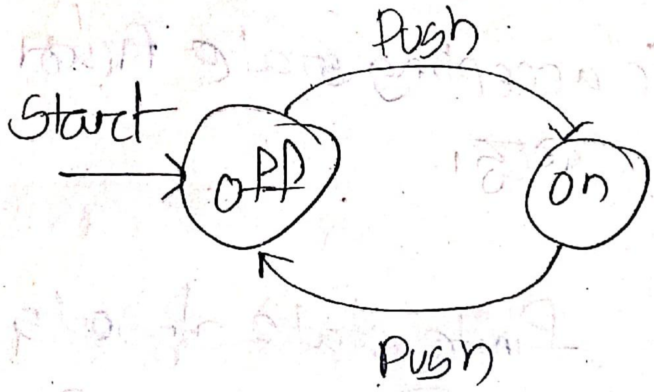
Automata theorry

Finite

automata - plutral

automatgon - singular



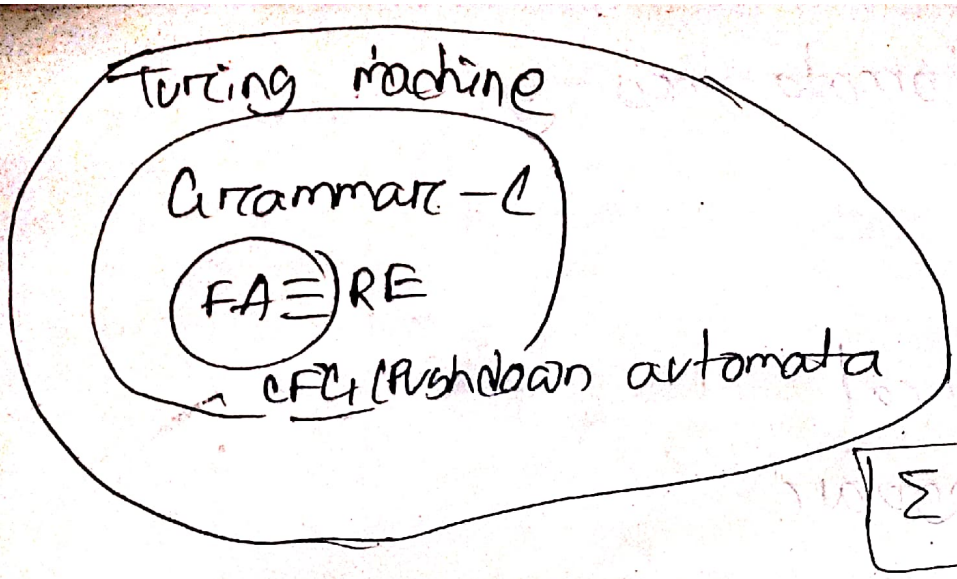
A finite automaton modeling an on/off switch

[Pdf-44]

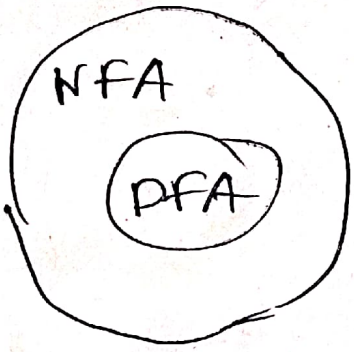
An alphabet is a finite, noempty set of symbols.

→ (. . .) কোনটা ও দিয়ে বোঝায় and so on

প্রবলেম solve করতে ⊙→ first এ Alpha bet set define করতে হবে।



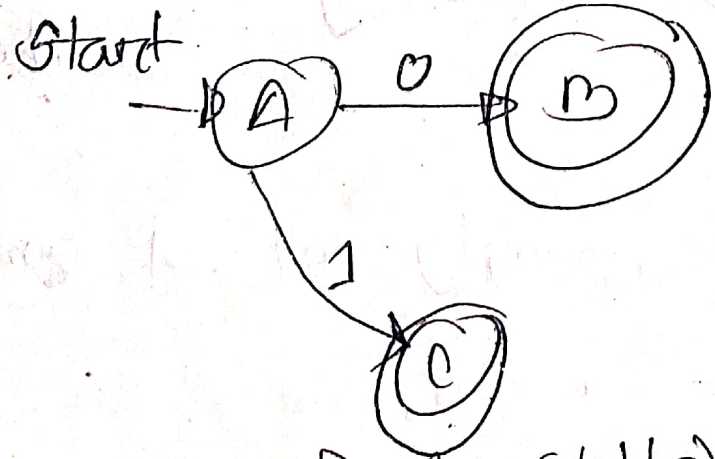
Σ → Alphabet



State circle, final state or accepting state থাকবে।

Finite state of sets

$Q = \{A, B, C\}$



set of input symbols

$\Sigma = \{0, 1\}$

Empty sets

A transition function (delta)

$\delta(A, 0) = B$

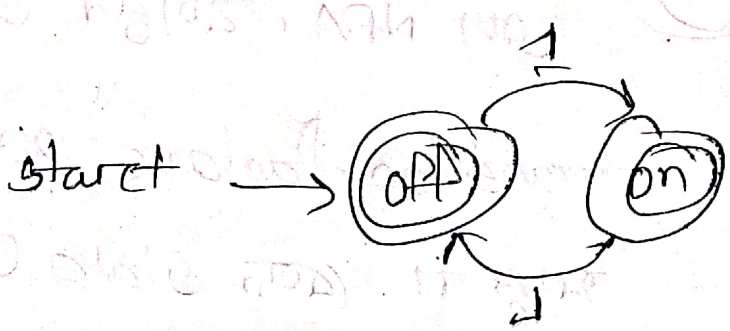
$\delta(A, 1) = C$

$\delta(B, 0) = \{\}$

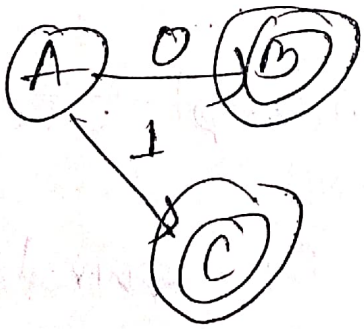
$\delta(B, 1) = \{\}$

*Multiple final set থাকতে পারে তবে start state একটাই।

*Starting state ও final state হতে পারে।

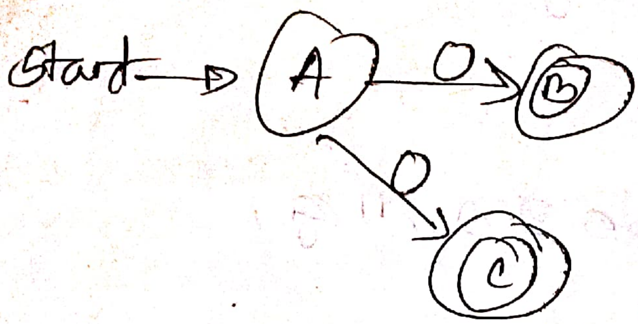


String 001



String 0, 1

∴যেকোনো Automata দিয়ে String Accept করানো, দিয়ে reject করানো।



$\delta(A,0) = \{B, C\}$

এটা NFA। কারণ এটার transition factor থেকে বলা যায় না কোন state এ যাবে।

~~So δ থেকে~~

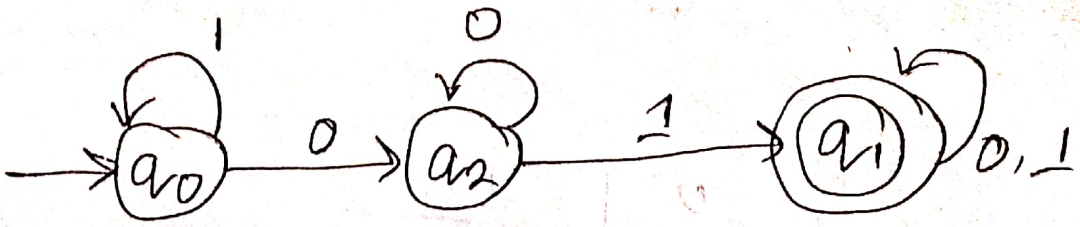
যদি কোনো transition function একাধিক state return করে তবে তা DFA থাকবে না।

FA accepts strings

\# দুটো Automata একই type strings Accept করে একই type strings reject করলে তা equivalent automata

