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Cost-Effective Operation and Tariff Strategies for Multi-Generation Electricity Production with Hydro Reservoir Management

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Part One: Electricity Supply Solely from Thermal Generators

When it comes to the supply of electricity through thermal generators, we can streamline the modeling process effectively by taking into account the available data and focusing on cost minimization as our primary objective. This can be achieved by clearly defining the objective function and specifying the relevant variables.

The costs associated with power plants can be categorized into three main groups:

1. **Generator Setup Costs**: These are the initial expenses incurred in establishing the generators.
2. **Fixed Electricity Production Costs**: These represent the fixed costs associated with generating electricity at a minimum level per hour of operation.
3. **Variable Electricity Production Costs**: This category encompasses the variable costs incurred when producing electricity beyond the minimum required capacity per hour of operation. It's important to note that this production falls within a range defined by minimum and maximum capacity levels.

By breaking down these cost components and incorporating them into our modeling framework, we can efficiently work towards the goal of minimizing expenses while maintaining a reliable supply of electricity from thermal generators.

The objective function of Part One

Setting up generators incurs various costs, and it's essential to understand these costs for effective planning. Let's break down these costs into three parts:

1. When setting up generators, it's natural that in the initial period, a certain number of generators of the three available types will become operational. As time progresses, this initial number may suffice, or additional generators might be required to meet the growing demand.
   To represent generators that start operating during any time interval, we can use the notation S_{ij}, where i denotes the generator type (i = 1,2,3), and j indicates the time interval (j = 1,2,3,4,5). Now, the initial cost component can be expressed based on the setup cost data in the table as follows:

$$2000\sum_{j} S_{1j} + 1000\sum_{j} S_{2j} + 500\sum_{j} S_{3j}$$

   To simplify this representation, we can use the fixed setup costs for each generator type denoted as E_i (initial Establish). For example, E_2 equals 1000. Therefore, a more concise form of this constraint is:

$$E_i \sum_{i,j} S_{ij} = \sum_{i,j} E_i S_{ij}$$

2. Moving on to the second cost component, which is the cost of generating electricity at the minimum level per hour of operation. We represent the number of type i

generators active in time period j as G_{ij} (Generator start). For instance, G_{23} represents the count of type 2 generators operating in the third time interval. In each interval, we have G_{ij} generators of each type running at the minimum level. So, the cost can be expressed as:

$$1000\sum_{j} G_{1j} + 2600\sum_{j} G_{2j} + 3000\sum_{j} G_{3j}$$

To simplify this section, we can use the minimum fixed costs for each generator type denoted as F_i (initial Fixed Cost). For example, F_3 equals 3000. Therefore, a more compact form of this constraint is:

$$F_i\sum_{i,j} G_{ij} = \sum_{i,j} F_i G_{ij}$$

3. Finally, for the last cost component, we consider the output of active generators of each type in each interval, denoted as O_{ij} (Output). Again, i represents the generator type ($i = 1,2,3$), and j indicates the time interval ($j = 1,2,3,4,5$). Each O_{ij} is the sum of the output of all generators of type i active in interval j and falls within the range of the minimum and maximum production limits, multiplied by the number of generators of that type (G_{ij}). Based on the data in Table 2, the cost calculation is as follows:

$$2\sum_{j}(O_{1j} - 850\,G_{1j}) + 1.3\sum_{j}(O_{2j} - 1250\,G_{2j}) + 3\sum_{j}(O_{3j} - 1500\,G_{3j})$$

To formalize this part further, we can use the variable costs for the minimum activity of each generator type, denoted as V_i (initial Variable Cost). For example, V_1 equals 2. Additionally, the minimum limit for each generator type can be represented as m_i (initial minimum). For example, m_2 is equal to 1250. Therefore, a simplified form of this constraint becomes:

$$V_i\sum_{i,j} O_{ij} - m_i G_{ij} = \sum_{i,j} V_i(O_{ij} - m_i G_{ij})$$

By summing up these three cost components, we arrive at the final objective function:

$$Min\ Z = \sum_{i,j} E_i S_{ij} + \sum_{i,j} F_i G_{ij} + \sum_{i,j} V_i(O_{ij} - m_i G_{ij})$$

Constraints of Part One

1. The initial set of restrictions pertains to fulfilling the subscribers' demands during specific time periods:

$$\left.\begin{array}{l}
1)\ O_{11} + O_{21} + O_{31} \geq 15000 \\
2)\ O_{12} + O_{22} + O_{32} \geq 30000 \\
3)\ O_{13} + O_{23} + O_{33} \geq 25000 \\
4)\ O_{14} + O_{24} + O_{34} \geq 40000 \\
5)\ O_{15} + O_{25} + O_{35} \geq 27000
\end{array}\right\} \longrightarrow$$

You can think of the subscriber demand within each interval, denoted as D_j (where D stands for Demand), as a fixed quantity. To summarize this set of constraints for each interval, you can do the following:

$$\sum_{i} O_{ij} \geq D_j$$

2. These limitations fall within the realm of technical constraints associated with generators, specifically pertaining to their minimum and maximum production thresholds:

6) $O_{1j} \geq 850G_{1j}$

6′) $O_{1j} \leq 2000G_{1j}$

The production capacity for type 1 generators during period j has reached its limits.

7) $O_{2j} \geq 1250G_{2j}$

7′) $O_{2j} \leq 1750G_{2j}$

The production capacity for type 2 generators during period j has reached its limits.

8) $O_{3j} \leq 1500G_{3j}$

8′) $O_{3j} \geq 4000G_{3j}$

The production capacity for type 3 generators during period j has reached its limits.

$$O_{ij} \geq G_{ij} \times m_i$$

$$O_{ij} \leq G_{ij} \times M_i$$

Here, m_i represents the initial minimum limit for generator type i, and M_i represents the initial maximum limit for generator type i.

3. If the network load increases by 15% in any period of time, the working generators should be able to respond to this increase in demand, and there is no need to start a new generator, and at the same time, each generator should work within its production limits. So:

9) $2000G_{11} + 1750G_{21} + 4000G_{31} \geq 1.15 \times 15000$

10) $2000G_{12} + 1750G_{22} + 4000G_{32} \geq 1.15 \times 30000$

11) $2000G_{13} + 1750G_{23} + 4000G_{33} \geq 1.15 \times 25000$

12) $2000G_{14} + 1750G_{24} + 4000G_{34} \geq 1.15 \times 40000$

13) $2000G_{15} + 1750G_{25} + 4000G_{35} \geq 1.15 \times 27000$

To encapsulate these restrictions succinctly:

$$\sum_i M_i G_{ij} \geq 1.15D_j$$

4. In each time interval (j), the number of generators for each type must be adjusted such that it equals the difference between the active generators in the previous and current intervals. However, to account for the constraint that S_{ij} cannot become negative, we express this set of restrictions using a greater-than-or-equal-to relationship. This approach ensures that there won't be any issues even if the current interval requires fewer generators of type (i) than the previous interval:

