Golden Section Search Method

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Equal Interval Search Method

- Choose an interval [a, b] over which the optima occurs

- Compute $f\left(\frac{a+b}{2}+\frac{\varepsilon}{2}\right)$ and $f\left(\frac{a+b}{2}-\frac{\varepsilon}{2}\right)$

- If $f\left(\frac{a+b}{2}+\frac{\varepsilon}{2}\right) > f\left(\frac{a+b}{2}-\frac{\varepsilon}{2}\right)$
then the interval in which the maximum occurs is $\left[\frac{a+b}{2}-\frac{\varepsilon}{2}, b\right]$ otherwise it occurs in $\left[a, \frac{a+b}{2}+\frac{\varepsilon}{2}\right]$

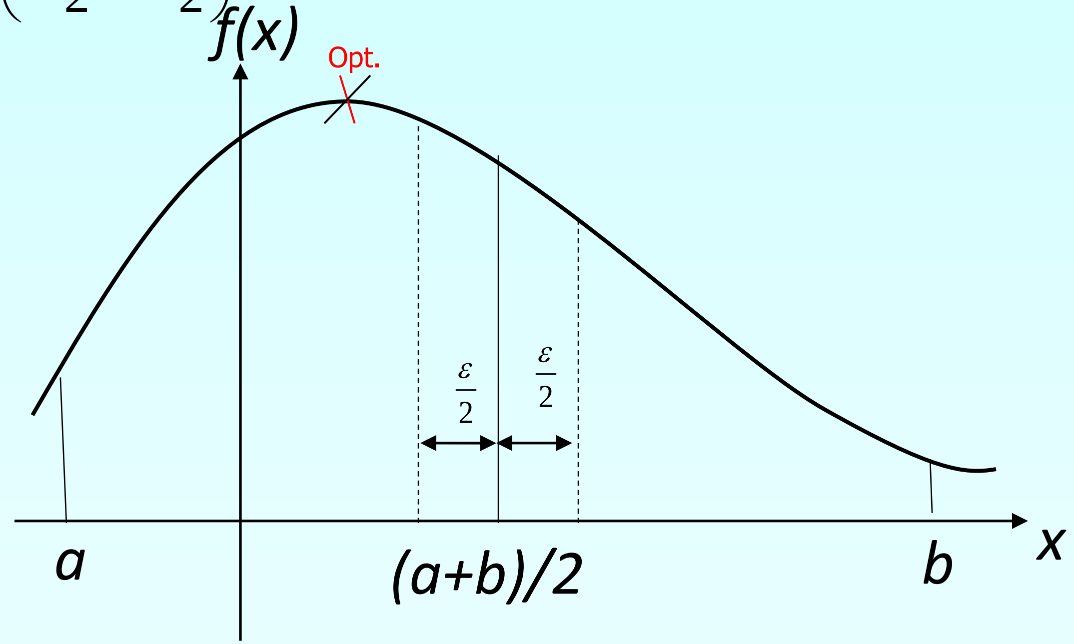


Figure 1 Equal interval search method.

Golden Section Search Method

- The Equal Interval method is inefficient when ε is small.
- The Golden Section Search method divides the search more efficiently closing in on the optima in fewer iterations.

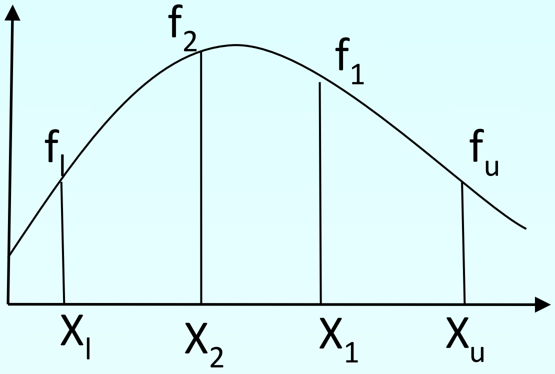
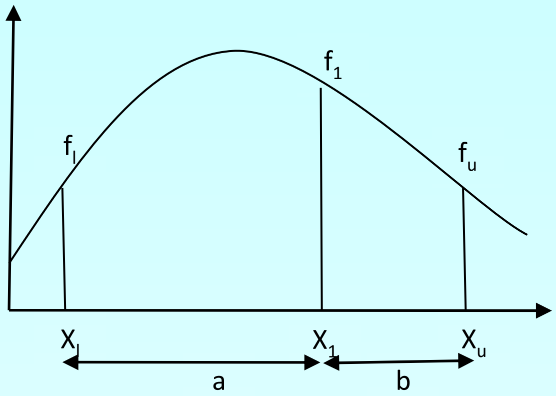