26-07-22

Ex: 2.1



~~DFA~~

NFA

$\delta(q_0, 0) = q_2$

$\delta(q_0, 1) = q_0$

and 

same 

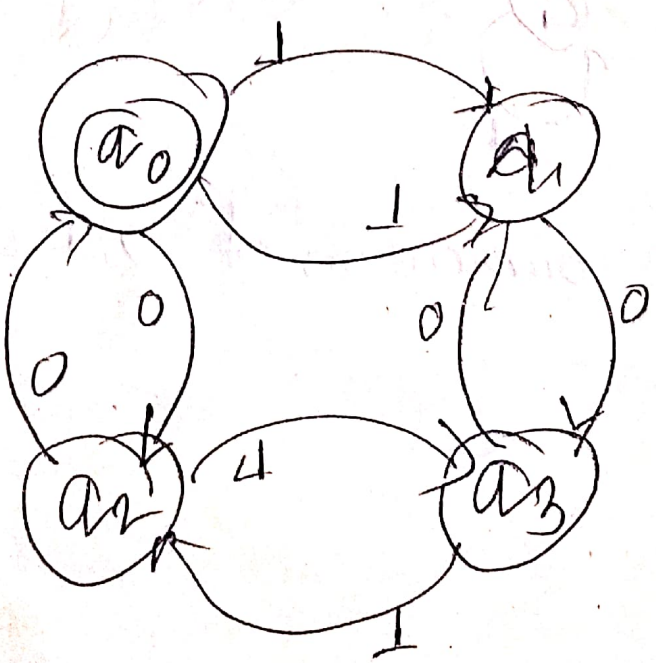
↑
এটা তে All possible combination of 0, 1 accept করবে।

Transition Table

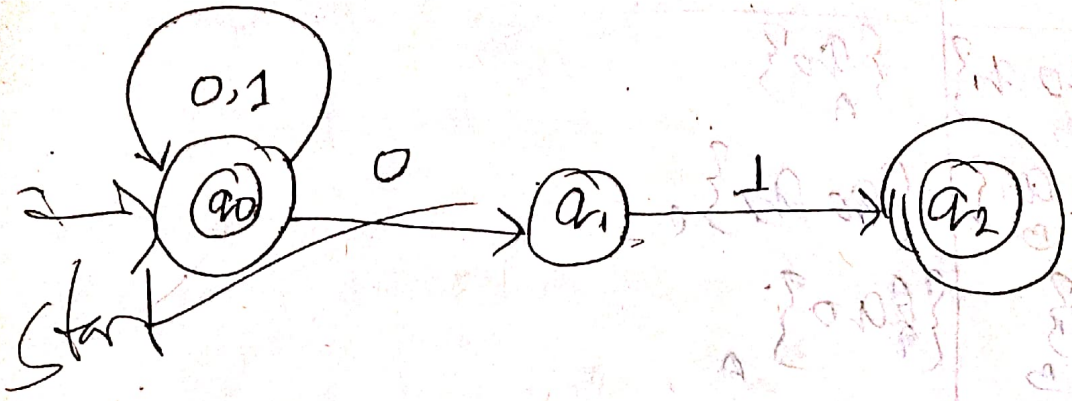
	0	1
→ $\{q_0\}$	$\{q_2\}$	$\{q_0\}$
$\{q_2\}$	$\{q_2\}$	$\{q_1\}$
* $\{q_1\}$	$\{q_1\}$	$\{q_1\}$

(*) ↖ sign দ্বারা final state mean করা হয়ে।

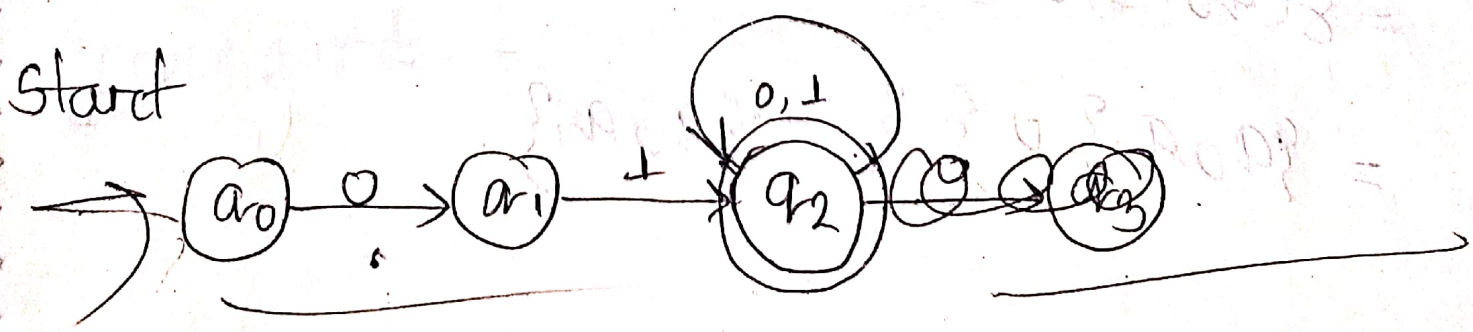
Ex: 2.4



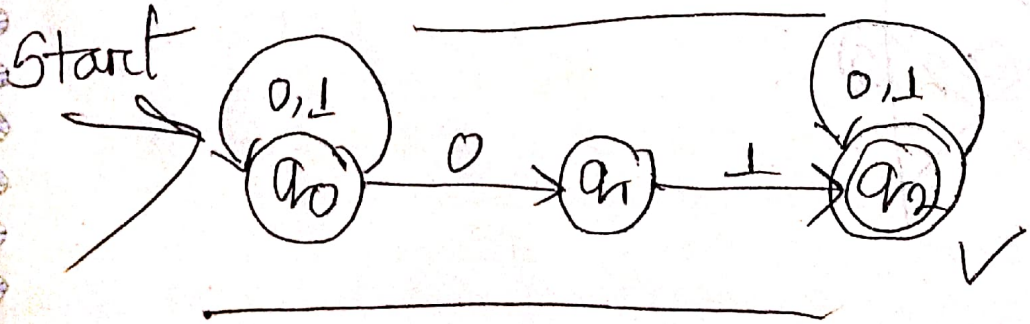
2.) An NFA accepting all strings that end in 01



(i) An NFA accepting all strings that ~~has a substring 01.~~ starts with 0,1



(ii) has a substring 0,1



04-07-2022

[2.0] Transition Table

	0	1
→ $\{q_0\}$ A	$\{q_0, q_1\}$ B	$\{q_0\}$ A
$\{q_0, q_1\}$ B	$\{q_0, q_1\}$ B	$\{q_0, q_2\}$ C
* $\{q_0, q_2\}$ C	$\{q_0, q_1\}$ B	$\{q_0\}$ A

$\delta(\{q_0, q_1\}, 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0)$

$= \{q_0, q_1\} \cup \{\} = \{q_0, q_1\}$

$\delta(\{q_0, q_1\}, 1) = \{q_0, q_2\}$

#

$\delta(\{q_1, q_2\}, 0)$

$= \delta(q_1, 0) \cup \delta(q_2, 0)$

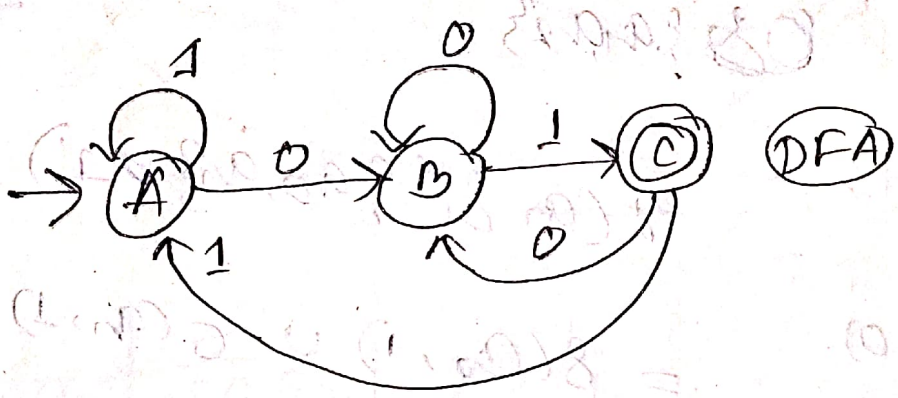
$= \{\} \cup \{\}$

Let,

$\{q_0\} = A$

$\{q_0 q_1\} = B$

$\{q_0, q_2\} = C$



[চিত্র 2.9 এর picture এর equivalent]
(NFA)

	0	1
A $\{q_0\}$	B $\{q_1\}$	C $\{\}$
C $\{\}$	B $\{q_1\}$	D $\{q_1, q_2\}$
* E $\{q_0, q_1, q_2\}$	B $\{q_1\}$	D $\{q_1, q_2\}$

