Probability

Ex.1) A bag contains 10 white, 8 red and 5 black balls. Find the chance that three balls drawn at random are all white.

Ex.2) A bag contains 8 green and 10 white balls. Two drawings of 2 balls are made such that, the balls are **not replaced** before the second trial. Find the probability that the first drawing will give 2 green and the second 2 white balls.

Compound probability: Compound probability is when the problem statement asks for the likelihood of the occurrence of **more than one outcome**.

Formula for compound probability

- P (A or B) = P (A) + P (B) – P (A and B)

Where, A and B are any two events.

P (A or B) is the probability of the occurrence of **at least one** of the events.

P (A and B) is the probability of the occurrence of **both** A and B at the same time.

Mutually Exclusive Events: When two events cannot occur at the same time, they are considered mutually exclusive.

Note: For a mutually exclusive event, P (A and B) = 0.

- P (A or B) = P (A) + P (B)

Ex.1) What is the probability of getting a 2 **or** a 5 when a dice is rolled?

Solution:

Here, The probability of getting a 2, $P(2) = \frac{1}{6}$

and, The probability of getting a 5, $P(5) = \frac{1}{6}$

Applying the formula of compound probability,

Probability of getting a 2 **or** a 5,
P (2 or 5) = P (2) + P (5) – P (2 and 5)

$$= \frac{1}{6} + \frac{1}{6} - 0 = \frac{2}{6} = \frac{1}{3}$$

Ex.2) Find the probability of selecting a black card or a 6 from a deck of 52 cards.

Solution:
We need to find out P (B or 6),

Probability of selecting a black card, $P(B) = \frac{26}{52}$

Probability of selecting a 6, $P(6) = \frac{4}{52}$

Probability of selecting both a black card and a 6, $P(\text{B and } 6) = \frac{2}{52}$

Applying the formula of compound probability,

Probability of getting a Black Card **or** a 6,

P (B or 6) = P (B) + P (6) – P (B and 6)

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

Ex. 3) what is the probability of the occurrence of a number that is odd or less than 5 when a fair dice is rolled?

Solution:
Let, the event of the occurrence of a number that is odd=A

The event of the occurrence of a number that is less than 5 =B

We need to find P (A or B).

$P(A) = \frac{3}{6}(\text{odd numbers} = 1,3 \text{ and } 5)$

$P(B) = \frac{4}{6}$ (numbers less than $5 = 1,2,3$ and 4)

$P(\text{A and B}) = \frac{2}{6}(\text{numbers that are both odd and less than } 5 = 1 \text{ and } 3)$

Now, P (A or B) = P (A) + P (B) – P (A and B)

$$\Rightarrow \text{P(A or B)} = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$$

H.W:

1) What is the probability of getting a 3 or a 6 when a dice is rolled?
2) Find the probability of selecting a Red card or 8 from a deck of 52 cards.
3) Find the probability of selecting a King or a Heart from a deck of 52 cards.
4) What is the probability of the occurrence of a number that is even or less than or equal to 6 when a fair dice is rolled?

Conditional probability: Conditional probability is calculating the probability of an event given that another event has already occurred.

The formula for conditional probability P (A|B), read as P (A given B) is

P (A|B) = P (A and B) / P (B)

Ex.1) In a class, 40% of the students study math and science. 60% of the students study math. What is the probability of a student studying science given he/she is already studying math?

Solution:
P (M and S) = 0.40

P (M) = 0.60

$$\text{P(S|M)} = \text{P(S and M)/P(M)} = \frac{0.40}{0.60} = \frac{2}{3}$$

Ex.2) When two dice are rolled, find the probability of getting a greater number on the first dice than the one on the second, given that the sum should equal 8.

Solution:
Let the event of getting a greater number on the first dice be G.

There are 5 ways to get a sum of 8 when two dice are rolled = {(2,6),(3,5),(4,4), (5,3),(6,2)}.

And there are two ways where the number on the first dice is greater than the one on the second given that the sum should equal 8, G = {(5,3), (6,2)}.

Therefore, P (Sum equals 8) $= \frac{5}{36}$

and, P(G and sum equals 8) $= \frac{2}{36}$

Now, P (G and sum equals 8| Sum equals 8) $=$ P(G and sum equals 8)/P (Sum equals 8)

$= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$

Ex.3) let us consider a survey given to 52 students in a Basic Algebra course at ECC, with the following responses to the statement "I enjoy math."

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Total
Male	6	10	3	0	0	19
Female	8	14	7	4	0	33
Total	14	24	10	4	0	52

What is the probability that a student enjoys math (**Agree or Strongly Agree**) given that the student is a female?

Solution:
Let, the event of the student enjoys math $=$ E

Therefore, P (E and F) $= \frac{22}{52}$

Let, the event of the student be a female $=$ F

Therefore, P (F) $= \frac{33}{52}$

Now, P (E| F) $= P(E \text{ and } F)/P(F) = \frac{22}{33}$

Complement of an event: A complement of an event A can be stated as that which does NOT contain the occurrence of A.

A complement of an event is denoted as $P(A^c)$

$$\mathbf{P(A^c) = 1 - P(A)}$$

$$\mathbf{Or, it\ can\ be\ stated,\ P(A) + P(A^c) = 1}$$

Ex.1) A single coin is tossed 5 times. What is the probability of getting at least one head?

Solution:

Probability of getting no head $= P(\text{all tails}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$P(\text{at least one head}) = 1 - P(\text{all tails}) = 1 - \frac{1}{32} = \frac{31}{32}$

Ex.2) A problem of statistics is given to three students A, B and C whose chance of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{2}$ respectively. What is the probability that the problems will be solved?

Solution:

Probability that student A will fail to solve the problem $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability that student B will fail to solve the problem $= 1 - \frac{1}{4} = \frac{3}{4}$

Probability that student C will fail to solve the problem $= 1 - \frac{1}{2} = \frac{1}{2}$

Since, the events are independent the probability that all students will fail to solve the problem $= \frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} = \frac{1}{4}$

Therefore, Probability that the problem will be solved $= 1 - \frac{1}{4} = \frac{3}{4}$