University of Asia Pacific (UAP)

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Ans to the qus no 1 (a)

$x_2 = 94 \times 0.1 = 9.4$

$$\begin{bmatrix} 2 & 8 & -11 \\ 1 & 6 & 4 \\ 16 & 9.4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 10 \end{bmatrix}$$

Finding [U] matrix,
using forward elemination process of gauss elemination.

$$\begin{bmatrix} 2 & 8 & -11 \\ 1 & 6 & 4 \\ 16 & 9.4 & 3 \end{bmatrix}$$

Step 1: $\frac{1}{2} = 0.5$; Row 2 - Row 1 (0.5)

$$= \begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 16 & 9.4 & 3 \end{bmatrix}$$

$\frac{16}{2} = 8$; Row 3 - Row1 (8)

$$= \begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & -54.6 & 91 \end{bmatrix}$$

Step 2

$\frac{-54.6}{2} = -27.3$; Row 3 - Row2 (27.3)

$$[U] = \begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & 0 & 350.35 \end{bmatrix}$$

Finding [L] matrix

~~Here~~

$$\begin{bmatrix} 2 & 8 & -11 \\ 1 & 6 & 4 \\ 16 & 9.4 & 3 \end{bmatrix}$$

Here,

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{2} = 0.5$$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{16}{2} = 8$$

From second step of forward elemination:

$$\begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & -54.6 & 91 \end{bmatrix}$$

$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{-54.6}{2}$$
$$= -27.3$$

$$\therefore [L] = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 8 & -27.3 & 1 \end{bmatrix}$$

Now,

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 8 & -27.3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & 0 & 350.35 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 8 & -27.3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 10 \end{bmatrix}$$

So, $\therefore x_1 = 7$

$0.5x_1 + x_2 = -6$

$\Rightarrow x_2 = -6 - (0.5 \times 7) = -9.5$

$\cancel{x_3 =}\ 8x_1 - 27.3x_2 + x_3 = 10$

$\Rightarrow x_3 = 10 - 8(7) + 27.3(-9.5)$

$= 10 - 56 \cancel{+} - 259.35$

$= -305.35$

$$\therefore [x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ -305.35 \end{bmatrix}$$

Now,

$$[U]\,[x] = [Z]$$

$$\begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & 0 & 350.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -9.5 \\ -305.35 \end{bmatrix}$$

so,

$$2a_1 + 8a_2 - 11a_3 = 7$$

$$2a_2 + 9.5a_3 = -9.5$$

$$350.35\,a_3 = -350.35$$

From, 3rd eq,

$$350.35a_3 = -350.35$$

$$a_3 = \frac{-350.35}{350.35}$$

$$= -1.$$

put a_3 on eq, 2nd,

$$2a_2 + 9a_3 = -9.5$$

$$\Rightarrow 2a_2 = -9.5 - 9a_3$$

$$\Rightarrow 2a_2 = -9.5 - 9(-1)$$

$$\Rightarrow 2a_2 = -9.5 + 9$$

$$\Rightarrow 2a_2 = -0.5$$

$$\Rightarrow a_2 = \frac{-0.5}{2} = -0.25$$

Now, Subsituting a_3 and a_2 on 1st equation,

$$2a_1 + 8(-0.25) - 11(-1) = 7$$

$$\Rightarrow 2a_1 - 2 + 11 = 7$$

$$\Rightarrow 2a_1 = 7 + 2 - 11$$

$$\Rightarrow 2a_1 = -2$$

$$\Rightarrow a_1 = \frac{-2}{2} = -1$$

So, the solution vector is,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -0.25 \\ -1 \end{bmatrix}$$

Ans to the qus no. 1 (b)

Yes LU decomposition can find the inverse of above coefficient matrix.

$$[A] = [L][U]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 8 & -27.3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 6 & 0 & 350.35 \end{bmatrix}$$