$\delta(\{q_0, q_1\}, 0)$

$= \delta(q_0, 0) \cup \delta(q_1, 0)$

$= \{q_1\} \cup \{q_1\}$

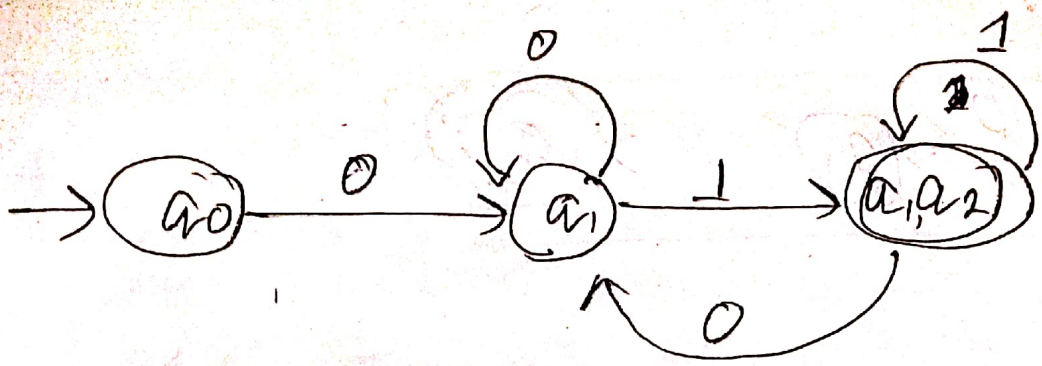
$\delta(\{q_0, q_1\}, 1)$

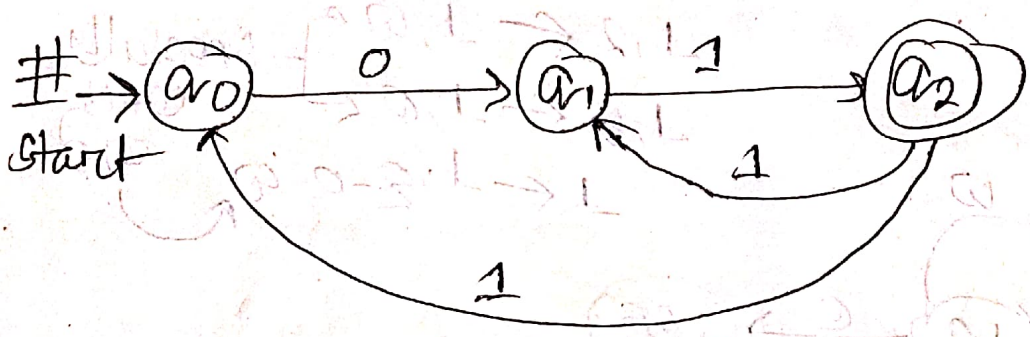
$= \delta(q_0, 1) \cup \delta(q_1, 1)$

$= \{\} \cup \{q_1, q_2\}$

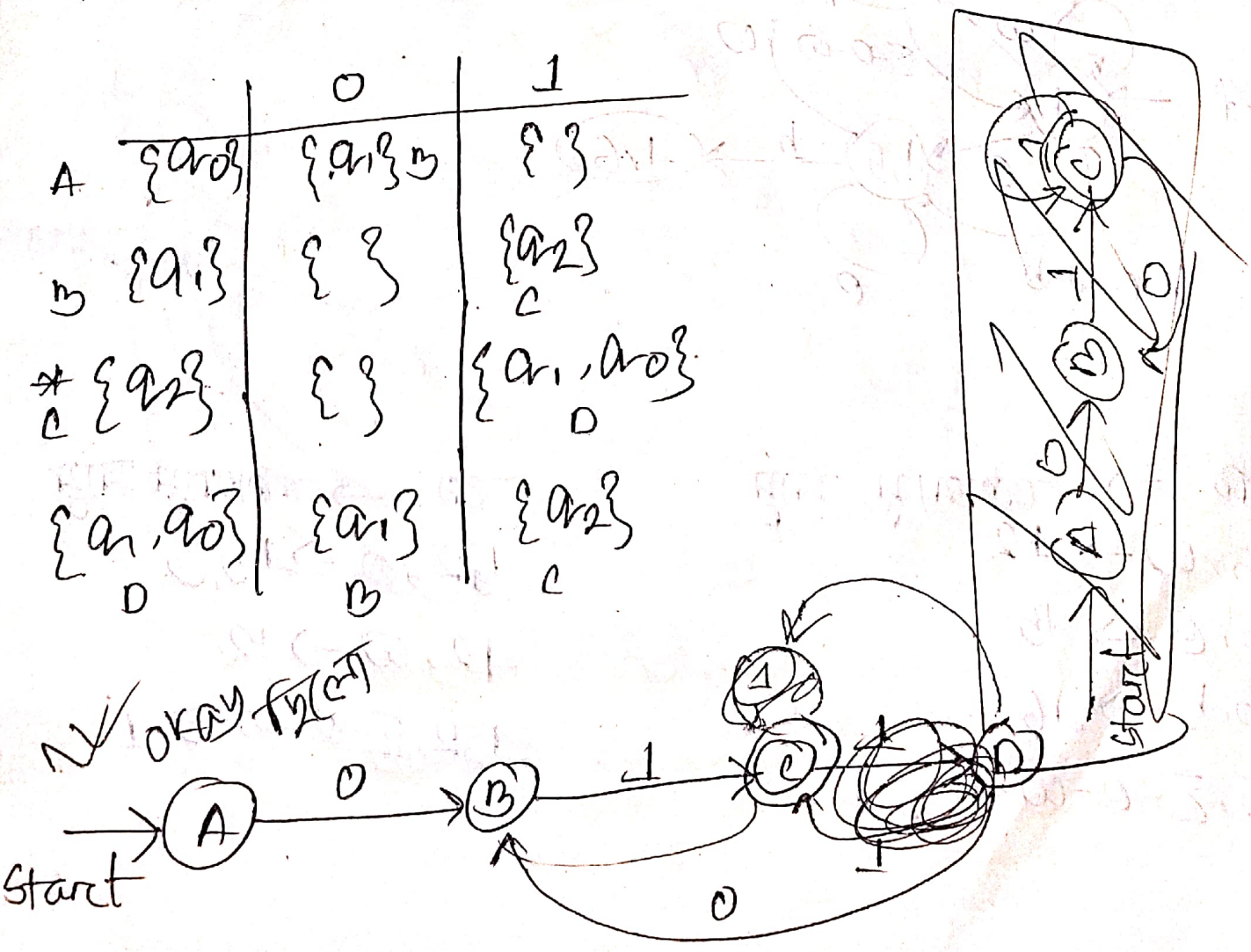
$\delta(q_0, 0) \cup \delta(q_2, 0)$

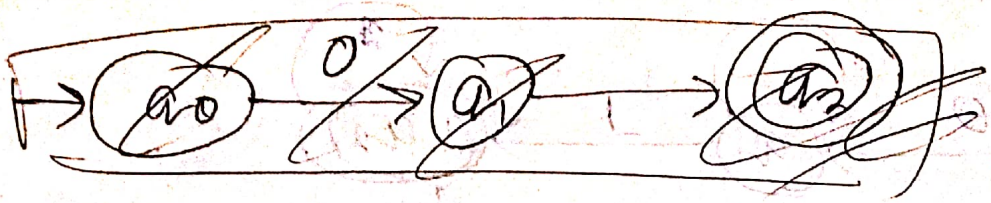
$= \{q_1\}$



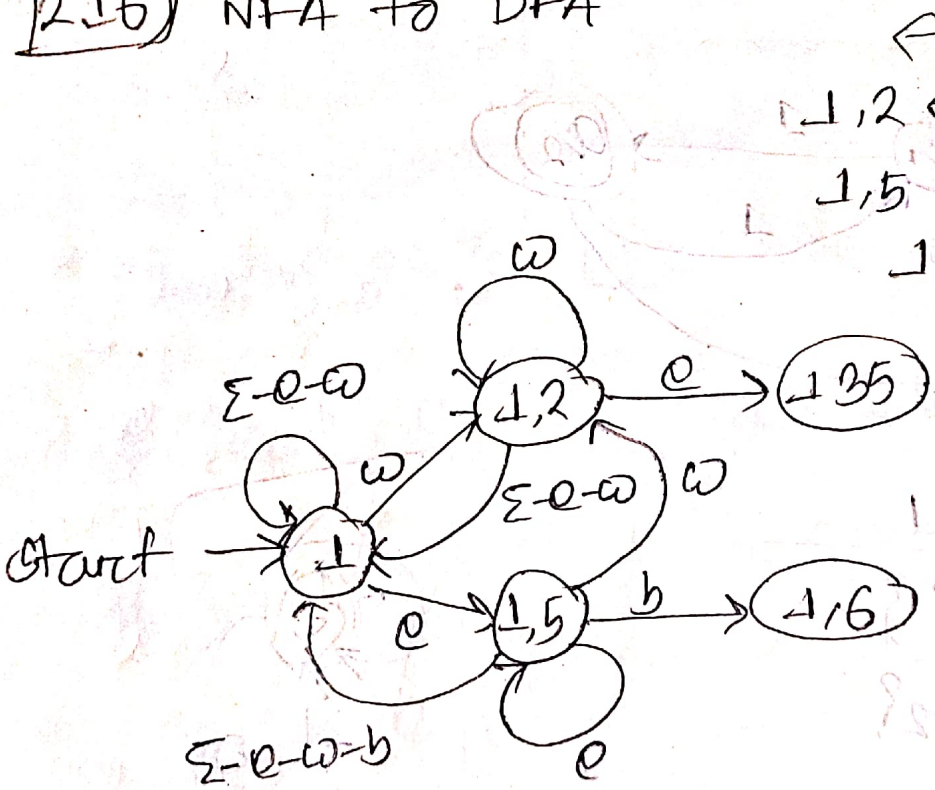


	0	1
A $\{q_0\}$	$\{q_1\}$ B	$\{\}$
B $\{q_1\}$	$\{\}$	$\{q_2\}$ C
* C $\{q_2\}$	$\{\}$	$\{q_1, q_0\}$ D
$\{q_1, q_0\}$ D	$\{q_1\}$ B	$\{q_2\}$ C