14) $S_{1j} \geq G_{1j} - G_{1j-1}$

15) $S_{2j} \geq G_{2j} - G_{2j-1}$

16) $S_{3j} \geq G_{2j} - G_{2j-1}$

Summarizing this set of restrictions as:

$$S_{ij} \geq G_{ij} - G_{ij-1}$$

5. Additionally, the quantity of generators utilized for electricity generation should not surpass the total count of generators at our disposal:

17) $G_{1j} \leq 12$ 20) $S_{1j} \leq 12$

18) $G_{2j} \leq 10$ 21) $S_{2j} \leq 10$

19) $G_{3j} \leq 5$ 22) $S_{3j} \leq 5$

When we define A_i (initial Available) as the count of type-i generators that are initially available, the corresponding limits will be as follows:

$$G_{ij} \leq A_i$$
$$S_{ij} \leq A_i$$

6. All the modeling variables are non-negative and except for the variable O_{ij} (which is related to the volume of electricity production and is a continuous value), the rest are all integers:

$$23)\ G_{ij}, S_{ij} \geq 0 \ \&\ Int \qquad\qquad 24)\ O_{ij} \geq 0$$

With all the necessary variables defined, the objective function explained, and the limits set, let's now summarize and model the final section as follows:

$$Min\ Z = \sum_{i,j} E_i S_{ij} + \sum_{i,j} F_i G_{ij} + \sum_{i,j} V_i (O_{ij} - m_i G_{ij})$$

$s.t.$

$$1)\ \sum_i O_{ij} \geq D_j \qquad for\ j = 1,2,3,4,5$$

$$2)\ O_{ij} \geq G_{ij} \times m_i \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

$$3)\ O_{ij} \leq G_{ij} \times M_i \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

$$4)\ \sum_i M_i G_{ij} \geq 1.15 D_j \qquad for\ j = 1,2,3,4,5$$

$$5)\ S_{ij} \geq G_{ij} - G_{ij-1} \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

$$6)\ G_{ij}, S_{ij} \leq A_i \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

$$7)\ G_{ij}, S_{ij} \geq 0 \ \&\ Int \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

$$8)\ O_{ij} \geq 0 \qquad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$$

Part Two: Electricity Supply from Both Thermal and Hydro Generators

In this section, as we delve into the data with a primary aim of cost minimization, our approach begins with the establishment of key variables, followed by a thorough elucidation of the objective function. Subsequently, we will proceed to construct and outline the remaining constraints. Within the realm of our analysis, it's crucial to distinguish between two main cost categories associated with water generators: the initial setup cost and the ongoing cost of electricity production (should they be constructed). Now, let's delve into the modeling and description of these two distinct cost components.

The objective function of Part Two

1. Let's start by considering the cost of construction. To do this, we can introduce binary variables, denoted as n_{ij} (initially new). These variables take the value of 1 if a generator of type i needs to be constructed in interval j, and 0 otherwise. Now, if we assume the cost of constructing a generator of type i as $E_i{}'$ (where the prime notation signifies a distinction from a cost mentioned in Part One), we can express this as:

$$1500 \sum_j n_{1j} + 1200 \sum_j n_{2j} = \sum_{i,j} E_i' n_{ij}$$

2. Moving on to the two types of water generators, which can be either on or off during any time period. We can define binary variables, denoted as H_{ij} (initially Hydro Power), such that H_{ij} equals 1 if water generator type i is operational in period j, and 0 otherwise. Additionally, if the generator is on during one interval, it remains on for the entire duration of that interval. Consequently, the cost calculation for this section becomes:

$$90(6H_{11} + 3H_{12} + 6H_{13} + 3H_{14} + 6H_{15}) + 150(6H_{21} + 3H_{22} + 6H_{23} + 3H_{24} + 6H_{25})$$

We can represent the duration of each interval as T_j (beginning of Time), where, for instance, T_3 equals 6. Furthermore, let C_i (initial Cost) denote the cost per working hour of generator type i. Thus, the simplified expression for this section is:

$$C_1 \sum_j T_j H_{1j} + C_2 \sum_j T_j H_{2j} = \sum_{i,j} C_i T_j H_{ij}$$

Finally, we can formulate the objective of this department and aim to minimize its cost as follows:

$$\min z = \sum_{i,j} E_i' n_{ij} + \sum_{i,j} C_i T_j H_{ij}$$

Constraints of Part Two

1. The first limitation pertains to demand, and it's worth noting that the availability of a water generator can influence the adjustments made to the demand limit in Part One. An important consideration here is that thermal generators have the capacity to allocate a portion of their output for water pumping into storage tanks, thereby ensuring they remain within the desired height range. For instance, if we introduce a variable called P_j (initial Pump) to represent the electricity consumption for water pumping within interval j, we can express the total power output received by consumers from thermal generators as the overall production minus the power consumed for pumping water. Consequently, the demand limit for each time period can be defined as follows:

$$\sum_i O_{ij} + \left(900(H_{1j}) + 1400(H_{2j})\right) - P_j \geq D_j$$

To clarify this constraint, let's define R_i (initial Rate) as the electricity production rate of a hydro generator of type i:

$$\sum_i O_{ij} + \sum_i R_i H_{ij} - P_j \geq D_j$$

2. Moving on to the next set of constraints, these are associated with the water levels in the reservoirs. Let's define the variable h_j as the initial height of water in reservoir j, which is essentially the water level at the start of interval j. These constraints can be summarized as follows:

$$h_1 = 16 \ , \ \begin{cases} h_j \leq 20 \\ h_j \geq 15 \end{cases} for\, j > 1$$

Furthermore, considering that the water level at time $j+1$ depends on factors like the water level at time j, water input and extraction during interval j, and electricity generation from water during interval j, we can establish a recursive relationship as follows:

$$h_{j+1} = h_j + \frac{P_j T_j}{3000} - 0.31(H_{1j} T_j) - 0.47(H_{2j} T_j)$$

We can also define d_i (depletion rate at the start) as the rate at which the water height decreases per hour due to generator type i. In this case, the upper limit can be expressed as:

$$h_{j+1} - h_j - \frac{P_j T_j}{3000} + \sum_i d_i T_j H_{ij} = 0$$

3. Moving on to the next constraint, it relates to responding to a 15% increase in demand. In this context, the production rate R_i of water generators (previously considered) becomes relevant. Consequently, the form of the third constraint in Part One can be adjusted as follows:

$$\sum_i M_i G_{ij} + \sum_i R_i \geq 1.15 D_j$$

4. Finally, the last constraint pertains to construction generators. The number of generators of each type added in each time interval, denoted as n_{ij}, must equal the difference between the active generators in the current and previous intervals. To ensure feasibility and avoid issues when there are fewer generators of type i required in the current interval compared to the previous one, we express this set of constraints as follows:

$$n_{ij} \geq H_{ij} - H_{ij-1}$$

It's worth noting that both n_{ij} and H_{ij} are binary variables (either 0 or 1). If both H_{ij} and H_{ij-1} are equal to 1, n_{ij} can be either 0 or 1. However, this doesn't pose a problem because our objective is cost minimization, and the objective function itself ensures that n_{ij} becomes 0 if such a situation arises.