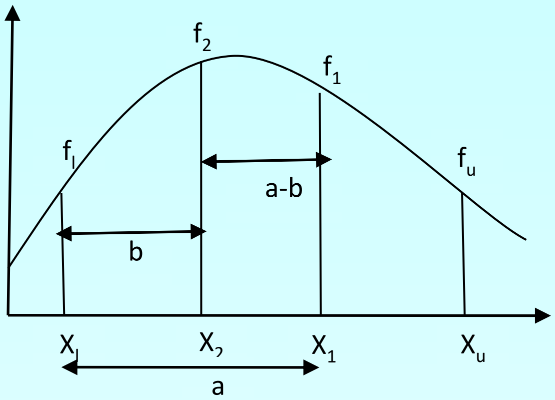
Figure 2. Golden Section Search method

Golden Section Search Method-Selecting the Intermediate Points



Determining the first intermediate point

$$\frac{a}{a+b}=\frac{b}{a}$$

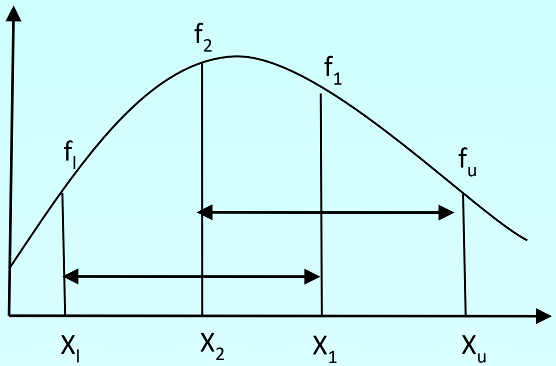


Determining the second intermediate point

$$\frac{b}{a}=\frac{a-b}{b}$$

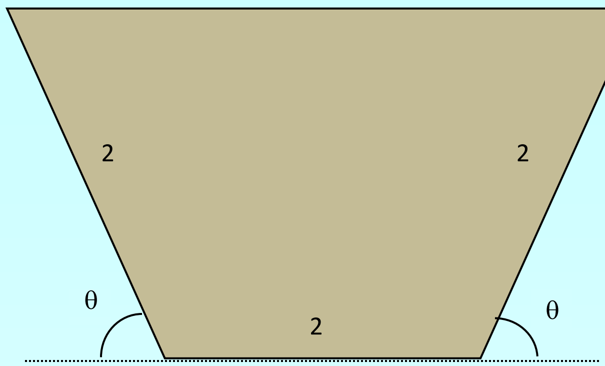
Golden Ratio=> $\frac{b}{a}=0.618\ldots$

Golden Section Search- Determining the new search region



- If $f(x_2) > f(x_1)$ then the new interval is $[x_l, x_2, x_1]$
- If $f(x_2) < f(x_1)$ then the new interval is $[x_2, x_1, x_u]$
- All that is left to do is to determine the location of the second intermediate point.

Example



The cross-sectional area A of a gutter with equal base and edge length of 2 is given by

$$A = 4\sin\theta(1+\cos\theta)$$

Find the angle θ which maximizes the cross-sectional area of the gutter. Using an initial interval of $[0,\pi/2]$ find the solution after 2 iterations. Use an initial $\varepsilon = 0.05$.

