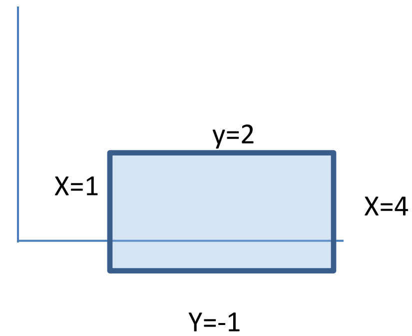
1. If R be a region bounded by x=1, x=4, y=-1 and y=2 then evaluate $\iint (2x+6x^2y)dydx$

Solution: Here R is a bounded parallelogram area with x=1, x=4, y=-1 and y=2.

So, $R = \{(x,y): 1 \le x \le 4, -1 \le y \le 2$



$$\iint (2x+6x^2y)dydx$$

$$= \int_1^4 \int_{-1}^2 (2x+6x^2y)dydx$$

$$= \int_1^4 \left[2xy + 3x^2y^2\right]_{-1}^2 dx$$

$$= \int_1^4 (6x+9x^2)dx$$

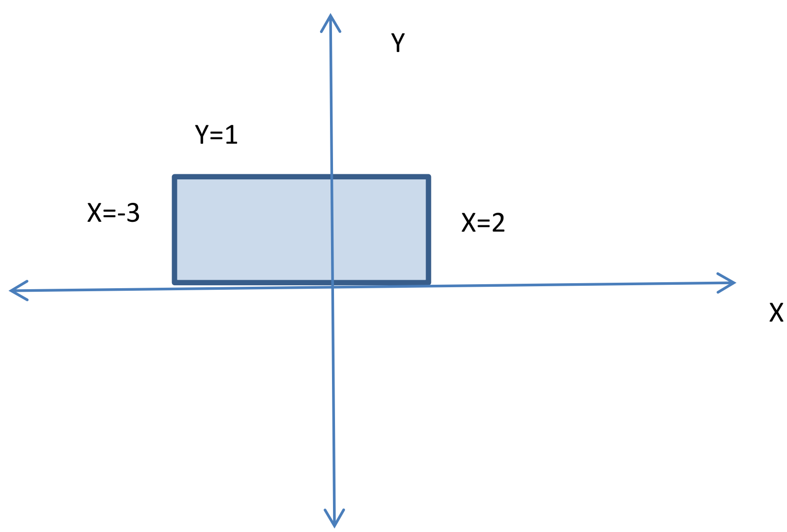
$$= \left[3x^2 + 3x^3\right]_1^4$$

$$= 234$$

2. Evaluate the following double integral over the rectangular region $R = \{(x,y): -3 \le x \le 2, 0 \le y \le 1\}$ and $\iint y^2 x dA$

Solution: Given that $R = \{(x,y): -3 \le x \le 2, 0 \le y \le 1\}$

So the region R is bounded by x=-3, x=2, y=0 and y=1



$$\iint y^2 x dA$$

$$= \int_{-3}^2 \int_0^1 (y^2x)dydx$$

$$= \int_{-3}^2 \left[\frac{y^3}{3}\right]_0^1 xdx$$

$$= \frac{1}{3}\int_{-3}^2 xdx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}\right]_{-3}^2$$

$$= \frac{1}{3}(2-\frac{9}{2})$$

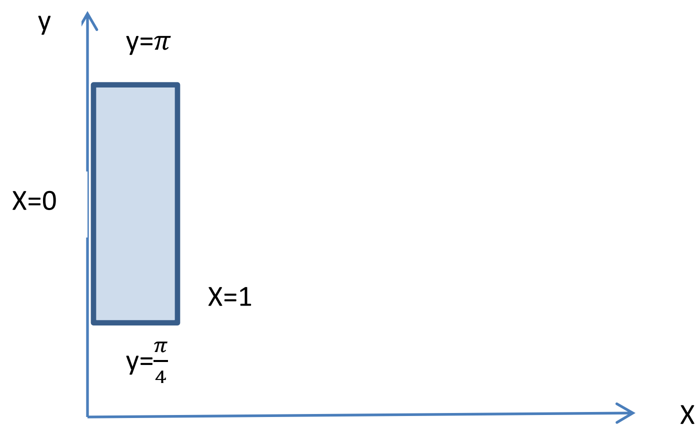
$$= -\frac{5}{6}$$

3. Evaluate the following double integral $\iint e^x \cos y dA$ over the rectangular region

$R = \{(x, y) : 0 \le x \le 1, \frac{\pi}{4} \le y \le \pi\}$

Solution: Given that $R = \{(x, y) : 0 \le x \le 1, \frac{\pi}{4} \le y \le \pi\}$

So the region R is bounded by x=0, x=1, y= $\frac{\pi}{4}$ and y= π



$$\iint e^x \cos y dA$$

$$= \int_0^1 \int_{\frac{\pi}{4}}^{\pi} e^x \cos y dy dx$$

$$= \int_0^1 [\sin y]_{\frac{\pi}{4}}^{\pi} e^x dx$$

$$= \int_0^1 (0 - \frac{1}{\sqrt{2}}) e^x dx$$

$$= -\frac{1}{\sqrt{2}} \int_0^1 e^x dx$$

$$= -\frac{1}{\sqrt{2}} \left[e^x\right]_0^1$$

$$= -\frac{1}{\sqrt{2}} (e - 1)$$

$$= \frac{1}{\sqrt{2}} (1 - e)$$

4. Evaluate $\iint (x + y) d(x, y)$ over the region bounded by the lines x=1, x=2, y=-x+2 and y=x+1.

Solution: as R is bounded by the lines x=1, x=2, y=-x+2 and y=x+1.