Since this part was proposed alongside Part One, obtaining the final objective function is as simple as combining the objective function of Part One with the objective function of this part. Additionally, regarding the constraints, two scenarios have emerged. Firstly, there were alterations made to some of the constraints in Part One, which are elaborated upon and applied in the first and third constraints of this section. Secondly, a set of new constraints has been introduced to the problem. These constraints encompass the specific constraints from Part One, the specific constraints from Part Two, as well as the common constraints that have undergone changes. Taken together, these constraints collectively represent all the constraints for the problem. Based on these scenarios, the final modeling can be illustrated as follows:

$$Min\ Z = \sum_{i,j} E_i S_{ij} + \sum_{i,j} F_i G_{ij} + \sum_{i,j} V_i (O_{ij} - m_i G_{ij}) + \sum_{i,j} E_i^{'} n_{ij} + \sum_{i,j} C_i T_j H_{ij}$$

$s.t.$ 1) $\sum_i O_{ij} + \sum_i R_i H_{ij} - P_j \geq D_j$ $\quad for\ j = 1,2,3,4,5$

2) $O_{ij} \geq G_{ij} \times m_i$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

3) $O_{ij} \leq G_{ij} \times M_i$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

4) $\sum_i M_i G_{ij} + \sum_i R_i \geq 1.15 D_j$ $\quad for\ j = 1,2,3,4,5$

5) $S_{ij} \geq G_{ij} - G_{ij-1}$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

6) $n_{ij} \geq H_{ij} - H_{ij-1}$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

7) $G_{ij}\ , S_{ij} \leq A_i$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

8) $h_1 = 16$

9) $h_j \geq 15$ $\quad for\ j = 2,3,4,5$

10) $h_j \leq 20$ $\quad for\ j = 2,3,4,5$

11) $h_{j+1} - h_j - \frac{P_j T_j}{3000} + \sum_i d_i T_j H_{ij} = 0$ $\quad for\ j = 1,2,3,4,5$

12) $G_{ij}, S_{ij} \geq 0\ \&\ Int$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

13) $O_{ij}, h_j, P_j \geq 0$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

14) $H_{ij}, n_{ij} = 0,1$ $\quad for\ i = 1,2,3\ and\ j = 1,2,3,4,5$

* * *

Phase Two; Coding and Sensitivity Analysis with GAMS

Optimal Solution of the Models

Electricity Supply Solely from Thermal Generators

First, part of the problem data was written as Set and Parameter in GAMS. Additional explanations of each set and parameter are given in quotation marks to facilitate understanding and increase the readability of the code. In the following, the explanations and details related to the types of thermal generators were recorded in the table *data (g, k)*. In the previous section, the set k is defined as the columns of the *data* table. Next, to access each column of this table, you can put the name of the corresponding column instead of k.

```
Set
   t 'periods'       / 12pm-6am, 6am-9am, 9am-3pm, 3pm-6pm, 6pm-12pm /
   g 'generators'    / type-1, type-2, type-3 /
   k 'parameters'    / min-pow,  max-pow,  cost-min,  cost-inc,  start,   number/
;

Parameter
   dem(t)  'demand     (1000mw)' / 12pm-6am 15, 6am-9am 30, 9am-3pm 25, 3pm-6pm 40, 6pm-12pm 27 /
   dur(t)  'duration    (hours)' / 12pm-6am  6, 6am-9am  3, 9am-3pm  6, 3pm-6pm  3, 6pm-12pm  6 /
;

Table data(g,k) 'generation data'
            min-pow  max-pow  cost-min  cost-inc  start   number
   type-1       .85      2.0      1000       2.0   2000       12
   type-2      1.25     1.75      2600       1.3   1000       10
   type-3      1.5      4.0       3000       3.0    500        5
;
```

Then according to the model presented in the first phase, appropriate variables have been defined. Some variables such as Gen (representing the number of active generators in each demand interval) and S, according to their definition in mathematical modeling, are a special type of variables (integer variable and positive variable, respectively), which are also included in Gems code. After these, the objective function and each equation are defined with their names, appropriate descriptions, and corresponding limits.

```
Variable
   O(g,t)      'generator output'
   Gen(g,t)    'number of generators in use'
   S(g,t)      'number of generators started up'
   Z           'total operating cost'
;

Integer Variable Gen;
Positive Variable S;

Equation
   dem_pow(t)    'demand for power                (1000mw)'
   reserve(t)    'spinning reserve requirements (1000mw)'
   start_up(g,t) 'start-up definition'
   min_out(g,t)  'minimum generation level        (1000mw)'
   max_out(g,t)  'maximum generation level        (1000mw)'
   cost          'cost definition'
;

dem_pow(t)..  sum(g, O(g,t)) =g= dem(t);

reserve(t)..  sum(g, data(g,"max-pow")*Gen(g,t)) =g= 1.15*dem(t);

start_up(g,t).. S(g,t) =g= Gen(g,t) - Gen(g,t--1);

min_out(g,t).. O(g,t) =g= data(g,"min-pow")*Gen(g,t);

max_out(g,t).. O(g,t) =l= data(g,"max-pow")*Gen(g,t);

cost ..     Z =e=   sum((g,t), dur(t)*data(g,"cost-min")*Gen(g,t) + data(g,"start")*S(g,t)
                    + 1000*dur(t)*data(g,"cost-inc")*(O(g,t)-data(g,"min-pow")*Gen(g,t)));

Gen.up(g,t) = data(g,"number");
```

Finally, after completing the modeling by Games, the execution code and its output were registered. The summary of the solved output of the model is given below.

```
----      69 VARIABLE O.L  generator output

          12pm-6am     6am-9am     9am-3pm     3pm-6pm    6pm-12pm

type-1      10.200      16.000      11.000      21.250      11.250
type-2       4.800      14.000      14.000      15.750      15.750
type-3                                           3.000


----      69 VARIABLE Gen.L  number of generators in use

          12pm-6am     6am-9am     9am-3pm     3pm-6pm    6pm-12pm

type-1      12.000      12.000      12.000      12.000      12.000
type-2       3.000       8.000       8.000       9.000       9.000
type-3                                           2.000


----      69 VARIABLE S.L  number of generators started up

           6am-9am     3pm-6pm

type-2       5.000       1.000
type-3                   2.000


----      69 VARIABLE Z.L                   =   988540.000  total operating cost
```

The minimum cost of electricity production by thermal generators is **988,540** euros.