[2.16] NFA to DFA

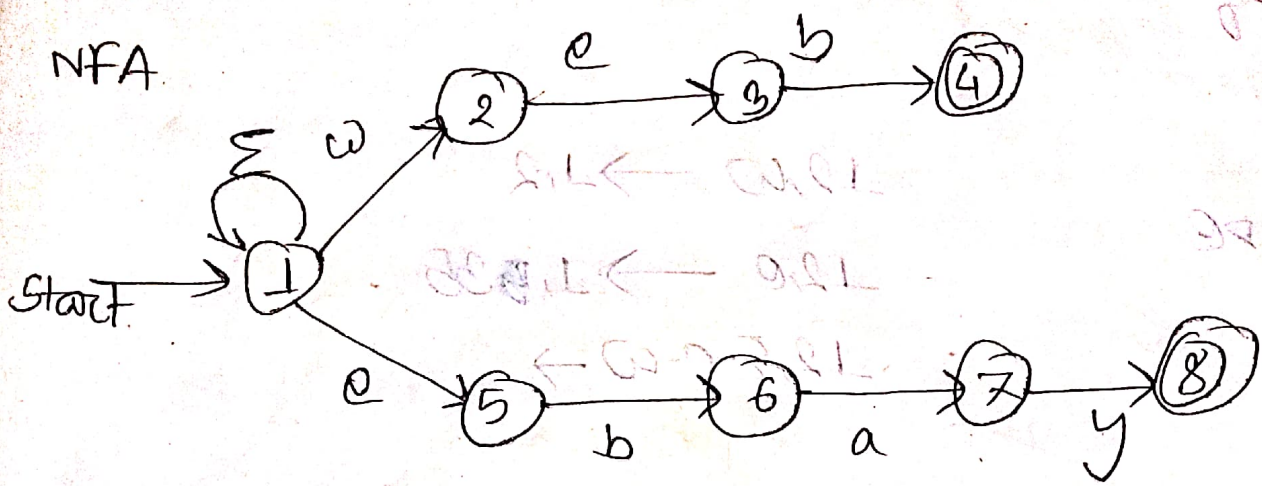


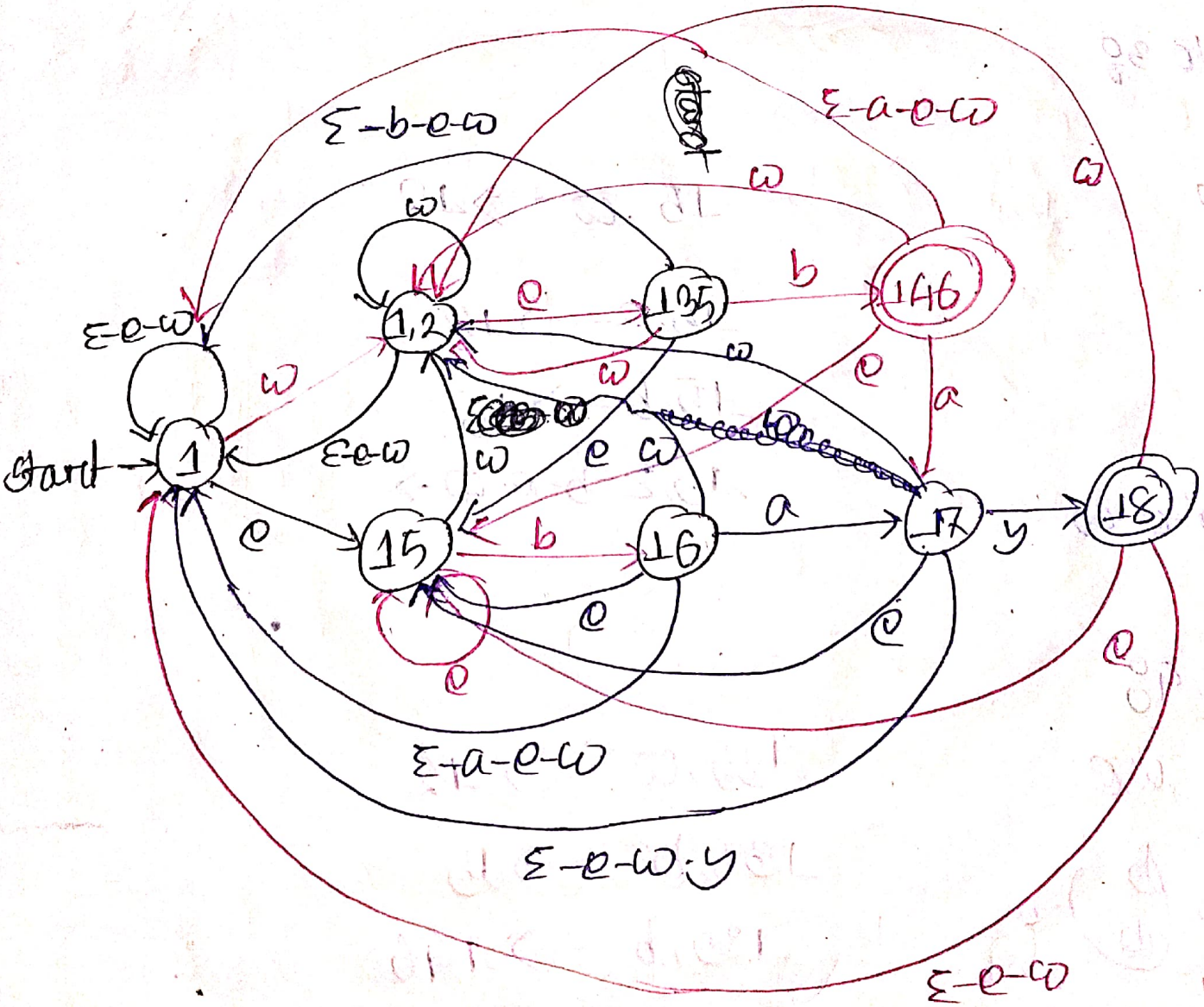
1,2 ← 1, a
1,5 ← 1, a
1 ← 1, ε-a-b
Specially আছে তাই

বেস → কোথায় যায়
1, a → 12
1, a → 15
1, b → 16
1, ε-a-b → 1

বেস → কোথায় যায়
12, a → 15, 3
12, a → 12
12, ε-a-b → 1

NFA


Start
1
2
3
4
5
6
7
8
ε
ω
e
b
a
y


Start
1
1,2
135
146
15
16
17
18
ε-b-e-ω
ε-a-e-ω
ε-e-ω
ε-a-e-ω
ε-e-ω.y
ε-e-ω
ω
e
a
b
y

State 2:

- a
- e, e → e
- Σ - e - a

12, a → 12

12, e → 135

12, Σ - e - a →

State 3:

- a
- e
- b
- Σ - b - e - a

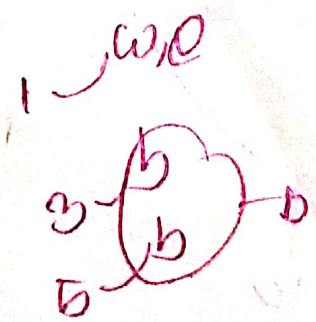
15, a → 12

15, e → 15

15, b → 46

15, Σ - b - e - a →

State 4:



135, a → 12

135, b → 15

135, b → 146

135, Σ - b - a →

state 5:

I6, ω → I2
I6, e → I5
I6, a → I7
I6, Σ-a-e-ω → 1

state 6:

