Lecture-3

Home Work Solution

1. Solution:-

The limiting value along x-axis

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{x\to 0}\lim_{y\to 0}\frac{x^2y^2}{x^2+y^2}$$

$$= \lim_{x\to 0}\frac{x^2 \cdot 0}{x^2+0}$$

$$= \lim_{x\to 0}\frac{0}{x^2} = 0$$

The limiting value along Y-axis

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0}\frac{x^2y^2}{x^2+y^2}$$

$$= \lim_{y\to 0}\frac{0\cdot y^2}{0+y^2} = \lim_{y\to 0}\frac{0}{y^2}$$

$$= 0$$

Now the limiting value along $y=mx$

$$\lim_{x\to 0} f(x,mx) = \lim_{x\to 0} \frac{x^2\cdot(mx)^2}{x^2+(mx)^2}$$

$$= \lim_{x\to 0} \frac{m^2x^4}{x^2+m^2x^2}$$

$$= \lim_{x\to 0} \frac{m^2x^4}{x^2(1+m^2)}$$

$$= \lim_{x\to 0} \frac{m^2}{1+m^2}\cdot x^2$$

$$= \frac{m^2}{1+m^2} \lim_{x\to 0} x^2$$

$$= \frac{m^2}{1+m^2}\cdot 0$$

$$= 0$$

Again, the limiting value along $y=x^2$

$$\lim_{x\to 0} f(x,x^2) = \lim_{x\to 0} \frac{x^2\cdot(x^2)^2}{x^2+(x^2)^2} = \lim_{x\to 0} \frac{x^6}{x^2+x^4}$$

$$= \lim_{x\to 0} \frac{x^6}{x^2(1+x^2)} = \lim_{x\to 0} \frac{x^4}{1+x^2}$$

$$= \frac{0}{1+0} = 0$$

Since the limiting value along all direction are same, which is 0.

So Limiting value $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

Now, functional value $f(0,0) = 0$.

Since $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$

$\therefore$ The function $f(x,y)$ is continuous at $(0,0)$. [Showed]

2. Solution:-

The limiting value along x-axis,

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{x\to 0}\lim_{y\to 0} \frac{xy}{x^2+y^2}$$

$$= \lim_{x\to 0} \frac{x\cdot 0}{x^2+0} = \lim_{x\to 0} \frac{0}{x^2}$$

$$= 0$$

The limiting value along y-axis

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0} \frac{xy}{x^2+y^2}$$

$$= \lim_{y\to 0} \frac{0\cdot y}{0+y^2} = \lim_{y\to 0} \frac{0}{y^2}$$

$$= 0.$$

Now the limiting value along $y = mx$

$$\lim_{x\to 0} f(x,mx) = \lim_{x\to 0} \frac{x\cdot mx}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2(1+m^2)}$$

$$= \lim_{x\to 0} \frac{m}{1+m^2} = \frac{m}{1+m^2}$$

The limiting value along x and y-axis are same but along $y=mx$ the value is different.

So $\lim_{(x,y)\to(0,0)} f(x,y)$ does not ~~ex~~ exist.

Thus the function $f(x,y)$ is discon-tinuous at $(0,0)$ [showed]

3. Solution:-

The limiting value along x-axis

$$\lim_{x\to0}\lim_{y\to0} f(x,y) = \lim_{x\to0}\lim_{y\to0}\frac{x^3y}{2x^6+y^2}$$

$$= \lim_{x\to0}\frac{x^3\cdot 0}{2x^6+0}$$

$$= \lim_{x\to0}\frac{0}{2x^6} = 0$$

The limiting value along y-axis

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0}\frac{x^3 y}{2x^6+y^2}$$

$$= \lim_{y\to 0}\frac{0\cdot y}{0+y^2} = 0$$

Now, the limiting value along $y=mx$

$$\lim_{x\to 0} f(x,mx) = \lim_{x\to 0}\frac{x^3 \cdot mx}{2x^6+m^2x^2} = \lim_{x\to 0}\frac{mx^4}{x^2(2x^4+m^2)}$$

$$= \lim_{x\to 0}\frac{mx^2}{2x^4+m^2}$$

$$= 0$$

Again, the limiting value along $y=x^3$

$$\lim_{x\to 0} f(x,x^2) = \lim_{x\to 0}\frac{x^3\cdot x^3}{2x^6+x^6}$$

$$= \lim_{x\to 0}\frac{x^6}{3x^6} = \lim_{x\to 0}\frac{1}{3}$$

$$= \frac{1}{3}$$

So, the limiting value along ~~x-ax~~ X-axis, Y-axis and $y = mx$ are same but along $y = x^3$ the value is different.

So, $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

$\therefore f(x,y)$ is discontinuous at $(0,0)$.

[showed]