Probability Distributions

Random Variable: If each value of a variable is associated with a defined probability, that variable is called random variable.

For example, If 5 coins are tossed 0, 1, 2, 3, 4 and 5; flowers may appear. The numbers of flower is a random variable.

Probability Distribution: If the values of a random variable are arranged according to their probabilities, the distribution is called probability distribution of the random variable.

Binomial Distribution: Let, p be the probability of occurrence and q be the probability of not occurrence of a particular event in a single trial so that $p + q = 1$.

If the experiment is repeated for n independent trials, the probability of occurrence of an event x times (and not occurring $n - x$ times) may be expressed as,

The probability density function, $P(x) = \binom{n}{x} p^x q^{n-x}, \qquad x = 0,1,2, \dots \dots \dots \dots, n$

Conditions of Binomial Distribution:

- There is a fixed number of trials.
- The trials are independent.
- There are only two outcomes for each trial such as success and failure.
- Probability of success remains constant from trial to trial.

Properties of Binomial Distribution:

- It is a discrete probability distribution with parameters n and p.

  (A discrete distribution describes the probability of occurrence of each value of a discrete random variable. A discrete random variable is a random variable that has countable values, such as a list of non-negative integers. Thus, a discrete probability distribution is often presented in tabular form.)

- It's $\text{Mean} = np, \text{Variance} = npq$. Mean is greater than the variance.

- Binomial Distribution tends to Poisson Distribution if the number of trials, n is very large ($n \to \infty$) and the probability of success, p is very small ($p \to 0$).

Ex. 1) An unbiased coin is tossed 6 times. Find the probability of getting (a) exactly 3 flowers, (b) at least 5 flowers, (c) at best 3 flowers, using binomial distribution.

Solution: Number of trials in the experiment, $n = 6$

Let, number of flowers, $x = 0,1,2,3,4,5,6$

The probability density function, $P(x) = \binom{n}{x} p^x q^{n-x} = \binom{6}{x} p^x q^{6-x}$ ……………………… (i)

Since the coin is unbiased, $p = q = \frac{1}{2}$

From (i),

$$P(x) = \binom{6}{x} p^x q^{6-x} = \binom{6}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{6-x} = \binom{6}{x}\left(\frac{1}{2}\right)^6$$

(a) Probability of getting exactly 3 flowers:

$$P(x = 3) = \binom{6}{3}\left(\frac{1}{2}\right)^6 = \frac{6!}{(6-3)!} \times \frac{1}{64} = \frac{5}{16}$$

(b) Probability of getting at least 5 flowers:

$$P(x \geq 5) = P(x = 5) + P(x = 6) = \binom{6}{5}\left(\frac{1}{2}\right)^6 + \binom{6}{6}\left(\frac{1}{2}\right)^6$$

$$= 6\left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \frac{6}{64} + \frac{1}{64} = \frac{7}{64}$$

(c) Probability of getting at best 3 flowers:

$$P(x \leq 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$= \binom{6}{0}\left(\frac{1}{2}\right)^6 + \binom{6}{1}\left(\frac{1}{2}\right)^6 + \binom{6}{2}\left(\frac{1}{2}\right)^6 + \binom{6}{3}\left(\frac{1}{2}\right)^6$$

$$= \left(\frac{1}{2}\right)^6\left[\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3}\right] = \frac{1}{64} \times 42 = \frac{21}{32}$$

Ex. 2) An unbiased coin is tossed 6 times. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively.
Find the probability density funtion of at best 1 flower.

Solution: Given, , mean, $np = 4$ and variance, $npq = \frac{4}{3}$

Now, $\frac{npq}{np} = \frac{\frac{4}{3}}{4} = \frac{1}{3}$

$q = \frac{1}{3}$

$\therefore p = 1 - \frac{1}{3} = \frac{2}{3}$

again, $np = 4 \Rightarrow n\left(\frac{2}{3}\right) = 4 \Rightarrow n = 6$

The probability density function, $P(x \leq 1) = P(x = 0) + P(x = 1)$

$$= \binom{6}{0}\left(\frac{2}{3}\right)^{0}\left(\frac{1}{3}\right)^{6-0} + \binom{6}{1}\left(\frac{2}{3}\right)^{1}\left(\frac{1}{3}\right)^{6-1}$$

$$= \frac{1}{729} + 6 \times \frac{2}{3} \times \frac{1}{243} = \frac{13}{729}$$

<u>H.W:</u>

1) 80% of people those who purchase pet insurance are women. If the owners of 9 pet insurance are randomly selected, then find the probability density function, that exactly 6 out of them are women. Also find mean and variance.

$n = 9$ $x = 6$ $p = 0.80$ $q = 0.20$

2) Using binomial distribution, find the probability density function if a coin is tossed 10 times then what are the chances of getting exactly 6 heads?

$n = 10$ $x = 6$ $p = q = \frac{1}{2}$

3) You sell sandwiches. 70% of people choose chicken, the rest choose something else. What is the probability of selling 2 chicken sandwiches to the next 3 customers?

$n = 3$ $x = 2$ $p = 0.70$ $q = 0.30$