(c) z এর সাপেক্ষে ইন্টিগ্রেশন করার [illegible] হবে।

✪ পর্যায়ক্রমিক ইনটিগ্রেশনের ক্রম ঃ

$$\int_{u_1}^{u_2}\int_{v_1}^{v_2}\int_{w_1}^{w_2} f(x,\ y,\ z)\ dx\ dy\ dz$$

প্রথম

দ্বিতীয়

তৃতীয়

অর্থাৎ $\int_{u_1}^{u_2}\int_{v_1}^{v_2}\int_{w_1}^{w_2} f(x,\ y,\ z)\ dx\ dy\ dz$ আকারের ইনটিগ্র্যাল এর ক্ষেত্রে, প্রথমে x এর সাপেক্ষে, দ্বিতীয় বারে y এর সাপেক্ষে এবং শেষে z এর সাপেক্ষে ইনটিগ্রেশন করা হয়।

সমাধানকৃত উদাহরণমালা
[Solved Examples]

উদাহরণ–1. দেখাও যে $\int_2^3\int_1^2 xy^2\,dy\,dx = \frac{35}{6}$

সমাধান ঃ এখানে $= \int_2^3\int_1^2 xy^2\,dy\,dx$

$$= \int_2^3 \left[\frac{xy^3}{3}\right]_{y=1}^{2} dx$$

$$= \frac{1}{3}\int_2^3 x(2^3 - 1^3)dx$$

$$= \frac{1}{3}\int_2^3 7x\,dx$$

$$= \frac{7}{3}\left[\frac{x^2}{2}\right]_2^3 = \frac{7}{6}[9-4)$$

$$= \frac{35}{6}.$$

উদাহরণ–5. $\int_1^2 \int_0^x \frac{dy\,dx}{x^2+y^2}$ এর মান নির্ণয় কর।

সমাধান ঃ এখানে $\int_1^2 \int_0^x \frac{dy\,dx}{x^2+y^2} = \int_1^2 \frac{1}{x}\left[\tan^{-1}\frac{y}{x}\right]_{y=0}^{x} dx$

$$= \int_1^2 \frac{1}{x}[\tan^{-1}(1) - \tan^{-1}(0)]dx$$

$$= \frac{\pi}{4}\int_1^2 \frac{dx}{x} = \frac{\pi}{4}[\ln x]_1^2$$

$$= \frac{\pi}{4}\ln 2.$$

উদাহরণ–6. $\int_1^2 \int_y^{3y} (3x^2+y^2)dx\,dy$ এর মান নির্ণয় কর।

সমাধান ঃ $\int_1^2 \int_y^{3y} (3x^2+y^2)dx\,dy$

$$= \int_1^2 \left[\frac{3x^3}{3} + y^2x\right]_{x=y}^{3y} dy$$

$$= \int_1^2 [\{(3y)^3 + y^2.3y\} - (y^3 + y^2.y)]dy$$

$$= \int_1^2 (27y^3 + 3y^3 - y^3 - y^3)dy$$

$$= \int_1^2 28y^3\,dy$$

$$= 28\left[\frac{y^4}{4}\right]_1^2$$

$$= \frac{28}{4}[2^4 - 1^4]$$

$$= 7(16-1)$$

$$= 7\times 15$$

$$= 105.$$

উদাহরণ–7. $\int_1^2 \int_x^{e^x} \left(\frac{x}{y}\right) dy\,dx$ এর মান নির্ণয় কর।

সমাধান ঃ এখানে $\int_1^2 \int_x^{e^x} \left(\frac{x}{y}\right) dy\,dx$

$$= \int_1^2 [x \ln y]_{y=x}^{e^x} dx$$

$$= \int_1^2 x\,[ln\, e^x - ln\, x]dx$$

$$= \int_1^2 x[x - ln\, x]dx \quad [\because ln\, e = 1]$$

$$= \int_1^2 x^2\, dx - \int_1^2 x\, ln\, x\, dx$$

$$= \left[\frac{x^3}{3}\right]_1^2 - \left[\frac{x^2}{2} ln\, x\right]_1^2 + \int_1^2 \frac{x}{2}\, dx$$

$$= \frac{1}{3}[2^3 - 1^3] - [2\, ln\, 2 - 0] + \left[\frac{x^2}{4}\right]_1^2$$

$$= \frac{7}{3} - 2\, ln\, 2 + \frac{1}{4}(4 - 1)$$

$$= \frac{7}{3} - 2\, ln\, 2 + \frac{3}{4}$$

$$= \frac{37}{12} - 2\, ln\, 2.$$

উদাহরণ-8. $\int_0^2 \int_y^{y^2} (x + 2y)dx\, dy$ এর মান নির্ণয় কর।

সমাধান ঃ এখানে $\int_0^2 \int_y^{y^2} (x + 2y)dx\, dy$

$$= \int_0^2 \left[\frac{x^2}{2} + 2yx\right]_{x=y}^{y^2} dy$$

$$= \int_0^2 \left\{\left(\frac{y^4}{2} + 2y^3\right) - \left(\frac{y^2}{2} + 2y^2\right)\right\} dy$$

$$= \int_0^2 \left(\frac{y^4}{2} + 2y^3 - \frac{5}{2} y^2\right) dy$$

$$= \left[\frac{y^5}{10} + \frac{2y^4}{4} \frac{5y^3}{6}\right]_{y=0}^2$$

$$= \left(\frac{32}{10} + \frac{16}{2} - \frac{40}{6}\right) - (0)$$

$$= \frac{16}{5} + 8 - \frac{20}{3} = \frac{48 + 120 - 100}{15}$$

$$= \frac{68}{15}.$$

উদাহরণ–16. $\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\, dr\, d\theta$ এর মান নির্ণয় কর।

[NUH(NM)-2...