~~I7, ω~~
~~I7 I8~~

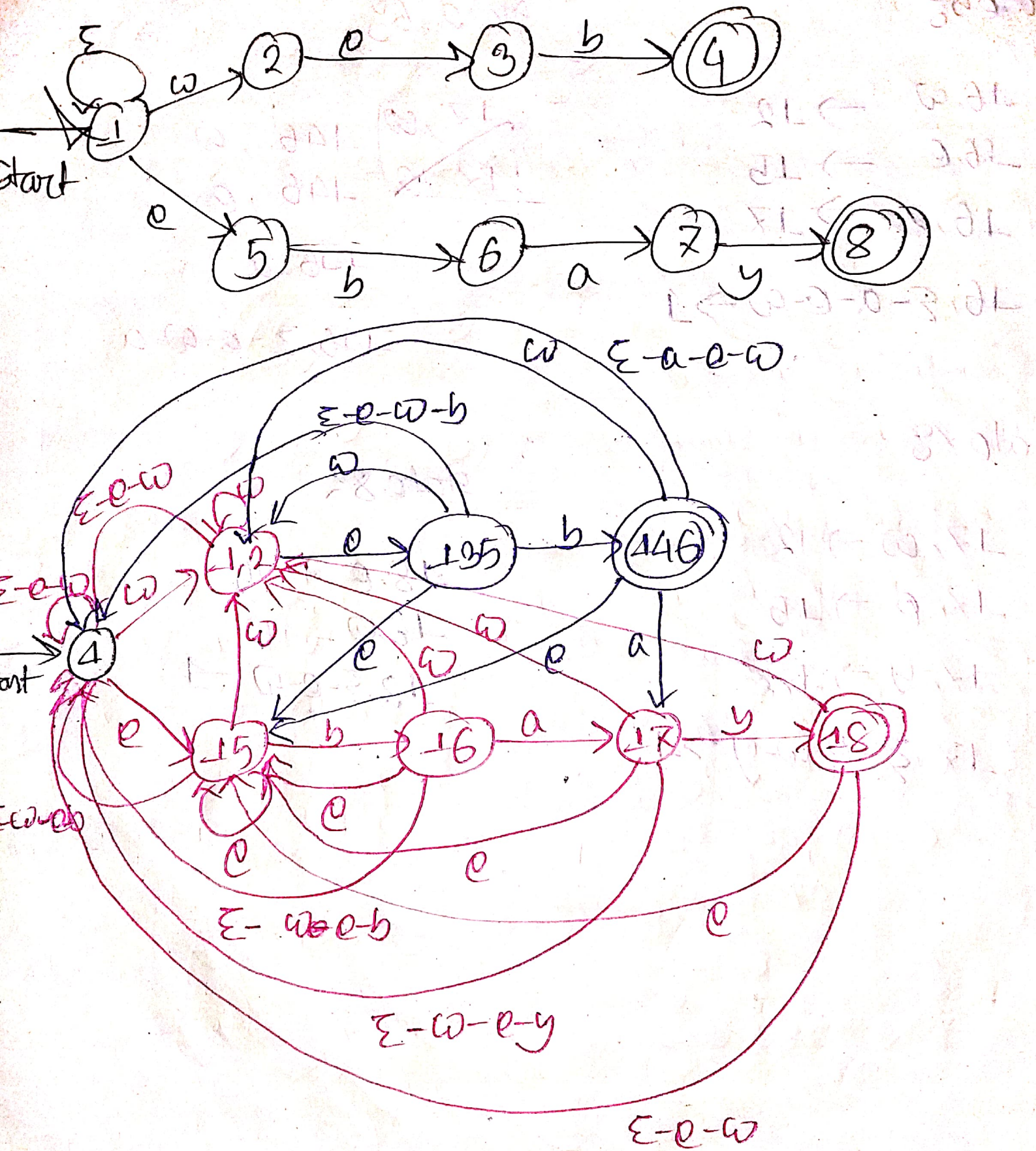
I46, ω
I46, e
I46, a
I46, Σ-e-ω-a

state 7:

I7, ω → 12
I7, e → I5
I7, y → I8
I7, Σ-e-ω-y → 1

state 8:

I8, e → I5
I8, ω → 12
I8, Σ-e-ω → 1


ε
ω
2
e
3
b
4
1
start
e
5
b
6
a
7
y
8
ω
ε-a-e-ω
ε-e-ω-b
ε-e-ω
ω
e
1,2
1,3,5
b
1,4,6
ε-e-ω
ω
4
start
ω
e
ω
e
ω
a
ω
e
e
15
b
16
a
17
y
18
ε-ω-e
e
e
e
ε-ω-e-b
e
ε-ω-e-y
ω-e-ε
ε-e-ω

Notes:

[1]
$q_1, w \to q_2$
$q_1, e \to q_5$
$q_1, \varepsilon\text{-}e\text{-}w \to f$

[2]
$q_2, w \to q_2$
$q_2, e \to q_5 q_3 5$
$[q_2, e]$
$q_2, \varepsilon\text{-}e\text{-}w \to f$

[3]
$q_5, w \to q_2$
$q_5, e \to q_5$
$q_5, b \to q_6$
$q_5, \varepsilon\text{-}w\text{-}e\text{-}b \to f$

[4]
$q_3 5, w \to q_2$
$q_3 5, e \to q_5$
$q_3 5, b \to q_4 6$
$q_3 5, \varepsilon\text{-}w\text{-}e\text{-}b \to f$

[5]
$q_6, w \to q_2$
$q_6, e \to q_5$
$q_6, a \to q_7$
$q_6, \varepsilon\text{-}a\text{-}e\text{-}w \to f$

[6]
$q_4 6, w \to q_2$
$q_4 6, e \to q_5$
$q_4 6, a \to q_7$
$q_4 6, \varepsilon\text{-}a\text{-}e\text{-}w \to f$

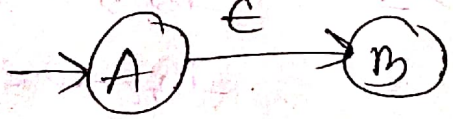
[7]
$q_7, w \to q_2$
$q_7, e \to q_5$
$q_7, y \to q_8$
$q_7, \varepsilon\text{-}w\text{-}e\text{-}y \to f$

[8]
$q_8, w \to q_2$
$q_8, e \to q_5$
$q_8, \varepsilon\text{-}e\text{-}w \to f$

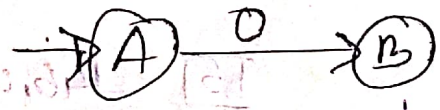
11-08-22

- DNFA Allows → ϵ transition ; এটাকে ϵ-NFA বলা যায়।

↳ এটা থেকেও DFA বানানো সম্ভব।

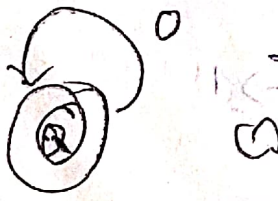


↳ যেখানে Input symbol টা ছাড়াতে যাওয়া যায়, এখানে ϵ থাকা না থাকা Same.

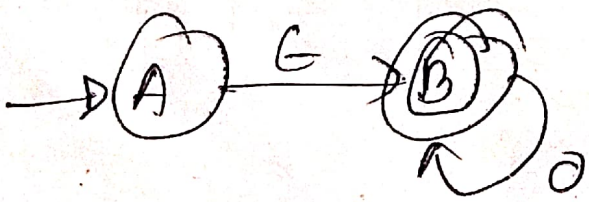


↳ এখানে 0 থাকলেই কেবল B তে যাওয়া যাবে।

এখানে A তে থাকা যা B তে যাবেও তাই।
তবে B তে যেতে যেকোনো A তে যাওয়া যাবে না।

 → δ(A, ϵ) = A

এখানে ϵ অথবা 0 Input symbol কতবার ঘুরেফিরে Final state.