So,

$$\iint (x+y)\,d(x,y)$$

$$= \int_1^2 \int_{-x+2}^{x+1} (x+y)\,dy\,dx$$

$$= \int_1^2 \left[xy + \frac{y^2}{2} \right]_{-x+2}^{x+1} dx$$

$$= \int_1^2 \left[x^2 + x + \frac{x^2+2x+1}{2} + x^2 - 2x - \frac{x^2-4x+4}{2} \right] dx$$

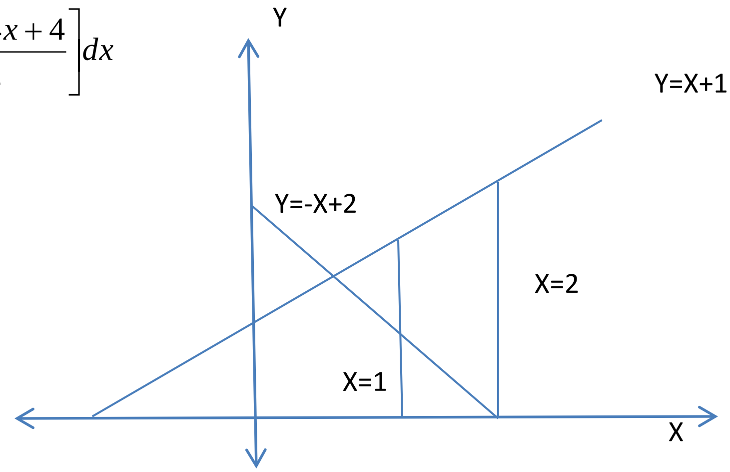
$$= \frac{1}{2}\int_1^2 (4x^2 + 4x - 3)\,dx$$

$$= \frac{1}{2}\left[\frac{4x^3}{3} + 2x^2 - 3x \right]_1^2$$

$$= \frac{1}{2}\left[(\frac{32}{3} + 8 - 6) - (\frac{4}{3} + 2 - 3) \right]$$

$$= \frac{1}{2}(\frac{38}{3} - \frac{1}{3})$$

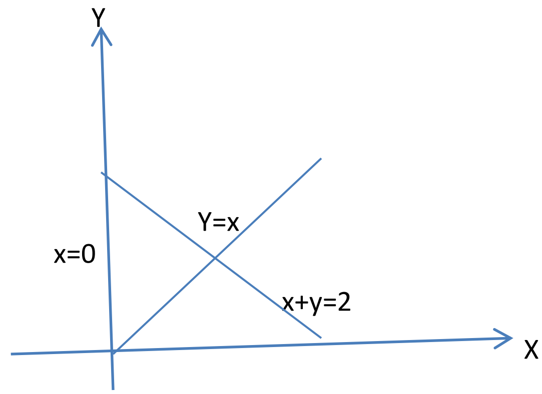
$$= \frac{37}{6}$$



5. Evaluate $\iint \frac{dA}{1+x+y}$ over the region bounded by the lines x=0, y=-x andx+y=2.

Solution: solving y=x and x+y=2 we get x=1,y=1

So, R is bounded by the lines x=1, x=0, y=x and y=2-x.



