# Probability

## Probability of n mutually exclusive events:

$P(A_1 + A_2 + \cdots\cdots\cdots\cdots\cdots\cdots + A_n) = P(A_1) + P(A_2) + \cdots\cdots\cdots\cdots\cdots + P(A_n)$

**Ex. 1) A card is randomly drawn from a well shuffled pack. What is the probability that the card will be either an Ace or a Queen of diamond?**

**Solution**: Let, the event that the card will be an ace $= A$

and, the event that the card will be a queen of diamond $= B$

$\therefore$ Probability of A, $P(A) = \frac{4}{52} = \frac{1}{13}$

and Probability of B, $P(B) = \frac{1}{52}$

Since, A and B are mutually exclusive, $P(A \text{ and } B) = 0$

Therefore, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= \frac{1}{13} + \frac{1}{52} - 0 = \frac{5}{52}$

**Ex. 2) An urn contains 4 red and 6 white balls. Two balls are drawn at random. What is the probability that both the balls are red.**

**Solution**: Total number of balls are $= 6 + 4 = 10$

2 balls can be drawn from 10 balls in $= 10_{C_2} = 45$ ways

2 balls can be drawn from 4 red balls in $= 4_{C_2} = 6$ ways

Therefore, probability $= \frac{6}{45} = \frac{2}{15}$

**Ex. 3) An urn contains 17 balls of which 8 white , 6 red and 3 blue.**
**Two balls are drawn at random. Find the pobability that,**
**(i) the balls are of the same color.**
**(ii) the balls are of different colors.**

**Solution**: From 17 balls, 2 balls can be drawn in $= 17_{C_2} = 136$ ways

(**i**) Let, the event that both the balls are white $= A$

The event that both the balls are red $= B$

and, the event that both the balls are blue $= C$

$\therefore P(A) = \frac{8_{C_2}}{136} = \frac{28}{136}, \qquad P(B) = \frac{6_{C_2}}{136} = \frac{15}{136}, \qquad P(C) = \frac{3_{C_2}}{136} = \frac{3}{136}$

Since, the events A, B and C are independent,
Hence, Probability that both balls are of same color is,

$$P(A + B + C) = P(A) + P(B) + P(C) = \frac{28}{136} + \frac{15}{136} + \frac{3}{136} = \frac{46}{136} = \frac{23}{68}$$

(**ii**) Two balls can be of different colors if,
One white and one red $=$ (WR)
One white and one blue $=$ (WB)
One red and one blue $=$ (RB)

One white and one red ball can be drawn in $= 8_{C_1} \times 6_{C_1} = 48$ ways

Hence, the probability, $P\ (WR) = \frac{48}{136}$

Similarly,

Probability of one white and one blue ball, $P\ (WB) = \frac{8_{C_1} \times 3_{C_1}}{136} = \frac{24}{136}$

Probability of one red and one blue ball, $P\ (RB) = \frac{6_{C_1} \times 3_{C_1}}{136} = \frac{18}{136}$

Hence, Probability that the two balls will be of different color is,

$$P\ (WR) + P\ (WB) + P\ (RB) = \frac{48}{136} + \frac{24}{136} + \frac{18}{136} = \frac{90}{136} = \frac{45}{68}$$

**Ex. 4) There are 20 tickets numbered 1, 2, ⋯ ⋯ ⋯ ⋯ , 20. A ticket is chosen at random. Find the probability that the serial number of the ticket will be multiple of 2 or 5.**

**Solution**: The number of equally likely cases is 20.

Let, A and B be the event that the ticket number is a multiple of 2 and 5 respectively.

Numbers multiple of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

$\therefore P(A) = \frac{10}{20}$

Numbers multiple of 5 are 5, 10, 15, 20

$\therefore P(B) = \frac{4}{20}$

Numbers multiple of both 2 and 5 are 10, 20

$\therefore$ $P(\text{A and B}) = \frac{2}{20}$

Probability that the chosen ticket will be multiple of 2 or 5,

$P(\text{A or B}) = P(A) + P(B) - P(\text{A and B}) = \frac{10}{20} + \frac{4}{20} - \frac{2}{20} = \frac{12}{20} = \frac{3}{5}$

**Ex. 5) Four Biologists, Two Economists, Three Mathematicians and one Engineer are working in a certain organization. A four member team is to be formed for a certain purpose. Find the probability that the team will include,**
**(i) at least one Mathematician (ii) the Engineer**

**Solution**: Total number of employees $= 4 + 2 + 3 + 1 = 10$
4 members can be selected from these 10 in $= 10_{C_4} = 210$ ways

(**i**) If no mathematician is included,
4 members can be selected from remaining 7 in $= 7_{C_4} = 35$ ways

Hence, the probability that there will be no mathematician is $= \frac{35}{210} = \frac{1}{6}$

Therefore, the probability that there will be at least one mathematician is $= 1 - \frac{1}{6} = \frac{5}{6}$

(**ii**) If the engineer is included,
3 members can be selected from remaining 9 in $= 9_{C_3} = 84$ ways
So, favourable cases with one engineer and three others will form the team $= 1 \times 84 = 84$ ways
$\therefore$ The probability $= \frac{84}{210} = \frac{2}{5}$

**<u>H.W:</u>**
1) An urn contains 5 red and 6 white balls. Three balls are drawn at random. What is the probability that all the three balls are white.

2) A card is randomly drawn from a well shuffled pack. What is the probability that the card will be either an Honors card or a spade?

3) There are 20 tickets numbered $1,2,\cdots\cdots\cdots\cdots,20$. A ticket is chosen at random.
Find the probability that the serial number of the ticket will be multiple of 3 or 5.

4) An urn contains 12 white, 10 black and 8 red balls. Another urn contains 10 white, 9 black and 10 red balls. One ball is randomly drawn from each urn. Find the probability that
(i) both the balls will be white. (ii) both marbles will be of same color

5) A random draw of two balls are made from an urn containing 2 red, 3 blue and 4 white balls. Find the probability that
(i) the balls are of the same color.
(ii) the balls are of different colors.