Electricity Supply from Both Thermal and Hydro Generators

In this section, we extend the model developed in the previous phase by incorporating water generators into it. To achieve this, we made necessary adjustments to the code, adding variables, tables, and constraints related to water generators. Just as in the previous section, we use the set v to represent the column names of the data table for the hydro generators. In the following code, you can simply replace v with the desired column name to read and utilize the desired value. Any new elements introduced compared to the previous code are highlighted in red.

```
Set
   t 'periods'              / 12pm-6am, 6am-9am, 9am-3pm, 3pm-6pm, 6pm-12pm /
   g 'generators'           / type-1, type-2, type-3 /
   h 'Hydro Generators'     / type-A, type-B /
   k 'parameters'           / min-pow,  max-pow,  cost-min,  cost-inc,  start,   number/
   v 'hydro parameters'     / rate, cost ph, depletion, st cost /
;

Parameter
   dem(t)  'demand     (1000mw)' / 12pm-6am 15, 6am-9am 30, 9am-3pm 25, 3pm-6pm 40, 6pm-12pm 27 /
   dur(t)  'duration    (hours)' / 12pm-6am  6, 6am-9am  3, 9am-3pm  6, 3pm-6pm  3, 6pm-12pm  6 /
;

Table data(g,k) 'generation data'
            min-pow  max-pow  cost-min  cost-inc  start   number
   type-1       .85      2.0      1000       2.0   2000       12
   type-2      1.25     1.75      2600       1.3   1000       10
   type-3      1.5      4.0       3000       3.0    500        5
;

Table hydro_data  (h,v)
                  rate     cost_ph        depletion        st_cost
   type-A         0.9         90             0.31            1500
   type-B         1.4        150             0.47            1200
;
```

Furthermore, when it comes to electricity generation with water generators, we require additional variables and constraints to account for factors like water tank depth and water pumping.

```
Variable
   O(g,t)       'generator output'
   Gen(g,t)     'number of generators in use'
   S(g,t)       'number of generators started up'
   Z            'total operating cost'
   N  (h,t)     'new hydro generator'
   Hyd  (h,t)   'working generator'
   P  (t)       'Pump'
   ht  (t)      'water level'
;

Integer Variable Gen,S;

Positive Variable O,ht,P;

Binary Variable N,Hyd;


Equation
   dem_pow(t)              'demand for power                (1000mw)'
   reserve(t)              'spinning reserve requirements  (1000mw)'
   start_up(g,t)           'start-up definition'
   min_out(g,t)            'minimum generation level       (1000mw)'
   max_out(g,t)            'maximum generation level       (1000mw)'
   cost                    'cost definition                    (1)'
   ht1  (t)                'initial height                     (m)'
   ht_low  (t)             'minimum of height                  (m)'
   ht_up  (t)              'maximum of height                  (m)'
   heit  (t)               'relation between differenet heights in different time periods'
   h_start_up  (h,t)       'start-up definition'
;
```

Given that the power plant has only one generator of each type, we consider the variables N and Hyd, which pertain to the activation of new generators and operational hydro generators, respectively, as binary. This reflects the reality that each generator is either entirely inactive or operational during specific time periods, as previously explored in phase one of our modeling.

```
dem_pow(t)..   sum(g, O(g,t))+sum(h,(hydro_data(h, "rate"))*Hyd(h,t))-P(t) =g= dem(t);

reserve(t)..   sum(g, data(g,"max-pow")*Gen(g,t))+sum(h,hydro_data(h,'rate')) =g= 1.15*dem(t);

start_up(g,t).. S(g,t) =g= Gen(g,t) - Gen(g,t--1);

min_out(g,t).. O(g,t) =g= data(g,"min-pow")*Gen(g,t);

max_out(g,t).. O(g,t) =l= data(g,"max-pow")*Gen(g,t);

ht1(t)..        ht('12pm-6am') =e= 16;

ht_low (t) ..   ht(t) =g= 15;
ht_up (t) ..  ht(t) =l= 20;

heit (t)    ..  ht(t++1)-ht(t)-((P(t)*dur(t))/3)+sum (h,hydro_data(h,'depletion')*dur(t)*Hyd(h,t))=e=0;

h_start_up(h,t) ..  N(h,t)=g=Hyd(h,t)-Hyd(h,t--1);

cost ..     Z =e=   sum((g,t), dur(t)*data(g,"cost-min")*Gen(g,t) + data(g,"start")*S(g,t)
                    + 1000*dur(t)*data(g,"cost-inc")*(O(g,t)-data(g,"min-pow")*Gen(g,t)))
                    +sum((h,t), hydro_data(h, 'st_cost')*N(h,t) + hydro_data(h, 'cost_ph')*dur(t)*Hyd(h,t));

Gen.up(g,t) = data(g,"number");

* now to sum it up:

Model HydroPower_Geneneration / all /;
HydroPower_Geneneration.optCr = 0;
```

After completing the modeling using mathematical games, we recorded the execution code and its output. Below is a summary of the model's solved output:

```
----    100 VARIABLE O.L  generator output

           12pm-6am     6am-9am     9am-3pm     3pm-6pm    6pm-12pm

type-1       10.200      14.250      10.200      21.350      10.565
type-2        5.250      15.750      15.750      15.750      15.750
type-3                                            1.500


----    100 VARIABLE Gen.L  number of generators in use

           12pm-6am     6am-9am     9am-3pm     3pm-6pm    6pm-12pm

type-1       12.000      12.000      12.000      12.000      12.000
type-2        3.000       9.000       9.000       9.000       9.000
type-3                                            1.000


----    100 VARIABLE S.L  number of generators started up

            6am-9am     3pm-6pm

type-2        6.000
type-3                    1.000


----    100 VARIABLE Z.L                   =   986630.000  total operating cost
```

```
----    100 VARIABLE N.L  new hydro generator

            3pm-6pm

type-B        1.000


----    100 VARIABLE Hyd.L  working generator

            3pm-6pm    6pm-12pm

type-B        1.000       1.000


----    100 VARIABLE P.L  Pump

12pm-6am 0.450,    9am-3pm  0.950,    6pm-12pm 0.715


----    100 VARIABLE ht.L  water level

12pm-6am 16.000,    6am-9am  16.900,    9am-3pm  16.900,    3pm-6pm  18.800
6pm-12pm 17.390


EXECUTION TIME       =        0.000 SECONDS      3 MB  24.1.2 r40979 WEX-VS8
```

The minimum cost of electricity production using both thermal and water generators is 986,630 euros. It is reasonable to expect that the final cost obtained by incorporating water generators will be equal to or lower than the cost in the previous section, where only thermal generators were used. As mentioned earlier, Model B is an extension of Model A, so the optimal solution for A also serves as one of the valid (though not necessarily optimal) solutions for B.

Sensitivity analysis by GAMS software

Electricity Supply Solely from Thermal Generators

Based on the training material provided, the *objrng* and *rhsrng* codes were utilized to ascertain the permissible range of variations in both the coefficients of the objective function and the values on the right-hand side.