 → δ(A, ϵ) = (A, B)

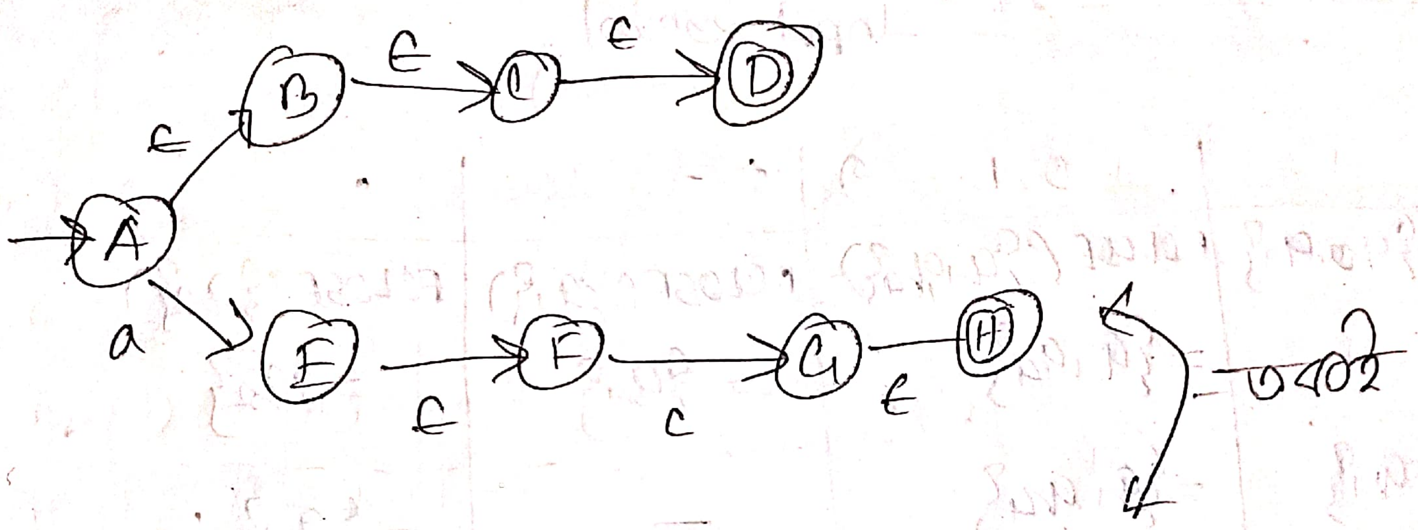
↑
একাধিক So NFA.

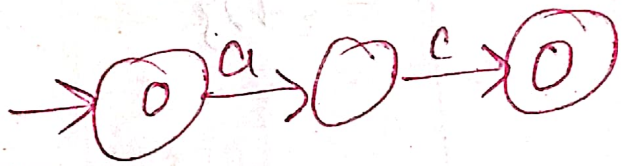
$a\epsilon c\epsilon = ac$

↓

concatenated

ε যদি concatenated অবস্থায় থাকে তাহলে সেটাই return করবে।





ECLOSE (A) = {A, B, C, D}   ECLOSE (E) = {E, F}

ECLOSE (B) = {B, C, D}   ECLOSE (F) = {F}

ECLOSE (C) = {C, D}

ECLOSE (D) = {D}

* প্রতিটি state এই  আছে (লুকানো)

* ephsilon closer কে সহজে close লেখা যায়।

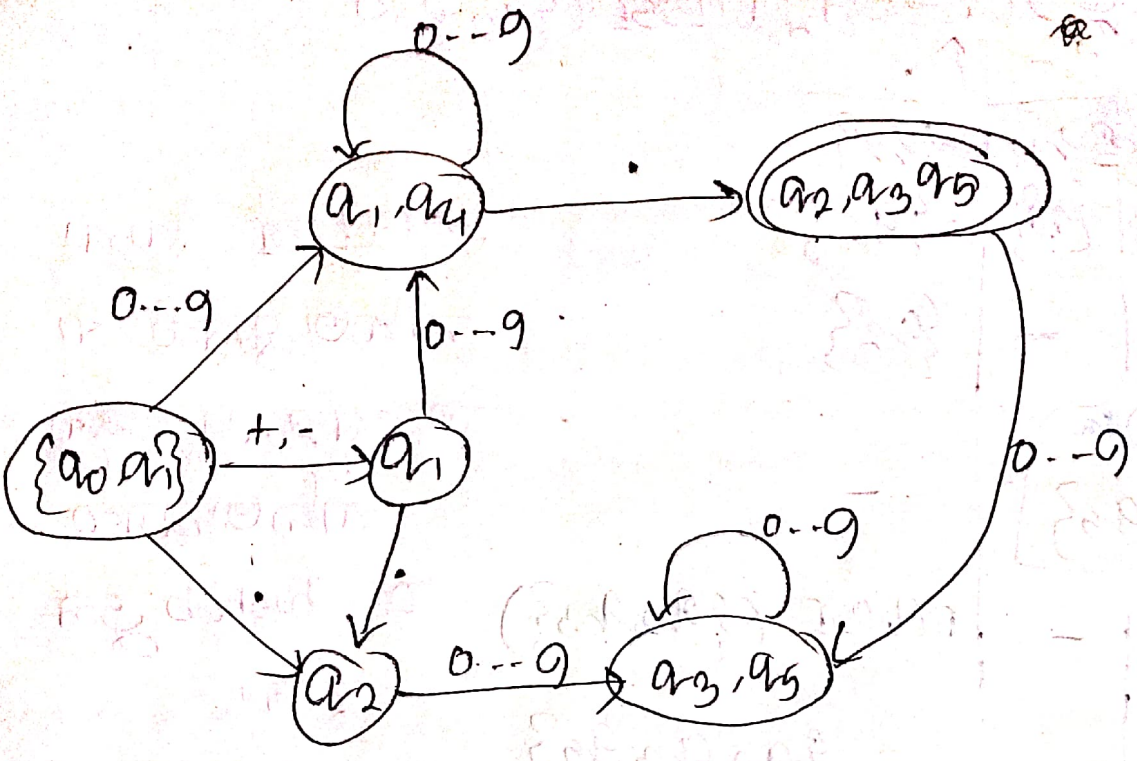
2.18

$$ECLOSE(q_0) = \{q_0, q_1\}$$

Transition table

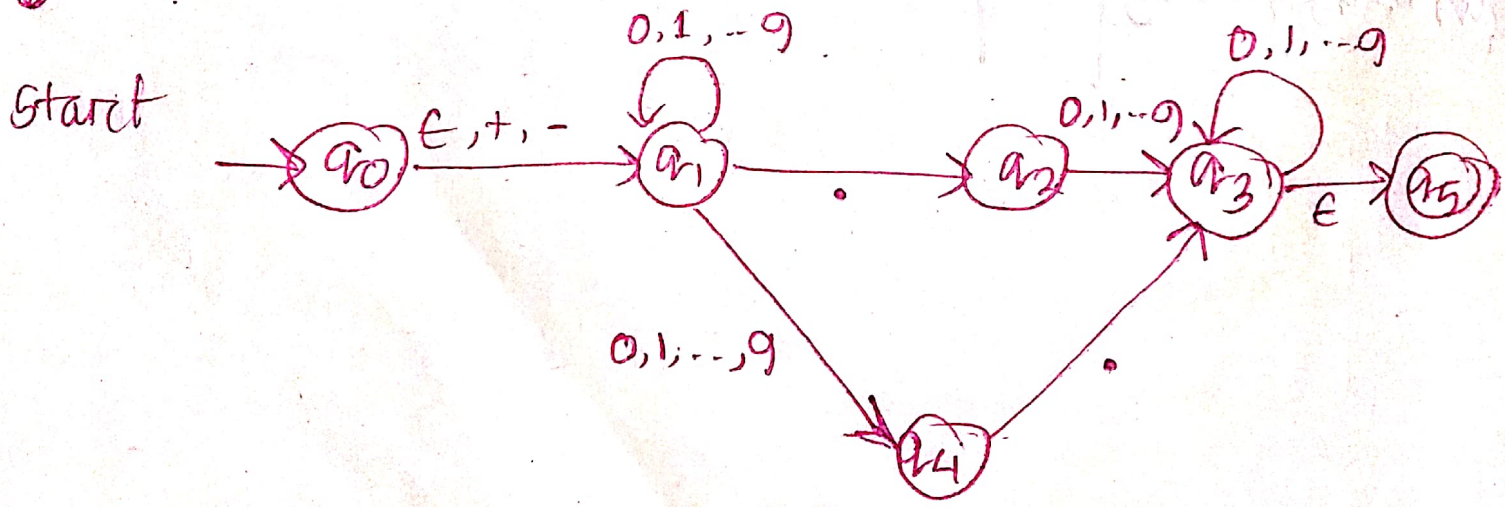
Input Symbol

	0, 1, ... 9	+ -	.
→$\{q_0, q_1\}$	ECLOSE($\{q_1, q_4\}$) $= \{q_1, q_4\}$	ECLOSE($\{q_1\}$) $= \{q_1\}$	ECLOSE($\{q_2\}$) $= \{q_2\}$
$\{q_1\}$	$= \{q_1, q_4\}$	—	$= \{q_2\}$
$\{q_2\}$	ECLOSE($\{q_2\}$) $= \{q_3, q_5\}$	—	—
$\{q_1, q_4\}$	$= \{q_1, q_4\}$	—	ECLOSE($\{q_2, q_3\}$) $= \{q_2, q_3 q_5\}$
*$\{q_3, q_5\}$	ECLOSE($\{q_3\}$) $= \{q_3, q_5\}$	—	—
$q_3 q_5 q_2\}$	$\{q_3, q_5\}$	—	—