সমাধান : $\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\, dr\, d\theta$

$$= \int_0^{\pi/2}\left[\frac{r^2}{2}\right]_0^{\sin\theta}\cos\theta\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2}(\sin\theta)^2\, d(\sin\theta)$$

$$= \frac{1}{2}\left[\frac{\sin^3\theta}{3}\right]_0^{\pi/2} = \frac{1}{6}$$

উদাহরণ–17. $\int_0^{\pi/2}\int_0^{\sin y} e^x\cos y\, dx\, dy$ এর মান নির্ণয় কর।

[NUH(NM)-200...

সমাধান : $\int_0^{\pi/2}\int_0^{\sin y} e^x\cos y\, dx\, dy$

$$= \int_0^{\pi/2}\left[e^x\right]_0^{\sin y}\cos y\, dy$$

$$= \int_0^{\pi/2}(e^{\sin y}-1)\, d(\sin y)$$

$$= \left[e^{\sin y}-\sin y\right]_0^{\pi/2}$$

$$= (e-1)-(1-0) = e-2.$$

উদাহরণ–18. $\int_0^{ln2}\int_0^1 xy\, e^{xy^2}\, dy\, dx$ এর মান নির্ণয় কর। [NUH(NM)-...

সমাধান : $\int_0^{ln2}\int_0^1 xy\, e^{xy^2}\, dy\, dx$

$$= \int_0^{ln2}\left[\int_0^1 ye^{xy^2}\, dy\right]x\, dx$$

$$= \int_0^{ln2}\left[\int_0^1 e^{xy^2}\, d(y^2)\right]\frac{x}{2}\, dx$$

$$= \int_0^{\ln 2} \left[\frac{e^{xy^2}}{x} \right]_0^1 \frac{x}{2} dx$$

$$= \int_0^{\ln 2} \left[\frac{e^x}{x} - \frac{1}{x} \right] \frac{x}{2} dx$$

$$= \frac{1}{2} \int_0^{\ln 2} (e^x - 1)\, dx$$

$$= \frac{1}{2} \left[e^x - x \right]_0^{\ln 2}$$

$$= \frac{1}{2} [(2 - \ln 2) - (1 - 0)]$$

$$= \frac{1}{2} (1 - \ln 2)$$

উদাহরণ–19. $\int_0^{\pi/4} \int_0^{2a\cos\phi} \int_0^{2\pi} \rho^2 \sin\phi \, d\theta \, d\rho \, d\phi$ এর মান নির্ণয় কর।

[NUH–2008]

সমাধান ঃ $\int_0^{\pi/4} \int_0^{2a\cos\phi} \int_0^{2\pi} \rho^2 \sin\phi \, d\theta \, d\rho \, d\phi$

$$= \int_0^{\pi/4} \int_0^{2a\cos\phi} [\theta]_0^{2\pi} \rho^2 \sin\phi \, d\rho \, d\phi$$

$$= 2\pi \int_0^{\pi/4} \int_0^{2a\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi$$

$$= 2\pi \int_0^{\pi/4} \left[\frac{\rho^3}{3} \right]_0^{2a\cos\phi} \sin\phi \, d\phi$$

$$= 2\pi \int_0^{\pi/4} \frac{8a^3}{3} (\cos\phi)^3 \sin\phi \, d\phi$$

$$= -\frac{16\pi a^3}{3} \int_0^{\pi/4} (\cos\phi)^3 \, d(\cos\phi)$$

$$= -\frac{16\pi a^3}{3} \left[\frac{\cos^4\phi}{4} \right]_0^{\pi/4}$$

$$= -\frac{16\pi a^3}{3} \left[\frac{1}{16} - \frac{1}{4} \right] = \pi a^3.$$

উদাহরণ-20. $\int_0^1 \int_0^{1-x} \int_0^{1-y^2} z\,dz\,dy\,dx$ এর মান নির্ণয় কর। [NUH-...