This analysis was conducted with the primary objective of ensuring that the optimal foundation remains unchanged. Subsequently, the outcomes of this analysis were documented in the form of an *.inc* file. Detailed information about the range of variations for each specific value can be found in the accompanying tables. It's worth noting that the code pertaining to the sensitivity analysis of electricity supply from thermal generators, as well as the code for the integration of water generators (Part Two), follows the same procedure.

```
* Now we do the Sensitivity Analysis: (the result of this step is available in the "Solution Report" provided by GAMS)

$onecho > cplex.opt
objrng all
rhsrng all
$offecho
Power_Geneneration.optfile = 1;

* and at last to solve the model we have:

solve Power_Geneneration minimizing Z using mip;
display O.l,Gen.l,S.l,Z.l;
```

You can find the results of the code above in the "Solution Report" section of the Games, which is also briefly summarized here.

EQUATION NAME	LOWER	CURRENT	UPPER
dem_pow(12pm-6am)	13.95	15	15.45
dem_pow(6am-9am)	24.2	30	38
dem_pow(9am-3pm)	24.2	25	38
dem_pow(3pm-6pm)	28.95	40	42.75
dem_pow(6pm-12pm)	25.95	27	39.75
reserve(12pm-6am)	-INF	17.25	29.25
reserve(6am-9am)	-INF	34.5	38
reserve(9am-3pm)	-INF	28.75	38
reserve(3pm-6pm)	-INF	46	47.75
reserve(6pm-12pm)	-INF	31.05	39.75
start_up(type-1, 12pm-6am)	-INF	0	0
start_up(type-1, 6am-9am)	-INF	0	0
start_up(type-1, 9am-3pm)	-INF	0	0
start_up(type-1, 3pm-6pm)	-INF	0	0
start_up(type-1, 6pm-12pm)	-INF	0	0
start_up(type-2, 12pm-6am)	-INF	0	6

min_out(type-1, 9am-3pm)	-INF	0	0.8
min_out(type-1, 3pm-6pm)	-INF	0	11.05
min_out(type-1, 6pm-12pm)	-INF	0	1.05
min_out(type-2, 12pm-6am)	-INF	0	1.05
min_out(type-2, 6am-9am)	-INF	0	4
min_out(type-2, 9am-3pm)	-INF	0	4
min_out(type-2, 3pm-6pm)	-INF	0	4.5
min_out(type-2, 6pm-12pm)	-INF	0	4.5
min_out(type-3, 12pm-6am)	-0.45	0	0
min_out(type-3, 6am-9am)	-8	0	0
min_out(type-3, 9am-3pm)	-13	0	0
min_out(type-3, 3pm-6pm)	-2.75	0	5
min_out(type-3, 6pm-12pm)	-12.75	0	0
max_out(type-1, 12pm-6am)	-13.8	0	+INF
max_out(type-1, 6am-9am)	-8	0	+INF
max_out(type-1, 9am-3pm)	-13	0	+INF
max_out(type-1, 3pm-6pm)	-2.75	0	+INF
max_out(type-1, 6pm-12pm)	-12.75	0	+INF

VARIABLE NAME	LOWER	CURRENT	UPPER
O(type-1, 12pm-6am)	7800	1.2e+004	+INF
O(type-1, 6am-9am)	3900	6000	9000
O(type-1, 9am-3pm)	7800	1.2e+004	1.8e+004
O(type-1, 3pm-6pm)	3900	6000	9000
O(type-1, 6pm-12pm)	7800	1.2e+004	1.8e+004
O(type-2, 12pm-6am)	0	7800	1.2e+004
O(type-2, 6am-9am)	-INF	3900	6000
O(type-2, 9am-3pm)	-INF	7800	1.2e+004
O(type-2, 3pm-6pm)	-INF	3900	6000
O(type-2, 6pm-12pm)	-INF	7800	1.2e+004
O(type-3, 12pm-6am)	7800	1.8e+004	+INF
O(type-3, 6am-9am)	6000	9000	+INF
O(type-3, 9am-3pm)	1.2e+004	1.8e+004	+INF
O(type-3, 3pm-6pm)	6000	9000	+INF
O(type-3, 6pm-12pm)	1.2e+004	1.8e+004	+INF
Gen(type-1, 12pm-6am)	-INF	-4200	+INF
Gen(type-1, 6am-9am)	-INF	-2100	+INF
Gen(type-1, 9am-3pm)	-INF	-4200	+INF
Gen(type-1, 3pm-6pm)	-INF	-2100	+INF
Gen(type-1, 6pm-12pm)	-INF	-4200	+INF
Gen(type-2, 12pm-6am)	-INF	5850	+INF
Gen(type-2, 6am-9am)	-INF	2925	+INF
Gen(type-2, 9am-3pm)	-INF	5850	+INF
Gen(type-2, 3pm-6pm)	-INF	2925	+INF
Gen(type-2, 6pm-12pm)	-INF	5850	+INF
Gen(type-3, 12pm-6am)	-INF	-9000	+INF
Gen(type-3, 6am-9am)	-INF	-4500	+INF
Gen(type-3, 6am-9am)	-INF	-4500	+INF
Gen(type-3, 9am-3pm)	-INF	-9000	+INF
Gen(type-3, 3pm-6pm)	-INF	-4500	+INF
Gen(type-3, 6pm-12pm)	-INF	-9000	+INF
S(type-1, 12pm-6am)	0	2000	+INF
S(type-1, 6am-9am)	0	2000	+INF
S(type-1, 9am-3pm)	0	2000	+INF
S(type-1, 3pm-6pm)	0	2000	+INF
S(type-1, 6pm-12pm)	0	2000	+INF
S(type-2, 12pm-6am)	0	1000	+INF
S(type-2, 6am-9am)	0	1000	+INF
S(type-2, 9am-3pm)	0	1000	+INF
S(type-2, 3pm-6pm)	0	1000	+INF
S(type-2, 6pm-12pm)	0	1000	+INF
S(type-3, 12pm-6am)	0	500	+INF
S(type-3, 6am-9am)	0	500	+INF
S(type-3, 9am-3pm)	0	500	+INF
S(type-3, 3pm-6pm)	0	500	+INF
S(type-3, 6pm-12pm)	0	500	+INF
Z	-INF	1	+INF