0...9
q_1, q_4
$q_2, q_3 q_5$
0...9
0...9
$\{q_0 q_1\}$
+,-
q_1
0...9
0...9
q_2
0...9
q_3, q_5

IDFA

6


Start
q_0
ϵ,+,-
0,1,..9
q_1
q_2
0,1,..9
0,1,..9
q_3
ϵ
q_5
0,1,..,9
q_4

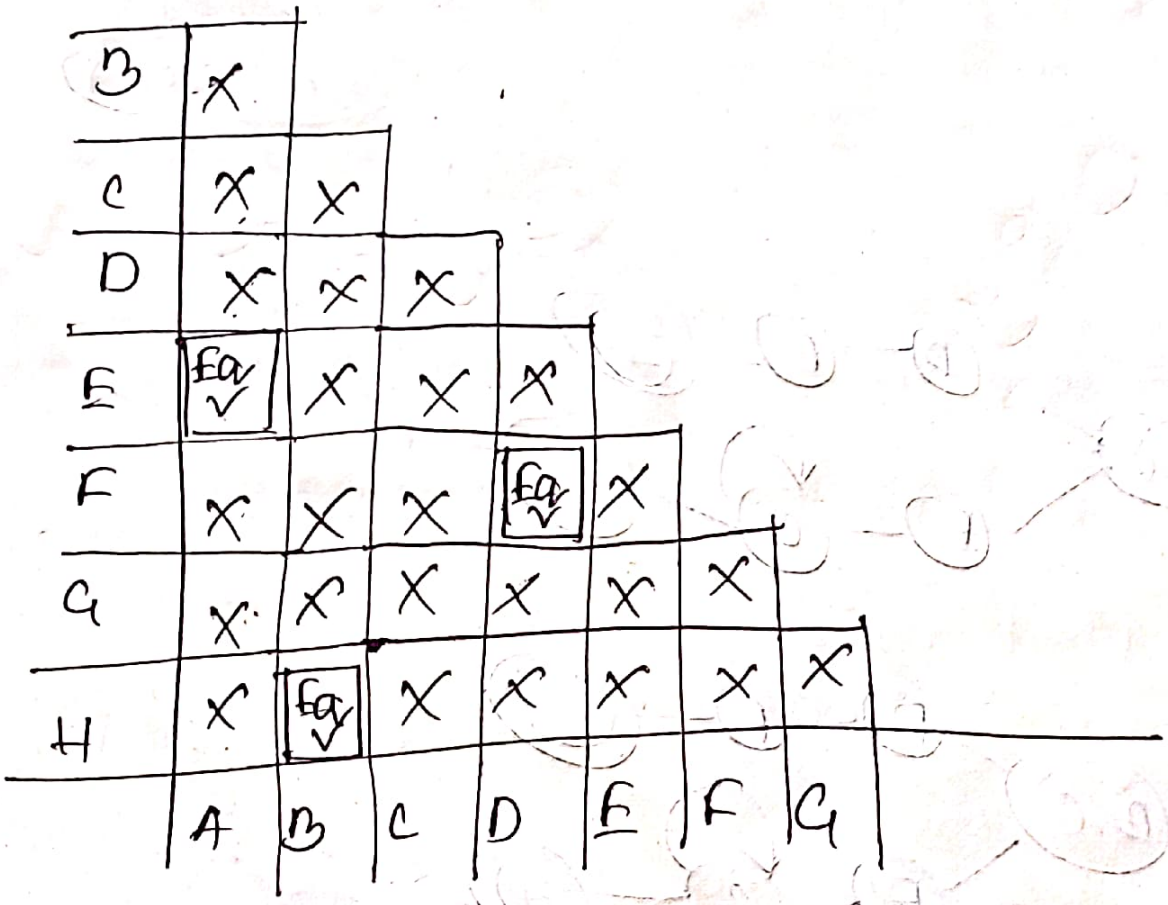
NFA

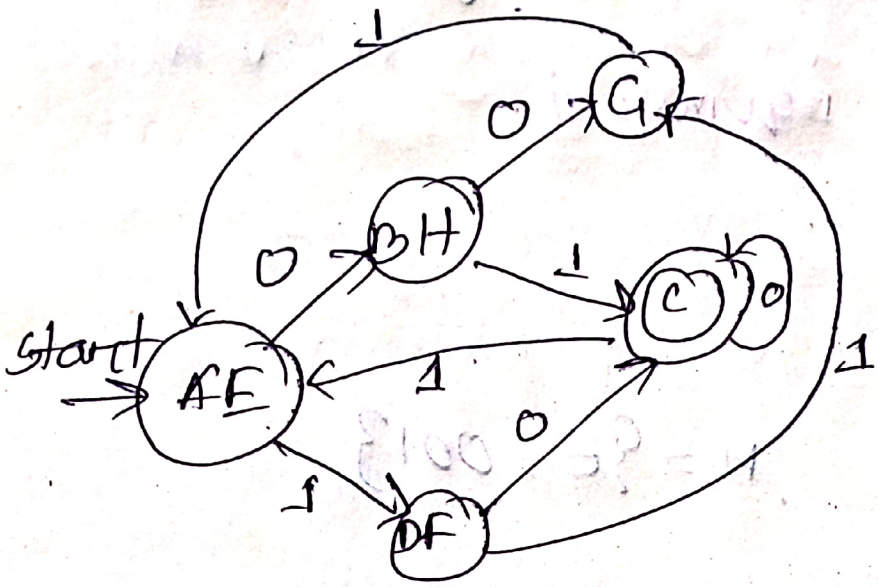
21/8/22

Equivalence and Minimization of Automata

*Final State কখনোই Equivalent হবে না।

Fig: 4.8



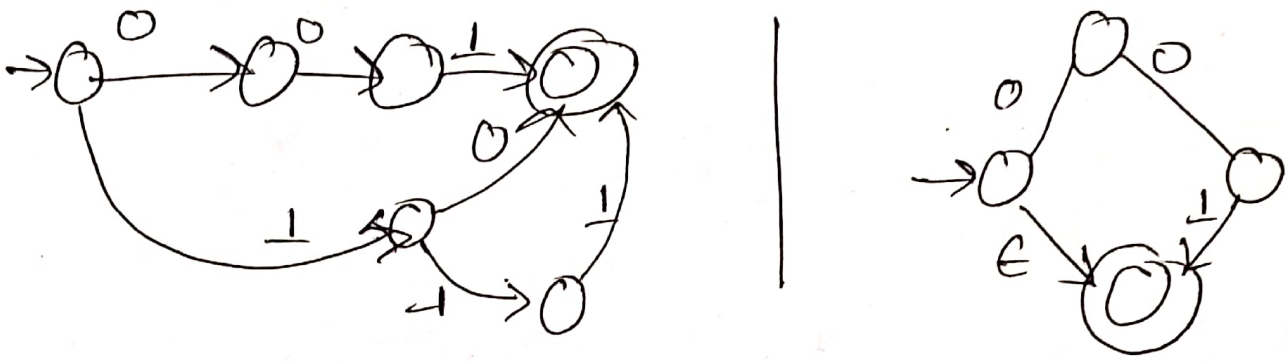

1
0
G
0
H
1
C
0
Start
AE
1
1
0
1
DF

28-08-22

যেকোনো Regular expression আছে যেকোনো Finite Automata আছে যারা Equivalent.