সমাধান ঃ $\int_0^1 \int_0^{1-x} \int_0^{1-y^2} z\,dz\,dy\,dx$

$$= \int_0^1 \int_0^{1-x} \frac{1}{2}\left[z^2\right]_0^{1-y^2} dy\,dx$$

$$= \frac{1}{2}\int_0^1 \int_0^{1-x} (1-y^2)^2\,dy\,dx$$

$$= \frac{1}{2}\int_0^1 \int_0^{1-x} (1-2y^2+y^4)dy\,dx$$

$$= \frac{1}{2}\int_0^1 \left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^{1-x} dx$$

$$= \frac{1}{2}\int_0^1 \left[1 - x - \frac{2}{3}(1-x)^3 + \frac{1}{5}(1-x)^5\right] dx$$

$$= \frac{1}{2}\left[x - \frac{x^2}{2} + \frac{2}{12}(1-x)^4 - \frac{1}{30}(1-x)^6\right]_0^1$$

$$= \frac{1}{2}\left[\left(1 - \frac{1}{2} + \frac{1}{6}\times 0 - \frac{1}{30}\times 0\right) - \left(0 - 0 + \frac{1}{6} - \frac{1}{30}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{4}{30}\right]$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{2}{15}\right)$$

$$= \frac{1}{2}\times\frac{11}{30} = \frac{11}{60}.$$

উদাহরণ–**27.** $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \sin\theta\, dr\, d\theta\, d\phi$ এ

সমাধান ঃ $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{3}\left[r^3\right]_0^a \sin\theta\, d\theta\, d\phi$$

$$= -\frac{a^3}{3}\int_0^{2\pi}\left[\cos\theta\right]_0^{\pi} d\phi$$

$$= -\frac{1}{3}a^3\int_0^{2\pi}(\cos\pi - \cos 0)d\phi$$

$$= -\frac{1}{3}a^3\int_0^{2\pi}(-1-1)d\phi$$

$$= \frac{2}{3}a^3\left[\phi\right]_0^{2\pi} = \frac{4\pi}{3}a^3.$$

উদাহরণ–**30.** $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$ এর মান নির্ণয় ক

[NUH-

সমাধান ঃ $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho^3 \sin\phi \cos\phi \, d\rho \, d\phi \, d\theta$

$$= \int_0^1 \rho^3 \, d\rho \cdot \int_0^{\pi/2} \sin\phi \cos\phi \, d\phi \cdot \int_0^{\pi/2} d\theta$$

$$= \left[\frac{\rho^4}{4}\right]_0^1 \cdot \frac{1}{2}\left[-\frac{\cos 2\phi}{2}\right]_0^{\pi/2} \cdot [\theta]_0^{\pi/2}$$

$$= \frac{1}{4} \cdot \frac{1}{4}(1+1) \cdot \frac{\pi}{2}$$

$$= \frac{\pi}{16}.$$

উদাহরণ-35. মান নির্ণয় কর (Evaluate) :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx.$$ [NUH-201

সমাধান ঃ $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} xy \left[\frac{z^2}{2}\right]_0^{\sqrt{1-x^2-y^2}} dy\, dx$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \frac{xy}{2} (1 - x^2 - y^2)\, dy\, dx$$

$$= \frac{1}{2} \int_0^1 \int_0^{\sqrt{1-x^2}} (xy - x^3y - xy^3)\, dy\, dx$$

$$= \frac{1}{2} \int_0^1 \left[x \cdot \frac{y^2}{2} - x^3 \cdot \frac{y^2}{2} - x \cdot \frac{y^4}{4}\right]_0^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2} \int_0^1 \left\{\frac{x}{2}(1 - x^2) - \frac{x^3}{2}(1 - x^2) - \frac{x}{4}(1 - x^2)^2\right\} dx$$

$$= \frac{1}{8} \int_0^1 \{2x(1 - x^2) - 2x^3(1 - x^2) - x(1 - 2x^2 + x^4)\}\, dx$$

$$= \frac{1}{8} \int_0^1 (x - 2x^3 + x^5)\, dx$$

$$=\frac{1}{8}\left[\frac{x^2}{2}-\frac{x^4}{2}+\frac{x^6}{6}\right]_0^1$$

$$=\frac{1}{8}\left[\frac{1}{2}-\frac{1}{2}+\frac{1}{6}\right]=\frac{1}{48}.$$

উদাহরণ–36. মান নির্ণয় কর (Evaluate) : $\int_1^2\int_0^z\int_0^{x\sqrt{3}} \frac{x\,dy\,dx\,dz}{\sqrt{x^2+y^2}}$.

[NUH-201

সমাধান ঃ $\int_1^2\int_0^z\int_0^{x\sqrt{3}} \frac{x\,dy\,dx\,dz}{\sqrt{x^2+y^2}}$

$$=\int_1^2\int_0^z x\left[ln\left(y+\sqrt{x^2+y^2}\right)\right]_0^{x\sqrt{3}} dx\,dz$$

$$=\int_1^2\int_0^z x\,[ln\,(\sqrt{3}+2)x - ln\,x]\,dx\,dz$$

$$= ln(2+\sqrt{3})\int_1^2\int_0^z x\,dx\,dz$$

$$= ln(2+\sqrt{3})\int_1^2\left[\frac{x^2}{2}\right]_0^z dz$$

$$= ln(2+\sqrt{3})\int_1^2\frac{z^2}{2}\,dz$$

$$=\frac{1}{2}\,ln\,(2+\sqrt{3})\left[\frac{z^3}{3}\right]_1^2$$

$$=\frac{7}{6}\,ln(2+\sqrt{3}).$$