

Composite Transformation :

As the name suggests itself Composition, here we combine two or more transformations into one single transformation that is equivalent to the transformations that are performed one after one over a 2-D object.

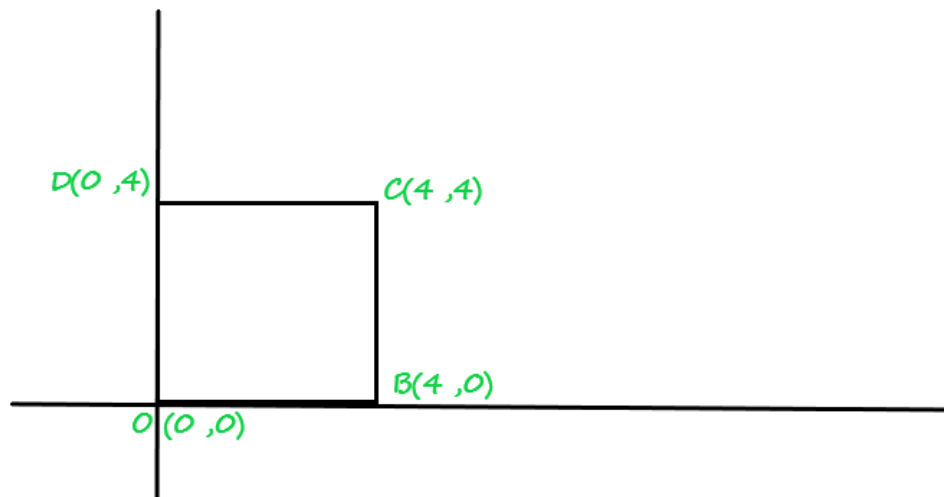
Example :

Consider we have a 2-D object on which we first apply transformation T_1 (**2-D matrix condition**) and then we apply transformation T_2 (**2-D matrix condition**) over the 2-D object and the object get transformed, the very equivalent effect over the 2-D object we can obtain by multiplying T_1 & T_2 (**2-D matrix conditions**) with each other and then applying the T_{12} (**resultant of $T_1 \times T_2$**) with the coordinates of the 2-D image to get the transformed final image.

Problem :

Consider we have a square $O(0, 0)$, $B(4, 0)$, $C(4, 4)$, $D(0, 4)$ on which we first apply T_1 (**scaling transformation**) given scaling factor is $S_x = S_y = 0.5$ and then we apply T_2 (**rotation transformation in clockwise direction**) it by 90° (angle), in last we perform T_3 (**reflection transformation about origin**).

Ans : The square O, A, C, D looks like :



First, we perform scaling transformation over a 2-D object :

Representation of scaling condition :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate O(0, 0) :

$$O \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B(4, 0) :

$$B \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

For coordinate C(4, 4) :

$$C \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

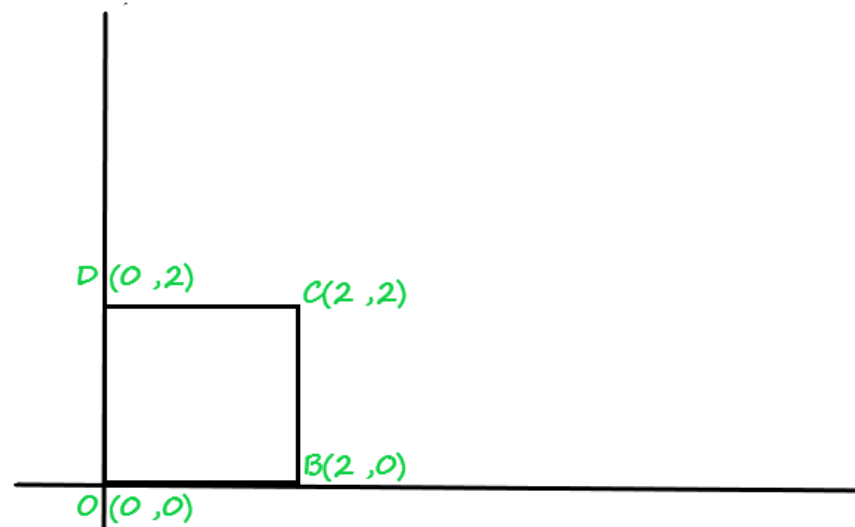
$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

For coordinate D(0, 4) :

$$D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

2-D object after scaling :



***Now, we'll perform rotation transformation in clockwise-direction on Fig.2 by 90°:**

The condition of rotation transformation of 2-D object about origin is :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\cos 90 = 0$$

$$\sin 90 = 1$$

For coordinate O(0, 0) :

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B(2, 0) :

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

For coordinate C(2, 2) :

$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

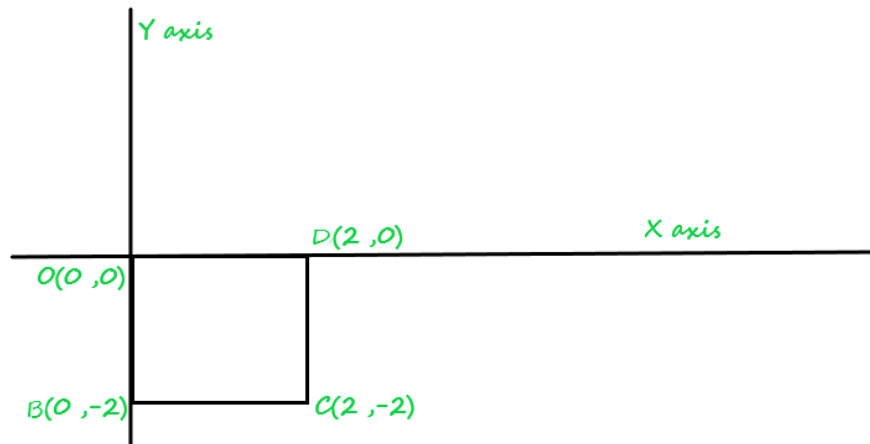
$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

For coordinate D(0, 2) :

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

2-D object after rotating about origin by 90° angle :



Now, we'll perform third last operation on Fig.3, by reflecting it about origin :

The condition of reflecting an object about origin is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate $O(0, 0)$:

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate $B'(0, 0)$:

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

For coordinate $C'(0, 0)$:

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

For coordinate D'(0, 0) :

$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The final 2-D object after reflecting about origin, we get :