Electricity Supply from Both Thermal and Hydro Generators

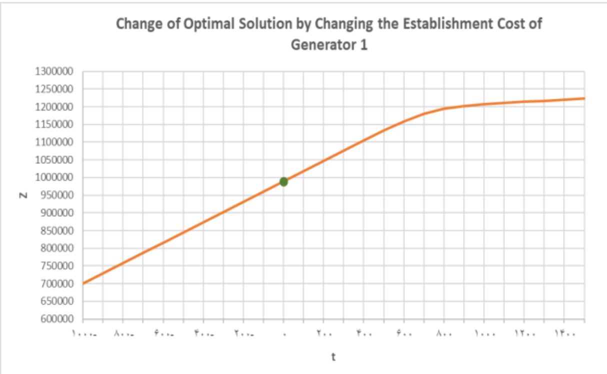
EQUATION NAME	LOWER	CURRENT	UPPER
dem_pow(12pm-6am)	14.63	15	15.45
dem_pow(6am-9am)	25.95	30	39.75
dem_pow(9am-3pm)	24.63	25	25.95
dem_pow(3pm-6pm)	28.85	40	42.65
dem_pow(6pm-12pm)	26.63	27	40.43
reserve(12pm-6am)	-INF	14.95	29.25
reserve(6am-9am)	-INF	32.2	39.75
reserve(9am-3pm)	-INF	26.45	39.75
reserve(3pm-6pm)	-INF	43.7	43.75
reserve(6pm-12pm)	-INF	28.75	39.75
start_up(type-1, 12pm-6am)	-INF	0	0
start_up(type-1, 6am-9am)	-INF	0	0
start_up(type-1, 9am-3pm)	-INF	0	0
start_up(type-1, 3pm-6pm)	-INF	0	0
start_up(type-1, 6pm-12pm)	-INF	0	0
start_up(type-2, 12pm-6am)	-INF	0	6
start_up(type-2, 6am-9am)	-INF	0	0
start_up(type-2, 9am-3pm)	-INF	0	0
start_up(type-2, 3pm-6pm)	-INF	0	0
start_up(type-2, 6pm-12pm)	-INF	0	0
start_up(type-3, 12pm-6am)	-INF	0	0
start_up(type-3, 6am-9am)	-INF	0	0
start_up(type-3, 9am-3pm)	-INF	0	0
start_up(type-3, 3pm-6pm)	-INF	0	0
start_up(type-3, 6pm-12pm)	-INF	0	1
min_out(type-1, 12pm-6am)	-0.45	0	0.365
min_out(type-1, 6am-9am)	-INF	0	4.05
min_out(type-1, 9am-3pm)	-0.95	0	0.365
min_out(type-1, 3pm-6pm)	-INF	0	11.15
min_out(type-1, 6pm-12pm)	-INF	0	0.365
min_out(type-2, 12pm-6am)	-INF	0	1.5
min_out(type-2, 6am-9am)	-INF	0	4.5
min_out(type-2, 9am-3pm)	-INF	0	4.5
min_out(type-2, 3pm-6pm)	-INF	0	4.5
min_out(type-2, 6pm-12pm)	-INF	0	4.5
min_out(type-3, 12pm-6am)	-INF	0	0
min_out(type-3, 6am-9am)	-INF	0	0
min_out(type-3, 9am-3pm)	-INF	0	0
min_out(type-3, 3pm-6pm)	-1.5	0	2.5
min_out(type-3, 6pm-12pm)	-INF	0	0
max_out(type-1, 12pm-6am)	-13.8	0	+INF
max_out(type-1, 6am-9am)	-9.75	0	+INF
max_out(type-1, 9am-3pm)	-13.8	0	+INF
max_out(type-1, 3pm-6pm)	-2.65	0	+INF
max_out(type-1, 6pm-12pm)	-13.43	0	+INF
max_out(type-2, 12pm-6am)	-0.45	0	0.365
max_out(type-2, 6am-9am)	-4.5	0	4.05
max_out(type-2, 9am-3pm)	-0.95	0	0.365
max_out(type-2, 3pm-6pm)	-2.65	0	11.15
max_out(type-2, 6pm-12pm)	-4.5	0	0.365
max_out(type-3, 12pm-6am)	0	0	+INF
max_out(type-3, 6am-9am)	0	0	+INF
max_out(type-3, 9am-3pm)	0	0	+INF
max_out(type-3, 3pm-6pm)	-2.5	0	+INF
max_out(type-3, 6pm-12pm)	0	0	+INF
cost	-INF	0	+INF
ht1(12pm-6am)	16	16	16
ht1(6am-9am)	16	16	16
ht1(9am-3pm)	16	16	16
ht1(3pm-6pm)	16	16	16
ht1(6pm-12pm)	16	16	16
ht_low(12pm-6am)	-INF	15	16
ht_low(6am-9am)	-INF	15	16.9
ht_low(9am-3pm)	-INF	15	16.9
ht_low(3pm-6pm)	-INF	15	18.8
ht_low(6pm-12pm)	-INF	15	17.39
ht_up(12pm-6am)	16	20	+INF
ht_up(6am-9am)	16.9	20	+INF
ht_up(9am-3pm)	16.9	20	+INF
ht_up(3pm-6pm)	18.8	20	+INF
ht_up(6pm-12pm)	17.39	20	+INF
heit(12pm-6am)	-1.9	0	0.73
heit(6am-9am)	-1.9	0	0.73
heit(9am-3pm)	-2.39	0	0.73
heit(3pm-6pm)	-2.39	0	0.73
heit(6pm-12pm)	-26.87	0	0.73
h_start_up(type-A, 12pm-6am)	-INF	0	0
h_start_up(type-A, 6am-9am)	-INF	0	0
h_start_up(type-A, 9am-3pm)	-INF	0	0
h_start_up(type-A, 3pm-6pm)	-INF	0	0
h_start_up(type-A, 6pm-12pm)	-INF	0	0
h_start_up(type-B, 12pm-6am)	-INF	0	1
h_start_up(type-B, 6am-9am)	-INF	0	0
h_start_up(type-B, 9am-3pm)	-INF	0	0
h_start_up(type-B, 3pm-6pm)	-INF	0	0
h_start_up(type-B, 6pm-12pm)	-INF	0	0