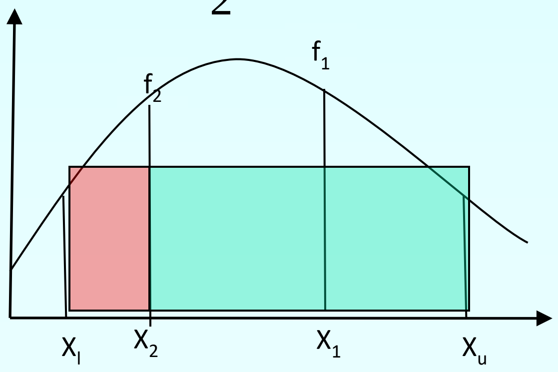
Solution

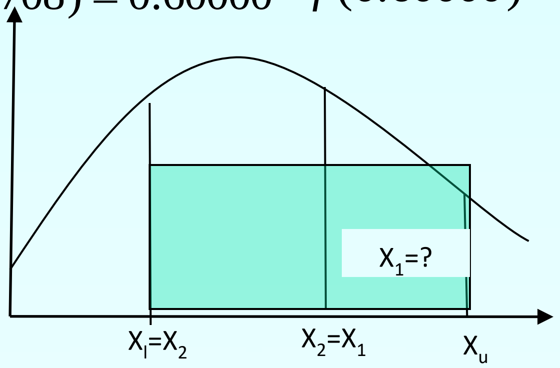
The function to be maximized is $f(\theta) = 4\sin\theta(1+\cos\theta)$

Iteration 1: Given the values for the boundaries of $x_l = 0\ and\ x_u = \pi/2$ we can calculate the initial intermediate points as follows:

$$x_1 = x_l + \frac{\sqrt{5}-1}{2}(x_u - x_l) = 0 + \frac{\sqrt{5}-1}{2}(1.5708) = 0.97080 \qquad f(0.97080) = 5.1654$$

$$x_2 = x_u - \frac{\sqrt{5}-1}{2}(x_u - x_l) = 1.5708 - \frac{\sqrt{5}-1}{2}(1.5708) = 0.60000 \qquad f(0.60000) = 4.1227$$





Solution Cont

$$x_1 = x_l + \frac{\sqrt{5}-1}{2}(x_u - x_l) = 0.60000 + \frac{\sqrt{5}-1}{2}(1.5708 - 0.60000) = 1.2000$$

To check the stopping criteria the difference between x_u and x_l is calculated to be

$$x_u - x_l = 1.5708 - 0.60000 = 0.97080$$

Solution Cont

Iteration 2

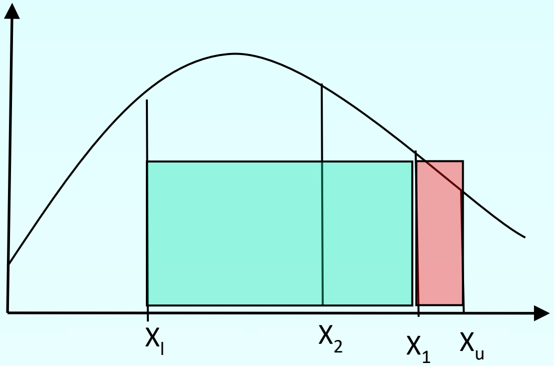
$$x_l = 0.60000$$

$$x_u = 1.5708$$

$$x_1 = 1.2000 \qquad f(1.2000) = 5.0791$$

$$x_2 = 0.97080 \qquad f(0.97080) = 5.1654$$

$$f(x_1) < f(x_2)$$



$$x_l = 0.60000$$

$$x_u = 1.2000$$

$$x_1 = 0.97080$$

$$x_2 = x_u - \frac{\sqrt{5}-1}{2}(x_u - x_l) = 1.2000 - \frac{\sqrt{5}-1}{2}(1.2000 - 0.6000) = 0.82918$$

$$\frac{x_u + x_l}{2} = 1.2000 + 0.6000 = 0.9000$$

Theoretical Solution and Convergence

Iteration	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	$\varepsilon = x_u - x_1$
1	0.0000	1.5714	0.9712	0.6002	5.1657	4.1238	1.5714
2	0.6002	1.5714	1.2005	0.9712	5.0784	5.1657	0.9712
3	0.6002	1.2005	0.9712	0.8295	5.1657	4.9426	0.6002
4	0.8295	1.2005	1.0588	0.9712	5.1955	5.1657	0.3710
5	0.9712	1.2005	1.1129	1.0588	5.1740	5.1955	0.2293
6	0.9712	1.1129	1.0588	1.0253	5.1955	5.1937	0.1417
7	1.0253	1.1129	1.0794	1.0588	5.1908	5.1955	0.0876
8	1.0253	1.0794	1.0588	1.0460	5.1955	5.1961	0.0541
9	1.0253	1.0588	1.0460	1.0381	5.1961	5.1957	**0.0334**

$$\frac{x_u + x_l}{2} = \frac{1.0253 + 1.0588}{2} = 1.0420 \qquad f(1.0420) = 5.1960$$

The theoretically optimal solution to the problem happens at exactly 60 degrees which is 1.0472 radians and gives a maximum cross-sectional area of 5.1962.

THE END