Step 1

$$[L][x] = [C]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 8 & -27.3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore x_1 = 1$

$0.5x_1 + x_2 = 0$

$\Rightarrow 0.5(1) + x_2 = 0$

$\Rightarrow x_2 = -0.5$

$\therefore x_2 = -0.5$

$8x_1 - 27.3x_2 + 1x_3 = 0$

$\Rightarrow x_3 = -8(1) + 27.3(-0.5)$

$= -8 - 13.65$

$= -21.65$

$$\therefore [x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ -21.65 \end{bmatrix}$$

Now, $[U][x] = [z]$ for $[x]$

$$\begin{bmatrix} 2 & 8 & -11 \\ 0 & 2 & 9.5 \\ 0 & 0 & 350.35 \end{bmatrix} \begin{bmatrix} b_{10} \\ b_{20} \\ b_{30} \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ -21.65 \end{bmatrix}$$

so,

$$2b_1 + 8b_2 - 11b_3 = 1$$

$$2b_2 + 9.5b_3 = -0.5$$

$$350.35\, b_3 = -21.65$$

so, using backward substitution,

$$b_3 = \frac{-21.65}{350.35}$$

$$= -0.0617$$

$$2b_2 = -0.5 - 9.5(b_3)$$

$$2b_2 = -0.5 - 0.5861$$

$$\Rightarrow b_2 = \frac{-0.5 - 0.5861}{2}$$

$$= -0.5430$$

Now,

$$2b_1 + 8b_2 - 11b_3 = 1$$

~~$\Rightarrow -11b_3 = 1 - 2($~~

$$\Rightarrow 2b_1 = 1 - 8(-0.5430) + 11(-0.0617)$$

$$\Rightarrow 2b_1 = 4.6653$$

$$\Rightarrow b_1 = \frac{4.6653}{2}$$

$$= 2.3326$$

$\therefore$ The First column of inverse [A]

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2.3326 \\ -0.5430 \\ -0.0617 \end{bmatrix}$$

∴ so the inverse of $\boxed{A}$

$$\begin{bmatrix} 2.3326 & \cdots & \cdots \\ -0.5430 & \cdots & \cdots \\ -0.0617 & \cdots & \cdots \end{bmatrix}$$

Here, I am calulate inverse of column 1.

Ans to the qus no 2(a)

Find the velocity of $t = 94 + 10$
$= 104$ s

$$\therefore v(t) = \sum_{i=0}^{1} L_i(t)\, v(t_i)$$

$$= L_0(t)\, v(t_0) + L_1(t)\, v(t_1)$$

$t_0 = 65.75$; $v(t_0) = 1902.249$

$t_1 = 95.5$ $v(t_1) = 2799.901$

$t_2 = 125.5$ $v(t_2) = \cancel{2799.9}$ 3697.553

$$L_0 = \left(\frac{t - t_1}{t_0 - t_1}\right)\left(\frac{t - t_2}{t_0 - t_2}\right)$$

$$L_1 = \left(\frac{t - t_0}{t_1 - t_0}\right)\left(\frac{t - t_2}{t_1 - t_2}\right)$$

$$L_2 = \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right)$$

$$v(t) = \left(\frac{t - t_1}{t_0 - t_1}\right)\left(\frac{t - t_2}{t_0 - t_2}\right) v(t_0)$$

$$+ \left(\frac{t - t_0}{t_1 - t_0}\right)\left(\frac{t - t_2}{t_1 - t_2}\right) +$$

$$v(t_1) + \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right) v(t_2)$$

$$y(104) = \left(\frac{104-95.5}{65.75-95.5}\right)\left(\frac{104-125.5}{65.75-125.5}\right)$$

$$(1902.249) + \left(\frac{104-65.75}{95.5-65.75}\right)$$

$$\left(\frac{104-125.5}{95.5-125.5}\right)(2799.901) + \left(\frac{104-65.75}{125.5-65.75}\right)$$

$$\left(\frac{104-95.5}{125.5-95.5}\right)(3697.553)$$

$$\cancel{= (-0.2857)(1902.249) + (0.3598)}$$

$$\cancel{= (-0.2857) \times (}$$

$$= (-0.1028)(1902.249) + (0.9219)$$

$$(2799.901) + (0.1813) \cdot (3697.553)$$

$= -195.5689 + 2570.9087 + 679.6649$

$= 3055.0047 \ m/s$

Ans to the qus no. 2(b)