$$\iint \frac{dA}{1+x+y}$$

$$= \int_0^1 \int_x^{2-x} \frac{dydx}{1+x+y}$$

$$= \int_0^1 [\ln(1+x+y)]_x^{2-x} dx$$

$$= \int_0^1 [\ln 3 - \ln(1+2x)] dx$$

$$= \ln 3 [x]_0^1 - [x \ln(1+2x)]_0^1 + \int_0^1 \frac{2x}{1+2x} dx$$

$$= \ln 3 - \ln 3 + \int_0^1 \frac{1+2x-1}{1+2x} dx$$

$$= [x - \frac{1}{2} \ln(1+2x)]_0^1$$

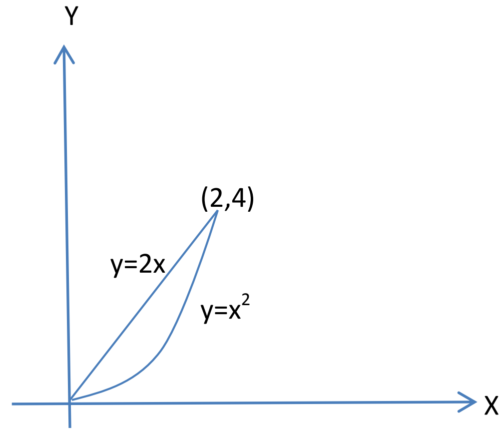
$$= 1 - \frac{1}{2} \ln 3$$

6. Evaluate $\iint (x^3+4y)dydx$ over the region bounded by the lines y=2x and y=x^2

Solution: solving y=2x and y=x^2

We get x=0,2 and y=0,4

So, R is bounded by the lines x=0,x=2, y=2x and y=x^2



$$\iint (x^3+4y)dydx$$

$$=\int_0^2\int_{x^2}^{2x}(x^3+4y)dydx$$

$$.=\int_0^2[x^3y+2y^2]_{x^2}^{2x}dx$$

$$=\int_0^2(8x^2-x^5)dx$$

$$=\frac{32}{3}$$

7. If dA is the elementary area in the region R enclosed by the lines $y^2+x=0$ and $y=x+2$ the evaluate $\iint dA$

Solution: solving $y^2+x=0$ and $y=x+2$ we get x=-1,-4 and y=1,-2

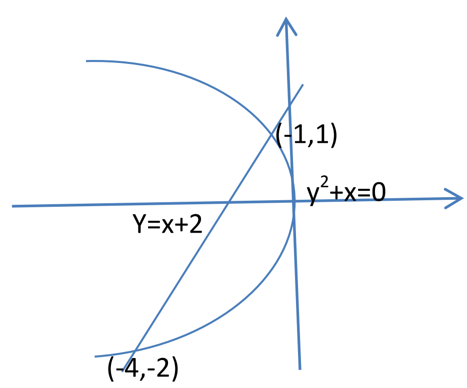
So, R is bounded by the lines x=y-2,x=-y^2, y=-1 and y=1

$$\iint dA$$

$$=\int_{-2}^{1}\int_{y-2}^{-y^2}dxdy$$

$$=\int_{-2}^{1}(-y^2-y+2)dy$$

$$=\left[2y-\frac{y^2}{2}-\frac{y^3}{3}\right]_{-2}^{1}$$



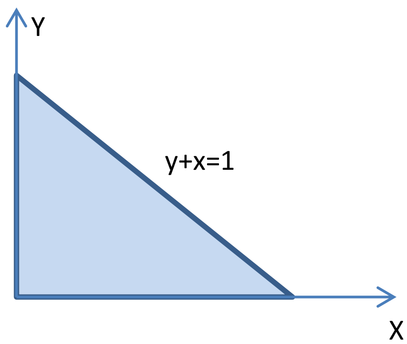
$$=\frac{7}{6}+\frac{10}{3}=\frac{9}{2}$$

8. Evaluate $\iint (x^2+y^2)dxdy$ where the region R is enclosed in first quadrant by $x+y \le 1$

Solution: given that R is enclosed in first quadrant by $x+y \le 1$

x+y=1 intersects X-axis and Y-axis at (1,0) 1nd (0.1) respectively.

So, R is bounded by the lines x=0,x=1, y=0 and y=1-x



$$\iint (x^2+y^2)dxdy$$

$$= \int_0^1 \int_0^{1-x} (x^2+y^2)dydx$$

$$= \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_0^{1-x} dx$$

$$= \frac{1}{3}\int_0^1 (x^2 - x^3 + \frac{1-3x+3x^3-x^3}{3})dx$$

$$= \frac{1}{3}\int_0^1 (1-3x+6x^2-4x^3)dx$$

$$= \frac{1}{3}\left[x - \frac{3x^2}{2} + 2x^3 - x^4 \right]_0^1$$

$$= \frac{1}{6}$$

9. Evaluate $\iint (x^2+y^2)dxdy$ where the region R is trangle area with vertices (0,0),(1,0) and (1.1).

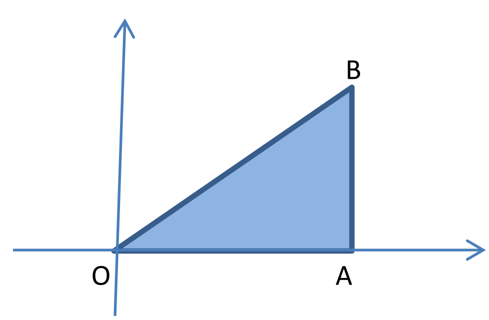
Solution: Let OAB is the required triangle where O(0,0), A(1,0) and B(1,1)

Equation of AB is x=1

Equation of OA is y-0

Equation of OB is y=x

So, R is bounded by the lines x=0,x=1, y=0 and y=x



$$\iint (x^2 + y^2)dxdy$$

$$= \int_0^1 \int_0^x (x^2 + y^2)dydx$$

$$= \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_0^x dx$$

$$= \frac{1}{3} \int_0^1 (x^3 + \frac{x^3}{3})dx$$

$$= \frac{4}{3} \left[\frac{x^4}{4} \right]_0^1$$

$$= \frac{1}{3}$$