Probability Distributions

Poisson Distribution: Like binomial distribution, Poisson Distribution is also a discrete probability distribution. The variance of this distribution assumes values 0, 1, 2… Etc. But there is no limit of the highest value. The distribution is a limiting form of binomial distribution and has only one parameter.

If the probability density function of a discrete non-negative random variable x is given by,

$$\text{The probability density function, } P(x) = \frac{e^{-m}\, m^x}{x!}\ , x = 0,1,2, \dots \dots \dots \dots \dots \dots \dots \infty \text{ and } m > 0$$

The probability distribution is known as Poisson distribution where, $e = 2.71828$ and m is the only parameter of the distribution.

Ex. 1) Find the probability density function of the following data by Poisson Distribution.
Results from 50 pots are given below:

No. of men (x_i)	0	1	2	3	4	5
No. of pot (f_i)	25	15	7	5	2	1

Solution: The probability density function, $P(x) = \frac{e^{-m}\, m^x}{x!}$

Here, $e = 2.71828, x = 6$

$$\text{Mean, } m = \sum \frac{f_i x_i}{N} = \frac{57}{50} = 1.14$$

$$\text{The probability density function, } P(x) = \frac{e^{-m}\, m^x}{x!} = \frac{(2.71828)^{-1.14}\,(1.14)^6}{6!} = 9.74 \times 10^{-4}$$

Ex. 2) Find the probability density function by poisson distribution of at most 4 defective fuses will be found in a box of 200 fuses, If experiment shows that 1 percent of such fuses is defective.

Solution: Here, given,

$$\text{Probability of defective fuses, } p = \frac{1}{100} = 0.01, n = 200$$

Mean, $m = np = 200 \times 0.01 = 2$

Now,

$$\text{The probability density function, } P(x) = \frac{e^{-m}\, m^x}{x!}$$

$$P(x \le 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$$

$$= \frac{(2.71828)^{-2}\, 2^0}{0!} + \frac{(2.71828)^{-2} 2^1}{1!} + \frac{(2.71828)^{-2}\, 2^2}{2!} + \frac{(2.71828)^{-2}\, 2^3}{3!} + \frac{(2.71828)^{-2}\, 2^4}{4!}$$

$$= (2.71828)^{-2} \left[1 + 2 + 2 + \frac{4}{3} + \frac{2}{3}\right] = (2.71828)^{-2} \times 7 = 0.9473$$

Ex. 3) The mean of death of cattle per day in a village is 2.5. What is the probability of no death in a particular day.

Solution: Here, given,
Mean, $m = 2.5$

The probability density function, $P(x) = \frac{e^{-m}\ m^x}{x!} = \frac{e^{-2.5}\ (2.5)^x}{x!}$

Probability of no death in a day,
$P(x = 0) = \frac{e^{-2.5}\ (2.5)^0}{0!} = e^{-2.5} = 0.0821$

Ex. 4) Suppose the average number of lions seen on a 1 day safari is 5.
What is the probability that tourists will see fewer than four lions on the next 1day safari
Find the probability density function by poisson distribution.

Solution: Here, given,

Mean, $m = 5$ (since 5 lions are seen per safari, on average)
x = 0, 1, 2, 3 (since we want to find the likelihood that tourists will see fewer than 4 lions)

The probability density function, $P(x) = \frac{e^{-m}\ m^x}{x!}$
$P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$$= \frac{(2.71828)^{-5}\ 5^0}{0!} + \frac{(2.71828)^{-5} 5^1}{1!} + \frac{(2.71828)^{-5}\ 5^2}{2!} + \frac{(2.71828)^{-2}\ 5^3}{3!}$$

$$= (2.71828)^{-5}\ \left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right] = 0.2650$$

Ex. 5) If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

Solution: Here,
The average number of failures per week, $m = \frac{3}{20} = 0.15$
x = 0, 1
The probability density function, $P(x) = \frac{e^{-m}\ m^x}{x!}$

$P(x \leq 1) = P(x = 0) + P(x = 1)$
$$= \frac{(2.71828)^{-0.15}\ 0.15^0}{0!} + \frac{(2.71828)^{-0.15} 0.15^1}{1!}$$
$$= (2.71828)^{-0.15}\ [1 + 0.15] = 0.98981$$

H.W:

1) A bank is interested in studying the number of people who use the ATM located outside its office late at night.
On average, 1.6 customers walk up to the ATM during any 10 minute interval between 9pm and midnight.

a. What is the probability of exactly 3 customers using the ATM during any 10 minute interval?
b. What is the probability of 3 or fewer people?

2) The Indiana Department of Transportation is concerned about the number of deer being struck by cars between Martinsville and Bloomington. They note the number of deer carcasses and other deer-related accidents over a 1-month period is 3.9 on average.
What is the probability of zero deer strike incidents between Martinsville and Bloomington?

3) A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's distribution to calculate the probability that in a given week he will sell

c. Not more than one policy.
d. 2 or more policies but less than 5 policies.
e. Assuming that there are 5 working days per week, what is the probability that in a given day he will sell one policy?

4) Twenty sheets of aluminum alloy were examined for surface flaws. The frequency of the number of sheets with a given number of flaws per sheet was as follows:

Number of flaws	0	1	2	3	4	5	6
Frequency	4	3	5	2	4	1	1

Find the probability by Poisson distribution of finding a sheet chosen at random which contains 3 or more surface flaws?

5) Vehicles pass through a junction on a busy road at an average rate of 300 per hour.

a. Find the probability that none passes in a given minute.
b. What is the expected number passing in two minutes?
c. Find the probability that this expected number actually pass through in a given two-minute period.

6) A company makes electric motors. The probability an electric motor is defective is 0.01.
What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?