For linear Interpolation, $t_1 = 125.5$

Here, $t_0 = 95.5$ $v(t_0) = 2799.901$, $v(t_1) = 3697.553$

$$v(t) = \frac{t - t_1}{t_0 - t_1} v(t_0) + \frac{t - t_0}{t_1 - t_0} v(t_1)$$

$$v(104) = \frac{104 - 125.5}{95.5 - 125.5} \cdot (2799.901)$$

$$+ \frac{104 - 95.5}{125.5 - 95.5} (3697.553)$$

$$= 2006.595 + 1047.69$$

$$= 3059.235 \text{ m/s}$$

So, absolute relative error,

$$|\epsilon_a| = \left| \frac{3055.0047 - 3059.235}{3055.0047} \right| \times 100$$

$$= \cancel{2.51\%}$$

$$= 0.02\%$$ Ans

Ans to the qus no. 4 (a)

$$A = 4\sin\theta\left(1+\cos\theta\right)$$

Iteration 1

Here,

$x_l = 0$

$x_u = 18.8$

$h = 18.8 - 0 = 18.8$

$x_1 = x_l + 0.618 \times h$

$= 0 + 0.618 \times 18.8$

$= 11.6189$

$x_2 = 18.8 - 0.618 \times 18.8$

$= 7.1816$

Now $f(x_2) = f(7.1816)$

$$= 4 \sin(7.1816)(1+\cos(7.1816))$$

$$= 0.9961$$

$f(x_1) = f(11.6189)$

$$= 4 \sin(11.6189)(1+\cos(11.6189))$$

$$= 1.5946$$

So, $f(x_1) > f(x_2)$,

Now,

$x_u = 1.5946$

$x_1 = \cancel{1.5946}\ 1.5946$

$x_l = 0.9961$

$x_2 = 1.5946$

Iteration 2

$x_u = 1.5946$

$x_1 = 1.5946$

$x_l = 0.9961$

$x_2 = 1.5946$

Ans to qus no. 3

a) $c_1 = 1$, $x_1 = -0.5773550269$

$c_2 = 1$ $x_2 = 0.5773550264$

$a = 0.2 \times 94 = 18.8$

$b = a + 3 = 21.8$

$\therefore \int_{18.8}^{21.8} x^3 e^x \, dx$

convert to limit from $[18.8, 21.8]$ to $[-1, 1]$

$\Rightarrow \int_{-1}^{1} f(t) \, dt$

$$\Rightarrow \frac{21.8 - 18.8}{2} \int_{-1}^{1} f\left(\frac{21.8 - 18.8}{2} x + \frac{21.8 + 18.8}{2} \right) dx$$

$\Rightarrow 1.5 \int_{-1}^{1} f(1.5x + 19.3)\, dx$

$\Rightarrow 1.5 \int_{-1}^{1} f(1.5x + 19.3)\, dx$

$\approx 1.5\, c_1 f(1.5x_1 + 19.3) + 1.5\, c_2 f(1.5x_2 + 19.3)$ $c_1 = 1,\ c_2 = 1,\ x_1 = -0.577350269$

$\therefore f(18.4) = x^3 e^x$

$= (18.4)^3 e^{(18.934)}$

$= 6.41$

$f(20.17) = (20.17)^3 \times e^{(20.17)}$

$= 4.7$

$\Rightarrow 1.5(6.41) + 1.5(4.7)$

$\Rightarrow 16.66$