Lecture-2
Home Work Solution

1. Solution:-

$$\lim_{(x,y)\to(1,3)} (4xy^2 - x)$$

$$= 4\cdot 1\cdot (3)^2 - 1$$

$$= 36 - 1$$

$$= 35$$

2. Solution:-

$$\lim_{(x,y)\to(0,0)} \frac{4x-y}{\sin y - 1}$$

$$= \frac{4\cdot 0 - 0}{\sin 0 - 1}$$

$$= \frac{0}{-1} = 0$$

3. Solution:-

$$\lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y}$$

$$= \frac{(-1)(2)^3}{-1+2}$$

$$= -8$$

4. Solution:

$$\lim_{(x,y)\to(1,-3)} e^{2x-y^2}$$

$$= e^{2\cdot 1 - (-3)^2}$$

$$= e^{2-9} = e^{-7}$$

$$= \frac{1}{e^7}$$

5. Solution:-

$$\lim_{(x,y)\to(0,0)} \ln(1+x^2y^3)$$

$$= \ln(1+0\cdot0)$$

$$= \ln(1)$$

$$= 0$$

6. Solution:-

$$\lim_{(x,y)\to(4,-2)} x\sqrt[3]{y^3+2x}$$

$$= 4\cdot\sqrt[3]{(-2)^3+2\cdot4}$$

$$= 4\cdot\sqrt[3]{-8+8}$$

$$= 4\cdot0$$

$$= 0$$

7.(a)

Solution:-

The limiting value along x-axis

$$\lim_{x\to0}\lim_{y\to0}\frac{3}{x^2+2y^2} = \lim_{x\to0}\frac{3}{x^2+2\cdot0}$$

$$= \lim_{x\to0}\frac{3}{x^2}$$

$$= \frac{3}{0}$$

$$= \infty$$

Again,

The limiting value along ~~y~~ y-axis

$$\lim_{y\to 0} \lim_{x\to 0} \frac{3}{x^3+2y^2} = \lim_{y\to 0} \frac{3}{0+2y^2}$$

$$= \lim_{y\to 0} \frac{3}{2y^2}$$

$$= \frac{3}{0}$$

$$= \infty$$

As, the limiting value along x-axis and y axis are ∞. But ∞ is not an exact number.

So $\lim_{(x,y)\to(0,0)} \frac{3}{x^3+2y^2}$ does not exist.

7.(b) Solution:-

The limiting value along x-axis

$$\lim_{x\to 0}\lim_{y\to 0}\frac{x+y}{2x^2+y^2} = \lim_{x\to 0}\frac{x+0}{2x^2+0}$$

$$= \lim_{x\to 0}\frac{x}{2x^2} = \lim_{x\to 0}\frac{1}{2x}$$

$$= \frac{1}{0} = \infty$$

Again,

the limiting value along y-axis

$$\lim_{y\to 0}\lim_{x\to 0}\frac{x+y}{2x^2+y^2} = \lim_{y\to 0}\frac{0+y}{0+y^2}$$

$$= \lim_{y\to 0}\frac{y}{y^2} = \lim_{y\to 0}\frac{1}{y}$$

$$= \frac{1}{0} = \infty$$

The limiting value along x-axis and y-axis ~~is~~ are ∞, which is not an exact number.

So $\lim_{(x,y)\to(0,0)}\frac{x+y}{2x^2+y^2}$ does not exist.

8. (a) Solution:-

The limiting value along x-axis

$$\lim_{x\to 0}\lim_{y\to 0}\frac{x-y}{x^2+y^2} = \lim_{x\to 0}\frac{x}{x^2} = \lim_{x\to 0}\frac{1}{x}$$

$$= \frac{1}{0} = \infty$$

Again,

The limiting value along y-axis

$$\lim_{y\to 0}\lim_{x\to 0}\frac{x-y}{x^2+y^2} = \lim_{y\to 0}\frac{0-y}{0+y^2} = \lim_{y\to 0}\frac{-y}{y^2}$$

$$= -\lim_{y\to 0}\frac{1}{y} = -\frac{1}{0} = -\infty$$

Since, the limiting value along x-axis and y-axis are not same. ~~so~~

So $\lim_{(x,y)\to(0,0)}\frac{x-y}{x^2+y^2}$ does not exist.

8(b). Solution:-

The limiting value along x-axis,

$$\lim_{x\to 0}\lim_{y\to 0}\frac{\cos xy}{x^2+y^2}=\lim_{x\to 0}\frac{\cos x\cdot 0}{x^2+0}$$

$$=\lim_{x\to 0}\frac{\cos 0}{x^2}=\lim_{x\to 0}\frac{1}{x^2}$$

$$=\frac{1}{0}=\infty$$

Now, the limiting value along y-axis

$$\lim_{y\to 0}\lim_{x\to 0}\frac{\cos xy}{x^2+y^2}=\lim_{y\to 0}\frac{\cos 0\cdot y}{0+y^2}$$

$$=\lim_{y\to 0}\frac{\cos 0}{y^2}=\lim_{y\to 0}\frac{1}{y^2}$$

$$=\frac{1}{0}=\infty$$

Since, the limiting value along x-axis and y-axis are ∞, which is not an exact value.

so, $\lim_{(x,y)\to(0,0)}\frac{\cos xy}{x^2+y^2}$ does not exist.