_2 + 3R_3, \;\; R_4' \rightarrow R_2 - 3R_4$$

Therefore, Rank of A, $R(A) = 2$ [Number of non − zero rows]

Rank of A + Nullity of A = 4

Nullity of A = 4 − 2 = 2

Ex. 2) Find the Rank of the matrix $\mathbf{A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}}$ **reducing it to echelon form.**

Solution: Given,

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 5 & -3 \end{bmatrix} \qquad R_2' \rightarrow R_2 - 2R_1, R_3' \rightarrow R_3 + R_1$$

$$\Rightarrow A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_3' \to R_2 - R_3$$

Therefore, Rank of A, $R(A) = 2$

Rank of A + Nullity of A = 4

Nullity of A = 4 − 2 = 2

Ex. 3) Find the inverse of the matrix $A = \begin{bmatrix} \mathbf{3} & \mathbf{0} & \mathbf{2} \\ \mathbf{2} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{bmatrix}$ **using Identity matrix and reducing it to Row Echelon form.**

Solution: Given,

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\right]$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 5 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\right] \quad R_1' \to R_1 + R_2$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\right] \quad R_1' \to \frac{R_1}{5}$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 1/5 & 0 \\ 0 & 0 & -2 & -2/5 & 3/5 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\right] \quad R_2' \to R_2 - 2R_1$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 1/5 & 0 \\ 0 & 0 & 1 & 1/5 & -3/10 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\right] \quad R_2' \to \frac{R_2}{-2}$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 1/5 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1/5 & -3/10 & 0 \end{array}\right)\right] \quad \text{Interchanging } R_2 \text{ and } R_3$$

$$\Rightarrow A = \left[\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 1/5 & 0 \\ 0 & 1 & 0 & -1/5 & 3/10 & 1 \\ 0 & 0 & 1 & 1/5 & 3/10 & 0 \end{array}\right)\right] \quad R_2' \to R_2 - R_3$$

$$\therefore A^{-1} = \begin{bmatrix} 1/5 & 1/5 & 0 \\ -1/5 & 3/10 & 1 \\ 1/5 & 3/10 & 0 \end{bmatrix}$$

H.W:

1) Find the Rank and Nullity of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ reducing it to echelon form.

2) Find the Rank of the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -1 & 3 \\ 1 & -1 & -3 & 1 & 1 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & -1 \end{bmatrix}$ reducing it to echelon form.

3) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$ using Identity matrix

and reducing it to Row Echelon form.