Probability

Bayes' Theorem: Let us consider n mutually exclusive events $E_1, E_2, \dots\dots\dots\dots\dots\dots, E_n$ such that $P(E_i) > 0$ for all $i = 1,2, \dots\dots\dots\dots, n$ and if E be an event such that, $P(E) > 0$, then

$$P(E_i|E) = \frac{P(E_i)\ P(E|E_i)}{\sum_{i=1}^{n}\{P(E_i)\ P(E|E_i)\}}$$

Ex.1) In a class of 60 students, 15 are girls, office record indicates that 80 percent of the girl students and 75 percent of the boy students got star marks in HSC. One student is randomly chosen and found to have secured star marks. Find the probability that the chosen student is a girl.

Solution: Let us suppose that,
E_1 be the event that a student is a boy.
E_2 be the event that a student is a girl.
E be the event that a student secures star mark.
Given,

$$P(E_1) = \frac{60-15}{60} = 0.75$$

$$P(E_2) = \frac{15}{60} = 0.25$$

$$P(E|E_1) = 0.75$$

$$P(E|E_2) = 0.80$$

We need to find, $P(E_2|E)$
According to Byes' Theorem,

$$P(E_2|E) = \frac{P(E_2)\ P(E|E_2)}{P(E_1)\ P(E|E_1) + P(E_2)\ P(E|E_2)} = \frac{0.25 \times 0.80}{0.75 \times 0.75 + 0.25 \times 0.80} = 0.26$$

Ex.2) In an MCQ system a question has 4 alternative answers of which only one is correct. An examinee is known to be aware of the correct answers of 60% of the questions and for the rest 40% questions he uses guessing. On examining a randomly selected question, it is found that the examinee has answered correctly. What is the probability that he knows it?

Solution: Let us suppose that,
E_1: the student knows the answer.
E_2: he guesses it.
E: he correctly answers a question.
Given,

$$P(E_1) = 0.60$$

$$P(E_2) = 0.40$$

$$P(E|E_1) = 1$$

$$P(E|E_2) = \frac{1}{4}$$

We need to find, $P(E_1|E)$
According to Byes' Theorem,

$$P(E_1|E) = \frac{P(E_1)\ P(E|E_1)}{P(E_1)\ P(E|E_1) + P(E_2)\ P(E|E_2)} = \frac{0.60 \times 1}{0.60 \times 1 + 0.40 \times \frac{1}{4}} = 0.857$$

Ex.3) In the month of monsoon 60% of the days are rainy and 40% of the days are sunny. It is experienced that meteorology department wrongly predicts 10% of the times in rainy days and 20% of the times on sunny days. Weather forecast indicates a certain day to be sunny. What is the probability that the forecast will be proved wrong?

Solution: Let us suppose that,
E_1: a day is rainy.
E_2: a day is sunny.
E: forecast indicates sunny day.
$\overline{E}$: forecast indicates rainy day.
Given,
$P(E_1) = 0.60$
$P(E_2) = 0.40$
$P(E|E_1) = 0.10$
$P(\overline{E}|E_2) = 0.20$
$P(E|E_2) = 1 - P(\overline{E}|E_2) = 1 - 0.20 = 0.80$
We need to find, $P(E_1|E)$
According to Byes' Theorem,

$$P(E_1|E) = \frac{P(E_1)\,P(E|E_1)}{P(E_1)\,P(E|E_1) + P(E_2)\,P(E|E_2)} = \frac{0.60 \times 0.10}{0.60 \times 0.10 + 0.40 \times 0.80} = 0.1579$$

H.W:
1) A doctor can give correct diagnosis in 70% cases. Past records indicate that 20% of his patients die after correct diagnosis while the chance of death by wrong diagnosis is 90%. One of his patients died. Find the probability that the diagnosis was wrong.

2) In 2021 there will be three candidates for the position of principal- Mr. Chatterji, Mr. Ayangar and Dr. Singh- whose chances of getting the appointment are in the proportion $4:2:3$ respectively. The probability that Mr. Chatterji if selected would introduce co-education in the college is 0.3. The probabilities of Mr. Ayangar and Dr. Singh doing the same are respectively 0.5 and 0.8.
If there is co-education in the college in 2022, what is the probability that Dr. Singh is the principal?

3) The probabilities of X, Y and Z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively.
If the bonus scheme has been introduced, what is the probability that the manager appointed was X?

4) A factory produces a certain type of outputs by three types of machines. The respective daily production figures are:
Machine 1: 3000 units Machine 2: 2500 units Machine 3: 4500 units
Past experience shows that, 1% percent of the output produced by Machine 1 is defective. The corresponding fractions of defectives for the other two machines are 1.2% and 2% respectively.
An item is drawn at random from the day's production and is found to be defective. What is the probability that it comes from the output of,
(i) Machine 1 (ii) Machine 2 (iii) Machine 3