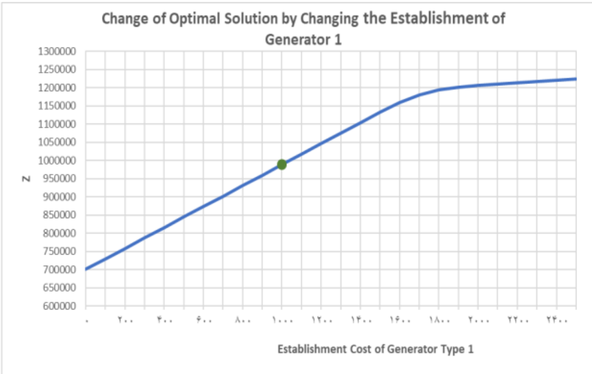
VARIABLE NAME	LOWER	CURRENT	UPPER
O(type-1, 12pm-6am)	1.2e+004	1.2e+004	+INF
O(type-1, 6am-9am)	6000	6000	9000
O(type-1, 9am-3pm)	1.2e+004	1.2e+004	+INF
O(type-1, 3pm-6pm)	6000	6000	9000
O(type-1, 6pm-12pm)	7800	1.2e+004	1.2e+004
O(type-2, 12pm-6am)	-INF	7800	1.2e+004
O(type-2, 6am-9am)	-INF	3900	6000
O(type-2, 9am-3pm)	-INF	7800	1.2e+004
O(type-2, 3pm-6pm)	-INF	3900	6000
O(type-2, 6pm-12pm)	-INF	7800	1.2e+004
O(type-3, 12pm-6am)	1.2e+004	1.8e+004	+INF
O(type-3, 6am-9am)	6000	9000	+INF
O(type-3, 9am-3pm)	1.2e+004	1.8e+004	+INF
O(type-3, 3pm-6pm)	6000	9000	+INF
O(type-3, 6pm-12pm)	1.2e+004	1.8e+004	+INF
Gen(type-1, 12pm-6am)	-INF	-4200	+INF
Gen(type-1, 6am-9am)	-INF	-2100	+INF
Gen(type-1, 9am-3pm)	-INF	-4200	+INF
Gen(type-1, 3pm-6pm)	-INF	-2100	+INF
Gen(type-1, 6pm-12pm)	-INF	-4200	+INF
Gen(type-2, 12pm-6am)	-INF	5850	+INF
Gen(type-2, 6am-9am)	-INF	2925	+INF
Gen(type-2, 9am-3pm)	-INF	5850	+INF
Gen(type-2, 3pm-6pm)	-INF	2925	+INF
Gen(type-2, 6pm-12pm)	-INF	5850	+INF
Gen(type-3, 12pm-6am)	-INF	-9000	+INF
Gen(type-3, 6am-9am)	-INF	-4500	+INF
Gen(type-3, 9am-3pm)	-INF	-9000	+INF
Gen(type-3, 3pm-6pm)	-INF	-4500	+INF
Gen(type-3, 6pm-12pm)	-INF	-9000	+INF
S(type-1, 12pm-6am)	-INF	2000	+INF
S(type-1, 6am-9am)	-INF	2000	+INF
S(type-1, 9am-3pm)	-INF	2000	+INF
S(type-1, 3pm-6pm)	-INF	2000	+INF
S(type-1, 6pm-12pm)	-INF	2000	+INF
S(type-2, 12pm-6am)	-INF	1000	+INF
S(type-2, 6am-9am)	-INF	1000	+INF
S(type-2, 9am-3pm)	-INF	1000	+INF
S(type-2, 3pm-6pm)	-INF	1000	+INF
S(type-2, 6pm-12pm)	-INF	1000	+INF
S(type-3, 12pm-6am)	-INF	500	+INF
S(type-3, 6am-9am)	-INF	500	+INF
S(type-3, 9am-3pm)	-INF	500	+INF
S(type-3, 3pm-6pm)	-INF	500	+INF
S(type-3, 6pm-12pm)	-INF	500	+INF
Z	-INF	1	+INF
N(type-A, 12pm-6am)	-INF	1500	+INF
N(type-A, 6am-9am)	-INF	1500	+INF
N(type-A, 9am-3pm)	-INF	1500	+INF
N(type-A, 3pm-6pm)	-INF	1500	+INF
N(type-A, 6pm-12pm)	-INF	1500	+INF
N(type-B, 12pm-6am)	-INF	1200	+INF
N(type-B, 6am-9am)	-INF	1200	+INF
N(type-B, 9am-3pm)	-INF	1200	+INF
N(type-B, 3pm-6pm)	-INF	1200	+INF
N(type-B, 6pm-12pm)	-INF	1200	+INF
Hyd(type-A, 12pm-6am)	-INF	540	+INF
Hyd(type-A, 6am-9am)	-INF	270	+INF
Hyd(type-A, 9am-3pm)	-INF	540	+INF
Hyd(type-A, 3pm-6pm)	-INF	270	+INF
Hyd(type-A, 6pm-12pm)	-INF	540	+INF
Hyd(type-B, 12pm-6am)	-INF	900	+INF
Hyd(type-B, 6am-9am)	-INF	450	+INF
Hyd(type-B, 9am-3pm)	-INF	900	+INF
Hyd(type-B, 3pm-6pm)	-INF	450	+INF
Hyd(type-B, 6pm-12pm)	-INF	900	+INF
P(12pm-6am)	-0	-0	4200
P(6am-9am)	-0	-0	+INF
P(9am-3pm)	-0	-0	4200
P(3pm-6pm)	-0	-0	+INF
P(6pm-12pm)	-4200	-0	0
ht(12pm-6am)	-INF	-0	+INF
ht(6am-9am)	-0	-0	2100
ht(9am-3pm)	-0	-0	2100
ht(3pm-6pm)	-0	-0	2100
ht(6pm-12pm)	-0	-0	2100

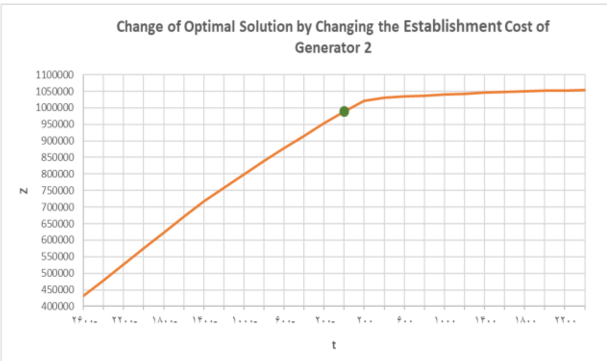
The Effect of Changes in the Values of Selected Coefficients on the Objective Function of Electricity Supply by Thermal Generators

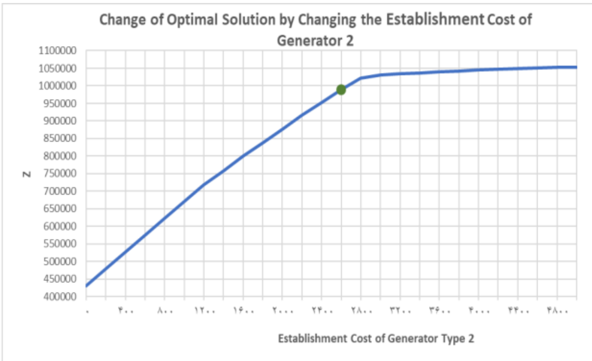
Coefficient of the Objective Function: Investigating the Effect of Changes in the Coefficient E (Fixed Startup Cost) on the Value of the Objective Function

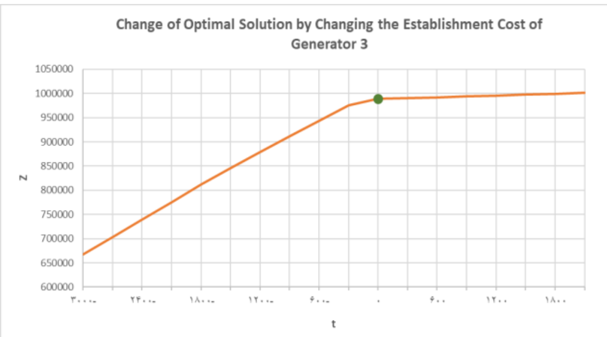
Assuming all other factors remain constant, for each step, we consider the fixed cost of establishing a thermal generator as $E_i + t$. Here, E_i represents the fixed setup cost for generator i as specified in the problem statement, while t is a parameter that undergoes incremental changes at each step. We then utilize *Microsoft Excel* software to calculate the objective function's value based on the varying values of t and E_i. The subsequent results are presented below. Within each graph, the green dot signifies the fixed setup cost, as per the provided query data.

