Probability

Definition: If an event can happen in **A** ways and fail to happen in **B** ways and each of this ways is equally likely, the probability of its happening is $\frac{A}{A+B}$ and that of its failing to happen is $\frac{B}{A+B}$.

$$\text{Therefore, Probability} = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

Some Basic Concepts of Probability:

- An **event** is any collections or outcomes of a procedure.
- A **Simple Event** is an outcome or an event that cannot be further broken down into simpler components.
- The **sample space** for a procedure consists of all possible simple events.

Probability Sample Math

Ex.1) In a fair sample of the population 1500 male and 1400 female babies were reported. What is the probability of a child being born as a female?

Solution: Number of favorable cases=1400

Total number of cases=1400+1500=2900

$$\text{Therefore, Probability} = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}} = \frac{1400}{2900} = \frac{14}{29} = 0.483$$

Ex.2) Two coins are tossed, find the probability that two heads are obtained.

Solution: The sample space S is given by.
S = {(H, T), (H, H), (T, H), (T, T)}

Let E be the event "two heads are obtained".
E = {(H, H)}

Therefore, the probability is, P $= \frac{1}{4}$

Ex.3) Two dice are rolled, find the probability that the sum is

a) Equal to 1
b) equal to 4
c) less than 13

Solution
a) The sample space S of two dice is shown below.
S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Let E be the event.

a) There are no outcomes which correspond to a sum equal to 1.
Therefore, the probability is, P $= \frac{0}{36} = 0$

b) Three possible outcomes give a sum equal to 4.

E = {(1,3),(2,2),(3,1)}
Therefore, the probability is, P $= \frac{3}{36} = \frac{1}{12}$

c) All possible outcomes, E = S, give a sum less than 13, hence.
Therefore, the probability is, P $= \frac{36}{36} = 1$

Ex.4) A dice is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

Solution

The sample space S is,

S = { (1,H),(2,H),(3,H),(4,H),(5,H),(6,H)

(1,T),(2,T),(3,T),(4,T),(5,T),(6,T)}

Let E be the event .

E={(1,H),(3,H),(5,H)}

Therefore, the probability is, $P = \frac{3}{12} = \frac{1}{4}$

Ex.5) A class consists of 80 students. 10 of them are girls and 70 boys. 45 of them are rich and remaining poor. 20 of them are fair complexioned. What is the probability of selecting a fair complexioned rich girl?

Solution:

$$\text{Probability of selecting a fair complexioned person} = \frac{20}{80}$$

$$\text{Probability of selecting a rich person} = \frac{45}{80}$$

$$\text{Probability of selecting a girl} = \frac{10}{80}$$

$$\text{Therefore, Probability of selecting a fair complexioned rich girl} = \frac{20}{80} \times \frac{45}{80} \times \frac{10}{80} = \frac{9}{512}$$

Exercise:

1) A bag contains 20 red and 40 green balls. A ball is drawn at random. What is the probability that it is green?

2) A coin is tossed 3 times. What is the probability of getting a Head in all the three tosses?

3) An urn contains 10 black and 6 white balls. Find the probability that a blind folded person in one draw shall obtain a white ball.

<u>Independent and Dependent Events</u>

Independent Event

When multiple events occur, if the outcome of one event <u>DOES NOT</u> affects the outcome of the other events, they are called independent events.
Say, a die is rolled twice. The outcome of the first roll doesn't affect the second outcome. These two are independent events.

Dependent Events

When two events occur, if the outcome of one event affects the outcome of the other, they are called dependent events.

Ex.1) Consider a pack contains 4 blue, 2 red and 3 black pens.
If a pen is drawn at random from the pack**, replaced** and the process repeated 2 more times, what is the probability of drawing 2 blue pens and 1 black pen?

Solution:

Here, total number of pens = 9

$$\text{Probability of drawing 1 blue pen } = \frac{4}{9}$$

$$\text{Probability of drawing another blue pen} = \frac{4}{9}$$

$$\text{Probability of drawing 1 black pen} = \frac{3}{9}$$

$$\text{Probability of drawing 2 blue pens and 1 black pen} = \frac{4}{9} \times \frac{4}{9} \times \frac{3}{9} = \frac{48}{729} = \frac{16}{243}$$

Ex.2) A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, **not replaced** and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen?

Solution:

Probability of drawing 1 blue pen $= \dfrac{4}{9}$

Probability of drawing another blue pen $= \dfrac{3}{8}$

Probability of drawing 1 black pen $= \dfrac{3}{7}$

Probability of drawing 2 blue pens and 1 black pen $= \dfrac{4}{9} \times \dfrac{3}{8} \times \dfrac{3}{7} = \dfrac{48}{729} = \dfrac{1}{14}$

Ex.3) What is the probability of drawing a king and a queen consecutively from a deck of 52 cards**, without** replacement?

Solution: Probability of drawing a king $= \frac{4}{52} = \frac{1}{13}$

After drawing one card, the number of cards are 51.

Probability of drawing a queen $= \dfrac{4}{51}$

Now, the probability of drawing a king and queen consecutively is $\dfrac{1}{13} \times \dfrac{4}{51} = \dfrac{4}{663}$

Ex.4) A box contains 4 chocobars and 4 ice creams. Tom eats 3 of them, by randomly choosing. What is the probability of choosing 2 chocobars and 1 icecream?

Solution:

Probability of choosing 1 chocobar $= \dfrac{4}{8}$

After taking out 1 chocobar, the total number is 7.

Probability of choosing 2nd chocobar $= \dfrac{3}{7}$

Probability of choosing 1 icecream out of a total of 6 $= \dfrac{4}{6}$

So the final probability of choosing 2 chocobars and 1 icecream $= \frac{4}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{1}{7}$

H.W:

1) A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?

2) The blood groups of 200 people is distributed as follows: 50 have type **A** blood, 65 have **B** blood type, 70 have **O** blood type and 15 have type **AB** blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

3) A dice is rolled, find the probability that the number obtained is greater than 4.

4) Two dice are rolled, find the probability that the sum is equal to 5.

5) A card is drawn at random from a deck of cards. Find the probability of getting the King of heart.