The union

$L = \{001, 10, 111\}$, $M = \{\epsilon, 001\}$



$L \cup M = \{001, 10, 111, \epsilon, 001\}$

$L \cup M = M \cup L$

The concatenation:

L এবং M concatenation হলে তৈরি হবে,

001 ∈ = 001
001 001

10 ∈ = 10
10001

111 ∈ = 111
111001

~~উদাহরণ~~ LM ≠ ML

এখানে sequence matter করে।

[Union এ concatenation fix number of string generate করে]

The closure:

$L = \{0, 11\}$

$L^0 = \{e\}$

$L^1 = L = \{0, 11\}$

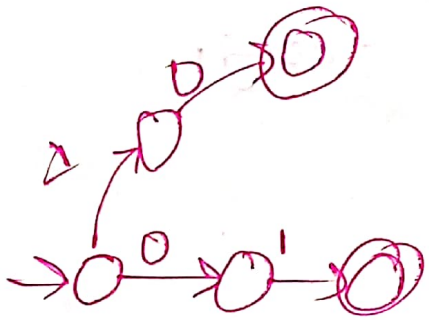
$L^2 = LL = \{00, 011, 110, 1111\}$

$L^3 = LLL = \{000, 0011, 0110, 01111, \ldots\}$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots\ldots$

*যেসব ক্ষেত্রে infinite সংখ্যক of string পাওয়া যাবে,

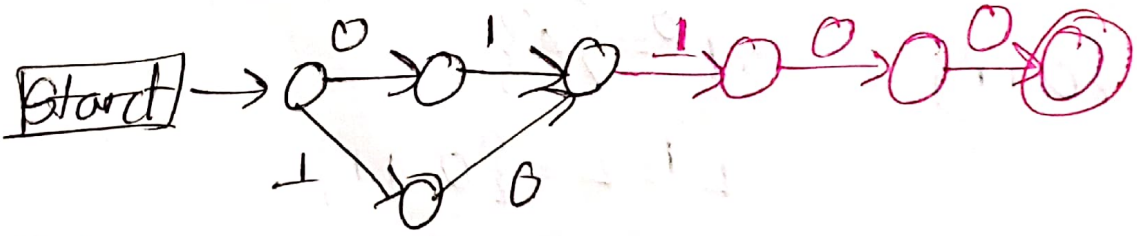
$RE = 001 + 10 + 111 + \epsilon$



$RE = (01 + 10)100$

যে কোন একটি 01 থাকবে OR 10 থাকবে কিন্তু শেষে থাকবে 100।

$L = \{01100, 10100\}$

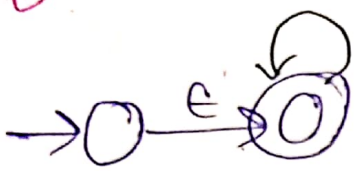


FA

RE = a^*

Strings = $\epsilon, a, aa, aaa, \cdots$

RE = 0^*



← NFA

এর equivalent 

← DFA

Strings: $\epsilon, 0, 01, 10, 11, 100, \cdots$ RE = $(0+1)^*$

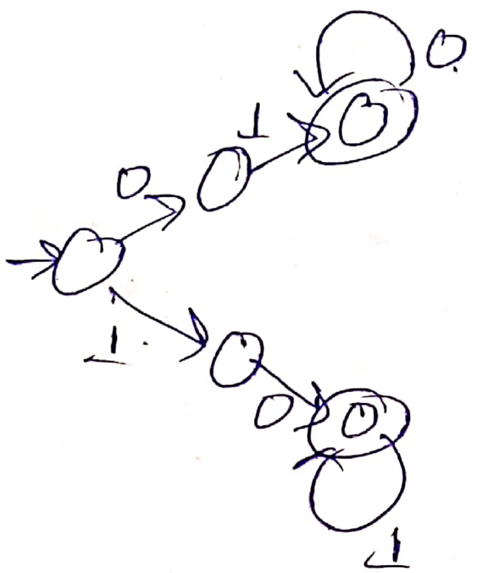
Length: 0, 1, 2, 3

ϵ, $\boxed{0+1}$, $\boxed{(0+1)(0+1)}$, $\boxed{(0+1)(0+1)(0+1)}$

$\boxed{0+1}$ → 0, 1

$\boxed{(0+1)(0+1)}$ → 00, 01, 10, 11

000, 001 ...



এখানে 0 closer

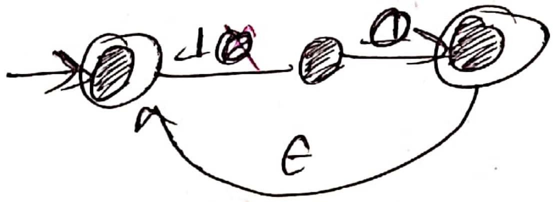
RE = $010^* + 101^*$

এখানে 1 closer

এটা F.A.

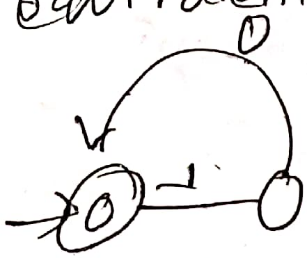
RE = $(10)^*$.

Strings: ε, 10, 1010, 10100, ...

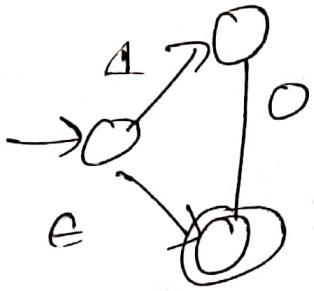


10ε10ε10

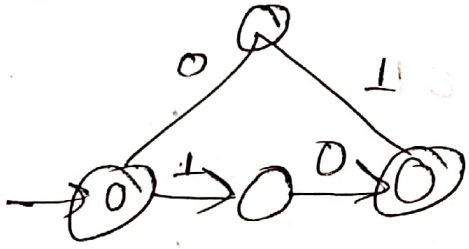
or equivalent



or equivalent

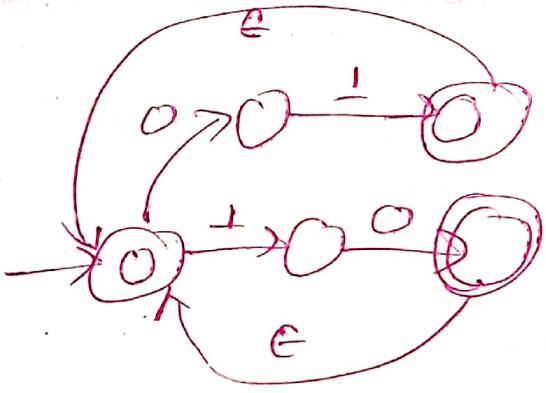


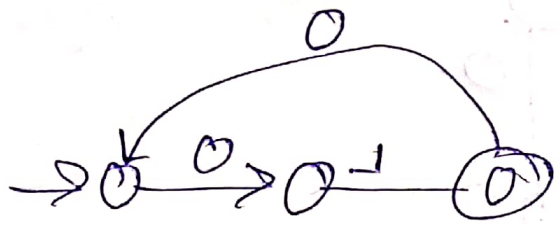
or equivalent



Now suppose

$RE = (10)^* + (01)^*$





String: 01

01001

01001001

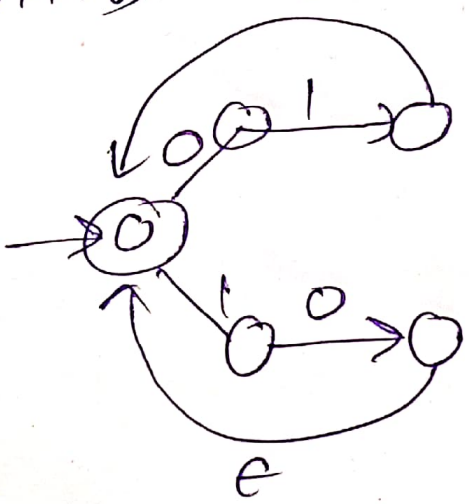
RE = 01(001)*

↑

কারণ এই অংশটা একবারে আসতে পারে বারবারও আসতে পারে,

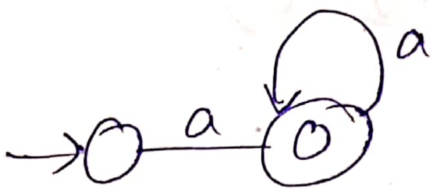
আবার একেবারে নাও আসতে পারে,

L = (01+10)*



$a^+ = aa^* = a, aa, aaa$ → a দিয়ে start হবে ε থাকবে না।

strings = a, aa, aaa, ...



$a? = \epsilon + a$ → কোনো মানে final state এ কোথাও ε দিয়ে যাবে or a দিয়ে

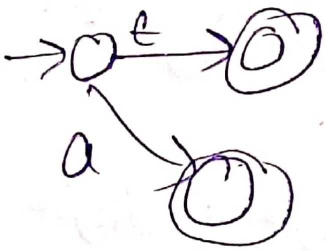


Fig 32.18

$\Sigma = \{0, 1\}$

$\Sigma = \{+, -, 0, 1, \ldots 9, .\}$ → alphabet

RE = (ε + '+' + '−') $(0+1+2+\ldots 9)^*$ ('.' (0+1+ ... 9)

union   symbol

$(0+1+2+\cdots 9)^* e$

$+$

$(0+1+2+\cdots+9)'.'(0+1+2+\cdots+9)^* e)$

symbol $= (+' \; +'-')$

digit $= (0+1+\cdots+9)$

dot $= .$

digit+ ← (ऐसा भी हो सकता है)

RE $= (\epsilon + \text{symbol})\,\text{digit}^*(\text{dot}\,\boxed{\text{digit digit}^*} +$

$\text{digit dot digit}^*)$

OR $= (\epsilon + \text{symbol})\,\text{digit}^*\,(\text{dot digit} + \text{digit dot})\,\text{digit}^*$