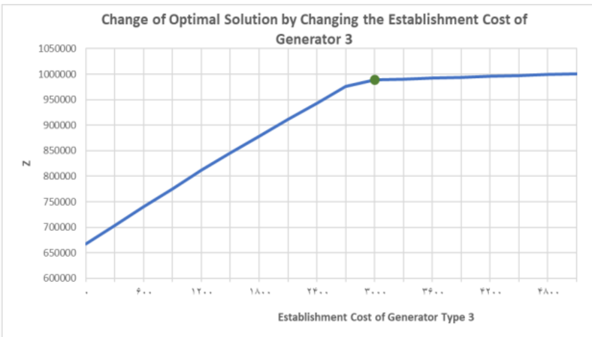








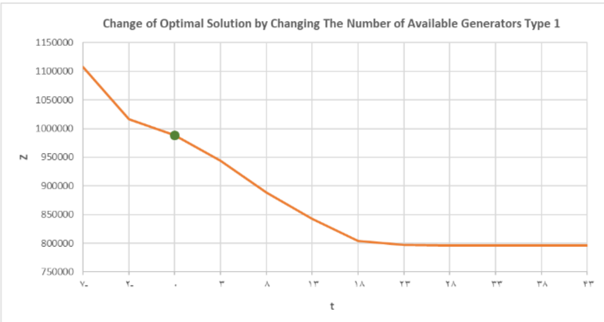


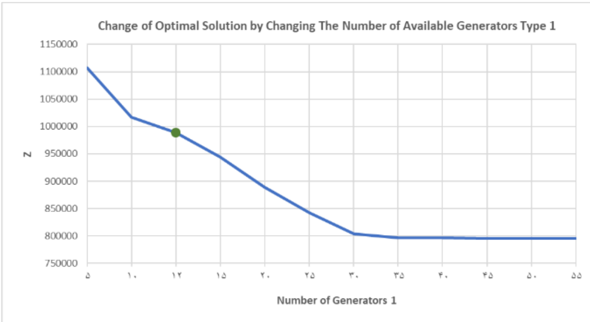


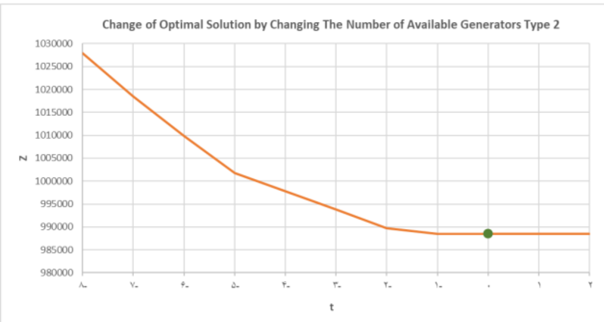
Across all the aforementioned curves, you'll observe a gradual decrease in slope from one point to the next, eventually reaching a state of relative stability. This behavior is due to the fact that as the setup cost for a particular type of generator increases, the power plant adapts by maximizing its response to customer demand using other available generator types, even minimizing the utilization of the more expensive type when feasible.

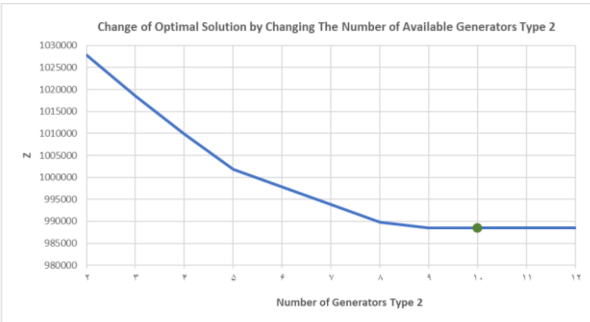
Value on the Right: Examining the Effect of Changes in Number (the Number of Available Thermal Generators) on the Value of the Objective Function

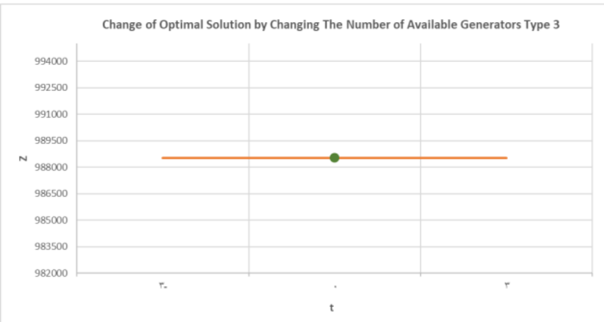
Similar to the previous part, assuming all other things to be constant, in each step the number of thermal generators is assumed as $Number_i + t$, where $Number_i$ is the number of thermal generators of type i available according to the problem and t is a parameter to explain the changes to it step by step Changes. Finally, the value of the objective function is drawn based on the value of t and $Number_i$ in *Excel* software and its results are given below. In each figure, the green dot represents the number of available generators of each type according to the query data. (To avoid misunderstanding, it should be mentioned that in the model written in GAMS, the number of existing thermal generators is displayed with $Number$ to increase the readability of the code, but in the mathematical modeling presented in phase one, the name A was chosen for it.)

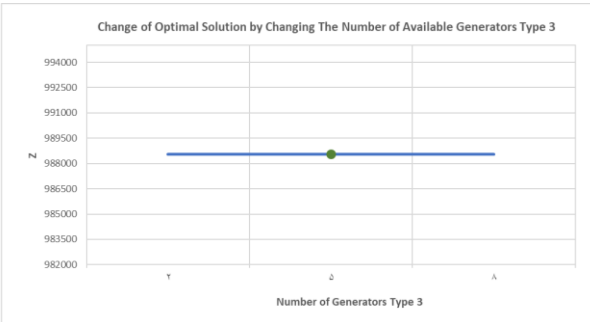












As can be seen, for all types of generators from one point onwards, the value of the objective function always remains constant. For example, in the optimal model, we found that the number of generators that will be working of type 3 is at most two; So having 10 generators of type 3 is no different than having 2 of them. But in relation to the number of generators, if the number of one type of a certain value is less than a specified limit, the rest of the generators of other types will not be able to respond properly to the needs of the customers, and the model will not respond, and the output of games will be infeasible.